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GRADE 10

KEVIN SMITH

SUITABLE FOR IEB AND NATIONAL CURRICULUM STUDENTS

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🟦	Headings and Sub-headings
🟩	Rules and Steps
🟥	Explanations
Bold	**Questions and Emphasis**
Text	Definitions and Explanations
Text	Formulas
💡	Edison - ideas, reminders and suggestions
🧢	Recap of previous work
💡	Homework exercises to complete

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BSc Eng (Electrical) WITS

New and updated with additional examples, exercises
and worked through solutions

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ABOUT THE AUTHOR

KEVIN SMITH

Kevin Smith is a young Electrical Engineer who is passionate about Education, specifically Maths and Science. He has been giving extra lessons in Maths and Science for the past 8 years and is the founder of **The Lab Maths and Science Academy** which is attended by students across Johannesburg.

As an extra lesson teacher, Kevin has established the critical areas where students struggle and has written the **Maths Handbook and Study Guides** with these areas in mind.
The books are written in a clear, simple, visual and logical manner. The colour coding facilitates explanations, definitions, formulas, recaps of previous work, hints and ideas.
They are easy to read, easy to understand and easy to apply what has been learnt. They work in conjunction with all other Maths Books. They are welcome additions to the **Handbook and Study Guide** series.

Kevin's objective is for the **Maths Handbook and Study Guides** to demystify Maths and help students to reach their potential in this challenging subject. The subtitles of the books are 'Maths made Easy' and this is what he aims to do. Kevin ensures that his work is up to date at all times and that it is suitable for IEB and National Curriculum students.

Beryl Lutrin – Berlut Books
Author of the **Afrikaans Handbook and Study Guide**
Co-author of the **English Handbook and Study Guide**
December 2011

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A very special thanks to my past Grade 10 teacher, Mrs Tracy Perkiss, for making Mathematics come alive in her outstanding Maths classes and for her contribution towards the information contained within this textbook.

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I would like to thank my family for their continual support and encouragement. My mother Vera Smith deserves a special mention for her assistance in the graphic design of this textbook. I could not have done it without her.

Lastly, I would like to dedicate this book to all of my past and present students, wishing them the best of luck in Mathematics and in life.

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TIPS ON HOW TO STUDY WITH THIS BOOK

1. **Read slowly**. A maths book does not flow like a novel - even missing a single word could cause a misunderstanding in a fundamental concept.

2. Take note of the **colours** used. If something is in colour it is there for a reason.

3. The **environment** in which a student studies is extremely important. **A quiet room with no distractions** is conducive to learning. A student will get very little out of a maths book if he/she cannot concentrate.

4. **Ensure that each sentence or line** of an equation **is understood** before proceeding.

5. **Do not leave out anything**. Each example illustrates a specific concept and these are rarely repeated. There is very little chance of understanding missed information by reading further in the textbook.

6. All **symbols** in mathematics have a very specific meaning. If there is any uncertainty about the meaning of a term or symbol, do not set it aside, look it up or ask a teacher to explain it to you. The **glossary** in the back of this book will explain most of the important terms.

7. Always study with a **pencil and paper**. Work out proofs and derivations yourself, using the textbook as a guide. **After reading an example, cover it up and attempt the problem yourself, taking note of any errors you made.**

8. Mathematics is a hands-on subject. A sportsman doesn't become a professional by watching on the side-lines. He/she **practises** for hours every day. In the same way, you cannot become a mathematician by watching others do maths. **Practice**, **practice** and more **practice** is the only way to truly master maths.

9. Your brain also needs to **rest**. Do not study for hours at a time without a break. Effective studying requires concentration. If you are tired take a **nap**.

10. When **preparing** for an exam or test, make sure that you do as many **past papers** as you can. If you are unsure of how to answer a question, try to find the method in this book before looking at the answer at the back. This will prove very satisfying and build your confidence. Before your exam, practise at least two papers under exam conditions and according to the allocated time.

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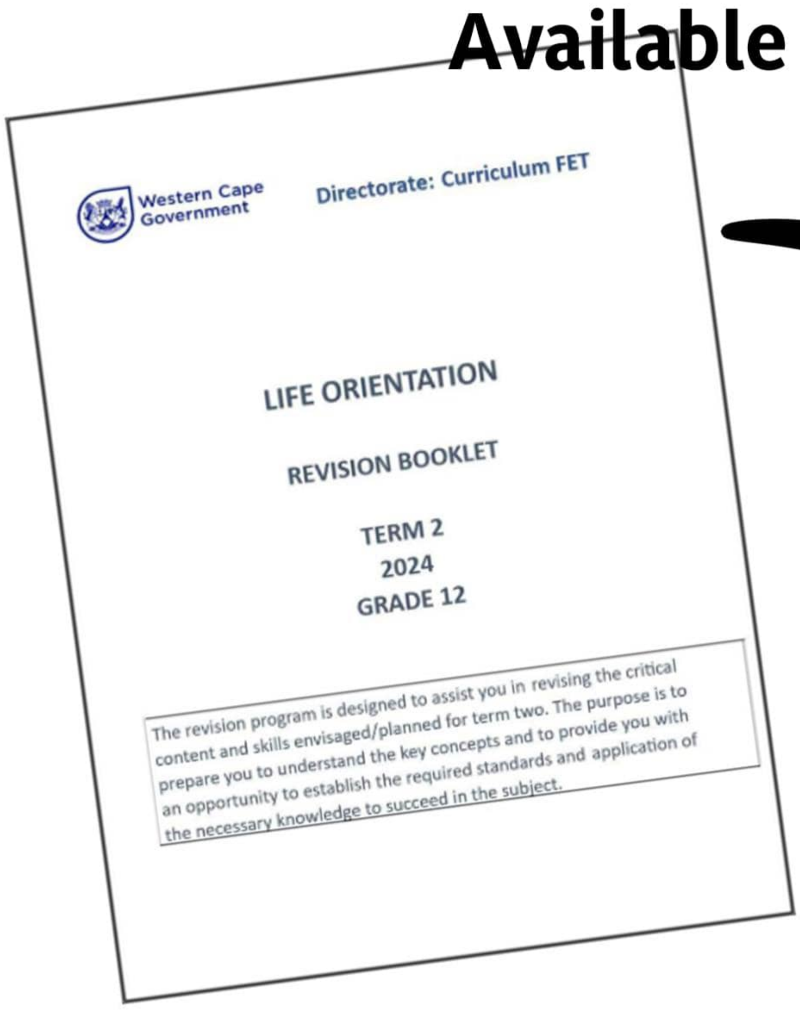

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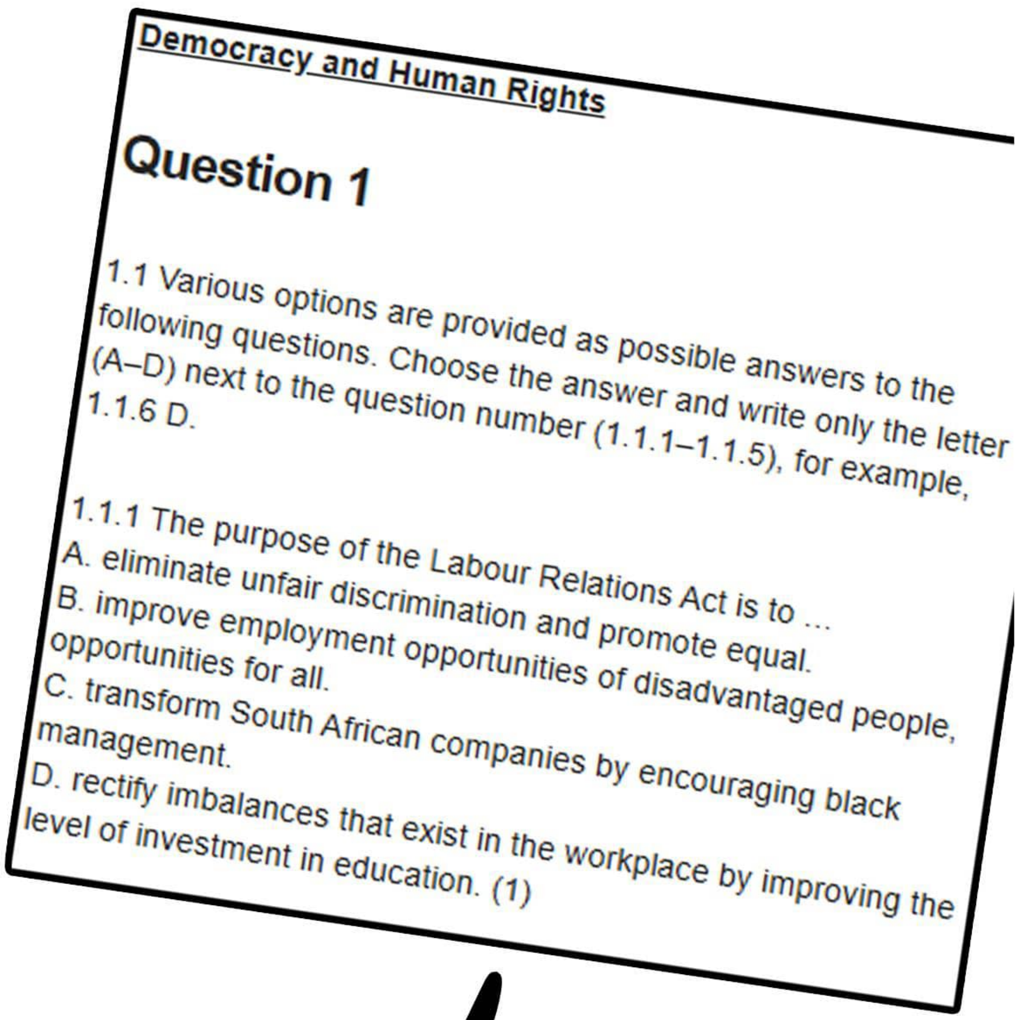
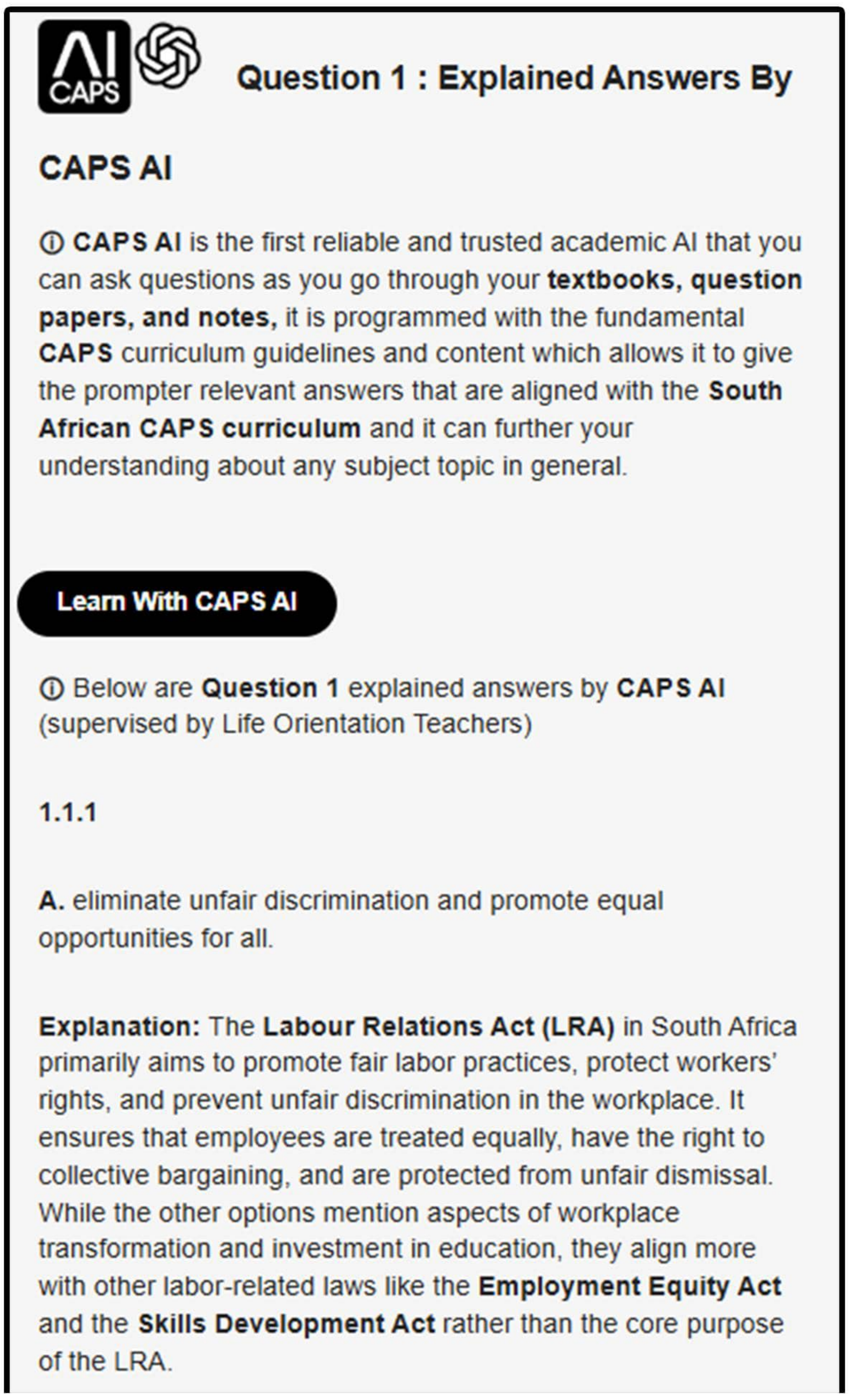


Democracy and Human Rights

Question 1

1.1 Various options are provided as possible answers to the following questions. Choose the answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5), for example, 1.1.6 D.

1.1.1 The purpose of the Labour Relations Act is to ...
A. eliminate unfair discrimination and promote equal opportunities for all.
B. improve employment opportunities of disadvantaged people.
C. transform South African companies by encouraging black management.
D. rectify imbalances that exist in the workplace by improving the level of investment in education. (1)



Question 1 : Explained Answers By CAPS AI

ⓘ **CAPS AI** is the first reliable and trusted academic AI that you can ask questions as you go through your **textbooks, question papers, and notes,** it is programmed with the fundamental **CAPS** curriculum guidelines and content which allows it to give the prompter relevant answers that are aligned with the **South African CAPS curriculum** and it can further your understanding about any subject topic in general.

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ⓘ Below are **Question 1** explained answers by **CAPS AI** (supervised by Life Orientation Teachers)

1.1.1

A. eliminate unfair discrimination and promote equal opportunities for all.

Explanation: The **Labour Relations Act (LRA)** in South Africa primarily aims to promote fair labor practices, protect workers' rights, and prevent unfair discrimination in the workplace. It ensures that employees are treated equally, have the right to collective bargaining, and are protected from unfair dismissal. While the other options mention aspects of workplace transformation and investment in education, they align more with other labor-related laws like the **Employment Equity Act** and the **Skills Development Act** rather than the core purpose of the LRA.

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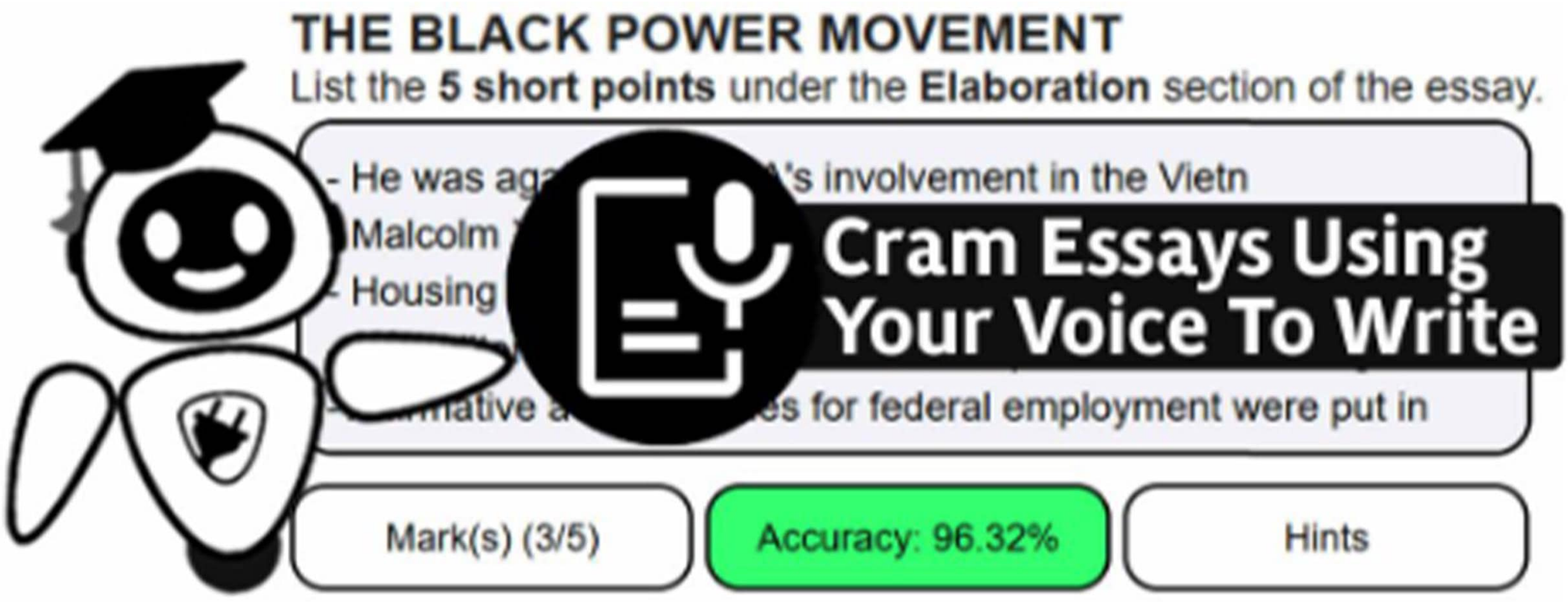

THE BLACK POWER MOVEMENT
List the **5 short points** under the **Elaboration** section of the essay.

- He was ag 's involvement in the Vietn
Malcolm
- Housing
- mative a es for federal employment were put in

Mark(s) (3/5)
Accuracy: 96.32%
Hints

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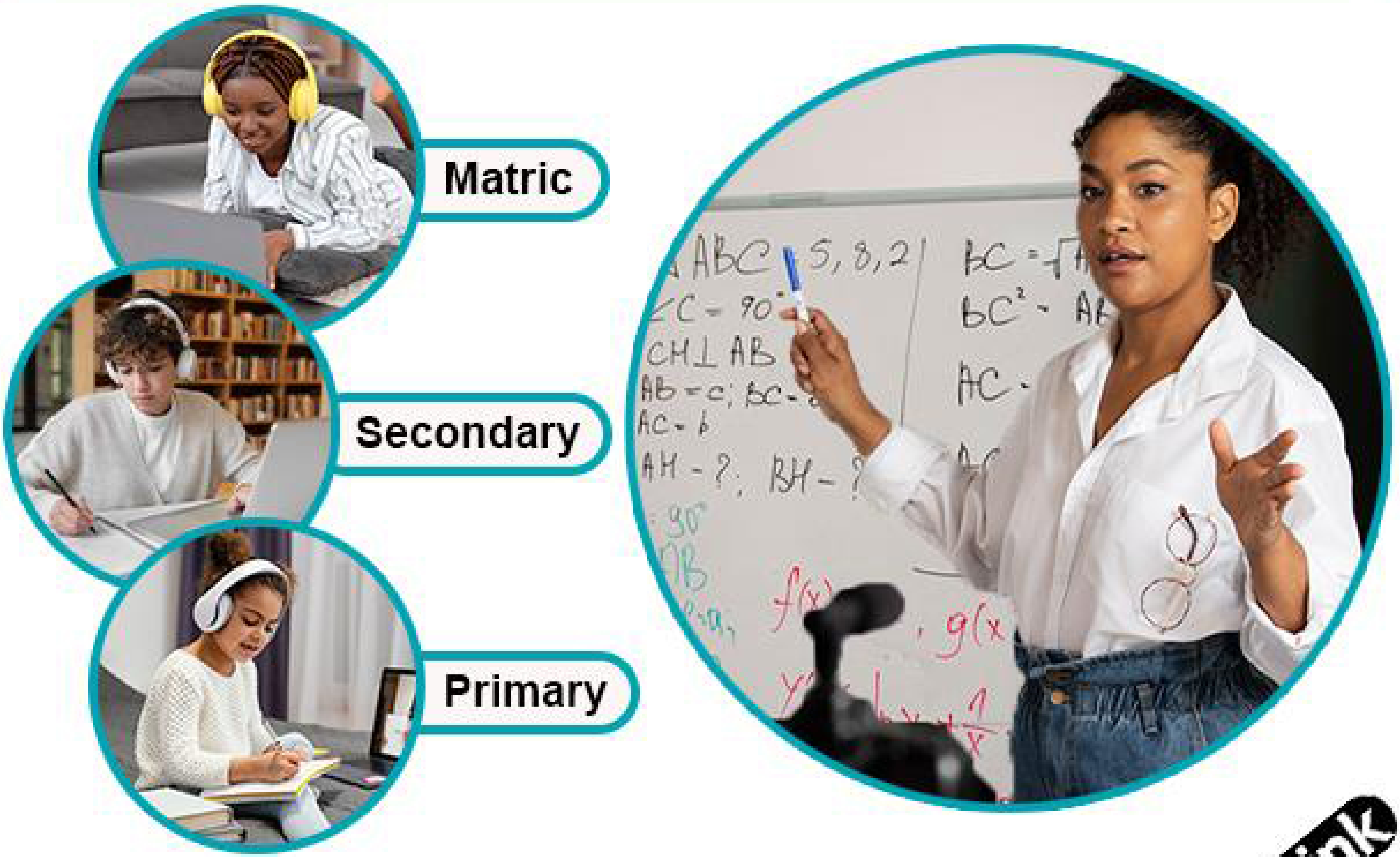

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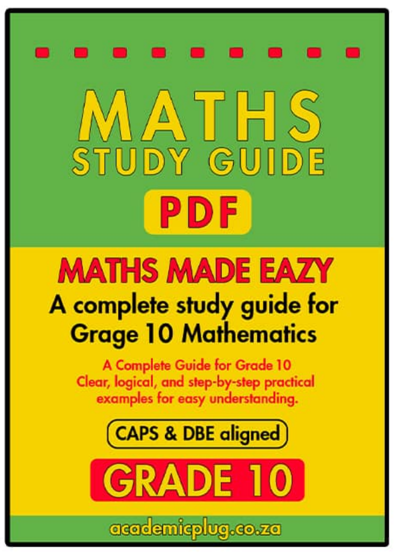

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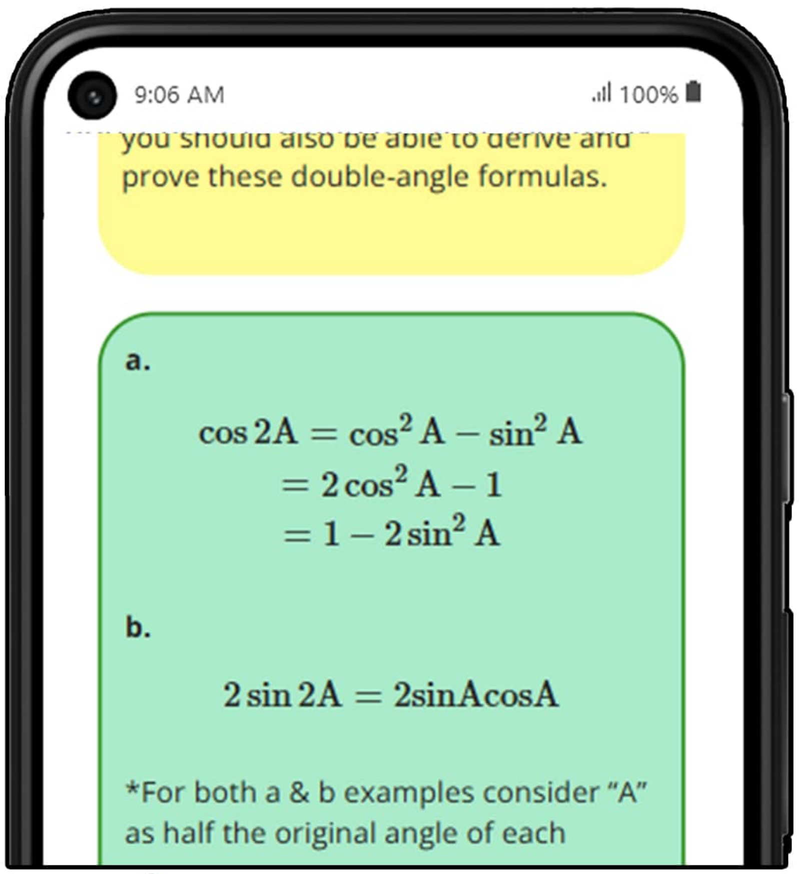

9:06 AM
100%

you should also be able to derive and prove these double-angle formulas.

a.

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

b.

$$2\sin 2A = 2\sin A\cos A$$

*For both a & b examples consider "A" as half the original angle of each

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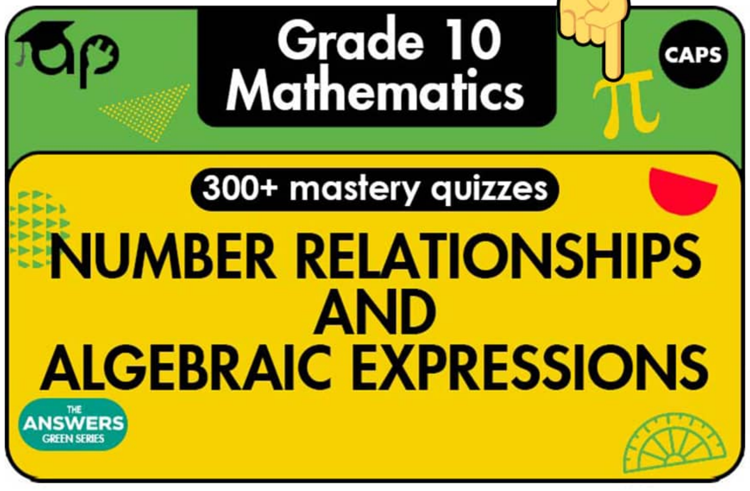

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CHAPTER 1

NUMBER RELATIONSHIPS AND ALGEBRAIC EXPRESSIONS

WHAT YOU WILL LEARN IN THIS CHAPTER

PAGE	CONTENT	CAPS	ASSESSMENTS YOU WILL BE ABLE TO:
3	**A. The Number System**	4	• Identify rational and irrational numbers. • Determine between which two integers a given simple surd lies. • Round off rational and irrational numbers to an appropriate degree of accuracy. • Convert terminating and recurring decimals to fractions.
3	• Natural Numbers		
3	• Whole Numbers		
3	• Integers		
3	• Rational Numbers		
3	• Irrational Numbers		
5	• Determining Between Which Two Integers an Irrational Number Lies		
6	**B. Rounding off**		
6	• Steps to Rounding Off		
7	**C. Rewriting Rational Numbers**		
7	• Steps to Writing Terminating Decimals as Fractions		
7	• Steps to Writing Recurring Decimals as Fractions		

A. THE NUMBER SYSTEM

The real number system consists of many different classifications.
This section recaps on the types of numbers that you have learnt so far and introduces the concepts of **rational** and **irrational numbers.**

NATURAL NUMBERS

{1; 2; 3; 4; ...}
Natural numbers start at 1 and increase in ones up to infinity (∞).
The symbol for a set of natural numbers is: $\mathbb{N}$

WHOLE NUMBERS

{0; 1; 2; 3; 4; ...}
Whole numbers start at 0 and increase in ones up to infinity (∞).
The symbol for a set of whole numbers is: $\mathbb{N}_0$

INTEGERS

{...; –4; –3; –2; –1; 0; 1; 2; 3; 4; ...}
Integers extend whole numbers to include negative numbers.
Integers start at –∞ and increase in ones up to +∞.
The symbol for a set of integers is: $\mathbb{Z}$

RATIONAL NUMBERS

Examples of rational numbers are: $\frac{3}{4}$; $0,\dot{3}$; $0,75$

Rational numbers are numbers which can be written as a **fraction** (where the numerator and denominator are integers) or as a **recurring decimal** or as a **decimal that terminates**. In order for a fraction to be a rational number, the numerator and denominator have to be integers and the denominator cannot be equal to zero.
The symbol for a set of rational numbers is: $\mathbb{Q}$

IRRATIONAL NUMBERS

Examples of irrational numbers are: π ; $\sqrt{2}$; $\sqrt[3]{9}$; 2,35408…

Irrational numbers **cannot be written as a fraction** or with **terminating or recurring decimals**. In other words irrational numbers are non-recurring and non-terminating. Examples of **irrational numbers** are **pi** and the **square root of numbers that are not perfect squares** or **cube roots of numbers that are not perfect cubes**.

An approximation for π is the rational number $\frac{22}{7}$. However, this is only an approximation. It is important to note that π still irrational.

Example: **Complete the following table by placing an 'x' in the correct column.**

Number	$\mathbb{N}$	$\mathbb{N}_0$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{Q}'$
$\frac{4}{5}$					
$0{,}825$					
$\sqrt{36}$					
$0{,}\dot{2}\dot{5}$					
$-\frac{2}{3}$					
π					
$\sqrt{13}$					
$0{,}212223\ldots$					
$0{,}2\dot{3}$					

Here are the answers. How did you do?

Number	$\mathbb{N}$	$\mathbb{N}_0$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{Q}'$
$\frac{4}{5}$				x	
$0{,}825$				x	
$\sqrt{36}$	x	x	x	x	
$0{,}\dot{2}\dot{5}$				x	
$-\frac{2}{3}$				x	
π					x
$\sqrt{13}$					x
$0{,}212223\ldots$					x
$0{,}2\dot{3}$				x	

Complete Exercise 1 on page 233.

DETERMINING BETWEEN WHICH TWO INTEGERS AN IRRATIONAL NUMBER LIES

Example 1: **Determine between which two integers $\sqrt{12}$ lies.**

$9 < 12 < 16 \rightarrow$ 9 and 16 are the first perfect square integers less than, and greater than 12

$\therefore \sqrt{9} < \sqrt{12} < \sqrt{16} \rightarrow$ square root all the numbers

$\therefore 3 < \sqrt{12} < 4$

$\therefore \sqrt{12}$ lies between the integers 3 and 4

Example 2: **Determine between which two integers $\sqrt[3]{50}$ lies.**

$27 < 50 < 64 \rightarrow$ 27 and 64 are the first perfect cube integers less than, and greater than 50

$\therefore \sqrt[3]{27} < \sqrt[3]{50} < \sqrt[3]{64} \rightarrow$ cube root all the numbers

$\therefore 3 < \sqrt[3]{50} < 4$

$\therefore \sqrt[3]{50}$ lies between the integers 3 and 4

Example 3: **Determine between which two integers $-\sqrt{28}$ lies.**

$25 < 28 < 36 \rightarrow$ 25 and 36 are the first perfect square integers less than, and greater than 28

$\therefore \sqrt{25} < \sqrt{28} < \sqrt{36} \rightarrow$ square root all the numbers

$\therefore 5 < \sqrt{28} < 6$

Now multiply all the numbers by -1

$\therefore -5 > -\sqrt{28} > -6 \rightarrow$ swap inequality signs when multiplying by a negative

$\therefore -\sqrt{28}$ lies between the integers -6 and -5

Example 4: **Determine between which two integers $\sqrt{58}$ lies.**

$49 < 58 < 64 \rightarrow$ 49 and 64 are the first perfect square integers less than and greater than 58

$\therefore \sqrt{49} < \sqrt{58} < \sqrt{64} \rightarrow$ square root all the numbers

$\therefore 7 < \sqrt{58} < 8$

$\therefore \sqrt{58}$ lies between the integers 7 and 8

Complete Exercise 2 on page 233.

B. ROUNDING OFF

It is often impractical to use numbers with many decimal places in calculations. In order to solve this problem, numbers are often **rounded off**. Depending on the application, rounding off to two decimal places is usually considered to be an appropriate degree of accuracy.

Unless otherwise stated, always round off numbers to two decimal places.

STEPS TO ROUNDING OFF

1. **After the decimal point, count the number of decimal places that you are required to round off to. Consider the number to the right of this point:**
   - **a. If the number is less than 5, round down by dropping all the digits to the right of this point.**
   - **b. If the number is greater than or equal to 5, round up by adding one to the number that you are rounding off to and dropping all the numbers to the right of it.**
2. **Ensure that you add or keep zeros as place holders.**

Example 1: Write 0,26382 to two decimal places.

$0,26382 \rightarrow$ 3 is the number to the right of two decimal places

$3 < 5$ (3 is less than 5) $\therefore$ drop the 3 and all the numbers after it

$\therefore 0,26382 \approx 0,26$ rounded off to two decimal places

Example 2: Write 0,26382 to three decimal places.

$0,26382 \rightarrow$ 8 is the number to the right of three decimal places

$8 > 5$ (8 is greater than 5) $\therefore$ add 1 to the 3 and drop all the numbers after it

$\therefore 0,26382 \approx 0,264$ rounded off to three decimal places

Example 3: Write 0,89631 to two decimal places.

$0,89631 \rightarrow$ 6 is the number to the right of two decimal places

$6 > 5$ (6 is greater than 5) $\therefore$ add 1 to the 9 and drop all the numbers after it

$\therefore 0,89631 \approx 0,90$ rounded off to two decimal places

If you are rounding to two decimal places there must be 2 numbers after the decimal point. Always keep the zero as a place holder.

Example 4: Determine the value of $\sqrt{12}$ correct to two decimal places.

$\sqrt{12} = 3,464101615... \rightarrow$ use a calculator

$3,464101615... \rightarrow$ 4 is the number to the right of two decimal places

$4 < 5$ (4 is less than 5) $\therefore$ drop the 4 and all the numbers after it

$\therefore \sqrt{12} \approx 3,46$ rounded off to two decimal places

C. REWRITING RATIONAL NUMBERS

In order for a number to be considered rational, it must be possible to write the number as a fraction. In Grade 10 you are expected to convert decimal numbers (terminating and recurring) into fractions and vice versa.

STEPS TO WRITING TERMINATING DECIMALS AS FRACTIONS

1. **Leave any whole number in front of the fraction**
2. **Write the number after the decimal point as the numerator of the fraction.**
3. **Count the number of places after the decimal point.**
4. **Write the denominator as 10, 100, 1000, etc. The number of zeros must be equal to the number of places after the decimal point.**
5. **Simplify the fraction (if possible).**

Example 1: Write 0,2 as a fraction.

$$0,2=\frac{2}{10}=\frac{1}{5}\rightarrow \text{one decimal place, } \therefore \text{ divide by 10}$$

Example 2: Write 0,25 as a fraction.

$$0,25=\frac{25}{100}=\frac{1}{4}\rightarrow \text{two decimal places, } \therefore \text{ divide by 100}$$

Example 3: Write 1,425 as a fraction.

$$1,425=1\frac{425}{1000}=1\frac{17}{40}\rightarrow \text{three decimal places, } \therefore \text{ divide by 1000}$$

STEPS TO WRITING RECURRING DECIMALS AS FRACTIONS

1. **Let the number that you are writing as a fraction equal *x*.**
2. **Make a new equation by multiplying both sides by 10, 100, etc. The number of zeroes must be equal to the number of recurring decimals.**
3. **Subtract the initial equation from the new equation.**
4. **Solve for *x* and simplify if possible.**

Example 1: Write $0,\dot{2}$ as a fraction.

$$0,\dot{2}=0,22222222\ldots\rightarrow \text{the dot on top of the 2 means recurring}$$

$$\text{let } x=0,\dot{2}\rightarrow \text{Equation 1}$$

$$\therefore 10x=2,\dot{2}\rightarrow \text{Equation 2}\rightarrow \text{one recurring decimal, } \therefore \text{ multiply } x \text{ by 10}$$

$$\text{Equation 2} - \text{Equation 1}$$

$$\therefore 10x-x=2,\dot{2}-0,\dot{2}$$

$$\therefore 9x=2$$

$$\therefore x=\frac{2}{9}$$

Example 2: **Write $0,\dot{3}\dot{5}$ as a fraction.**

$0,\dot{3}\dot{5} = 0,35353535...$

let $x = 0,\dot{3}\dot{5} \rightarrow$ Equation 1

$\therefore 100x = 35,\dot{3}\dot{5} \rightarrow$ Equation 2 $\rightarrow$ two recurring decimals, $\therefore$ multiply x by 100

Equation 2 – Equation 1

$\therefore 100x - x = 35,\dot{3}\dot{5} - 0,\dot{3}\dot{5}$

$\therefore 99x = 35$

$\therefore x = \frac{35}{99}$

Example 3: **Write $0,\dot{3}22\dot{5}$ as a fraction.**

$0,\dot{3}22\dot{5} = 0,322532253225... \rightarrow 0,\dot{3}22\dot{5}$ is short hand notation for $0,\dot{3}\dot{2}\dot{2}\dot{5}$ or $0,\overline{3225}$

let $x = 0,\dot{3}22\dot{5} \rightarrow$ Equation 1

$\therefore 10\,000x = 3225,\dot{3}22\dot{5} \rightarrow$ Equation 2 $\rightarrow$ four recurring decimals, $\therefore$ multiply x by $10\,000$

Equation 2 – Equation 1

$\therefore 10\,000x - x = 3225,\dot{3}22\dot{5} - 0,\dot{3}22\dot{5}$

$\therefore 9\,999x = 3\,225$

$\therefore x = \frac{3\,225}{9\,999} = \frac{1075}{3333}$

Example 4: **Write $0,2\dot{3}$ as a fraction.**

$0,2\dot{3} = 0,23333333333...$

let $x = 0,2\dot{3}$

Multiply both sides by 10 to get one recurring decimal point.

$\therefore 10\,x = 2,\dot{3} \rightarrow$ Equation 1

$\therefore 100\,x = 23,\dot{3} \rightarrow$ Equation 2 $\rightarrow$ one recurring decimal, $\therefore$ multiply $10x$ by 10

Equation 2 – Equation 1

$\therefore 100x - 10x = 23,\dot{3} - 2,\dot{3}$

$\therefore 90x = 21$

$\therefore x = \frac{21}{90}$

$\therefore x = \frac{7}{30}$

Complete Exercise 3 and 4 on pages 233 and 234 and the Mixed Exercise on page 234.

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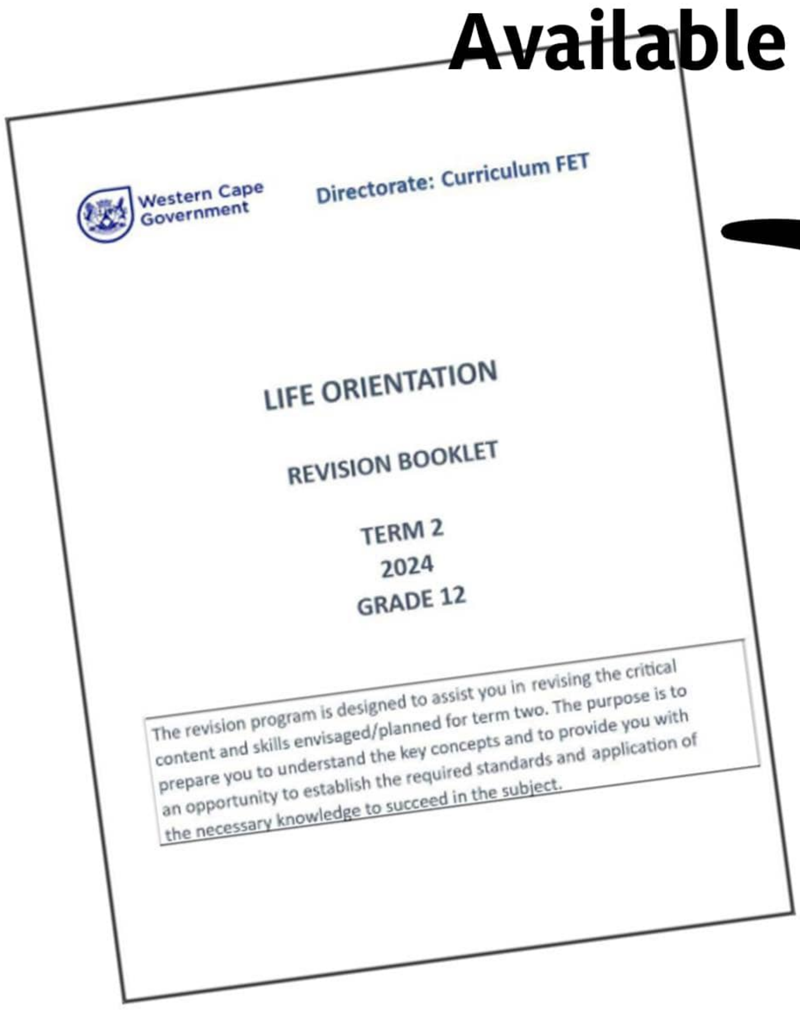

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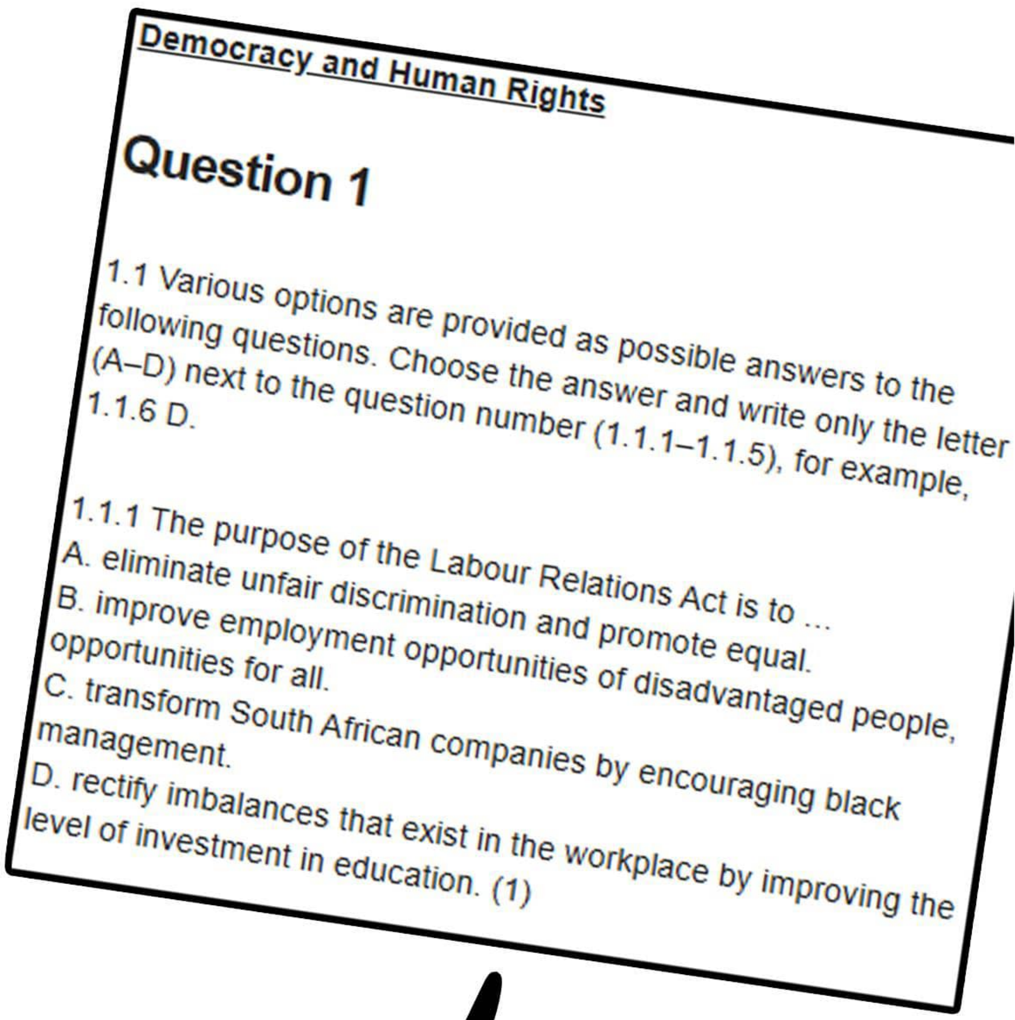
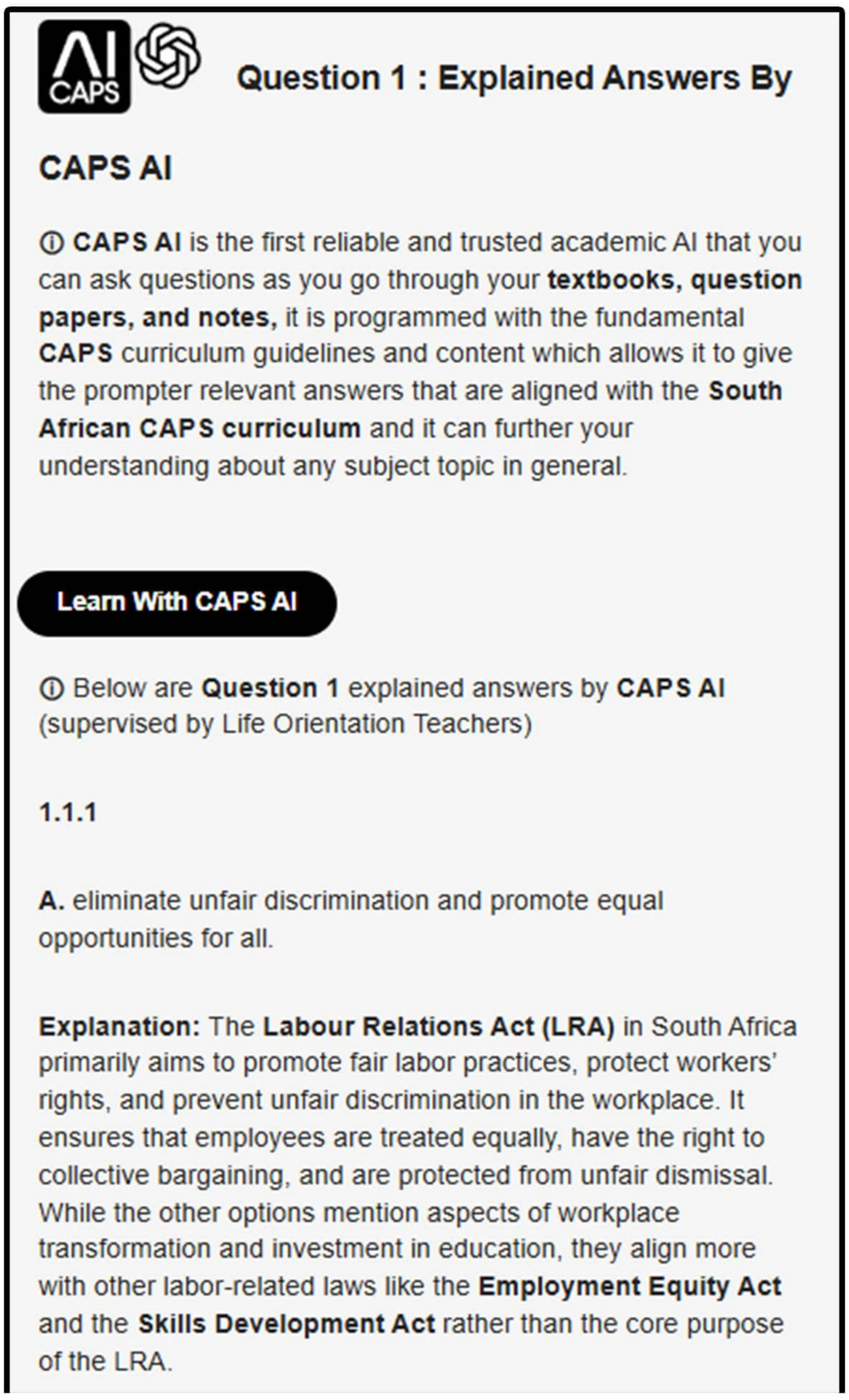


Democracy and Human Rights

Question 1

1.1 Various options are provided as possible answers to the following questions. Choose the answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5), for example, 1.1.6 D.

1.1.1 The purpose of the Labour Relations Act is to ...
A. eliminate unfair discrimination and promote equal opportunities for all.
B. improve employment opportunities of disadvantaged people.
C. transform South African companies by encouraging black management.
D. rectify imbalances that exist in the workplace by improving the level of investment in education. (1)



Question 1 : Explained Answers By CAPS AI

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1.1.1

A. eliminate unfair discrimination and promote equal opportunities for all.

Explanation: The **Labour Relations Act (LRA)** in South Africa primarily aims to promote fair labor practices, protect workers' rights, and prevent unfair discrimination in the workplace. It ensures that employees are treated equally, have the right to collective bargaining, and are protected from unfair dismissal. While the other options mention aspects of workplace transformation and investment in education, they align more with other labor-related laws like the **Employment Equity Act** and the **Skills Development Act** rather than the core purpose of the LRA.

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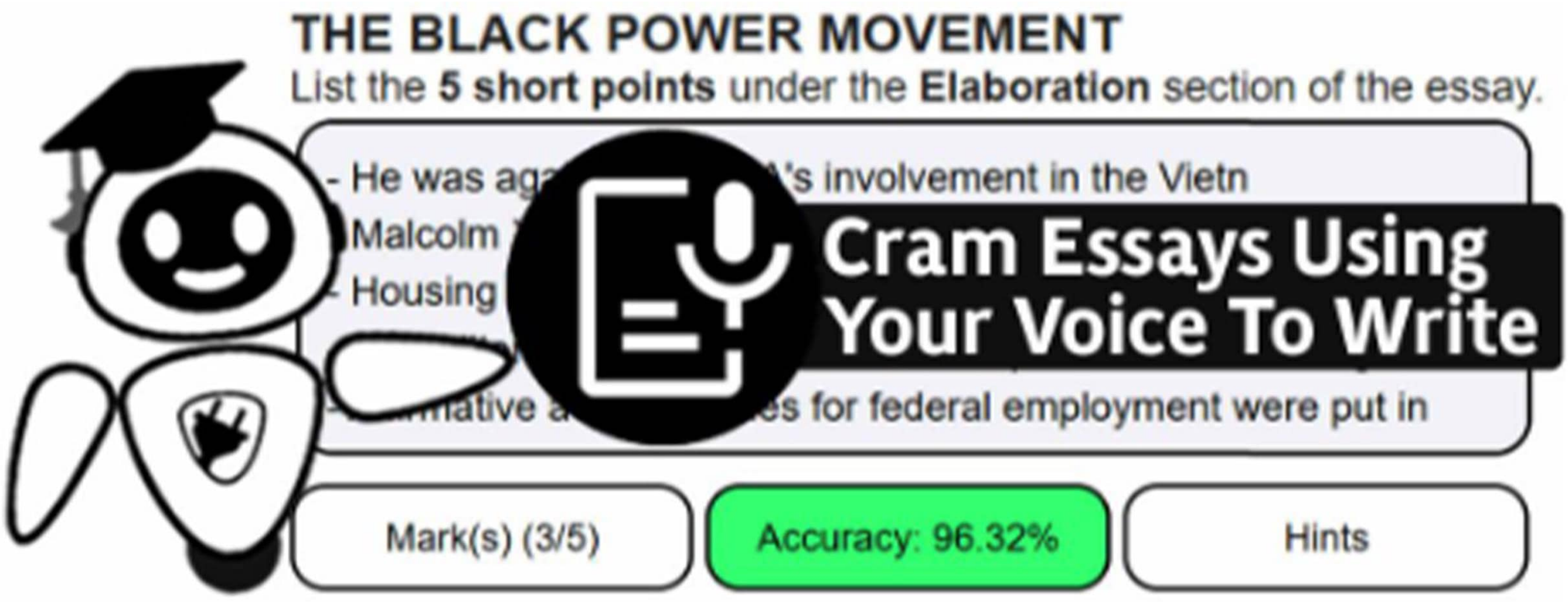

THE BLACK POWER MOVEMENT
List the **5 short points** under the **Elaboration** section of the essay.

- He was ag 's involvement in the Vietn
Malcolm
- Housing
- mative a es for federal employment were put in

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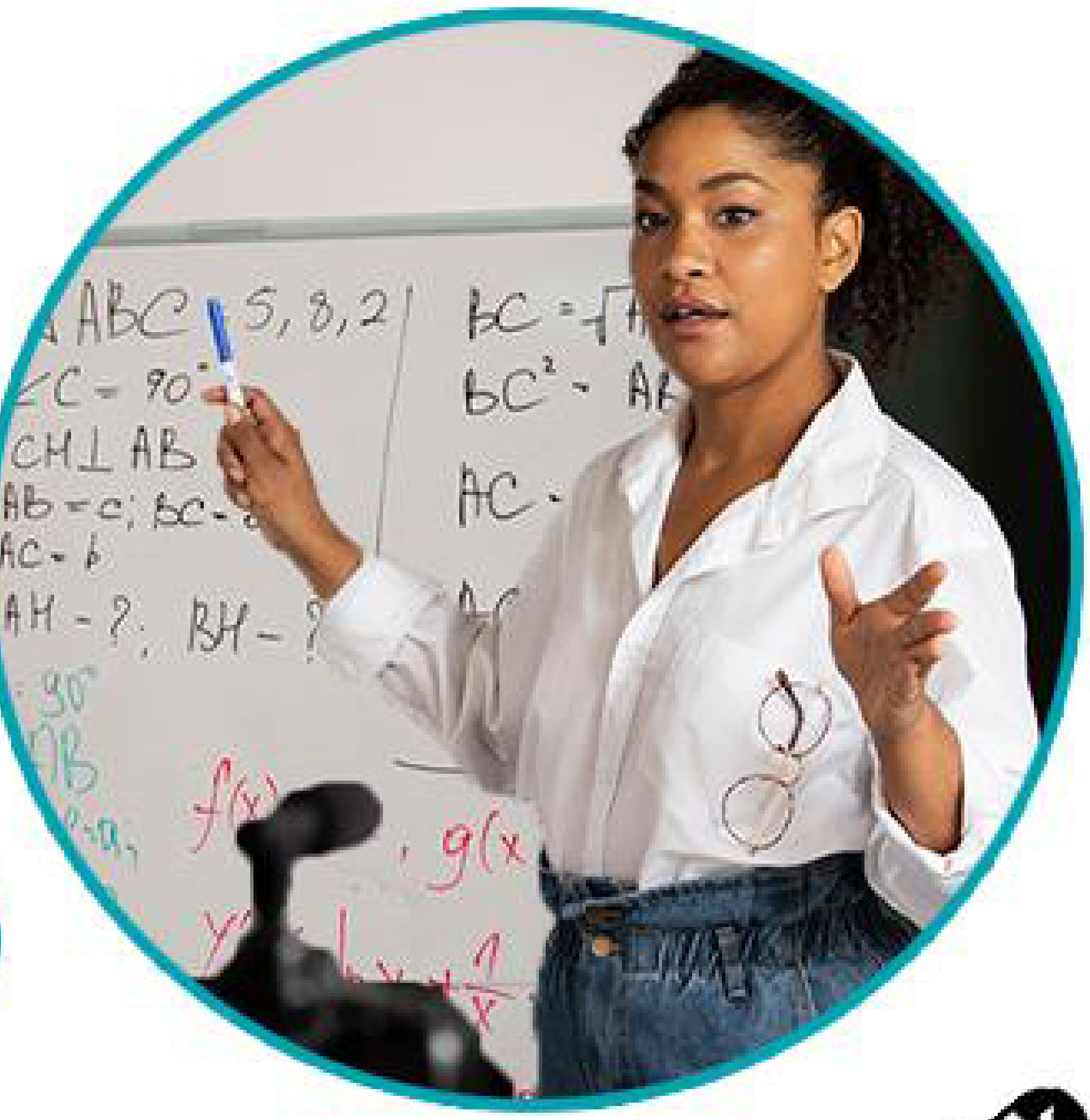

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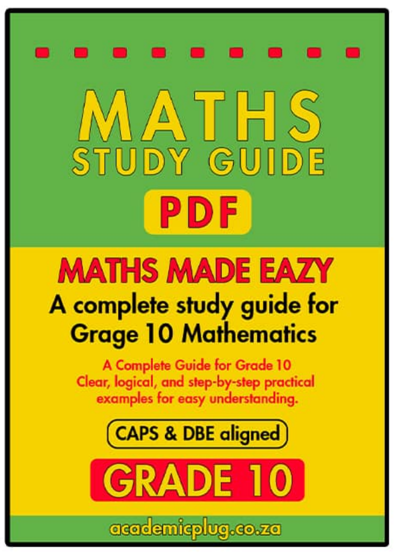

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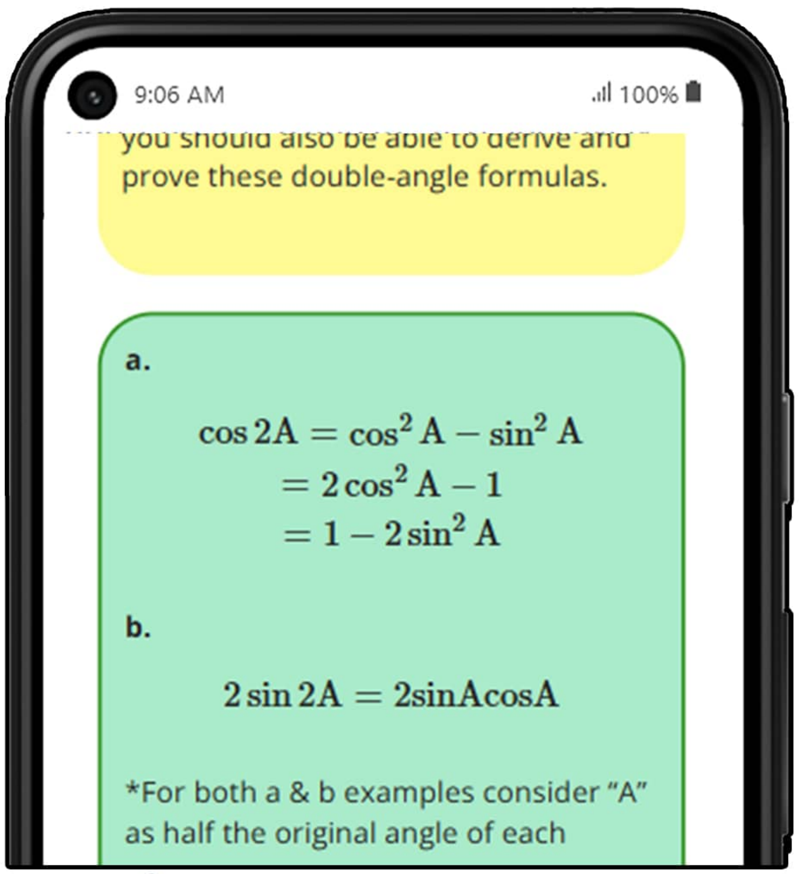

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you should also be able to derive and prove these double-angle formulas.

a.

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

b.

$$2\sin 2A = 2\sin A\cos A$$

*For both a & b examples consider "A" as half the original angle of each

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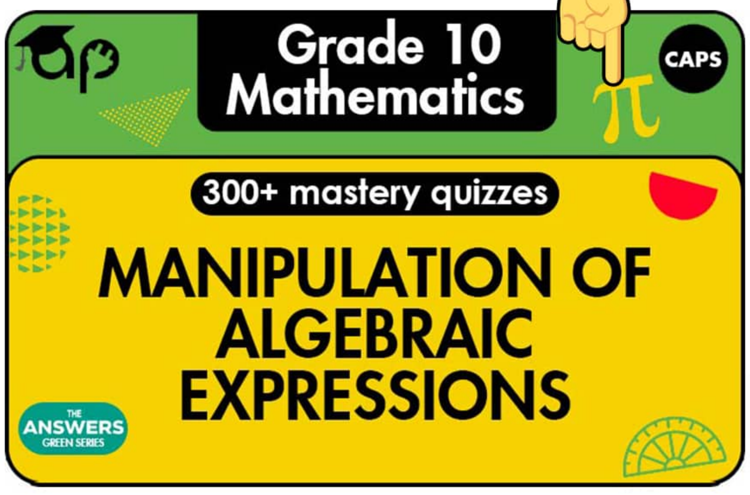

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CHAPTER 2

MANIPULATION OF ALGEBRAIC EXPRESSIONS

WHAT YOU WILL LEARN IN THIS CHAPTER

PAGE	CONTENT	CAPS	ASSESSMENTS YOU WILL BE ABLE TO:
10	**A. Grade 9 Revision**	4	• Multiply a binomial by a trinomial. • Factorise trinomials. • Factorise the difference of two cubes. • Factorise by grouping in pairs. • Simplify, add and subtract algebraic fractions.
10	• The Distributive Law		
11	• Expanding and Simplifying the Product of Binomials		
12	• Square Binomials		
13	• Difference of Square Binomials		
14	• Factorisation by taking out a Highest Common Factor		
15	• Factorisation of the Difference of Two Squares		
16	**B. Grade 10 Algebra**		
16	• Multiplying a Binomial by a Trinomial		
17	• Factorising Trinomials		
21	• The Cross Method		
22	• Factorisation by Grouping in Pairs		
24	• Factorisation of the Sum and Difference of Cubes		
25	**C. Simplifying Algebraic Expressions**		
25	• Rules of Fractions		
25	• Multiplication and Division of Fractions		
27	• Addition and Subtraction of Fractions		
29	• Examples Involving a Sign Change		
30	• Mixed Examples		

A. GRADE 9 REVISION

There are certain principles from Grade 9 that are essential to know in Grade 10:

1. **The Distributive Law**
2. **Expanding and simplifying the product of binomials**
3. **Factorisation by taking out a highest common factor**
4. **Factorisation of the difference of two squares**

1. THE DISTRIBUTIVE LAW

The distributive law states that: $a(b+c+d)=ab+ac+ad$

Example 1: **Simplify** $2(x^3+3x^2+8x+7)$.

$2(x^3+3x^2+8x+7) \rightarrow$ distribute the 2 into every term in the bracket

$=2\times x^3+2\times 3x^2+2\times 8x+2\times 7 \rightarrow$ you do not have to show this step

$=2x^3+6x^2+16x+14$

Example 2: **Simplify** $3-3(3x^2-2x+5)$.

$3-3(3x^2-2x+5) \rightarrow$ only distribute the -3

$=3-9x^2+6x-15 \rightarrow$ don't forget to change the signs when distributing a negative

$=\ -9x^2+6x-12 \rightarrow$ simplify by adding like terms $(3$ and $-15)$

Example 3: **Simplify** $3(2x^2-2x+1)+2x(3x^2-2x+5)$.

$3(2x^2-2x+1)+2x(3x^2-2x+5) \rightarrow$ use the distributive law

$=6x^2-6x+3+6x^3-4x^2+10x \rightarrow$ simplify by adding like terms

$=\ 6x^3+2x^2+4x+3$

Example 4: **Simplify** $-4x(2x^2-3x)-3x^2$.

$-4x(2x^2-3x)-3x^2 \rightarrow$ use the distributive law and don't forget to distribute the minus

$=-8x^3+12x^2-3x^2 \rightarrow$ change signs when distributing a minus, add like terms

$=\ -8x^3+9x^2$

Complete Exercise 1 number 1 on page 238.

2. EXPANDING AND SIMPLIFYING THE PRODUCT OF BINOMIALS

When expanding the product of binomials it is essential that each term in the first bracket is multiplied by each term in the second bracket.

$$(a+b)(c+d)=ac+ad+bc+bd$$

Example 1: Expand and simplify $(x+3)(x+1)$.

$(x+3)(x+1)$

$=(x)x+(x)1+(3)x+(3)1 \rightarrow$ you do not have to show this step

$=x^2+x+3x+3 \rightarrow$ add like terms together

$=x^2+4x+3$

Example 2: Expand and simplify $(x-3)(x-2)$.

$(x-3)(x-2)$

$=(x)(x)+(x)(-2)+(-3)(x)+(-3)(-2) \rightarrow$ you do not have to show this step

$=x^2-2x-3x+6 \rightarrow$ add like terms together

$=x^2-5x+6$

Example 3: Expand and simplify $(x-2)(x+5)$.

$(x-2)(x+5)$

$=(x)(x)+(x)(5)+(-2)(x)+(-2)(5) \rightarrow$ you do not have to show this step

$=x^2+5x-2x-10 \rightarrow$ add like terms together

$=x^2+3x-10$

Example 4: Expand and simplify $(2x-2y)(x+3y)$.

$(2x-2y)(x+3y)$

$=(2x)x+(2x)3y+(-2y)x+(-2y)3y \rightarrow$ you do not have to show this step

$=2x^2+6xy-2xy-6y^2 \rightarrow$ add like terms together

$=2x^2+4xy-6y^2$

Expanding the product of binomials is often taught as FOIL.
(First Outers Inners Last)

Complete Exercise 1 number 2 on page 238.

SQUARE BINOMIALS

When a binomial is squared it can be expanded as the product of two binomials. However, there is a standard form of a binomial that is squared which can provide you with a quicker method of squaring binomials.

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b) \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2\end{aligned} \qquad \begin{aligned}(a-b)^2 &= (a-b)(a-b) \\ &= a^2 - ab - ab + b^2 \\ &= a^2 - 2ab + b^2\end{aligned}$$

It can be seen that a general expression for squaring a binomial is:

$$(\text{term 1} + \text{term 2})^2 = (\text{term 1})^2 + 2(\text{term 1} \times \text{term 2}) + (\text{term 2})^2$$

There always has to be a middle term when squaring a binomial.

Example 1: **Expand and simplify $(x-3)^2$.**

$$(\overset{\times 2}{x-3})^2 = x^2 - 6x + 9$$

Example 2: **Expand and simplify $(3x+2y)^2$.**

$$(\overset{\times 2}{3x+2y})^2 = 9x^2 + 12xy + 4y^2$$

Example 3: **Expand and simplify $(2x+5)^2 - (3x-3)^2$.**

$$(\underset{\times 2}{2x+5})^2 - (\underset{\times 2}{3x-3})^2$$

$$= 4x^2 + 20x + 25 - (9x^2 - 18x + 9)$$

$$= 4x^2 + 20x + 25 - 9x^2 + 18x - 9 \rightarrow \text{distribute the minus into the bracket}$$

$$= -5x^2 + 38x + 16$$

Example 4: **Expand and simplify $(x+4)^2 + (x-3)(x-2)$.**

$$(\underset{\times 2}{x+4})^2 + (x-3)(x-2)$$

$$= x^2 + 8x + 16 + x^2 - 2x - 3x + 6$$

$$= 2x^2 + 3x + 22$$

Complete Exercise 1 number 3 on page 238.

DIFFERENCE OF SQUARE BINOMIALS

The product of two binomials which results in the difference of their squares, is known as the difference of two squares. This occurs when two identical binomials that have different signs are multiplied together.

In this case the sum of the middle terms is always equal to 0.

$$\begin{aligned}(a-b)(a+b) &= a^2+ab-ab-b^2 \\ &= a^2-b^2 \rightarrow \text{no middle term}\end{aligned}$$

Example 1: Expand and simplify $(x-2)(x+2)$.

$$\begin{aligned}&(x-2)(x+2) \\ &=x^2-4\end{aligned}$$

Example 2: Expand and simplify $(x-a)(x+a)$.

$$\begin{aligned}&(x-a)(x+a) \\ &=x^2-a^2\end{aligned}$$

Example 3: Expand and simplify $(3x+4)(3x-4)$.

$$\begin{aligned}&(3x+4)(3x-4) \\ &=9x^2-16\end{aligned}$$

Example 4: Expand and simplify $[x-(a-4)][x+(a-4)]$.

$$\begin{aligned}&[x-(a-4)][x+(a-4)] \rightarrow \text{these are still the same brackets with different signs} \\ &=x^2-(a-4)^2 \\ &=x^2-(a^2-8a+16) \rightarrow \text{don't forget the middle term when squaring binomials} \\ &=x^2-a^2+8a-16 \rightarrow \text{don't forget to distribute the minus}\end{aligned}$$

Complete Exercise 1 number 4 on page 238.

3. FACTORISATION BY TAKING OUT A HIGHEST COMMON FACTOR

So far in this chapter we have explored different forms of multiplication.
Factorisation is reversing the process of multiplication. The most basic form of factorisation is taking out a highest common factor (HCF).
If possible, the first step in factorisation is always to take out a HCF.

$ax+bx+cx \rightarrow x$ is common to all terms

$= x(a+b+c) \rightarrow$ as x is common to to all terms, a HCF of x can be taken out

i.e. divide each term by the HCF

If you distribute the HCF back into the bracket you must get the original expression.

Example 1: **Factorise $5x+10y+20z$.**

$5x+10y+20z \rightarrow 5$ is a factor that is common to all the terms

$= 5(x+2y+4z) \rightarrow$ take out a common factor of 5

Example 2: **Factorise $ax+bx+10x$.**

$ax+bx+10x \rightarrow x$ is a factor that is common to all the terms

$= x(a+b+10) \rightarrow$ take out a common factor of x

Example 3: **Factorise $8x^2+2x$.**

$8x^2+2x \rightarrow 2$ and x are factors that are common to all the terms

$= 2x(4x+1) \rightarrow$ take out a common factor of $2x$, you have to write the 1

Example 4: **Factorise $3b(x^2+1)-10(x^2+1)$.**

$3b(x^2+1)-10(x^2+1) \rightarrow$ in this case an entire bracket is common and can be taken out

$= (x^2+1)(3b-10) \rightarrow$ take out a common factor of (x^2+1)

Example 5: **Factorise $3y(x-1)+(-x+1)$.**

$3y(x-1)+(-x+1) \rightarrow$ change the signs in the second bracket to make a common bracket

$= 3y(x-1)-(x-1) \rightarrow$ take out a minus, to make a common bracket of $(x-1)$

$= (x-1)(3y-1) \rightarrow$ you have to write the 1

You can often take out a 'minus' in order to make a common bracket.

Complete Exercise 1 number 5 on page 238.

4. FACTORISATION OF THE DIFFERENCE OF TWO SQUARES

Factorisation of the difference of two squares is the opposite of expanding difference of square binomials (pg. 13). When factorising using the difference of two squares, rewrite the two squares as the product of their roots.

$$\left(x^2-a^2\right)=\left(\sqrt{x^2}-\sqrt{a^2}\right)\left(\sqrt{x^2}+\sqrt{a^2}\right)$$
$$=(x-a)(x+a)$$

Only use the difference of two sqaures if the two sqaures are separated by a minus sign.

Example 1: **Factorise x^2-25.**

x^2-25

$=(x-5)(x+5)$

Example 2: **Factorise $\frac{1}{9}x^2-\frac{1}{25}$.**

$\frac{1}{9}x^2-\frac{1}{25} \rightarrow \frac{1}{9}$ and $\frac{1}{25}$ are perfect squares

$=\left(\frac{1}{3}x-\frac{1}{5}\right)\left(\frac{1}{3}x+\frac{1}{5}\right)$

Example 3: **Factorise x^2+1.**

$x^2+1 \rightarrow$ this is the sum of two squares **not** the difference of two squares

$\therefore$ No solution $\rightarrow$ difference of two squares has to be separated by a minus sign

Example 4: **Factorise $18x^2-32$.**

$18x^2-32 \rightarrow 2$ is common to 18 and 32, always look for an HCF first

$=2\left(9x^2-16\right) \rightarrow$ now you can use difference of two squares

$=2(3x-4)(3x+4) \rightarrow$ do not drop the 2

Example 5: **Factorise $16x^4-1$.**

$16x^4-1$

$=\left(4x^2+1\right)\left(4x^2-1\right) \rightarrow$ this can be taken further as $4x^2-1$ is a difference of 2 squares

$=\left(4x^2+1\right)(2x+1)(2x-1) \rightarrow 4x^2+1$ cannot be factorised any further

Complete Exercise 1 number 6 on page 238.

MULTIPLYING A BINOMIAL BY A TRINOMIAL

When multiplying a binomial by a trinomial, each term in the binomial has to be multiplied by each term in the trinomial.

$$(x+y)(a+b+c)=ax+bx+cx+ay+by+cy$$

The methods for multiplying a binomial by a trinomial do not change from the method of multiplying binomials together (Pg. 11).

Example 1: Expand and simplify $(x+a)(2x-3a+1)$.

$$(x+a)(2x-3a+1)$$
$$=(x)(2x)+(x)(-3a)+(x)(1)+(a)(2x)+(a)(-3a)+(a)(1)$$
$$=2x^2-3ax+x+2ax-3a^2+a \rightarrow \text{add like terms}$$
$$=2x^2+x-ax-3a^2+a$$

Example 2: Expand and simplify $(x-3)\left(x^2-3x+2\right)$.

$$(x-3)\left(x^2-3x+2\right)$$
$$=(x)\left(x^2\right)+(x)(-3x)+(x)(2)+(-3)\left(x^2\right)+(-3)(-3x)+(-3)(2)$$
$$=x^3-3x^2+2x-3x^2+9x-6$$
$$=x^3-6x^2+11x-6$$

Example 3: Expand and simplify $(x-y)^3$.

$$(x-y)^3$$
$$=(x-y)(x-y)^2$$
$$=(x-y)\left(x^2-2xy+y^2\right) \rightarrow \text{don't forget the middle term}$$
$$=(x)\left(x^2\right)+(x)(-2xy)+(x)\left(y^2\right)+(-y)\left(x^2\right)+(-y)(-2xy)+(-y)\left(y^2\right)$$
$$=x^3-2x^2y+xy^2-x^2y+2xy^2-y^3$$
$$=x^3-3x^2y+3xy^2-y^3$$

Complete Exercise 2 on page 239.

FACTORISING TRINOMIALS

The general form of a quadratic trinomial is $ax^2 \pm bx \pm c$. There are two types of quadratic equations that you are expected to factorise:

1. **A quadratic trinomial where the coefficient of x^2 is equal to 1 ($a = 1$).**
2. **A quadratic trinomial where the coefficient of x^2 is not equal to 1 ($a \neq 1$).**

Quadratic binomials factorise into the product of two binomials.

Factorising Trinomials in the Form of $x^2 \pm bx + c$

$$x^2 \pm bx + c$$

→ if the sign before the constant is a '+' then you know both signs in the brackets will be the same

→ both signs in the brackets will be the same as the sign before the coefficient of x

Illustrative Example 1: Factorise $x^2 + 5x + 6$.

$x^2 + 5x + 6$

Both signs in the brackets will be the same because the sign of the constant is positive.

Both signs in the brackets will be a '+' as the coefficient of x is positive.

$(x + \square)(x + \square)$

In order to find the missing numbers you look at the factors of the constant (6).

The factors of 6 that can be added together that will result in the coefficient of x (5) will be the correct factors.

Factors of 6 are: $\begin{matrix} 6 \\ 1 \end{matrix} \quad \boxed{\begin{matrix} 3 \\ 2 \end{matrix}} \rightarrow$ 3 and 2 added together is equal to 5

$\therefore x^2 + 5x + 6 = (x + 3)(x + 2)$

Illustrative Example 2: Factorise $x^2 - 9x + 20$.

$x^2 - 9x + 20$

Both signs in the brackets will be the same because the sign of the constant is positive.

Both signs in the brackets will be a '−' as the coefficient of x is negative.

$(x - \square)(x - \square)$

In order to find the missing numbers you look at the factors of the constant (20).

The factors of 20 that can be added together, that will result in the coefficient of x (-9), will be the correct factors.

Factors of 20 are: $\begin{matrix} 20 & 10 \\ 1 & 2 \end{matrix} \quad \boxed{\begin{matrix} 5 \\ 4 \end{matrix}} \rightarrow$ 5 and 4 added together is equal to 9

$\therefore x^2 - 9x + 20 = (x - 5)(x - 4)$

Example 1: **Factorise $x^2-7x+12$.**

$x^2-7x+12 \rightarrow$ both signs in the bracket are the same, both signs are '$-$'

$=(x-4)(x-3) \rightarrow$ factors of 12: $\begin{array}{cc}12 & 6\\ 1 & 2\end{array}\boxed{\begin{array}{c}4\\ 3\end{array}} \rightarrow -4-3=-7$

Example 2: **Factorise $x^2+11x+24$.**

$x^2+11x+24 \rightarrow$ both signs in the bracket are the same, both signs are '$+$'

$=(x+8)(x+3) \rightarrow$ factors of 24: $\begin{array}{ccc}24 & 12 & 6\\ 1 & 2 & 4\end{array}\boxed{\begin{array}{c}8\\ 3\end{array}} \rightarrow 8+3=11$

Example 3: **Factorise $2x^2+18x+36$.**

$2x^2+18x+36 \rightarrow$ always look for a common factor first (2 is common to all the terms)

$=2\left(x^2+9x+18\right) \rightarrow$ both signs in the bracket are the same, both signs are '$+$'

$=2(x+6)(x+3) \rightarrow$ factors of 18: $\begin{array}{cc}18 & 9\\ 1 & 2\end{array}\boxed{\begin{array}{c}6\\ 3\end{array}} \rightarrow 6+3=9$

Example 4: **Factorise $3x^2-24x+45$.**

$3x^2-24x+45 \rightarrow$ always look for a common factor first (3 is common to all the terms)

$=3\left(x^2-8x+15\right) \rightarrow$ both signs in the bracket are the same, both signs are '$-$'

$=3(x-5)(x-3) \rightarrow$ factors of 15: $\begin{array}{c}15\\ 1\end{array}\boxed{\begin{array}{c}5\\ 3\end{array}} \rightarrow -5-3=-8$

Example 5: **Factorise $2x^2+12x+18$.**

$2x^2+12x+18 \rightarrow$ always look for a common factor first (2 is common to all the terms)

$=2\left(x^2+6x+9\right) \rightarrow$ both signs in the bracket are the same, both signs are '$+$'

$=2(x+3)(x+3) \rightarrow$ factors of 9: $\begin{array}{c}9\\ 1\end{array}\boxed{\begin{array}{c}3\\ 3\end{array}} \rightarrow 3+3=6$

$=2(x+3)^2$

When factorising, always look for a Highest Common Factor (HCF) first (pg. 14).

Complete Exercise 3 number 1 on page 239.

Factorising Trinomials in the Form of $x^2 \pm bx - c$

$x^2 \pm bx - c$

→ if the sign before the constant is a '−' then you know there will be opposite signs in the brackets

→ the larger of the two factors will get the sign before the coefficient of x

Illustrative Example 1: Factorise $x^2 - x - 6$.

$x^2 - x - 6$

The signs in the brackets will be opposite because the sign of the constant is negative.

The larger factor will get a '−' as the coefficient of x is negative.

$(x-\square)(x+\square)$

In order to find the missing numbers you look at the factors of the constant (6).

The correct factors will be the factors of 6, where the difference between the factors will result in the coefficient of x (-1).

Factors of 6 are: $\begin{matrix} 6 \\ 1 \end{matrix} \quad \boxed{\begin{matrix} 3 \\ 2 \end{matrix}} \rightarrow -3+2=-1$

$\therefore x^2 - x - 6 = (x-3)(x+2) \rightarrow$ 3 is the larger factor, $\therefore$ the 3 gets the '−'

Illustrative Example 2: Factorise $x^2 + 11x - 12$.

$x^2 + 11x - 12$

The signs in the brackets will be opposite because the sign of the constant is negative.

The larger factor will get a '+' as the coefficient of x is positive.

$(x-\square)(x+\square)$

In order to find the missing numbers you look at the factors of the constant (12).

The correct factors will be the factors of 12, where the difference between the factors will result in the coefficient of x (11).

Factors of 12 are: $\begin{matrix} 6 \\ 2 \end{matrix} \quad \boxed{\begin{matrix} 12 \\ 1 \end{matrix}} \rightarrow 12-1=11$

$\therefore x^2 + 11x - 12 = (x-1)(x+12) \rightarrow$ 12 is the larger factor, $\therefore$ the 12 gets the '+'

Example 1: Factorise x^2-2x-8.

$x^2-2x-8 \rightarrow$ opposite signs in the brackets, the larger factor gets a '$-$'

$$\text{factors of 8: } \begin{matrix}8\\1\end{matrix}\ \boxed{\begin{matrix}4\\2\end{matrix}} \rightarrow\ -4+2=-2$$

$=(x-4)(x+2) \rightarrow 4$ is the larger factor, $\therefore$ the 4 gets the '$-$'

Example 2: Factorise $x^2-2x-15$.

$x^2-2x-15 \rightarrow$ opposite signs in the brackets, the larger factor gets a '$-$'

$$\text{factors of 15: } \begin{matrix}15\\1\end{matrix}\ \boxed{\begin{matrix}5\\3\end{matrix}} \rightarrow\ -5+3=-2$$

$=(x-5)(x+3) \rightarrow 5$ is the larger factor, $\therefore$ the 5 gets the '$-$'

Example 3: Factorise $2x^2+14x-36$.

$2x^2+14x-36 \rightarrow$ always look for a common factor first (2 is common to all the terms)

$=2\left(x^2+7x-18\right) \rightarrow$ opposite signs in the brackets, the larger factor gets a '$+$'

$$\text{factors of 18: } \begin{matrix}18 & 6\\1 & 3\end{matrix}\ \boxed{\begin{matrix}9\\2\end{matrix}} \rightarrow\ 9-2=7$$

$=2(x+9)(x-2) \rightarrow 9$ is the larger factor, $\therefore$ the 9 gets the '$+$'

Example 4: Factorise $3x^2+6x-9$.

$3x^2+6x-9 \rightarrow$ always look for a common factor first (3 is common to all the terms)

$=3\left(x^2+2x-3\right) \rightarrow$ opposite signs in the brackets, the larger factor gets a '$+$'

$$\text{factors of 3: } \boxed{\begin{matrix}3\\1\end{matrix}} \rightarrow\ 3-1=2$$

$=3(x+3)(x-1) \rightarrow 3$ is the larger factor, $\therefore$ the 3 gets the '$+$'

Example 5: Factorise $5x^2+5x-10$.

$5x^2+5x-10 \rightarrow$ always look for a common factor first (5 is common to all the terms)

$=5\left(x^2+x-2\right) \rightarrow$ opposite signs in the brackets, the larger factor gets a '$+$'

$$\text{factors of 2: } \boxed{\begin{matrix}2\\1\end{matrix}} \rightarrow\ 2-1=1$$

$=5(x+2)(x-1) \rightarrow 2$ is the larger factor, $\therefore$ the 2 gets the '$+$'

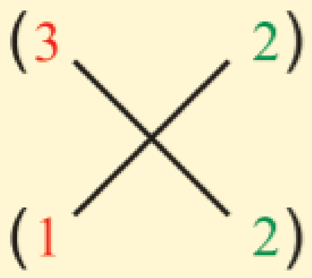
Complete Exercise 3 number 2 on page 239.

THE CROSS METHOD

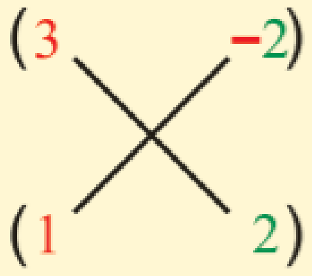
The cross method is used when the coefficient of x^2 is not equal to one.
The best way to illustrate the cross method is through an example.

Illustrative Example: **Factorise $3x^2+4x-4$**

Always look for an HCF first (in this case there isn't one).
Now place the factors of 3 on the left hand side of the cross and the factors of 4 on the right hand side. Then multiply diagonally across the lines to obtain two numbers, which when added or subtracted will make up the coefficient of x (which is 4).



When **multiplying diagonally across the lines** you get 6 and 2. With these numbers you can make up 4. However, the 2 has to be negative. Therefore, you make the number on the right hand side of the cross that makes up the 2 negative.

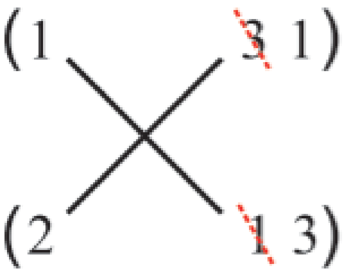


This gives you the two brackets. You always add the x to the number on the left hand side of the cross.

$$\therefore 3x^2+4x-4=(3x-2)(x+2)$$

Example 1: **Factorise $2x^2+5x+3$.**

$2x^2+5x+3 \rightarrow$ there is a coefficient of x^2 and there is no HCF $\therefore$ use the cross method

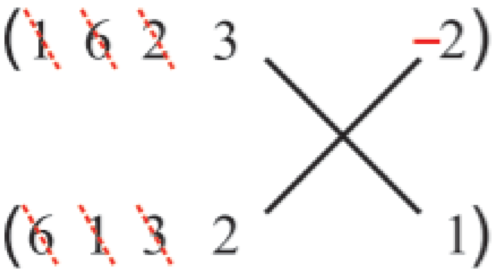


$\rightarrow$ 2x3=6 1x1=1 $\rightarrow$ You can obtain a '5' with a '6' and '1'. However, the '1' has to be negative and both signs have to be positive. Therefore, the first choice is incorrect.

$$\therefore 2x^2+5x+3=(x+1)(2x+3)$$

Example 2: **Factorise $6x^2-x-2$.**

$6x^2-x-2 \rightarrow$ there is a coefficient of x^2 and there is no HCF $\therefore$ use the cross method



$\rightarrow$ 3x1=3 2x2=4 $\rightarrow$ These are the only two factors that can be used to obtain a '-1' however the '4' has to be negative. Therefore one of the two's has to be negative.

$$\therefore 6x^2-x-2=(3x-2)(2x+1)$$

Complete Exercise 3 number 3 on page 239.

FACTORISATION BY GROUPING IN PAIRS

Grouping is generally used to factorise an expression with more than 3 terms. The idea behind grouping is to group terms together and then, if possible, take out a HCF in order to obtain a common bracket.

Example 1: Factorise $my+5ny+mx+5nx$.

$my+5ny+mx+5nx \rightarrow$ group the 1st and 2nd terms, and the 3rd and 4th terms

$=y(m+5n)+x(m+5n) \rightarrow$ you now have a bracket that is common

$=(m+5n)(y+x)$

This example could also have been done by grouping the first and third term together and grouping the second and fourth term together, as shown below:

$my+5ny+mx+5nx \rightarrow$ group the 1st and 3rd terms, and the 2nd and 4th terms

$=my+mx+5ny+5nx$

$=m(y+x)+5n(y+x) \rightarrow$ you now have a bracket that is common

$=(y+x)(m+5n)$

Example 2: Factorise $5ax-5bx-ay+by$.

$5ax-5bx-ay+by \rightarrow\rightarrow$ group the 1st and 2nd terms, and the 3rd and 4th terms

$=5x(a-b)-y(a-b) \rightarrow$ take out $'-y'$ in order to make a common bracket

$=(a-b)(5x-y)$

This example could also have been done by grouping the first and third term together and grouping the second and fourth term together, as shown below:

$5ax-5bx-ay+by \rightarrow$ group the 1st and 3rd terms, and the 2nd and 4th terms

$=a(5x-y)-b(5x-y) \rightarrow$ take out $'-b'$ in order to make a common bracket

$=(5x-y)(a-b)$

Example 3: Factorise $3px+6py+2qx+4qy$.

$3px+6py+2qx+4qy \rightarrow$ group the 1st and 2nd terms, and the 3rd and 4th terms

$=3p(x+2y)+2q(x+2y) \rightarrow$ you now have a bracket that is common

$=(x+2y)(3p+2q)$

This example could also have been done by grouping the first and third term together and grouping the second and fourth term together, as shown below:

$3px+6py+2qx+4qy \rightarrow$ group the 1st and 3rd terms, and the 2nd and 4th terms

$=x(3p+2q)+2y(3p+2q) \rightarrow$ you now have a bracket that is common

$=(3p+2q)(x+2y)$

Example 4: **Factorise $x^2+2xy+y^2-4$.**

$x^2+2xy+y^2-4 \rightarrow$ you have to recognise this as a perfect square trinomial (pg. 12)

$=(x+y)^2-4 \rightarrow$ difference of two squares

$=[(x+y)-2][(x+y)+2]$

$=(x+y-2)(x+y+2)$

Example 5: **Factorise $16-x^2-4y+yx$.**

$16-x^2-4y+yx \rightarrow$ difference of squares

$=(4-x)(4+x)-y(4-x) \rightarrow$ take out a negative y, $(4-x)$ is a HCF

$=(4-x)(4+x-y)$

Example 6: **Factorise a^2-a-b^2-b.**

$a^2-a-b^2-b \rightarrow$ group the first and third terms, and the second and fourth terms

$=a^2-b^2-(a+b) \rightarrow$ the grouping has obtained the difference of 2 squares

$=(a-b)(a+b)-(a+b) \rightarrow (a+b)$ is a HCF

$=(a+b)(a-b-1)$

Example 7: **Factorise $p-5-5p^4+p^5$.**

$p-5-5p^4+p^5 \rightarrow$ take out a HCF of p^4

$=p-5+p^4(-5+p) \rightarrow -5+p$ is the same as $p-5$

$=(p-5)+p^4(p-5) \rightarrow p-5$ is a HCF

$=(p-5)(1+p^4)$

Example 8: **Factorise $2-x-2x^2+x^3$.**

$2-x-2x^2+x^3 \rightarrow$ take out a HCF of $-x^2$

$=(2-x)-x^2(2-x) \rightarrow 2-x$ is a HCF

$=(2-x)(1-x^2) \rightarrow 1-x^2$ is the difference of two squares, $\therefore$ you must factorise further

$=(2-x)(1-x)(1+x)$

Be on the lookout for perfect square trinomials and the difference of two squares. Make sure that you always factorise as far as possible.

Complete Exercise 3 number 4 on page 239.

FACTORISATION OF THE SUM AND DIFFERENCE OF CUBES

The sum and difference of perfect cubes always factorise into the form shown below:

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

1st bracket : cube root each term and keep the same sign

2nd bracket : square the first term, change the sign and multiply the terms together

Square the last term (the sign of the last term is always positive)

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \qquad a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

The sum and difference of cubes always factorises into a binomial and a trinomial.

Example 1: Factorise $8x^3 + y^3$.

$8x^3 + y^3$

$= (2x + y)(4x^2 - 2xy + y^2)$

Example 2: Factorise $54x^3 - 16$.

$54x^3 - 16$ → always look for a HCF when factorising (2 is common to both terms)

$= 2(27x^3 - 8)$

$= 2(3x - 2)(9x^2 + 6x + 4)$ → you cannot drop the 2

Example 3: Factorise $(x - y)^3 - 64x^3$.

$(x - y)^3 - 64x^3$ → difference of two perfect cubes

$= [(x - y) - 4x][(x - y)^2 + 4x(x - y) + 16x^2]$

$= (x - y - 4x)(x^2 - 2xy + y^2 + 4x^2 - 4xy + 16x^2)$ → gather like terms

$= (-3x - y)(21x^2 - 6xy + y^2)$

In factorisation of the sum and difference of two cubes the trinomial, (in the second bracket) will never factorise further.

Complete Exercise 3 number 5 on page 240.

C. SIMPLIFYING ALGEBRAIC EXPRESSIONS

There are two ways that you will be expected to simplify algebraic fractions:

1. **Multiplication and division**
2. **Addition and subtraction**

RULES OF FRACTIONS

Rule		Explanation
1	$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$	**When multiplying fractions, multiply the numerators and the denominators.** **Factorise numerators and denominators if possible and try to cancel first.**
	Restrictions: $b\neq 0$ and $d\neq 0$	
2	$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$	**When dividing fractions, invert the fraction after the divide sign and change the operation from division to multiplication.** **This is known as the tip and times rule.**
	Restrictions: $b\neq 0$; $c\neq 0$ and $d\neq 0$	
3	$\frac{a}{b}\pm\frac{c}{b}=\frac{a\pm c}{b}$	**Fractions can only be added (or subtracted) if they have the same denominators. If the denominators are the same, keep the denominator and add the numerators together.**
	Restrictions: $b\neq 0$	

You cannot add or subtract fractions if the denominators are not the same.

MULTIPLICATION AND DIVISION OF FRACTIONS

Example 1: Simplify $\frac{12}{x}\times\frac{4x^2}{3}$**.**

$\frac{12}{x}\times\frac{4x^2}{3}$ → you can only cancel numerators with denominators

$=\frac{\overset{4}{\cancel{12}}}{\cancel{x}}\times\frac{4\overset{x}{\cancel{x^2}}}{\cancel{3}}$

$=\frac{4}{1}\times\frac{4x}{1}$

$=16x$

The denominator of a fraction can never be equal to zero.

Example 2: **Simplify** $\frac{3}{4x}\times\frac{12x}{11}\div\frac{3x}{22}$.

$$\frac{3}{4x}\times\left(\frac{12x}{11}\div\frac{3x}{22}\right)\rightarrow \text{always do division first}$$

$$\frac{3}{4x}\times\left(\frac{12x}{11}\times\frac{22}{3x}\right)\rightarrow \text{tip and times}$$

$$=\frac{3}{4x}\times\left(\frac{{}^{4}\cancel{12}\,\cancel{x}}{\cancel{11}}\times\frac{\cancel{22}^{\,2}}{\cancel{3}\,\cancel{x}}\right)$$

$$=\frac{3}{\cancel{4}x}\times\left(\frac{\cancel{8}^{\,2}}{1}\right)$$

$$=\frac{6}{x}$$

Example 3: **Simplify** $\frac{3}{4}\div\frac{1}{2}\times\frac{3}{2}$.

$$\left(\frac{3}{4}\div\frac{1}{2}\right)\times\frac{3}{2}\rightarrow \text{always do division first}$$

$$\left(\frac{3}{4}\times\frac{2}{1}\right)\times\frac{3}{2}\rightarrow \text{tip and times}$$

$$=\left(\frac{3}{{}^{2}\cancel{4}}\times\frac{\cancel{2}}{1}\right)\times\frac{3}{2}$$

$$=\frac{3}{2}\times\frac{3}{2}$$

$$=\frac{9}{4}$$

Example 4: **Simplify** $\frac{3}{5x}\times\frac{5(x+2)}{3}$.

$$\frac{3}{5x}\times\frac{5(x+2)}{3}\rightarrow \text{never cancel over a '+' or '$-$' sign}$$

$$=\frac{\cancel{3}}{\cancel{5}x}\times\frac{\cancel{5}(x+2)}{\cancel{3}}$$

$$=\frac{x+2}{x}\rightarrow \text{never cancel over a '+' or '$-$' sign}$$

Complete Exercise 4 on page 240.

ADDITION AND SUBTRACTION OF FRACTIONS

Fractions can only be added or subtracted if they have the same denominator. In order to add or subtract fractions you have to understand the concepts of a **Lowest Common Multiple (LCM)** and **Lowest Common Denominator (LCD)**.

LCM's and LCD's:

The **lowest common multiple** of two or more expressions is the smallest multiple that each expression can be divided into, without leaving a remainder.
The **lowest common denominator** is the LCM of all the denominators in an expression.

Illustrative Examples:

The **LCM** of $2,\ 3$ and 4 is **12**

The **LCM** of $3a,\ 6a^2$ and $12a^3$ is $\mathbf{12a^3}$ → the LCM of a variable is the variable with the largest exponent

The **LCD** of $\frac{1}{2},\ \frac{3}{x}$ and $\frac{5}{3x^2}$ is $\mathbf{6x^2}$

Each factor of the different terms must be represented in the LCM.

Example 1: Simplify $\mathbf{\frac{2x}{3}+\frac{5}{4}+\frac{3x}{12}}$.

$$\frac{2x}{3}+\frac{5}{4}+\frac{3x}{12} \rightarrow \text{LCD} = 12$$

$$=\frac{2x(4)}{3(4)}+\frac{5(3)}{4(3)}+\frac{3x}{12} \rightarrow \text{what you do to the bottom you do to the top}$$

$$=\frac{8x}{12}+\frac{15}{12}+\frac{3x}{12} \rightarrow \text{now that the denominators are the same you can add the numerators}$$

$$=\frac{8x+15+3x}{12}$$

$$=\frac{11x+15}{12}$$

Whatever is done to the denominator to obtain the LCD has to be done to the numerator. (Whatever you do to the bottom you do to the top).

Example 2: **Simplify** $\frac{2}{5x}+\frac{1}{3x^3}-\frac{3}{15x^2}$.

$$\frac{2}{5x}+\frac{1}{3x^3}-\frac{3}{15x^2} \rightarrow \text{LCD} = 15x^3$$

$$=\frac{2\left(3x^2\right)}{5x\left(3x^2\right)}+\frac{1(5)}{3x^3(5)}-\frac{3(x)}{15x^2(x)} \rightarrow \text{what you do to the bottom you do to the top}$$

$$=\frac{6x^2+5-3x}{15x^3}$$

Example 3: **Simplify** $\frac{x-3}{2}+\frac{4x-2}{3}-\frac{11x+2}{6}$.

$$\frac{x-3}{2}+\frac{4x-2}{3}-\frac{11x+2}{6} \rightarrow \text{LCD} = 6$$

$$=\frac{3(x-3)+2(4x-2)-(11x+2)}{6} \rightarrow \text{don't forget to distribute the minus}$$

$$=\frac{3x-9+8x-4-11x-2}{6}$$

$$=\frac{-15}{6}$$

$$=\frac{-5}{2}$$

Example 4: **Simplify** $\frac{8x+2}{4}+\frac{6x+7}{6}-\frac{9x+11}{3}$.

$$\frac{8x+2}{4}+\frac{6x+7}{6}-\frac{9x+11}{3} \rightarrow \text{LCD} = 12$$

$$=\frac{3(8x+2)}{4(3)}+\frac{2(6x+7)}{6(2)}-\frac{4(9x+11)}{3(4)} \rightarrow \text{what you do to the bottom you do to the top}$$

$$=\frac{24x+6+12x+14-36x-44}{12} \rightarrow \text{don't forget to distribute the minus}$$

$$=\frac{-24}{12}$$

$$=-2$$

Complete Exercise 5 on page 240.

EXAMPLES INVOLVING A SIGN CHANGE

When dealing with fractions it is useful to note the following:

1) $-\frac{x}{y}=\frac{-x}{y}=\frac{x}{-y}$

2) $-\frac{x-y}{y}=\frac{x-y}{-y}=\frac{-(x-y)}{y}=\frac{-x+y}{y}$

3) $(x-y)^2=(y-x)^2 \rightarrow (x-y)^2=\left[-(+y-x)\right]^2=(y-x)^2$

Example 1: **Simplify** $\frac{3(x-1)}{x-5}\div\frac{2(1-x)}{5-x}$.

$$\frac{3(x-1)}{x-5}\div\frac{2(1-x)}{5-x}$$

$$=\frac{3(x-1)}{x-5}\times\frac{5-x}{2(1-x)} \rightarrow \text{'wrong order', take out a minus}$$

$$=\frac{3(x-1)}{x-5}\times\frac{-(x-5)}{-2(x-1)}$$

$$=\frac{3}{2}$$

Example 2: **Simplify** $\frac{(x-y)^2}{2}\div\frac{2(y-x)^2}{3}$.

$$\frac{(x-y)^2}{2}\div\frac{2(y-x)^2}{3}$$

$$=\frac{(x-y)^2}{2}\times\frac{3}{2(y-x)^2} \rightarrow \text{tip and times}$$

$$=\frac{3(x-y)^2}{4(y-x)^2} \rightarrow (y-x)^2=(x-y)^2\text{, see note (3) in the above Edison}$$

$$=\frac{3\cancel{(x-y)^2}}{4\cancel{(x-y)^2}}$$

$$=\frac{3}{4}$$

MIXED EXAMPLES

Example 1: Simplify $\frac{4x^2+8x}{4x}$.

$$\frac{4x^2+8x}{4x} \rightarrow \text{never cancel over a '+' or '−' sign, always factorise}$$

$$=\frac{\cancel{4x}(x+2)}{\cancel{4x}} \rightarrow \text{a HCF of } 4x \text{ was taken out}$$

$$=x+2$$

Example 2: Simplify $\frac{(x-5)(x+1)+4(x+1)}{(x+1)}$.

$$\frac{(x-5)(x+1)+4(x+1)}{(x+1)} \rightarrow \text{never cancel over a '+' or '−' sign, always factorise}$$

$$=\frac{(x+1)[(x-5)+4]}{(x+1)} \rightarrow \text{a HCF of } (x+1) \text{ was taken out}$$

$$=\frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}} \rightarrow -5+4=-1$$

$$=x-1$$

Example 3: Simplify $\frac{x}{x-1}-\frac{12}{x^2-1}-\frac{6}{x+1}$.

$$\frac{x}{x-1}-\frac{12}{x^2-1}-\frac{6}{x+1} \rightarrow \text{factorise}$$

$$=\frac{x}{x-1}-\frac{12}{(x-1)(x+1)}-\frac{6}{x+1}$$

$$=\frac{x(x+1)-12-6(x-1)}{(x-1)(x+1)} \rightarrow \text{LCD}=(x-1)(x+1)$$

$$=\frac{x^2+x-12-6x+6}{(x-1)(x+1)}$$

$$=\frac{x^2-5x-6}{(x-1)(x+1)}$$

$$=\frac{(x-6)(x+1)}{(x-1)(x+1)}$$

$$=\frac{x-6}{x-1}$$

Example 4: **Simplify** $\dfrac{1}{x+1}+\dfrac{2x}{x^2-1}+\dfrac{1}{1-x}$.

$$\frac{1}{x+1}+\frac{2x}{x^2-1}+\frac{1}{1-x} \rightarrow \text{factorise}$$

$$=\frac{1}{x+1}+\frac{2x}{(x-1)(x+1)}+\frac{1}{1-x}$$

$$=\frac{1}{x+1}+\frac{2x}{(x-1)(x+1)}-\frac{1}{x-1} \rightarrow \text{take out a 'minus' to get a denominator of } x-1$$

$$=\frac{1(x-1)+2x-(x+1)}{(x-1)(x+1)} \rightarrow \text{LCD}=(x-1)(x+1)$$

$$=\frac{x-1+2x-x-1}{(x-1)(x+1)}$$

$$=\frac{2x-2}{(x-1)(x+1)} \rightarrow \text{factorise}$$

$$=\frac{2(x-1)}{(x-1)(x+1)}$$

$$=\frac{2}{x+1}$$

Example 5: **Simplify** $\dfrac{x}{x+2}-\dfrac{2}{x-1}-\dfrac{6}{x^2+x-2}$.

$$\frac{x}{x+2}-\frac{2}{x-1}-\frac{6}{x^2+x-2}$$

$$=\frac{x}{x+2}-\frac{2}{x-1}-\frac{6}{(x+2)(x-1)} \rightarrow \text{factorise}$$

$$=\frac{x(x-1)-2(x+2)-6}{(x+2)(x-1)} \rightarrow \text{LCD}=(x+2)(x-1)$$

$$=\frac{x^2-x-2x-4-6}{(x+2)(x-1)}$$

$$=\frac{x^2-3x-10}{(x+2)(x-1)}$$

$$=\frac{(x-5)(x+2)}{(x+2)(x-1)}$$

$$=\frac{x-5}{x-1}$$

Example 6: **Simplify** $\dfrac{x^2-3x}{x^2-2x-3} \div \dfrac{x^2-4x}{x^2+2x+1} \times \dfrac{12x-48}{1}$.

$$\frac{x^2-3x}{x^2-2x-3} \div \frac{x^2-4x}{x^2+2x+1} \times \frac{12x-48}{1}$$

$$= \frac{x^2-3x}{x^2-2x-3} \times \frac{x^2+2x+1}{x^2-4x} \times \frac{12x-48}{1} \rightarrow \text{tip and times}$$

$$= \frac{x(x-3)}{(x-3)(x+1)} \times \frac{(x+1)(x+1)}{x(x-4)} \times \frac{12(x-4)}{1} \rightarrow \text{factorise}$$

$$= \frac{\cancel{x}\,\cancel{(x-3)}}{\cancel{(x-3)}\,\cancel{(x+1)}} \times \frac{\cancel{(x+1)}\,(x+1)}{\cancel{x}\,\cancel{(x-4)}} \times \frac{12\,\cancel{(x-4)}}{1}$$

$$= \frac{12(x+1)}{1}$$

$$= 12x+12$$

Example 7: **Simplify** $\dfrac{a}{1-\dfrac{a}{b}} \div \dfrac{b}{1-\dfrac{b}{a}}$.

$$\frac{a}{1-\dfrac{a}{b}} \div \frac{b}{1-\dfrac{b}{a}}$$

$$= \left[a \div \left(1-\frac{a}{b}\right)\right] \div \left[b \div \left(1-\frac{b}{a}\right)\right] \rightarrow \text{always use this notation when you have a } \frac{\text{fraction}}{\text{fraction}}$$

$$= \left[a \div \left(\frac{b-a}{b}\right)\right] \div \left[b \div \left(\frac{a-b}{a}\right)\right] \rightarrow \text{find an LCD for each bracket}$$

$$= \left(a \times \frac{b}{b-a}\right) \div \left(b \times \frac{a}{a-b}\right) \rightarrow \text{tip and times}$$

$$= \frac{ab}{b-a} \div \frac{ba}{a-b} \rightarrow \text{tip and times again and note that } ba \text{ is the same as } ab$$

$$= \frac{ab}{-(a-b)} \times \frac{a-b}{ab} \rightarrow \text{a 'minus' was taken out}$$

$$= -1$$

Example 8: **Simplify** $\dfrac{x^4-x}{x^2-4x+3} \div \dfrac{x^3+x^2+x}{x^2-5x+6}$.

$$\frac{x^4-x}{x^2-4x+3} \div \frac{x^3+x^2+x}{x^2-5x+6} \rightarrow \text{factorise numerators and denominators}$$

$$= \frac{x\left(x^3-1\right)}{(x-3)(x-1)} \div \frac{x\left(x^2+x+1\right)}{(x-3)(x-2)} \rightarrow \text{difference of cubes}$$

$$= \frac{x(x-1)\left(x^2+x+1\right)}{(x-3)(x-1)} \times \frac{(x-3)(x-2)}{x\left(x^2+x+1\right)} \rightarrow \text{tip and times}$$

$$= \frac{\cancel{x}\,\cancel{\left(x^2+x+1\right)}\,\cancel{(x-1)}}{\cancel{(x-3)}\,\cancel{(x-1)}} \times \frac{\cancel{(x-3)}\,(x-2)}{\cancel{x}\,\cancel{\left(x^2+x+1\right)}}$$

$$= x-2$$

Example 9: **Simplify** $\dfrac{2x^3-16}{x^2-7x+10} \div \dfrac{(x+1)\left(x^2+2x+4\right)}{x^2-4x-5}$.

$$\frac{2x^3-16}{x^2-7x+10} \div \frac{(x+1)\left(x^2+2x+4\right)}{x^2-4x-5} \rightarrow \text{factorise numerators and denominators}$$

$$= \frac{2\left(x^3-8\right)}{(x-5)(x-2)} \div \frac{(x+1)\left(x^2+2x+4\right)}{(x-5)(x+1)} \rightarrow \text{difference of cubes}$$

$$= \frac{2(x-2)\left(x^2+2x+4\right)}{(x-5)(x-2)} \times \frac{(x-5)(x+1)}{(x+1)\left(x^2+2x+4\right)} \rightarrow \text{tip and times}$$

$$= \frac{2\,\cancel{(x-2)}\,\cancel{\left(x^2+2x+4\right)}}{\cancel{(x-5)}\,\cancel{(x-2)}} \times \frac{\cancel{(x-5)}\,\cancel{(x+1)}}{\cancel{(x+1)}\,\cancel{\left(x^2+2x+4\right)}}$$

$$= 2$$

Complete Exercise 6 on page 240 and
the Mixed Exercise on pages 241 and 242.

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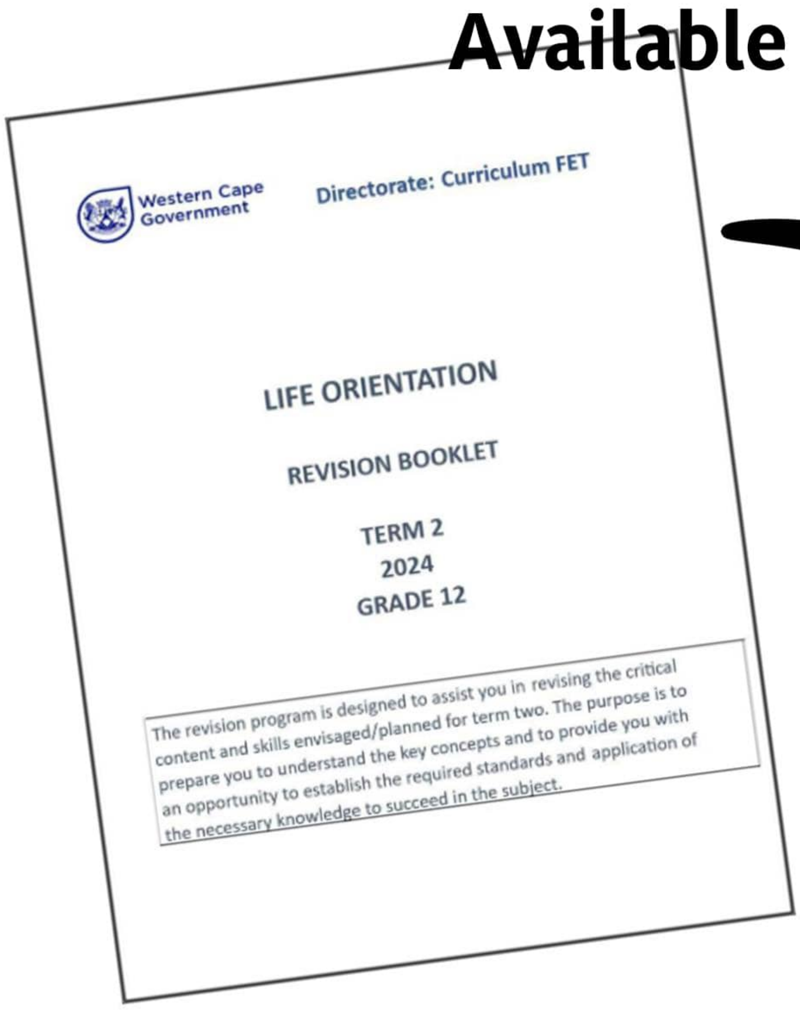

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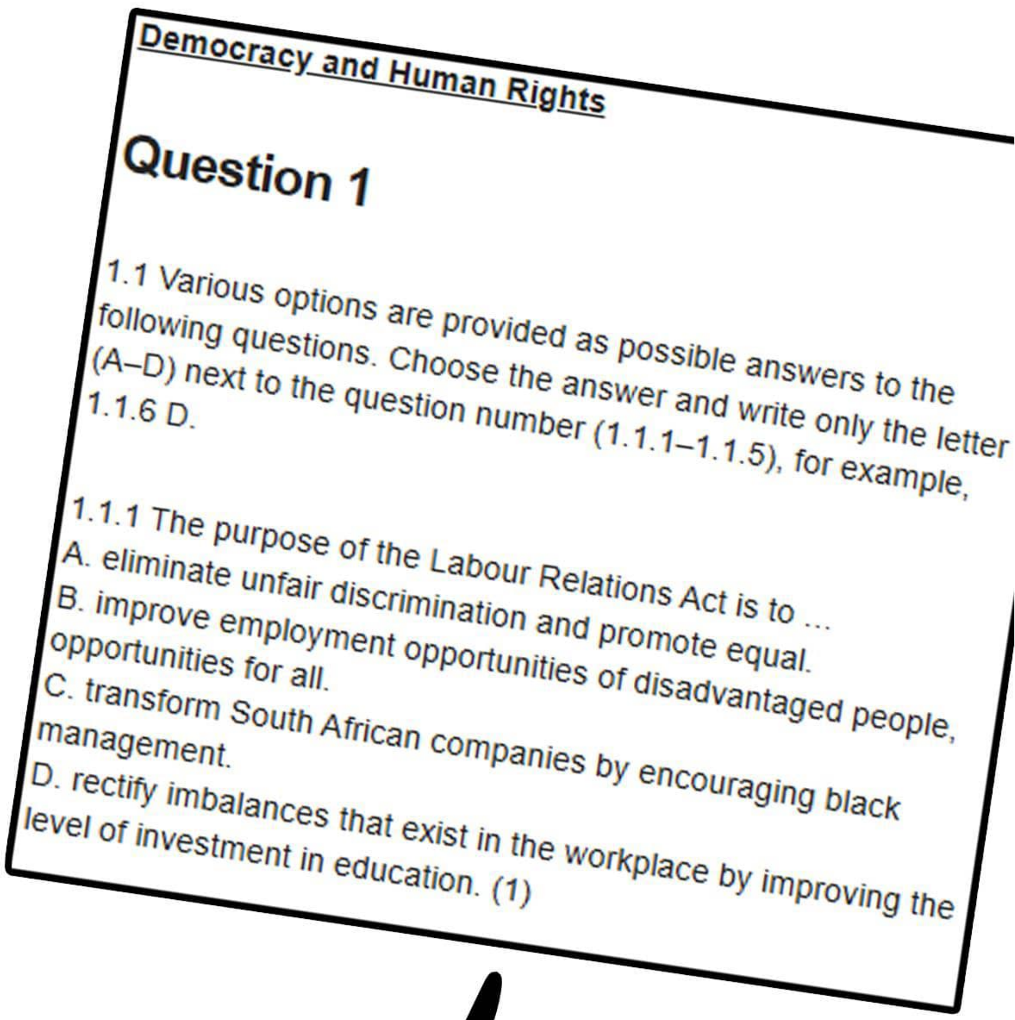
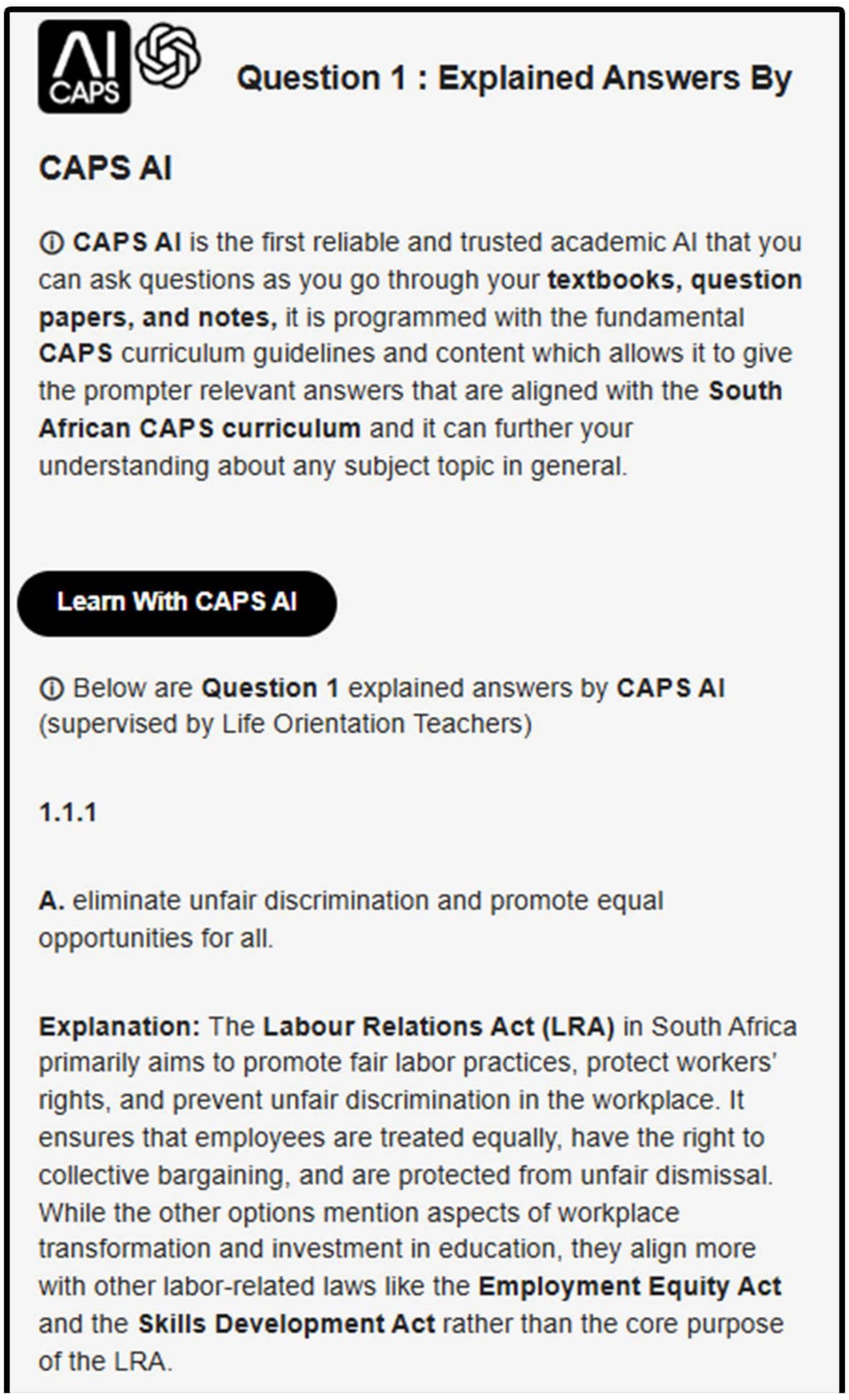


Democracy and Human Rights

Question 1

1.1 Various options are provided as possible answers to the following questions. Choose the answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5), for example, 1.1.6 D.

1.1.1 The purpose of the Labour Relations Act is to ...
A. eliminate unfair discrimination and promote equal opportunities for all.
B. improve employment opportunities of disadvantaged people,
C. transform South African companies by encouraging black management.
D. rectify imbalances that exist in the workplace by improving the level of investment in education. (1)



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ⓘ Below are **Question 1** explained answers by **CAPS AI** (supervised by Life Orientation Teachers)

1.1.1

A. eliminate unfair discrimination and promote equal opportunities for all.

Explanation: The **Labour Relations Act (LRA)** in South Africa primarily aims to promote fair labor practices, protect workers' rights, and prevent unfair discrimination in the workplace. It ensures that employees are treated equally, have the right to collective bargaining, and are protected from unfair dismissal. While the other options mention aspects of workplace transformation and investment in education, they align more with other labor-related laws like the **Employment Equity Act** and the **Skills Development Act** rather than the core purpose of the LRA.

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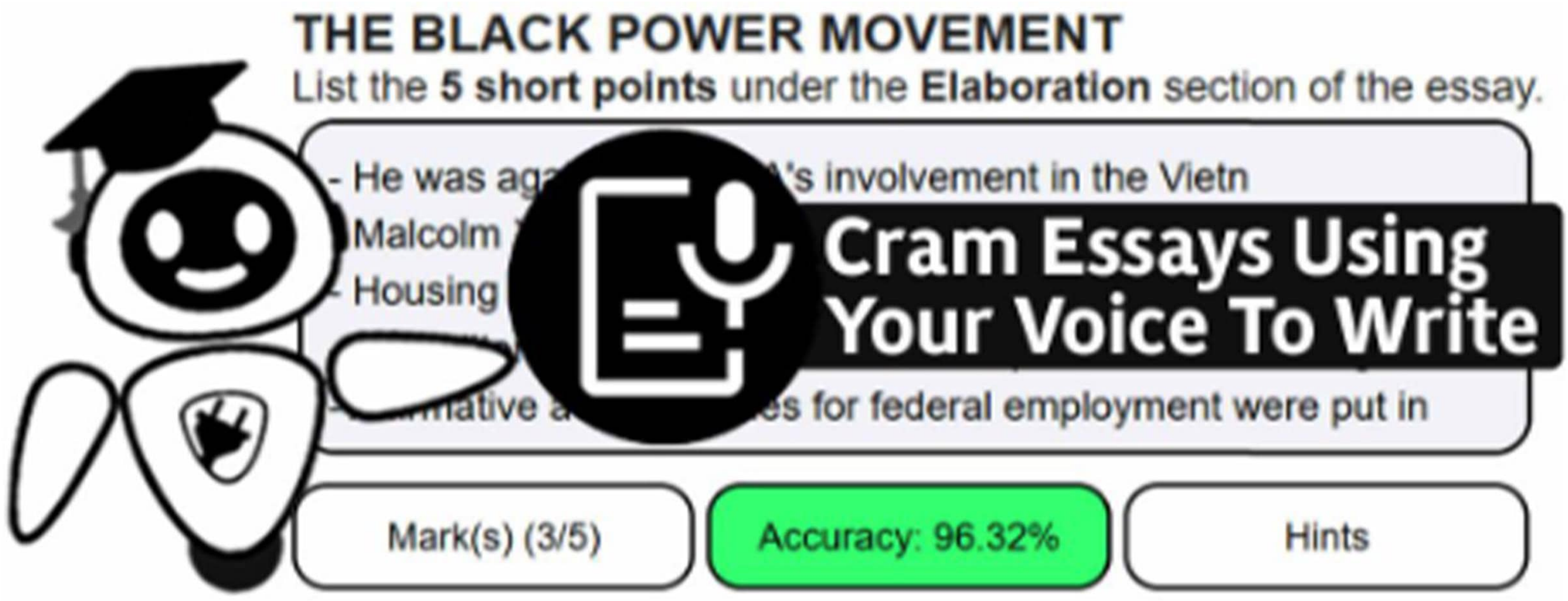

THE BLACK POWER MOVEMENT

List the **5 short points** under the **Elaboration** section of the essay.

- He was ag 's involvement in the Vietn
Malcolm
- Housing
- mative a es for federal employment were put in

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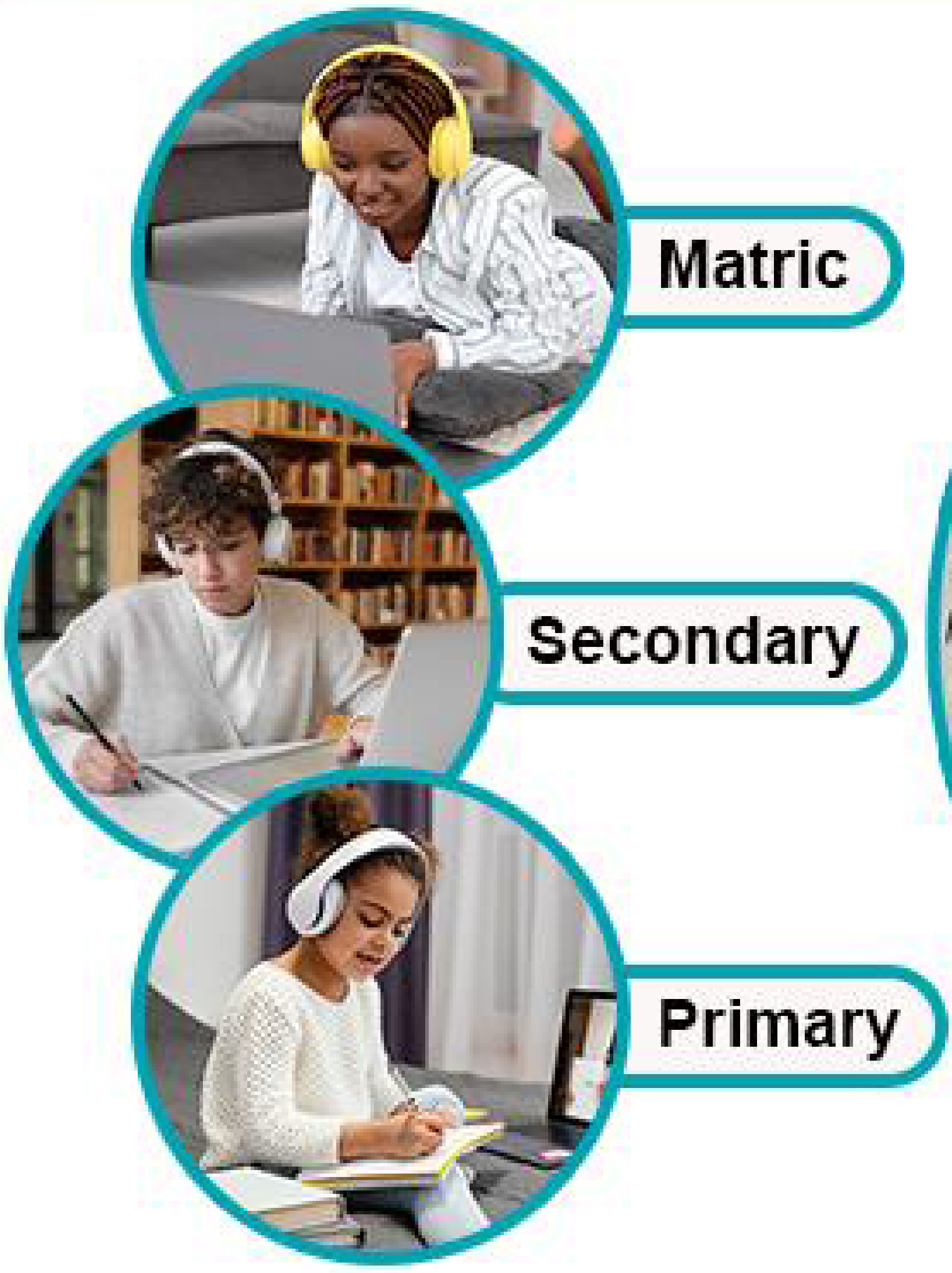

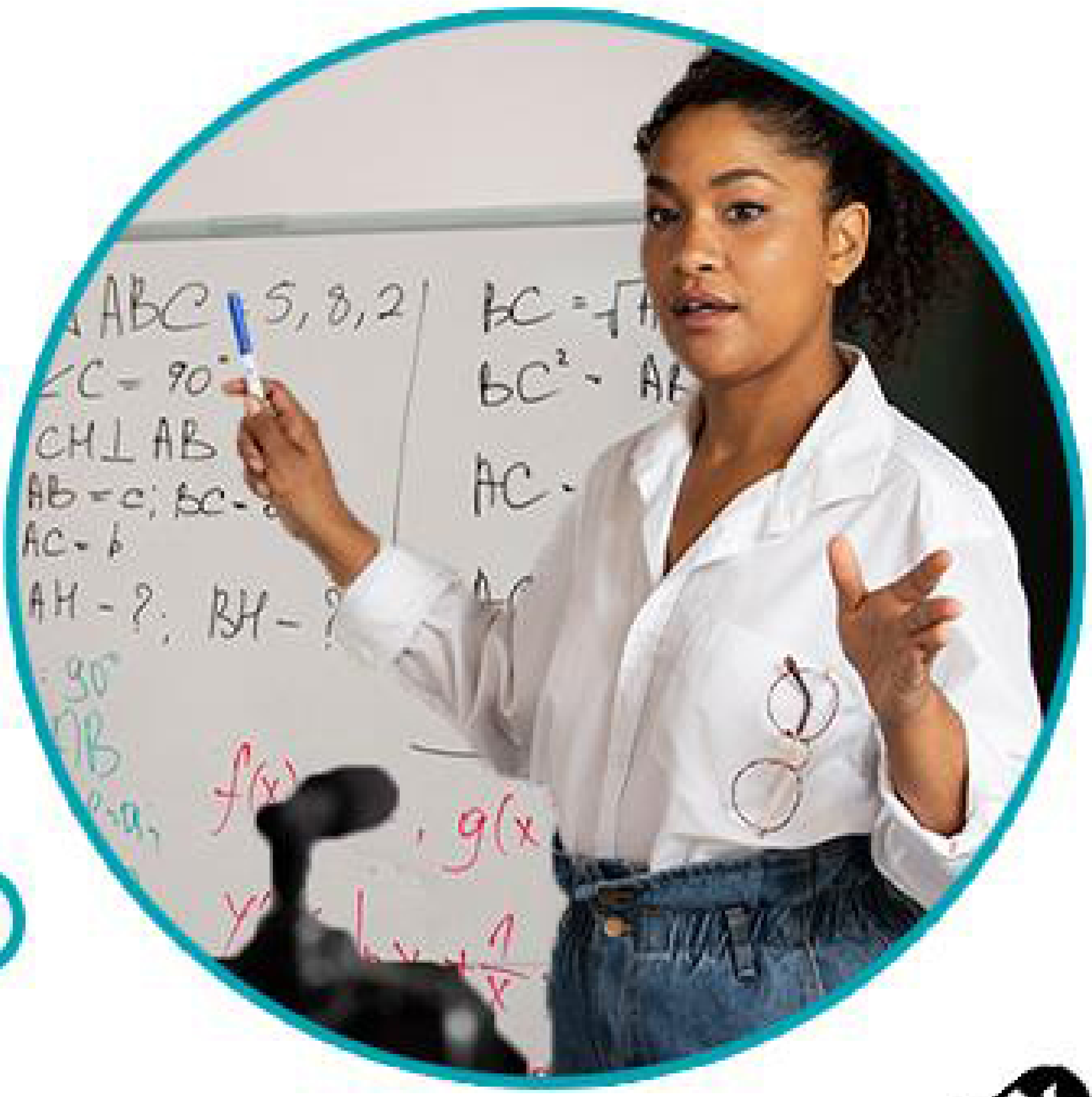

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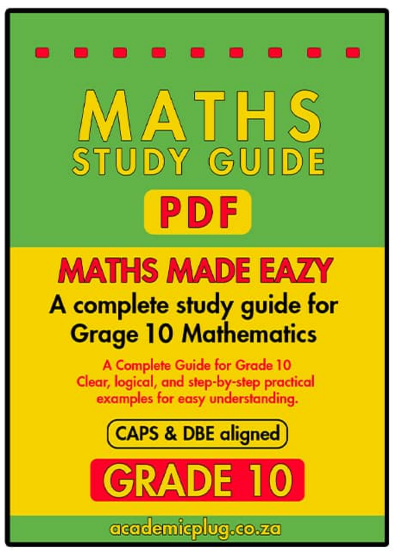
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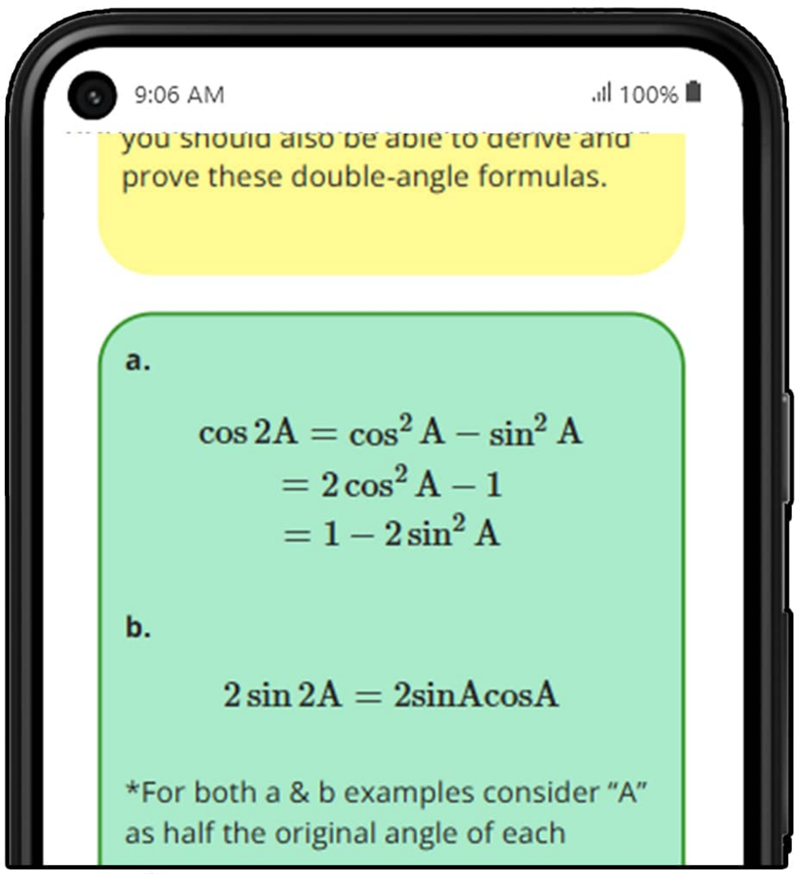
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you should also be able to derive and prove these double-angle formulas.

a.

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

b.

$$2\sin 2A = 2\sin A\cos A$$

*For both a & b examples consider "A" as half the original angle of each

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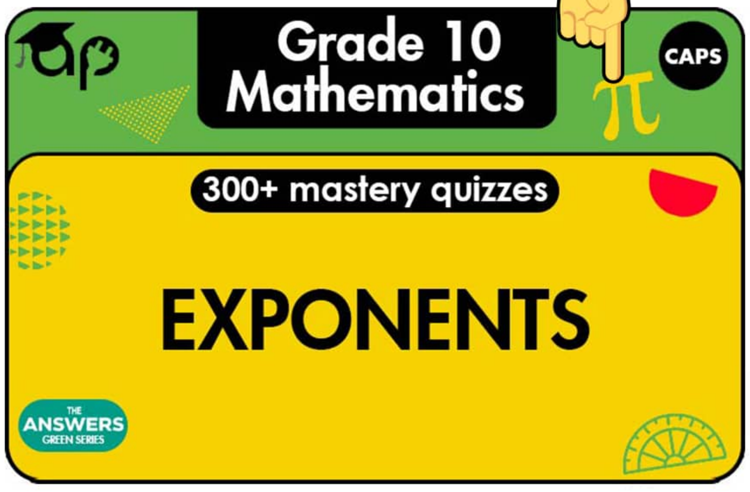
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CHAPTER 3
EXPONENTS

WHAT YOU WILL LEARN IN THIS CHAPTER

PAGE	CONTENT	CAPS	ASSESSMENTS YOU WILL BE ABLE TO:
35	**A. Exponents**	4	• Simplify expressions using the laws of exponents for integral and rational exponents. • Solve Exponential Equations
35	• Exponent Laws		
37	• Multiplication and Division with Variable Exponents		
38	• Addition and Subtraction with Variable Exponents		
40	• Rational Exponents Laws		
41	**B. Exponential Equations**		
41	• Exponential Equations Using Basic Algebra		
43	• Exponential Equations Using Trial and Error		

A. EXPONENTS

Exponents are a shorthand notation used in order to express the number of times a number is multiplied by itself. Exponents are often used to represent very large or small numbers. **An exponent is also referred to as a power or as an index.** **A base (number or variable)** is always **raised to an exponent**.

Notation:

$$\text{Power}\left\{\, 2^{3\,\longrightarrow\,\text{Exponent}}_{\rightarrow\,\text{Base}} \longrightarrow b^{x\,\longrightarrow\,\text{Exponent}}_{\rightarrow\,\text{Base}} \,\right\}\text{Power}$$

EXPONENT LAWS

Law	Example	Explanation
Law 1: $x^m x^n = x^{m+n}$	$a^3 a^2 = a^{3+2} = a^5$	**When multiplying like bases, keep the base and add the exponents.**
Law 2: $\dfrac{x^m}{x^n} = x^{m-n}$	$\dfrac{6a^8}{3a^3} = 2a^{8-3} = 2a^5$	**When dividing like bases, keep the base and subtract the exponents. (numbers divide normally)**
Law 3: $\left(x^m\right)^n = x^{m\times n}$	$\left(a^2\right)^3 = a^{2\times 3} = a^6$	**When raising exponents to a power, keep the base and multiply the exponents.**
Law 4: $(xyz)^m = x^m y^m z^m$	$\left(2a^2 b\right)^3 = 2^3 a^{2\times 3} b^{1\times 3} = 8a^6 b^3$	**When the product of different bases is raised to an exponent, each base must be raised to that exponent.**
Law 5: $\left(\dfrac{x}{y}\right)^m = \dfrac{x^m}{y^m}$	$\left(\dfrac{a}{b}\right)^3 = \dfrac{a^3}{b^3}$	**When a fraction is raised to an exponent, the numerator and denominator must be raised to that exponent.**
Law 6: $x^{-m} = \dfrac{1}{x^m}$	$a^{-3} = \dfrac{1}{a^3}$	**A base raised to a negative exponent is equal to its reciprocal raised to the same positive exponent.**
Law 7: $\dfrac{1}{x^{-m}} = x^m$	$\dfrac{1}{a^{-3}} = a^3$	**A base raised to a negative exponent is equal to its reciprocal raised to the same positive exponent.**
Law 8: $x^0 = 1$	$(25abc)^0 = 1$	**Any base raised to zero is equal to one. (there is no solution to zero raised to the zero)**

All of the exponent laws also apply in reverse.

Example 1: **Simplify** $\frac{6\left(x^3y\right)^2}{3}\times\frac{x}{y}\times\frac{y^2}{x^{-3}}$.

$$\frac{6\left(x^3y\right)^2}{3}\times\frac{x}{y}\times\frac{y^2}{x^{-3}}$$

$$=\frac{6x^6y^2}{3}\times\frac{x}{y}\times y^2x^3 \rightarrow \text{law 4, law 7}$$

$$=\frac{6x^6y^2.x^1.y^2x^3}{3y^1}$$

$$=2x^{6+1+3}.y^{2+2-1} \rightarrow \text{law 1, law 6 and law 1}$$

$$=2x^{10}y^3$$

If an exponent is not shown, the exponent is 1.
Always work with number separately to variables.

Example 2: **Simplify** $\left(\frac{3x}{2y}\right)^2\div\frac{3x}{2y}$.

$$\left(\frac{3x}{2y}\right)^2\div\frac{3x}{2y}$$

$$=\frac{9x^2}{4y^2}\div\frac{3x}{2y} \rightarrow \text{law 5}$$

$$=\frac{9x^2}{4y^2}\times\frac{2y}{3x} \rightarrow \text{'tip and times'}$$

$$=\frac{18x^2y^1}{12y^2x^1}$$

$$=\frac{3}{2}x^{2-1}.y^{1-2} \rightarrow \text{law 1, law 6 and law 1}$$

$$=\frac{3}{2}xy^{-1} \rightarrow \text{in most cases you have to write your answers with positive exponents}$$

$$=\frac{3x}{2y}$$

Recap:(also see pg. 25)
'Tip and times' when dividing fractions

$$\frac{a}{b}\div\frac{x}{y}=\frac{a}{b}\times\frac{y}{x}$$

You are usually required to write answers with positive exponents.

Complete Exercise 1 on page 254.

MULTIPLICATION AND DIVISION WITH VARIABLE EXPONENTS

When dealing with multiplication and division of variable exponents, write each base as a product of its prime factors and apply the laws of exponents (pg. 35).

Example 1: **Simplify** $\frac{27^{x+2}.9^{2x}}{3^{3x}.81^{x+1}}$.

$\frac{27^{x+2}.9^{2x}}{3^{3x}.81^{x+1}}$

$= \frac{\left(3^3\right)^{x+2}.\left(3^2\right)^{2x}}{3^{3x}.\left(3^4\right)^{x+1}}$ → convert all bases to their prime factors

$= \frac{3^{3x+6}.3^{4x}}{3^{3x}.3^{4x+4}}$ → law 3, don't forget to distribute the exponents

$= 3^{3x+6+4x-(3x+4x+4)}$ → law 1 and 7

$= 3^{3x+6+4x-3x-4x-4}$ → don't forget to distribute the negative into the bracket

$= 3^2$

$= 9$

Example 2: **Simplify** $\frac{9^{3x}.12^{2x+2}}{4^{2x+4}.9^{4x+1}}$.

$\frac{9^{3x}.12^{2x+2}}{4^{2x+4}.9^{4x+1}}$

$= \frac{\left(3^2\right)^{3x}.\left(2^2\times 3\right)^{2x+2}}{\left(2^2\right)^{2x+4}.\left(3^2\right)^{4x+1}}$ → convert all bases to their prime factors

$= \frac{3^{6x}.2^{4x+4}.3^{2x+2}}{2^{4x+8}.3^{8x+2}}$ → law 3 and 4

$= 3^{6x+2x+2-(8x+2)}.2^{4x+4-(4x+8)}$ → law 1, 6 and 7

$= 3^{6x+2x+2-8x-2}.2^{4x+4-4x-8}$ → don't forget to distribute the negative into the bracket

$= 3^0.2^{-4}$ → law 8, law 6

$= \frac{1}{2^4}$

$= \frac{1}{16}$

Complete Exercise 2 on page 254.

ADDITION AND SUBTRACTION WITH VARIABLE EXPONENTS

During addition and subtraction of bases with variable exponents, law 1 on pg. 35 is generally used in reverse to split apart exponents. In most cases factorisation can then be used to simplify the expression.

Example 1: Simplify $\dfrac{4.3^{2x}-9.3^{2x-2}}{3^{2x}-3^{2x-1}}$ **.**

$$\frac{4.3^{2x}-9.3^{2x-2}}{3^{2x}-3^{2x-1}}$$

$$=\frac{\left(2^2\right).3^{2x}-\left(3^2\right).3^{2x-2}}{3^{2x}-3^{2x-1}}\rightarrow \text{convert all bases to their prime factors}$$

$$=\frac{2^2.3^{2x}-3^2.3^{2x}.3^{-2}}{3^{2x}-3^{2x}.3^{-1}}\rightarrow \text{split exponents apart using law 1 in reverse}$$

$$=\frac{2^2.3^{2x}-3^2.3^{2x}.3^{-2}}{3^{2x}-3^{2x}.3^{-1}}\rightarrow \text{now there is a HCF of } 3^{2x} \text{ which can be taken out}$$

$$=\frac{\cancel{3^{2x}}\left(2^2-3^2.3^{-2}\right)}{\cancel{3^{2x}}\left(1-3^{-1}\right)}\rightarrow \text{a HCF of } 3^{2x} \text{ was taken out which can now be cancelled}$$

> **Recap: (also see pg. 14)**
>
> **If an expression has a factor that is common to all terms, you can take out the highest common factor (HCF)**
>
> $$ax+ab+2ac=a\left(x+b+2c\right)$$

$$=\frac{\left(4-9\times\frac{1}{9}\right)}{1-\frac{1}{3}}\rightarrow \text{law 6}$$

$$=\frac{4-1}{\frac{3}{3}-\frac{1}{3}}\rightarrow \text{find LCD's when adding fractions, } \frac{3}{3} \text{ is the same as 1}$$

$$=\frac{3}{\frac{2}{3}}$$

$$=3\div\frac{2}{3}$$

$$=3\times\frac{3}{2}\rightarrow \text{tip and times}$$

$$=\frac{9}{2}$$

Example 2: **Simplify** $\dfrac{5^{x-1}.11+5^{x+1}}{5^{x}+5^{x-1}}$.

$$\frac{5^{x-1}.11+5^{x+1}}{5^{x}+5^{x-1}}$$

$$=\frac{5^{x}.5^{-1}.11+5^{x}.5^{1}}{5^{x}+5^{x}.5^{-1}} \rightarrow \text{split like bases apart using law 1 and take out a HCF of } 5^{x}$$

$$=\frac{\cancel{5^{x}}\left(5^{-1}.11+5^{1}\right)}{\cancel{5^{x}}\left(1+5^{-1}\right)} \rightarrow \text{a HCF of } 5^{x} \text{ was taken out which can now be cancelled}$$

$$=\frac{\frac{11}{5}+5}{1+\frac{1}{5}} \rightarrow \text{law 6}$$

$$=\frac{\frac{11}{5}+\frac{25}{5}}{\frac{5}{5}+\frac{1}{5}} \rightarrow \text{find LCD's when adding fractions}$$

$$=\frac{\frac{36}{5}}{\frac{6}{5}}$$

$$=\frac{36}{5}\div\frac{6}{5}$$

$$=\frac{36}{5}\times\frac{5}{6} \rightarrow \text{tip and times}$$

$$=6$$

Recap:

When adding or subtracting fractions, the denominators have to be the same.

Complete Exercise 3 on page 254.

RATIONAL EXPONENTS LAWS

In this section you will learn the rules for when exponents are rational. In other words, the exponents can also contain fractions. All the rules which you have learnt for integer exponents still apply. Here you are merely adding in additional rules for exponents.

Law	Example
Law 1: $x^{\frac{1}{a}} = \sqrt[a]{x}$	$4^{\frac{1}{2}} = \sqrt[2]{4} = 2$
Law 2: $x^{\frac{b}{a}} = \sqrt[a]{x^b}$	$4^{\frac{3}{2}} = \sqrt[2]{4^3} = \sqrt[2]{64} = 8$
Law 3: $x^{-\frac{b}{a}} = \sqrt[a]{x^{-b}}$	$27^{-\frac{1}{3}} = \sqrt[3]{27^{-1}} = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$

Example 1: Simplify $16^{\frac{1}{2}} + 9^{\frac{3}{2}} - 5$.

$$16^{\frac{1}{2}} + 9^{\frac{3}{2}} - 5$$

$= \sqrt{16} + \sqrt{9^3} - 5 \rightarrow$ rational exponents law 1 and law 2

$$= 4 + \sqrt{729} - 5$$

$$= 4 + 27 - 5$$

$$= 26$$

Note that law 2 above could also be written as : $x^{\frac{b}{a}} = \sqrt[a]{x^b} = \left(\sqrt[a]{x}\right)^b$

Using this principle the example could be done using the alternate method shown below:

Alternate Method:

$$16^{\frac{1}{2}} + 9^{\frac{3}{2}} - 5$$

$= \sqrt{16} + \left(\sqrt{9}\right)^3 - 5 \rightarrow$ rational exponents law 1 and law 2

$$= 4 + (3)^3 - 5$$

$$= 4 + 27 - 5$$

$$= 26$$

Example 2: Show that $\sqrt{8} = 2\sqrt{2}$ without the use of a calculator.

$\sqrt{8} = (4 \times 2)^{\frac{1}{2}} \rightarrow$ break 8 apart into a perfect square and another number

$$= 4^{\frac{1}{2}} \times 2^{\frac{1}{2}}$$

$$= \sqrt{4} \times \sqrt{2}$$

$$= 2\sqrt{2}$$

Complete Exercise 4 on page 254.

B. EXPONENTIAL EQUATIONS

An exponential equation is an equation in which the variable you are solving for is an exponent.

In Grade 10 there are two methods used to solve exponential equations:

1. **Using basic algebra**
2. **Using trial and error**

Basic linear equations from Grade 9 will be used in this section and are revised on page 53.

1. EXPONENTIAL EQUATIONS USING BASIC ALGEBRA

When solving exponential equations it is important to take note of the following:

$$\text{If } x^a = x^b \text{ where } x > 0 \text{ and } x \neq 1$$
$$\text{Then } a = b$$

In order for the above statement to be true the bases have to be the same.

Example 1: **Solve for x if $6^x = 36$.**

$6^x = 36 \rightarrow$ get the same bases on both sides by expressing 36 as a power of 6

$\therefore 6^x = 6^2$

$\therefore x = 2$

Example 2: **Solve for x if $8^{x+4} = 4^{2x+9}$.**

$8^{x+4} = 4^{2x+9} \rightarrow$ get the same bases on both sides

$\therefore \left(2^3\right)^{x+4} = \left(2^2\right)^{2x+9}$

$\therefore 2^{3x+12} = 2^{4x+18} \rightarrow$ exponential law 3 (pg. 35)

$\therefore 3x + 12 = 4x + 18$

$\therefore -x = 6$

$\therefore x = -6$

Example 3: **Solve for x if $3^{2x} = \frac{1}{9}$.**

$3^{2x} = \frac{1}{9} \rightarrow$ get the same bases on both sides

$\therefore 3^{2x} = \frac{1}{3^2}$

$\therefore 3^{2x} = 3^{-2} \rightarrow$ exponential law 6 (pg. 35)

$\therefore 2x = -2$

$\therefore x = -1$

Example 4: Solve for x if $2(25)^x = 250$.

$2(25)^x = 250 \rightarrow$ divide both sides by 2

$\therefore 25^x = 125$

$\therefore (5^2)^x = 5^3 \rightarrow$ get the same base on both sides

$\therefore 5^{2x} = 5^3$

$\therefore 2x = 3$

$\therefore x = \frac{3}{2}$

Example 5: Solve for x if $4^{x+6} = 1$.

$4^{x+6} = 1$

$\therefore 4^{x+6} = 4^0 \rightarrow$ any number or variable raised to zero is equal to 1 (law 8 pg. 35)

$\therefore x + 6 = 0$

$\therefore x = -6$

Example 6: Solve for x if $125^x \times 25^{x-2} = 25$.

$125^x \times 25^{x-2} = 25$

$\therefore (5^3)^x \times (5^2)^{x-2} = 5^2 \rightarrow$ express all the numbers as a power of base 5

$\therefore 5^{3x} \times 5^{2x-4} = 5^2 \rightarrow$ don't forget to distribute the 2

$\therefore 5^{3x+2x-4} = 5^2$

$\therefore 3x + 2x - 4 = 2$

$\therefore 5x = 6$

$\therefore x = \frac{6}{5}$

Example 7: Solve for x if $3^x = 1{,}5 \times 2^x$.

$3^x = 1{,}5 \times 2^x \rightarrow$ divide both sides by 2^x in order to get all the x's on one side

$\therefore \frac{3^x}{2^x} = \frac{3}{2} \rightarrow 1{,}5 = \frac{3}{2}$

$\therefore \left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^1$

$\therefore x = 1$

Example 8: **Solve for x if $\frac{25^x - 1}{5^x + 1} = 24$.**

$$\therefore \frac{\left(5^2\right)^x - 1}{5^x + 1} = 24$$

$$\therefore \frac{5^{2x} - 1}{5^x + 1} = 24 \rightarrow \text{factorise using the difference of two squares}$$

$$\therefore \frac{\left(5^x - 1\right)\cancel{\left(5^x + 1\right)}}{\cancel{\left(5^x + 1\right)}} = 24$$

$$\therefore 5^x - 1 = 24$$

$$\therefore 5^x = 25$$

$$\therefore 5^x = 5^2$$

$$\therefore x = 2$$

Complete Exercise 5 number 1 on page 255.

2. EXPONENTIAL EQUATIONS USING TRIAL AND ERROR

It is not always possible to get the same bases on both sides. In this case you have to use **trial and error** in order to solve for the value of an unknown variable.

Example 1: **Solve for x correct to two decimal places if $2^x = 7$.**

By substituting values for x, work out two integer solutions of 2^x that 7 would lie between.

$2^2 = 4$

$2^3 = 8$

7 lies between 4 and 8

$\therefore x$ lies between 2 and 3

As 8 is closer to 7 than 4 is, the answer will lie closer to 3 than 2

$\therefore$ try a value such as $2{,}8$

$2^{2{,}81} = 6{,}964$

$6{,}964$ is less than 7, $\therefore$ try $2{,}9$

$2^{2{,}9} = 7{,}464$

You now know that 7 lies between $2^{2{,}8}$ and $2^{2{,}9}$

Now find the value for x between $2{,}8$ and $2{,}9$ that lies closest to 7

try $x = 2{,}81$

$2^{2{,}8} = 7{,}01$

$2^{2{,}82} = 7{,}06$

$7{,}01$ is closer to 7 than $7{,}06$

$\therefore x = 2{,}81 \rightarrow$ the question said correct to two decimal places

Example 2: **Solve for x correct to two decimal places if $3^x = 25$.**

By substituting values for x, work out two integer solutions of 3^x that 25 would lie between.

$3^2 = 9$

$3^3 = 27$

25 lies between 9 and 27

$\therefore x$ lies between 2 and 3

As 25 is closer to 27 than 9 is, the answer will lie closer to 3 than 2

$\therefore$ try a value such as 2,8

$3^{2,8} = 21{,}674$

$21{,}674$ is less than 25, $\therefore$ try 2,9

$3^{2,9} = 24{,}19$

You now know that 25 lies between $3^{2,9}$ and 3^3

Now find the value for x between 2,9 and 3 that lies closest to 25

try $x = 2{,}92$

$3^{2,92} = 24{,}73$

$3^{2,93} = 25{,}00$

$\therefore x = 2{,}93 \rightarrow$ the question said correct to two decimal places

Example 3: **The rate at which a rabbit population increases can be given by the equation $A = P\left(1 + \frac{i}{100}\right)^n$, where P is the number of rabbits in the population at the present time and A is the number of rabbits n years from now. The value of i is the percentage that the rabbit population increases by each year. If a rabbit population of 200 increases at a rate of 12% every year, how long would it take for the population to double? Give your answer correct to the nearest year.**

$\therefore 400 = 200\left(1 + \frac{12}{100}\right)^n \rightarrow$ divide both sides by 200

$\therefore 2 = (1{,}12)^n$

Substitute integers for n starting with a low number such as 4.

$1{,}12^4 = 1{,}57$

$1{,}12^6 = 1{,}97$

$1{,}12^7 = 2{,}21 \rightarrow$ this answer is too high so try a lower number such as 6,1

$1{,}12^{6,1} = 1{,}996$

$1{,}12^{6,12} = 2{,}00$

$\therefore n \approx 6$

$\therefore$ After 6 years the rabbit population would double.

Complete Exercise 5 number 2 on page 255 and the Mixed Exercise on pages 255 and 256.

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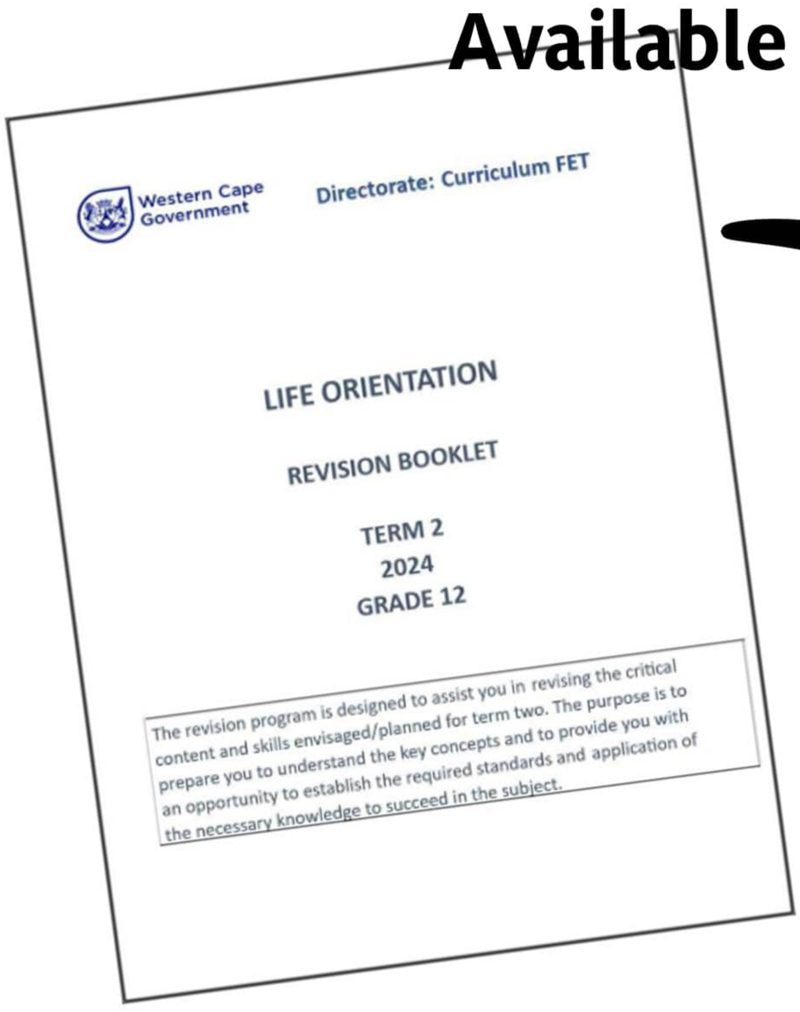

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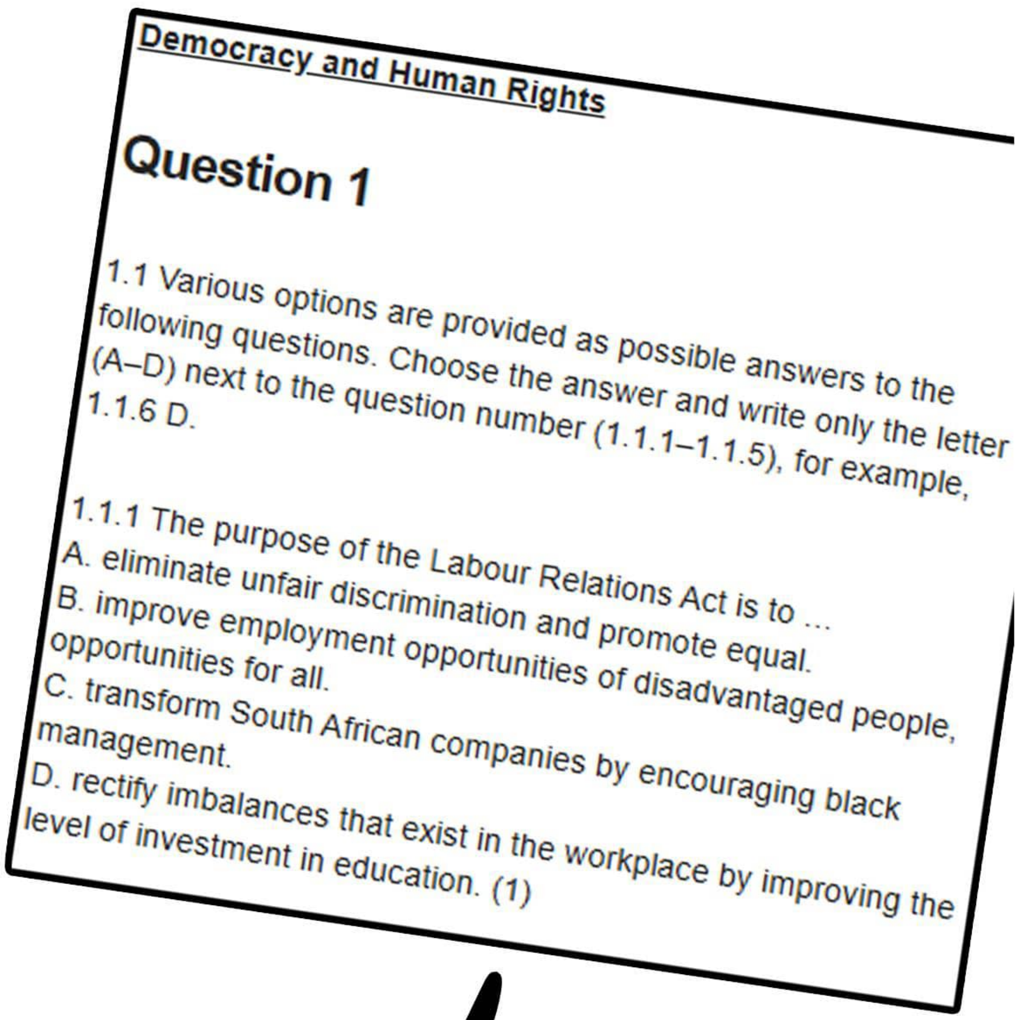
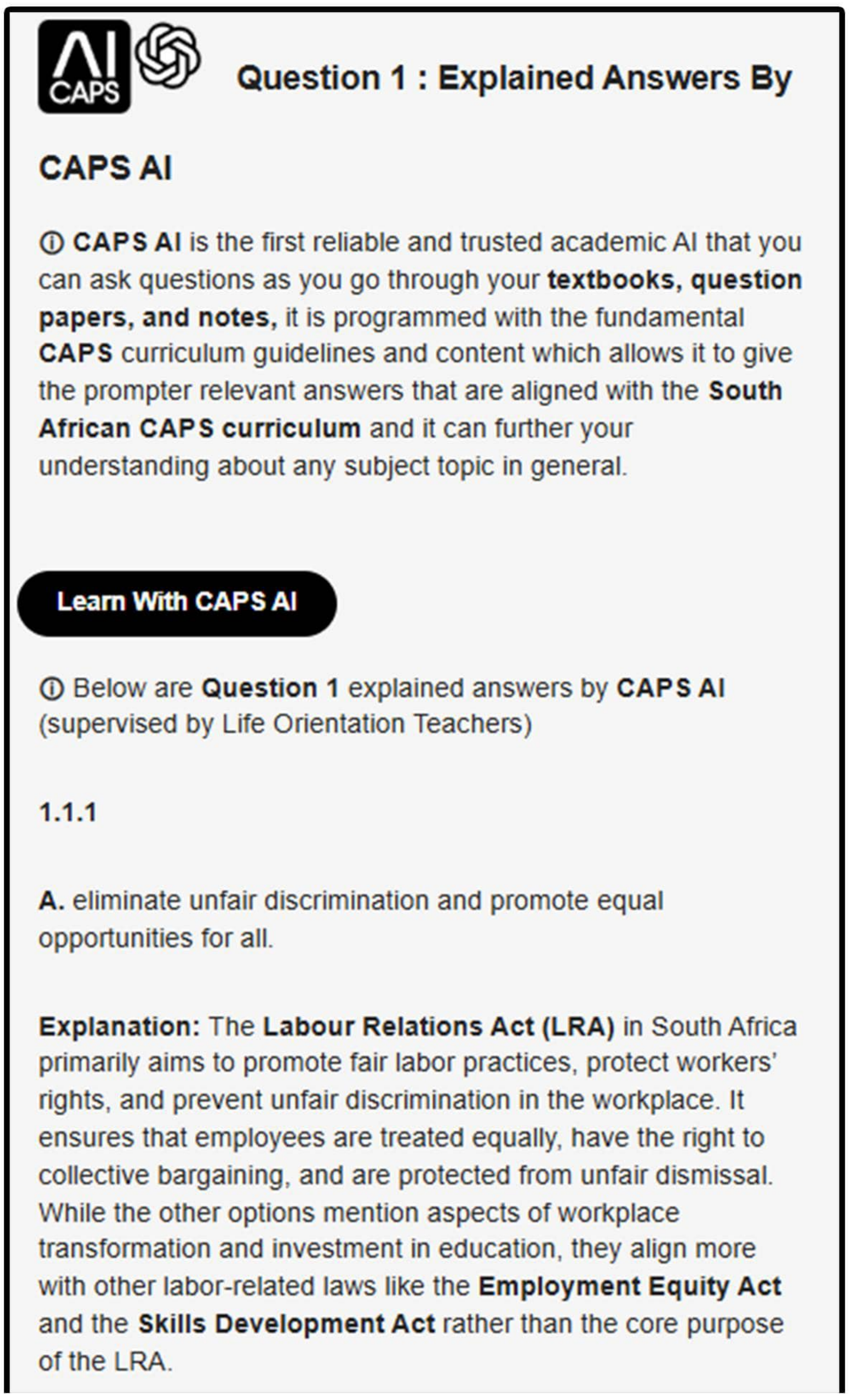


Democracy and Human Rights

Question 1

1.1 Various options are provided as possible answers to the following questions. Choose the answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5), for example, 1.1.6 D.

1.1.1 The purpose of the Labour Relations Act is to ...
A. eliminate unfair discrimination and promote equal opportunities for all.
B. improve employment opportunities of disadvantaged people.
C. transform South African companies by encouraging black management.
D. rectify imbalances that exist in the workplace by improving the level of investment in education. (1)



Question 1 : Explained Answers By CAPS AI

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1.1.1

A. eliminate unfair discrimination and promote equal opportunities for all.

Explanation: The **Labour Relations Act (LRA)** in South Africa primarily aims to promote fair labor practices, protect workers' rights, and prevent unfair discrimination in the workplace. It ensures that employees are treated equally, have the right to collective bargaining, and are protected from unfair dismissal. While the other options mention aspects of workplace transformation and investment in education, they align more with other labor-related laws like the **Employment Equity Act** and the **Skills Development Act** rather than the core purpose of the LRA.

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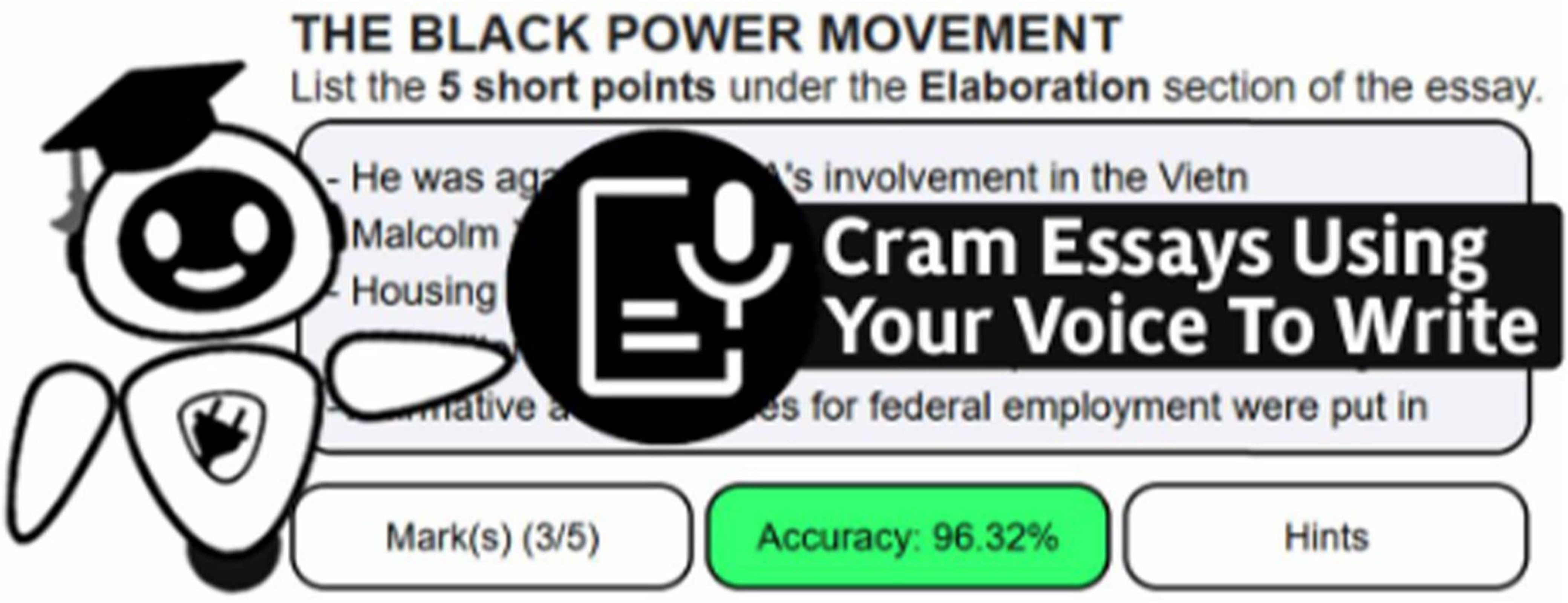

THE BLACK POWER MOVEMENT

List the **5 short points** under the **Elaboration** section of the essay.

- He was ag 's involvement in the Vietn
Malcolm
- Housing
- mative a es for federal employment were put in

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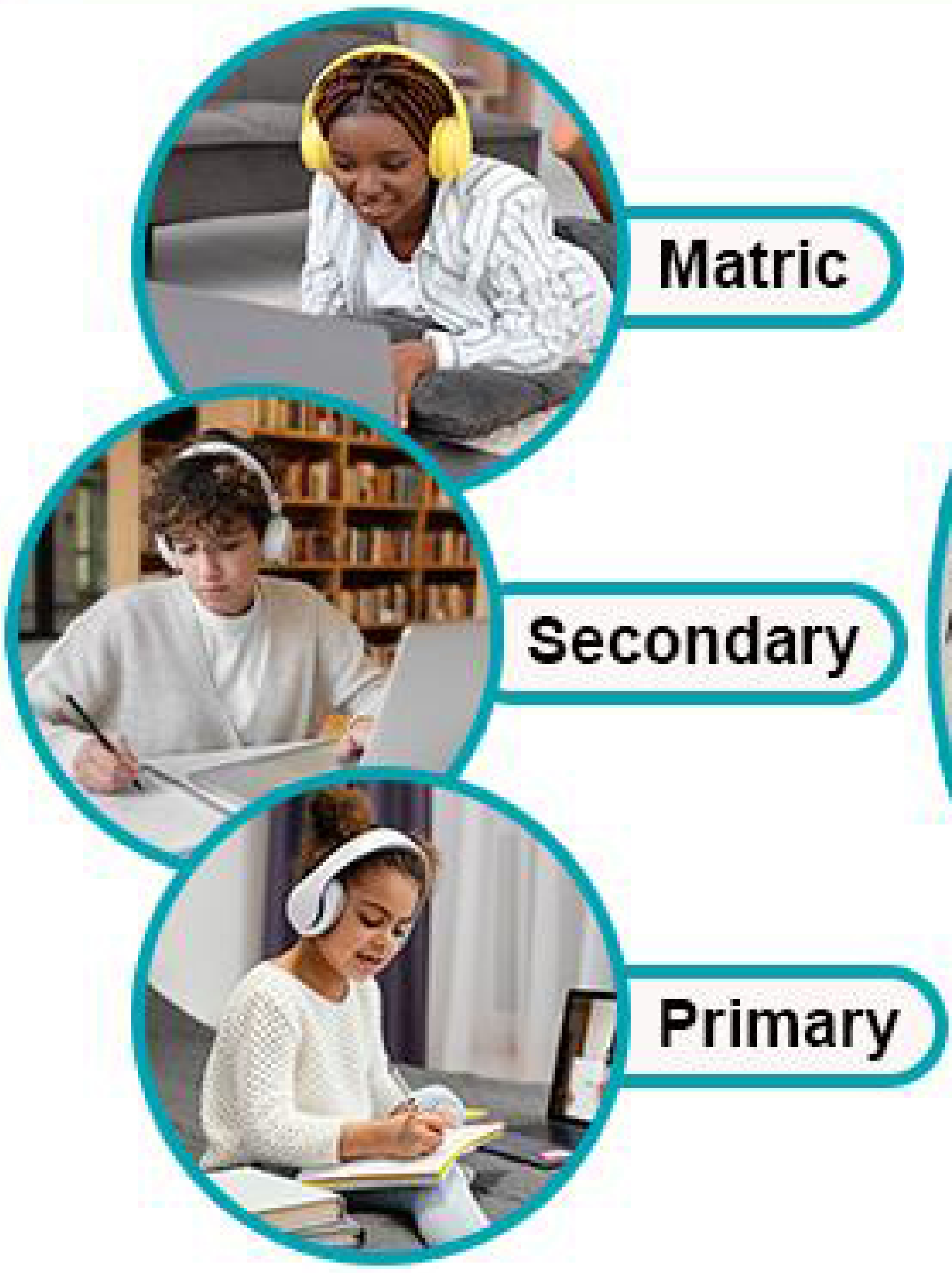

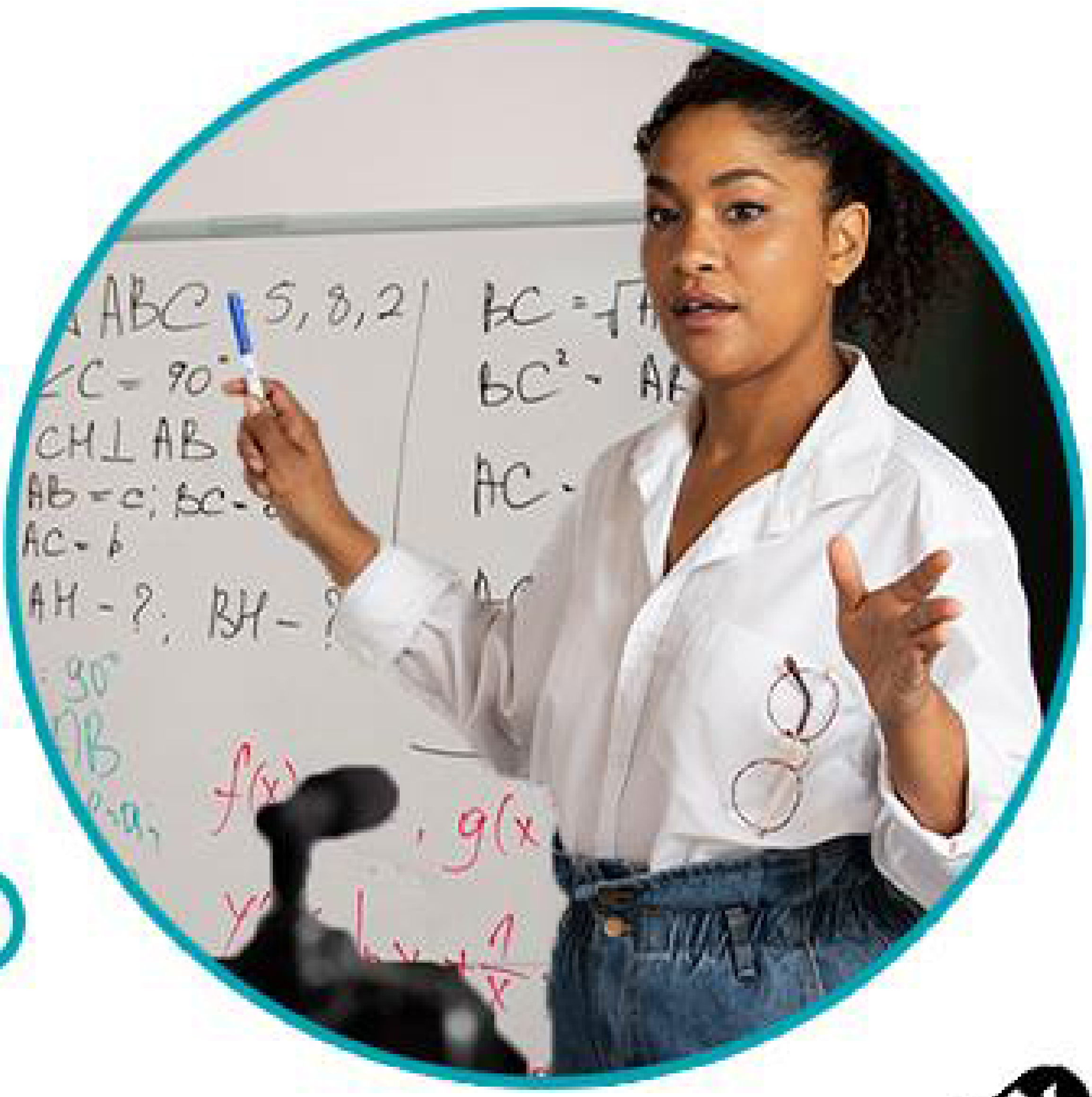

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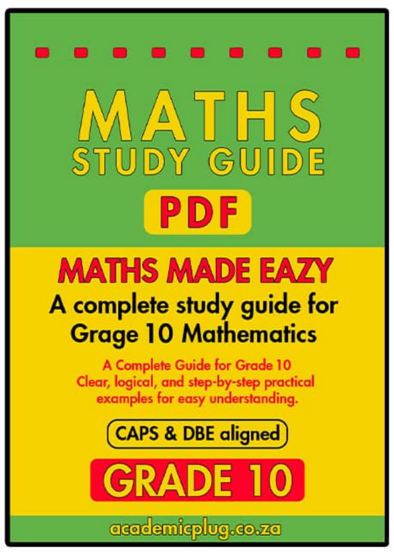

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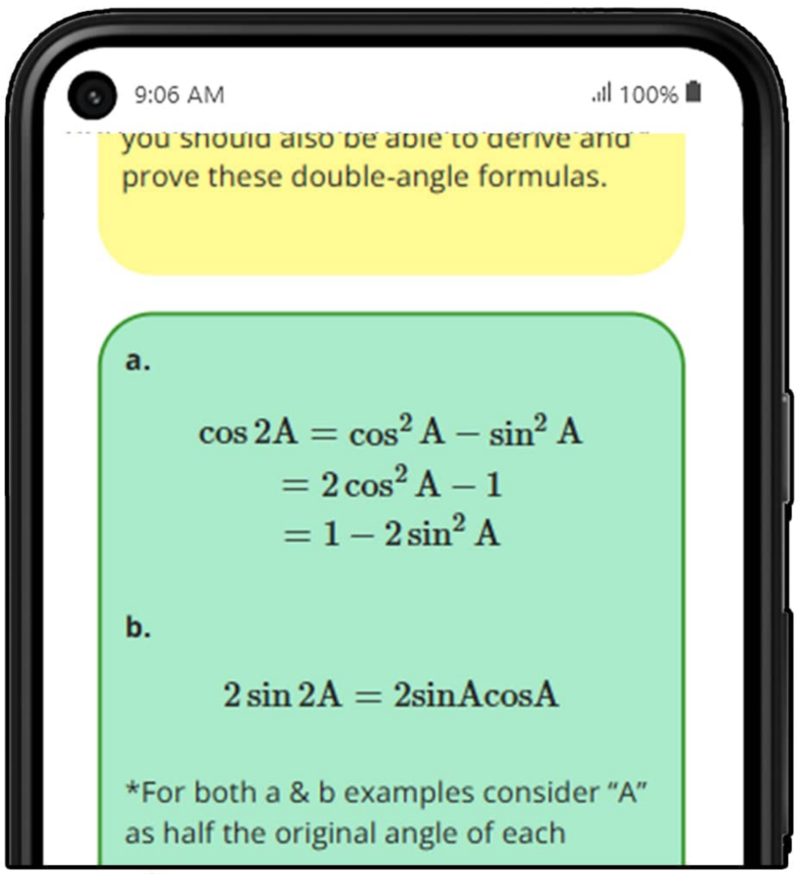

9:06 AM 100%

you should also be able to derive and prove these double-angle formulas.

a.

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

b.

$$2\sin 2A = 2\sin A\cos A$$

*For both a & b examples consider "A" as half the original angle of each

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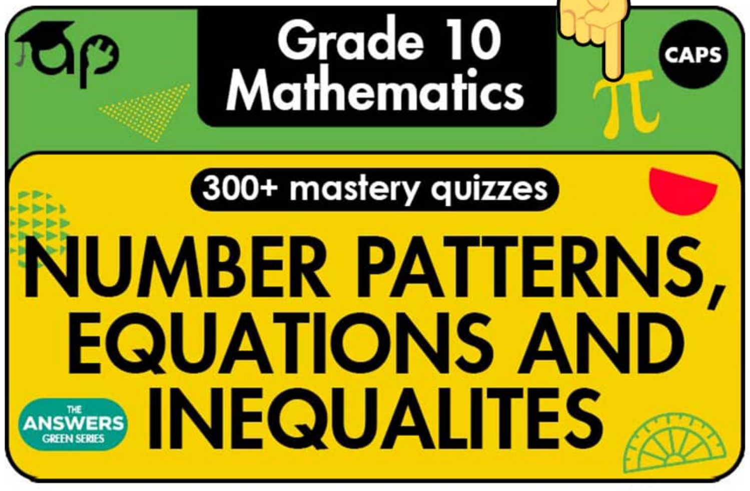

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CHAPTER 4
NUMBER PATTERNS, EQUATIONS AND INEQUALITIES

WHAT YOU WILL LEARN IN THIS CHAPTER

PAGE	CONTENT	CAPS	ASSESSMENTS YOU WILL BE ABLE TO:
46	**A. Number Patterns**	2	• Investigate number patterns leading to those where there is a common difference between consecutive terms and the general term is therefore linear.
46	• Linear Number Patterns		
50	• Unseen Number Patterns		
53	**B. Linear Equations**	4	
55	**C. Quadratic Equations**		• Solve linear equations.
55	• Steps to Solving Quadratic Equations		• Solve quadratic equations by factorisation.
57	• Tips to Solving Quadratic Equations		
58	**D. Literal Equations**		• Solve literal equations.
59	**E. Number Line Revision**		• Solve linear inequalities and illustrate the solution graphically (using a number line).
59	• Set Builder Notation		
60	• Interval Notation		
61	**F. Linear Inequalities**		• Solve linear equations in two variables (linear simultaneous equations).
63	**G. Simultaneous Equations**		
63	• Elimination		
65	• Substitution		• Use mathematical models to solve and investigate problems that arise in real life context (word problems).
67	**H. Mathematical Modelling**		
67	• Age/Number Problems		
68	• Shopping Problems		
70	• Speed, Distance and Time Problems		
73	• Mathematical Modelling with Simultaneous Equations		

A. NUMBER PATTERNS

In Grade 10 you are required to identify number patterns and obtain a general formula for any term in the pattern.

LINEAR NUMBER PATTERNS

In linear number patterns there is a common difference between consecutive terms. This means that the same value is either added to or subtracted from each preceding term.

In a linear number pattern the general formula for the n^{th} term in a pattern is:

$$T_n = dn + c$$

Where: $n \rightarrow$ the position of the term in the sequence $($*ie*: term 1, term 2, ...$)$

$d \rightarrow$ the common difference between sequential terms $(T_2 - T_1; T_3 - T_2; \ldots)$

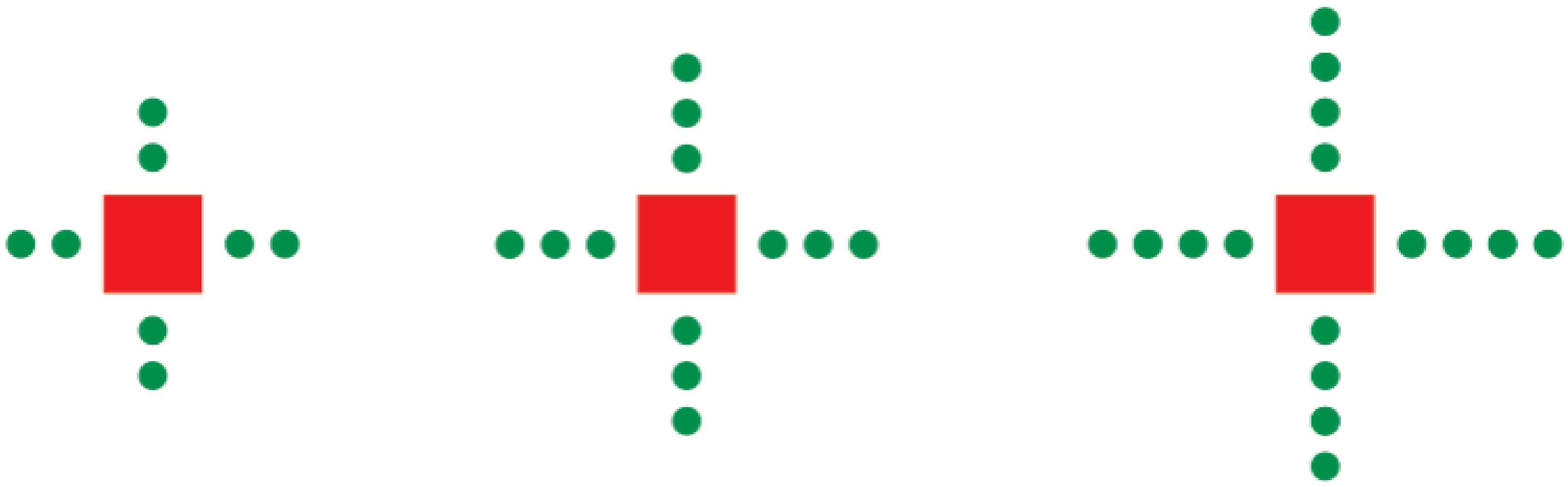
$c \rightarrow$ a constant term

If a number pattern is linear, the difference between any two sequential terms in the pattern will be the same. In other words: $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = T_n - T_{n-1}$

Steps to writing an expression for the nth term of a linear number pattern:

1. **Determine if the pattern is linear by checking that: $d = T_2 - T_1 = T_3 - T_2$**
2. **Substitute d into the general formula : $T_n = dn + c$**
3. **Substitute a term (usually the first term) into the formula and solve for c.**
4. **Write down the final expression.**

Example 1: **Seedlings on a farm are planted around an underground watering system in the following pattern:**



Initial plantation **Second plantation** **Third plantation**

a) **How many seedlings will there be in the fourth plantation?**

b) **Write a rule for the number of seedlings required for the nth plantation in words. This is known as a conjecture.**

c) **Write a mathematical formula for the number of seedlings required for the nth plantation.**

d) **How many seedlings will there be in the 12th plantation?**

e) **How many plantations will there be when there are more than 88 seedlings?**

a) Pattern: $8; 12; 16...$

$T_2 - T_1 = 12 - 8$ $\quad$ $T_3 - T_2 = 16 - 12$

$= 4$ $\quad$ $= 4 \rightarrow T_2 - T_1 = T_3 - T_2$ $\therefore$ pattern is linear

$\therefore d = 4$

As the pattern is linear with a common difference of 4, 4 seedlings are being added onto each plantation.

$\therefore$ in the fourth plantation there will be 20 seedlings.

b) Plantation 1: 8 seedlings $\rightarrow 8 = 4(1) + 4$

Plantation 2: 12 seedlings $\rightarrow 12 = 4(2) + 4$

Plantation 3: 16 seedlings $\rightarrow 16 = 4(3) + 4$

Conjecture: The number of seedlings in each plantation equals **four times the plantation number** (ie. plantation 1, 2 or 3) **plus four**.

c) Pattern is linear, $\therefore T_n = dn + c$

$d = 4$

$\therefore T_n = 4n + c$

In order to solve for c, substitute a term of the pattern into the equation.

Substitute the first term into the equation, $\therefore n = 1$ and $T_1 = 8$

$\therefore 8 = 4(1) + c$

$\therefore c = 4$

$\therefore T_n = 4n + 4$

d) In order to determine the number of seedlings in the 12th plantation, set $n = 12$

$\therefore T_{12} = 4(12) + 4$

$\therefore T_{12} = 52 \rightarrow$ there are 52 seedlings planted in the 12th plantation

e) In order to determine the plantation where there will be more than 88 seedlings you have to determine n when $T_n = 88$

$\therefore 88 = 4n + 4$

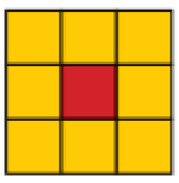
$\therefore 84 = 4n$

$\therefore n = 21 \rightarrow$ this means that in the 21st plantation there will be 88 seedlings

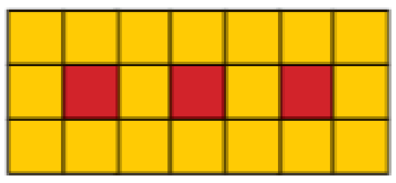
But the question asks when there will be **more** than 88 seedlings,

$\therefore$ There will be more than 88 seedlings in the **22nd** plantation

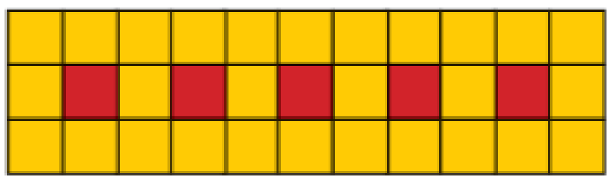
Example 2: **A carpenter is making a pattern from yellow and red tiles in order to tile a bathroom floor. He makes the pattern in steps as shown below.**



Step 1



Step 2



Step 3

a) How many tiles will there be in the pattern in step 4?

b) Write a rule for the number of yellow and red tiles required for the nth step in words. This is known as a conjecture.

c) Write a mathematical formula for the number of red and yellow tiles required for the nth step.

d) Write a mathematical formula for the number of red tiles required for the nth step.

e) Write a mathematical formula for the number of yellow tiles required for the nth step.

f) How many red tiles will be required for the 15th step?

a) Pattern: 9; 21; 33...

$T_2 - T_1 = 21 - 9$ $\quad$ $T_3 - T_2 = 33 - 21$

$= 12$ $\quad$ $= 12 \rightarrow T_2 - T_1 = T_3 - T_2 \ \therefore$ pattern is linear

$\therefore d = 12$

As the pattern is linear with a common difference of 12, 12 tiles are being added on in each step.

$\therefore$ in the fourth step there will 45 tiles.

b) *Conjecture*: In step 1 there are 9 tiles, thereafter 12 tiles are added in each step. Therefore the number of tiles required in each step equals **twelve times the step you are on minus three**. → minus three as you start with 9 tiles not 12

Step 1: 9 tiles $\rightarrow 9 = 12(1) - 3$

Step 2: 21 tiles $\rightarrow 21 = 12(2) - 3$

Step 3: 33 tiles $\rightarrow 33 = 12(3) - 3$

c) Pattern is linear, $\therefore T_n = dn + c$

$d = 12$

$\therefore T_n = 12n + c$

In order to solve for c, substitute a term of the pattern into the equation.

Substitute the first term into the equation, $\therefore n = 1$ and $T_1 = 9$

$\therefore 9 = 12(1) + c$

$\therefore c = -3$

$\therefore T_n = 12n - 3$

d) Red tile pattern: 1; 3; 5

$T_2 - T_1 = 3 - 1$ $\qquad$ $T_3 - T_2 = 5 - 3$

$= 2$ $\qquad$ $= 2 \rightarrow T_2 - T_1 = T_3 - T_2 \therefore$ pattern is linear

$\therefore d = 2$

$\therefore T_n = 2n + c$

In order to solve for c, substitute a term of the pattern into the equation.

Substitute the first term into the equation, $\therefore n = 1$ and $T_1 = 1$

$\therefore 1 = 2(1) + c$

$\therefore a = -1$

$\therefore T_n = 2n - 1$

e) Yellow tile pattern: 8; 18; 28

$T_2 - T_1 = 18 - 8$ $\qquad$ $T_3 - T_2 = 28 - 18$

$= 10$ $\qquad$ $= 10 \rightarrow T_2 - T_1 = T_3 - T_2 \therefore$ pattern is linear

$\therefore d = 10$

$\therefore T_n = 10n + c$

In order to solve for c, substitute a term of the pattern into the equation.

Substitute the first term into the equation, $\therefore n = 1$ and $T_1 = 8$

$\therefore 8 = 10(1) + c$

$\therefore c = -2$

$\therefore T_n = 10n - 2$

f) Red tiles: $T_n = 2n - 1$

15^{th} step: $n = 15$ and $\mathrm{T}_{15} = ?$

$T_{15} = 2(15) - 1$

$\therefore T_{15} = 29$

$\therefore$ 29 red tiles are required for the 15^{th} step.

Complete Exercise 1 numbers 1 to 4 on page 264.

UNSEEN NUMBER PATTERNS

In Grade 10 you will be given number pattern problems that do not have a routine method for solving. These problems require you to think logically and to work out the underlying pattern yourself.
In addition, you may have to devise and prove your own conjecture for such a pattern.

Example 1: Using your knowledge of number patterns, calculate the last digit of 3^{2012} and hence calculate the last digit of 3^{2013}, 3^{2014} and 3^{2015}.

In order to solve this problem you have to attempt to find a pattern relating to the last digit of 3^{2012}.

Start with small numbers when trying to identify the pattern.

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 81$

$3^5 = 243$

$3^6 = 729$

$3^7 = 2187$

$3^8 = 6561$

$3^9 = 19683$

It can be seen that the last digit of 3 raised to successive powers follows a pattern which repeats itself in fours. The pattern is: 3, 9, 7 , 1

2012 is divisible by 4, $\therefore$ the last digit will be the 4^{th} digit in the pattern.

$\therefore$ the last digit of 3^{2012} is 1

The pattern will then repeat itself.

$\therefore$ the last digit of 3^{2013} is 3

$\therefore$ the last digit of 3^{2014} is 9

$\therefore$ the last digit of 3^{2015} is 7

Example 2: **Investigate the number pattern below and answer the following questions:**

$$\mathbf{1^2 - 2^2 = -1 - 2}$$
$$\mathbf{2^2 - 3^2 = -2 - 3}$$
$$\mathbf{3^2 - 4^2 = -3 - 4}$$

a) Using the above pattern determine the value of $199^2 - 200^2$.
b) Write a rule in words for this pattern. (this is known as a conjecture)
c) Write your rule as a mathematical equation and prove that it is correct.

a) From the above pattern:

$$\begin{aligned} 199^2 - 200^2 &= -199 - 200 \\ &= -399 \end{aligned}$$

b) *Conjecture*: Any integer squared, less its next consecutive integer squared, equals the negative of that integer less its next consecutive integer.

c) $x^2 - (x+1)^2 = -x - (x+1) \rightarrow x$ is any integer, $x+1$ is the next consecutive integer

To prove that the conjecture is true, you have to prove that the LHS (left hand side) is equal to the RHS (right hand side).

$$\begin{aligned} LHS &= x^2 - (x+1)^2 \\ &= x^2 - \left(x^2 + 2x + 1\right) \rightarrow \text{don't forget the middle term} \\ &= x^2 - x^2 - 2x - 1 \\ &= -2x - 1 \end{aligned} \qquad RHS = -x - (x+1)$$

You can now simplify the *RHS* further:

$$\begin{aligned} RHS &= -x - (x+1) \\ &= -x - x - 1 \\ &= -2x - 1 = LHS \\ &\therefore LHS = RHS \end{aligned}$$

Recap: Expanding square binomials is revised on page 12.

Example 3: **Investigate the pattern below and answer the following questions:**

$$(3+4)^2-(3-4)^2=48$$
$$(5+8)^2-(5-8)^2=160$$
$$(10+2)^2-(10-2)^2=80$$

a) **Write a rule in words for this pattern.** (this is known as a conjecture)
b) **Write your rule as a mathematical equation and prove that it is correct.**

a) $(3+4)^2-(3-4)^2=48 \rightarrow 4(3)(4)$

$(5+8)^2-(5-8)^2=160 \rightarrow 4(5)(8)$

$(10+2)^2-(10-2)^2=80 \rightarrow 4(10)(2)$

Conjecture:

The square of the sum of two consecutive integers exceeds the square of their difference by four times their product.

b) $(x+y)^2-(x-y)^2=4xy$

To prove that the conjecture is true, you have to prove that the LHS (left hand side) is equal to the RHS (right hand side).

$LHS=(x+y)^2-(x-y)^2$ $\qquad\qquad$ $RHS=4xy$

$=x^2+2xy+y^2-\left(x^2-2xy+y^2\right) \rightarrow$ don't forget the middle term

$=x^2+2xy+y^2-x^2+2xy-y^2 \rightarrow$ don't forget to distribute the minus

$=4xy=RHS$

$\therefore LHS=RHS$

Complete Exercise 1 number 5 on page 264 and number 6 on page 265.

B. LINEAR EQUATIONS

A linear equation is an equation in which the highest power of the variable you are solving for is 1. A linear equation cannot have more than one solution.

Always place all of the *x*'s (or the variable that you are solving for) on one side of the equation. Remember that whatever you do to one side of the equation you have to do to the other side.

Example 1: **Solve for *x* if $x+2=5$.**

$x+2=5$

$\therefore x+2-2=5-2 \rightarrow$ isolate the variable by subtracting 2 from both sides

$\therefore x=3$

Example 2: **Solve for *x* if $5(x+1)=3(3x+2)-4$.**

$5(x+1)=3(3x+2)-4$

$\therefore 5x+5=9x+6-4 \rightarrow$ distributive law

$\therefore 5x+5=9x+2 \rightarrow$ isolate the variable by subtracting $5x$ and 2 from both sides

$\therefore 5x+5-5x-2=9x+2-5x-2 \rightarrow$ you do not have to show this step

$\therefore 3=4x$

$\therefore \frac{3}{4}=\frac{\cancel{4}x}{\cancel{4}} \rightarrow$ divide both sides by 4 to isolate the x

$\therefore x=\frac{3}{4}$

Example 3: **Solve for *x* if $4(x+5)=2(8x-3)-2$.**

$4(x+5)=2(8x-3)-2$

$\therefore 4x+20=16x-6-2 \rightarrow$ distributive law

$\therefore 4x+20=16x-8 \rightarrow$ isolate the variable by subtracting $4x$ and adding 8 to both sides

$\therefore 4x+20-4x+8=16x-8-4x+8 \rightarrow$ you do not have to show this step

$\therefore 28=12x$

$\therefore \frac{28}{12}=\frac{\cancel{12}x}{\cancel{12}} \rightarrow$ divide both sides by 12 to isolate the x

$$\therefore x=\frac{28}{12}$$
$$=\frac{7}{3}$$

Example 4: **Solve for x if $\frac{8}{x}+4=\frac{5}{x}+\frac{3}{x}$.**

$\frac{8}{x}+4=\frac{5}{x}+\frac{3}{x} \rightarrow$ LCD $= x$, $x \neq 0 \rightarrow$ you have to put in this restriction

$\therefore 8+4x=5+3 \rightarrow$ both sides were multiplied by the LCD

$\therefore 8+4x=8$

$\therefore 4x=0$

$\therefore \frac{\cancel{4}x}{\cancel{4}}=\frac{0}{4} \rightarrow$ divide both sides by 4 to isolate the x

$\therefore x=0$

but from the restriction above $x \neq 0$

$\therefore$ no solution

Example 5: **Solve for x if $\frac{24}{x+4}=\frac{8}{x+2}$.**

$\frac{24}{x+4}=\frac{8}{x+2} \rightarrow$ LCD $=(x+2)(x+4)$, $x \neq -2$ and $x \neq -4$

$\therefore \frac{24(x+2)}{(x+2)(x+4)}=\frac{8(x+4)}{(x+2)(x+4)} \rightarrow$ you do not have to show this step

You now multiply both sides by the LCD (drop the LCD).

You do not have to show this step, it is put in for explanation purposes.

$$\therefore \frac{24(x+2)}{\cancel{(x+2)}\,\cancel{(x+4)}}\times\cancel{(x+2)}\,\cancel{(x+4)}=\frac{8(x+4)}{\cancel{(x+2)}\,\cancel{(x+4)}}\times\cancel{(x+2)}\,\cancel{(x+4)}$$

$\therefore 24(x+2)=8(x+4) \rightarrow$ you can go straight from the 1st step to this step

$\therefore 24x+48=8x+32$

$\therefore 24x-8x=32-48$

$\therefore 16x=-16$

$\therefore x=-1$

You can only 'drop' a LCD if you have an equation. If you are simplifying a fraction and not solving for a variable you have to keep the LCD.

A denominator can never be equal to zero. If there is a variable in the denominator, you have to put in restrictions in order to ensure that the denominator will not be equal to zero.

Complete Exercise 2 on page 265.

C. QUADRATIC EQUATIONS

A quadratic equation is an equation in the form of:

$$ax^2+bx+c=0$$

Quadratic equations always have two solutions.

STEPS TO SOLVING QUADRATIC EQUATIONS

1. **RHS or LHS must be equal to zero.**
2. **Factorise (always look for a common factor first).**
3. **Solve for x.**

x-values are called the solutions or roots of an equation.

Example 1: Solve for x if $x^2=6x$.

$x^2=6x \rightarrow$ the LHS or RHS must be equal to 0

$\therefore x^2-6x=0$

$\therefore x(x-6)=0 \rightarrow$ take out the HCF of x (always look for a HCF first)

The only way that this equation can be true, is if : $x=0$ or $x-6=0$

$\therefore x=0$ or $x-6=0$

$x=6$

Example 2: Solve for x if $8x^2=8$.

$8x^2=8 \rightarrow$ the LHS or RHS must be equal to 0

$\therefore 8x^2-8=0$

$\therefore 8(x^2-1)=0 \rightarrow$ take out the HCF of 8 (always look for a HCF first)

$\therefore (x^2-1)=0 \rightarrow$ both sides were divided by $8 \rightarrow 0\div 8=0$

$\therefore (x-1)(x+1)=0 \rightarrow$ factorised using difference of two squares

$\therefore x-1=0$ or $x+1=0$

$\therefore x=1$ or $x=-1$

You can never divide both sides by a variable (drop a variable), as the value of the variable could be equal to zero. This would also result in a solution being lost.

Example 3: **Find the roots of $x^2+x=20$.**

$x^2+x=20 \rightarrow$ the LHS or RHS must be equal to 0

$\therefore x^2+x-20=0$

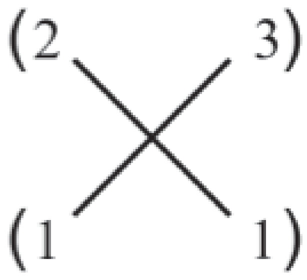
$\therefore (x+5)(x-4)=0$

$\therefore x+5=0$ or $x-4=0$

$\therefore x=-5$ or $x=4$

Example 4: **Solve for x if $2x^2+5x+3=0$.**

$2x^2+5x+3=0 \rightarrow$ there is no HCF, therefore use the cross method



$\therefore (2x+3)(x+1)=0$

$\therefore 2x+3=0$ or $x+1=0$

$\therefore 2x=-3$ or $x=-1$

$\therefore x=\frac{-3}{2}$

Example 5: **Solve for x if $3x^2+3x-18=0$.**

$3x^2+3x-18=0 \rightarrow$ 3 is common to all terms, always look for an HCF first

$\therefore 3\left(x^2+x-6\right)=0 \rightarrow$ divide both sides by 3

$\therefore x^2+x-6=0$

$\therefore (x+3)(x-2)=0$

$\therefore x+3=0$ or $x-2=0$

$\therefore x=-3$ or $x=2$

Example 6: **Solve for x if $(2x+5)(8x-3)=0$.**

$(2x+5)(8x-3)=0 \rightarrow$ if the RHS or LHS is equal to 0, don't multiply out the brackets

$\therefore 2x+5=0$ or $8x-3=0$

$\therefore 2x=-5$ or $8x=3$

$\therefore x=\frac{-5}{2}$ or $x=\frac{3}{8}$

Example 7: **Solve for x if $x(x+4)=32$.**

$x(x+4)-32=0 \rightarrow$ the LHS or RHS has to equal 0

$\therefore x^2+4x-32=0$

$\therefore (x+8)(x-4)=0$

$\therefore x+8=0$ or $x-4=0$

$\therefore x=-8$ or $x=4$

TIPS TO SOLVING QUADRATIC EQUATIONS

From the examples on the previous pages it can be seen that there is a **quick method** for solving for x once an equation is factorised.

$$(ax+b)(cx-d)=0$$

$\therefore ax+b=0$ or $cx-d=0$

$\therefore ax=-b$ or $cx=+d$

$\therefore x=-\frac{b}{a}$ or $x=+\frac{d}{c}$

keep the constant

divide by the coefficient of x

change the sign

This allows you to go from the factorised equation straight to the answer for x.

Example 1: Solve for x if $(x+5)(x+8)=0$.

$(1x+5)(1x+8)=0$

$\therefore x=-\frac{5}{1}$ or $x=-\frac{8}{1}$

$\therefore x=-5$ or $x=-8$

Example 2: Solve for x if $(3x+5)(7x-2)=0$.

$(3x+5)(7x-2)=0$

$\therefore x=-\frac{5}{3}$ or $x=+\frac{2}{7}$

Example 3: Solve for x if $(8x+2)(14x-7)=0$.

$(8x+2)(14x-7)=0$

$\therefore x=-\frac{2}{8}$ or $x=+\frac{7}{14}$

$\therefore x=-\frac{1}{4}$ or $x=+\frac{1}{2}$

Complete Exercise 3 on page 266.

D. LITERAL EQUATIONS (CHANGING THE SUBJECT OF THE FORMULA)

In literal equations you will be expected to solve for a variable in terms of numbers and other variables. This is also known as changing the subject of a formula.

Example 1: **The volume of a sphere is given as $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. Express the radius of a sphere in terms of V.**

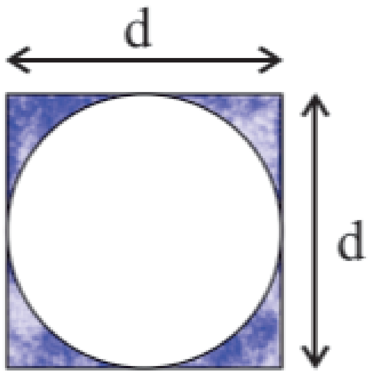
$V = \frac{4}{3}\pi r^3$

$\therefore 3V = 4\pi r^3$ → both sides were multiplied by 3

$\therefore \frac{3V}{4\pi} = r^3$ → both sides were divided by 4π

$\therefore r = \sqrt[3]{\frac{3V}{4\pi}}$ → cube root both sides

Example 2: **The area of the shaded region shown below is given as $A = d^2 - \frac{\pi d^2}{4}$.**



a) Make d the subject of the formula.
b) Determine the value of d if the shaded region has an area of 24cm^2.

a) $A = d^2 - \frac{\pi d^2}{4}$

$\therefore 4A = 4d^2 - \pi d^2$ → both sides were multiplied by 4

$\therefore 4A = d^2(4-\pi)$ → a common factor of d^2 was taken out

$\therefore d^2 = \frac{4A}{4-\pi}$ → both sides were divided by $4-\pi$

$\therefore d = \sqrt{\frac{4A}{4-\pi}}$ → both sides were square rooted, 1 positive answer as d is a length

b) $d = \sqrt{\frac{4A}{4-\pi}} = \sqrt{\frac{4(24)}{4-\pi}} = 10{,}58\ cm$ → always put in units

Complete Exercise 4 on page 266.

E. NUMBER LINE REVISION

In Grade 10 you are expected to represent numbers and inequalities on a number line. There are two ways to do this:

1. **Set Builder Notation**
2. **Interval Notation**

1. SET BUILDER NOTATION

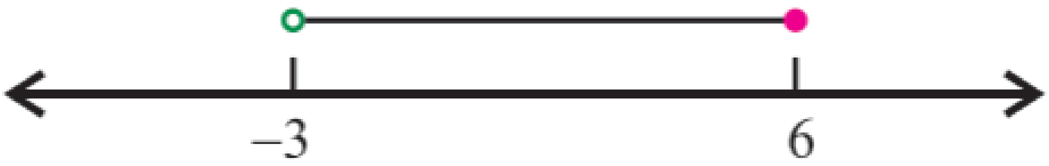
Set Builder Notation can be used to represent **all numbers** in the real number system.

Example 1: **Represent $\{x : -3 < x \leq 6; x \in \mathbb{R}\}$ on a number line.**

$\{x : -3 < x \leq 6; x \in \mathbb{R}\} \rightarrow x$ is greater than -3 but less than or equal to 6, $x \in \mathbb{R}$

open dot as x is **greater** than –3

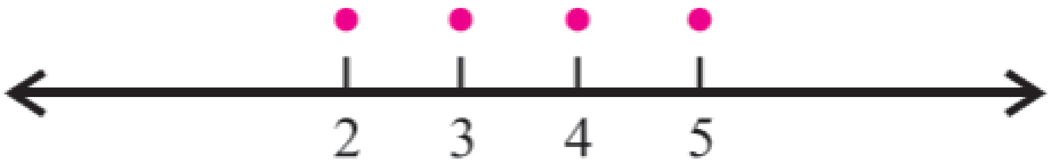
closed dot as x is **less** than or **equal** to 6



Example 2: **Represent $\{x : 2 \leq x \leq 5; x \in \mathbb{Z}\}$ on a number line.**

$\{x : 2 \leq x \leq 5; x \in \mathbb{Z}\} \rightarrow x$ is greater than or equal to 2 but less than or equal to 5, $x \in \mathbb{Z}$

You cannot join the dots with a line as x has to be an integer.

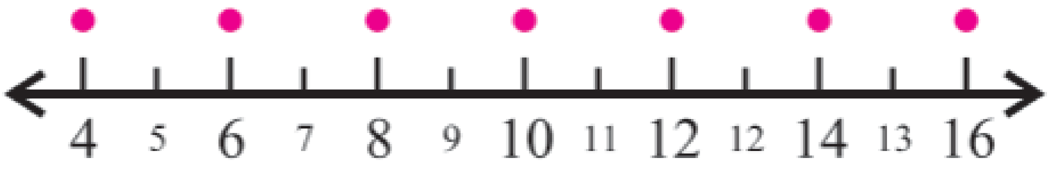


Example 3: **Represent $\{2x : 2 \leq x \leq 8;\ x \in \mathbb{Z}\}$ on a number line.**

$\{2x : 2 \leq x \leq 8;\ x \in \mathbb{Z}\} \rightarrow x$ is greater than or equal to 2 but less than or equal to 8, $x \in \mathbb{Z}$

$2x$: As it is $2x$ instead of x the values on the number line have to be multiplied by 2

The question is asking you to represent twice the values of x on the number line



If you are not told which set the numbers belong to, you can assume that the set is real. Real numbers are illustrated on number lines with solid lines.

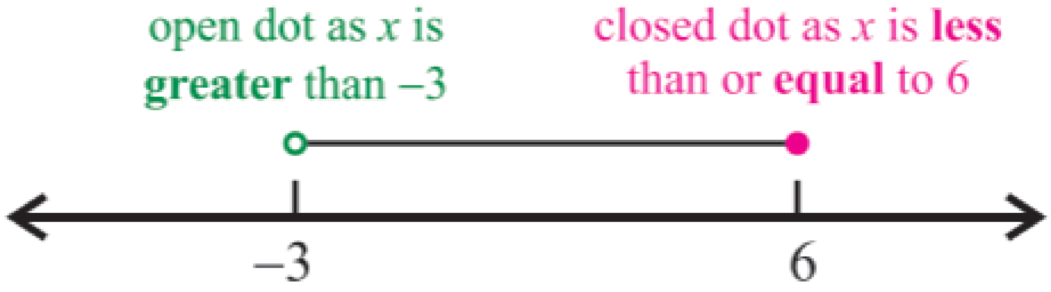
2. INTERVAL NOTATION

Interval Notation is another method of writing a set real of numbers.
The table below shows how interval notation relates to set builder notation.
Interval notation can only be used to represent real numbers.

Interval Notation	Set Builder Notation	Meaning
$x \in [4;10]$	$\{x : 4 \leq x \leq 10\}$	x is greater than or equal to 4 but less than or equal to 10
$x \in [-5;3)$	$\{x : -5 \leq x < 3\}$	x is greater than or equal to -5 but less than 3
$x \in (-\infty;20)$	$\{x : x < 20\}$	x is less than 20
$x \in (5;\infty)$	$\{x : x > 5\}$	x is greater than 5

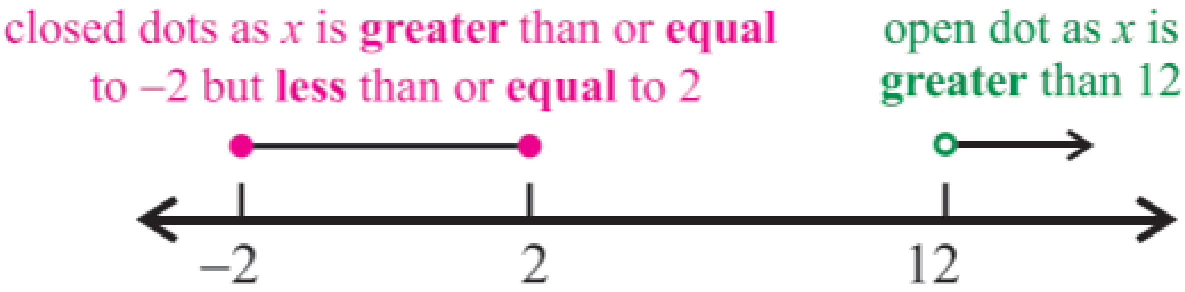
Example 1: Represent $x \in (-3;6]$ in Set Builder Notation and on a number line.

Set Builder Notation: $\{x : -3 < x \leq 6 ; x \in \mathbb{R}\}$

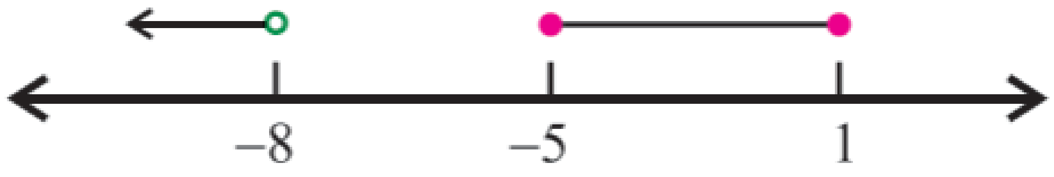

open dot as x is greater than –3
closed dot as x is less than or equal to 6
–3
6

Example 2: Represent $x \in [-2;2]$ or $(12;\infty)$ in Set Builder Notation and on a number line.

Set Builder Notation: $\{x : -2 \leq x \leq 2 \text{ or } x > 12 ; x \in \mathbb{R}\}$


closed dots as x is greater than or equal to –2 but less than or equal to 2
open dot as x is greater than 12
–2
2
12

Example 3: Represent the following number line in Interval and Set Builder Notation.



Interval Notation: $x \in (-\infty;-8) \cup [-5;1] \rightarrow \cup$ is the symbol for the word 'or'

Set Builder Notation: $\{x : x < -8 \text{ or } -5 \leq x \leq 1 ; x \in \mathbb{R}\}$

In interval notation a round bracket is equivalent to an open dot and a square bracket is equivalent to a closed dot. The symbol $\cup$ can be used instead of the word 'or', which is the union of two sets.

Complete Exercise 5 on pages 266 and 267.

F. LINEAR INEQUALITIES

Linear inequalities are equations which have an inequality sign ($<$, $>$, $\leq$, $\geq$) instead of an equals sign. Inequalities can be represented graphically on a number line.

If you multiply or divide both sides of an inequality by a negative number, the direction of the inequality sign has to change.

Adding or subtracting a number from both sides of an inequality does not affect the direction of the inequality sign.

Example 1: Solve for x if $5x+4\geq 3x+10$.
In addition, illustrate your answer on a number line.

$5x+4\geq 3x+10$

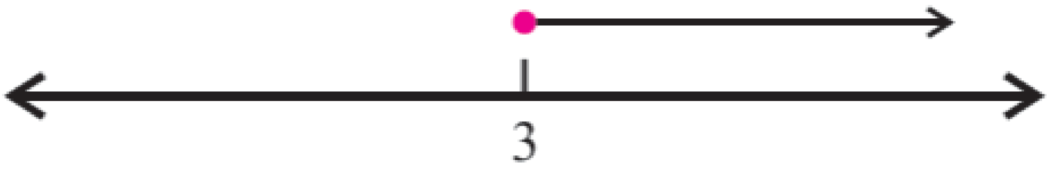
$\therefore 5x-3x\geq 10-4$

$\therefore 2x\geq 6$

$\therefore x\geq 3$

This can also be written as: $x\in[3;\infty)$

closed dot as x is greater than or **equal** to 3



Example 2: Solve for x if $5(2x+1)<3(5x-3)$.
In addition, illustrate your answer on a number line.

$5(2x+1)<3(5x-3)$

$\therefore 10x+5<15x-9 \rightarrow$ distributive law

$\therefore 10x-15x<-9-5$

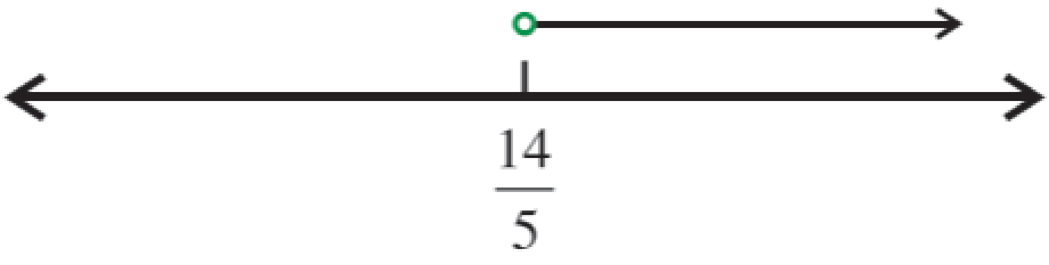
$\therefore -5x<-14$

$\therefore \frac{-5x}{-5}>\frac{-14}{-5} \rightarrow$ change the direction of the inequality when dividing by a negative

$\therefore x>\frac{14}{5}$

This can also be written as: $x\in\left(\frac{14}{5};\infty\right)$

open dot as x is **greater** than $\frac{14}{5}$



Example 3: **Solve for x if $\frac{2(x+1)}{3}-\frac{2(3x+2)}{6}<x-4$ and $x \in \mathbb{Z}$.**
In addition, illustrate your answer on a number line.

$$\frac{2(x+1)}{3}-\frac{2(3x+2)}{6}<\frac{x-4}{1}$$

$\therefore \frac{2x+2}{3}-\frac{6x+4}{6}<x-4 \rightarrow$ distributive law, LCD $=6$

$\therefore 2(2x+2)-(6x+4)<6(x-4) \rightarrow$ both sides were multiplied by the LCD

$\therefore 4x+4-6x-4<6x-24$

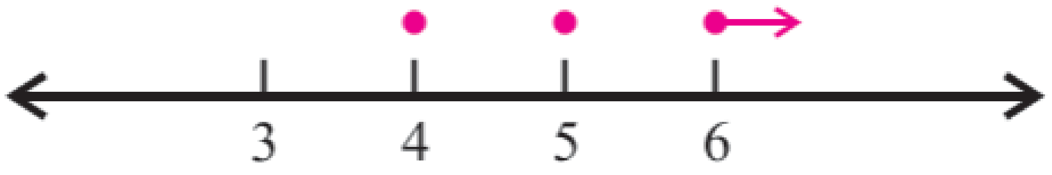
$\therefore -8x<-24$

$\therefore x>\frac{-24}{-8} \rightarrow$ don't forget to change the direction of the inequality

$\therefore x>3$

x is an element of integers. The next integer after 3 is 4. Therefore the answer can also be written as $x \geq 4$.

As x is greater than 3 there is no dot on top of 3
the next integer after 3 is 4 so closed dots start on 4



Always write at least three numbers to show the pattern.

Example 4: **Solve for x if $-5 \leq x-2<10$.**
In addition, illustrate your answer on a number line.

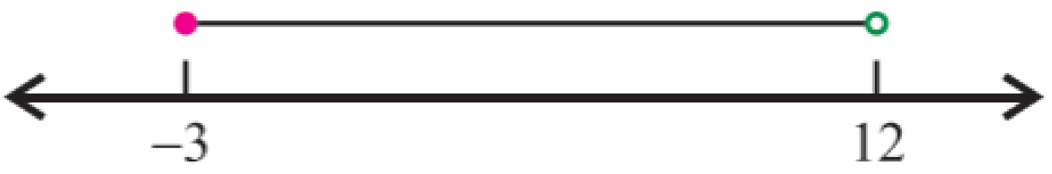
$-5 \leq x-2<10 \rightarrow$ you can deal with each side separately

$\therefore -5+2 \leq x<10+2 \rightarrow$ add 2 to both sides to get x on its own

$\therefore -3 \leq x<12$

closed dot as x is greater than or **equal** to -3

open dot as x is **less** than 12



You can only draw a solid line if the variable is a real number. If it was restricted to whole numbers, natural numbers, or integers, you would have to give exact answers.

Complete Exercise 6 on page 267.

G. SIMULTANEOUS EQUATIONS

Simultaneous equations are used to solve for more than one variable.
In Grade 10 there are two methods you will learn to solve simultaneous equations:

1. **Elimination**
2. **Substitution**

The solutions of linear simultaneous equations represent the intersection of two lines.

1. ELIMINATION

When using elimination to solve simultaneous equations, you have to **add or subtract equations** in order **to eliminate one variable**.

STEPS TO SOLVING SIMULTANEOUS EQUATIONS BY ELIMINATION:

1. **Label the equations as equation 1 and equation 2 respectively.**
2. **Manipulate the equations so that one of the variables can be eliminated.**
3. **Add or subtract the equations to/from each other in order to eliminate a variable.**
4. **Solve for the unknown variables.**

Example 1: **Solve for x and y if $2x+y=12$ and $5x+y=21$.**

$2x+y=12 \rightarrow$ equation ①

$5x+y=21 \rightarrow$ equation ②

It can be seen that if the above equations are subtracted from each other, y will be eliminated.

equation ② – equation ①:

$$\begin{array}{r} 5x+y\ =21 \\ -\left(2x+y=12\right) \\ \hline 3x+0\ =9 \end{array} \rightarrow \begin{array}{r} 5x+y=21 \\ -2x-y=-12 \\ \hline 3x+0=9 \end{array} \rightarrow \text{the minus has to be distributed}$$

$\therefore 3x=9$

$\therefore x=3 \rightarrow$ equation ③

Now that you have solved for x, you can substitute x back into equation 1 or 2 in order to solve for y.

subst ③ into ①

$\therefore 2(3)+y=12$

$\therefore 6+y=12$

$\therefore y=6$

$\therefore x=3$ and $y=6$

When using elimination to solve simultaneous equations and you have solved for one of the variables, you can substitute your answer into equation 1 or 2. Always chose the easiest option.

Example 2: **Solve for x and y if $4x+3y=30$ and $8x+2y=52$.**

$4x+3y=30 \rightarrow$ equation ①

$8x+2y=52 \rightarrow$ equation ②

In this case it can be seen that simply adding or subtracting the above equations will not eliminate a variable. Therefore, you have to manipulate one or both of the equations so that a variable can be eliminated by addition or subtraction.

equation ① $\times 2$:

$\therefore 8x+6y=60 \rightarrow$ equation ③ $\rightarrow$ multiply every term in equation ① by 2

You now subtract equation 3 from equation 2 in order to eliminate the variable x.

equation ② – equation ③:

$$\begin{array}{r} 8x+2y=52 \\ -\left(8x+6y=60\right) \\ \hline 0-4y=-8 \end{array} \rightarrow \begin{array}{r} 8x+2y=52 \\ -8x-6y=-60 \\ \hline 0-4y=-8 \end{array} \rightarrow \text{the minus has to be distributed}$$

$\therefore -4y=-8$

$\therefore y=2 \rightarrow$ equation ④

Now that you have solved for y, you can substitute y back into equation 1 or 2 in order to solve for x.

substitute ④ into ①:

$\therefore 4x+3(2)=30$

$\therefore 4x+6=30$

$\therefore 4x=24$

$\therefore x=6$

$\therefore x=6$ and $y=2$

For simultaneous equations which have to be manipulated before using elimination, it is often easier to use the method of substitution shown on the next page.

Complete Exercise 7 number 1 on page 267.

2. SUBSTITUTION

When using substitution to solve simultaneous equations, you have to make one of the variables the subject of the equation.

You then substitute the solution of that variable into the other equation in order to eliminate one of the variables.

STEPS TO SOLVING SIMULTANEOUS EQUATIONS BY SUBSTITUTION

1. **Label the equations as equation 1 and equation 2 respectively.**
2. **In one equation, make one of the variables the subject of the equation.**
3. **Substitute into the other equation.**
4. **Solve for the unknown variables.**

Example 1: Solve for x and y if $4x+8y=36$ and $2x+9y=28$.

$4x+8y=36 \rightarrow$ equation ①

$2x+9y=28 \rightarrow$ equation ②

From equation ①:

$4x=36-8y \rightarrow$ make x the subject of the equation

$\therefore x=9-2y \rightarrow$ equation ③, divide each term by 4 in order to solve for x

substitute ③ into ②:

$\therefore 2(9-2y)+9y=28 \rightarrow$ substitute $x=9-2y$, for all the x's in equation 2

$\therefore 18-4y+9y=28$

$\therefore 5y=10$

$\therefore y=2 \rightarrow$ equation ④

substitute ④ into ③:

$\therefore x=9-2(2) \rightarrow y=2$

$\therefore x=5$ and $y=2$

In a test or exam you may be told which method to use when solving simultaneous equations. If you use a different method from the one you are told to use, you will receive no marks.

Example 2: Solve for x and y if $5x+10y=80$ and $3x+4y=42$.

$5x+10y=80$ → equation ①

$3x+4y=42$ → equation ②

From equation ①:

$5x=80-10y$ → make x the subject of the equation

$\therefore x=16-2y$ → equation ③, divide each term by 5 in order to solve for x

substitute ③ into ②:

$\therefore 3(16-2y)+4y=42$ → substitute $x=16-2y$ for all the x's in equation 2

$\therefore 48-6y+4y=42$

$\therefore -2y=-6$

$\therefore y=3$ → equation ④

substitute ④ into ③:

$\therefore x=16-2(3)$ → $y=3$

$\therefore x=10$ and $y=3$

Example 3: Solve for x and y if $6x+12y=18$ and $9x+20y=28$.

$6x+12y=18$ → equation ①

$9x+20y=28$ → equation ②

From equation ①:

$6x=18-12y$ → make x the subject of the equation

$\therefore x=3-2y$ → equation ③, divide each term by 6 in order to solve for x

substitute ③ into ②:

$\therefore 9(3-2y)+20y=28$ → substitute $x=3-2y$ for all the x's in equation 2

$\therefore 27-18y+20y=28$

$\therefore 2y=1$

$\therefore y=\frac{1}{2}$ → equation ④

substitute ④ into ③:

$\therefore x=3-2\left(\frac{1}{2}\right)$ → $y=\frac{1}{2}$

$\therefore x=3-1$

$\therefore x=2$ and $y=\frac{1}{2}$

Complete Exercise 7 number 2 on page 267.

H. MATHEMATICAL MODELLING

In mathematical modelling you will be given a scenario in words. You will then be expected to derive your own equations from the given information in order to solve the problem.

STEPS TO SOLVING WORD PROBLEMS:

1. **Identify what you are asked to find and call it *x*.**
2. **Only work in one unit, eg. kg and g or minutes and seconds.**
3. **When you have two expressions in an equation, make sure you know which one is larger.**
4. **Make sure you understand what operation the question is asking you to perform, eg. The sum of is '+'.**
5. **Draw a table or sketch if possible.**
6. **Make sure that you state the answer clearly.**

Recap:

Sum of → +
Difference → −
Product → ×
Quotient → ÷

AGE/NUMBER PROBLEMS

Example 1: **Bob is 8 years older than Jane. In 6 years' time Bob will be twice as old as Jane. How old is Bob?**

Let Bob's age now $= x$

$\therefore$ Jane's age now $= x - 8 \rightarrow$ Jane is 8 years younger than Bob

You now have an expression for Jane's age and Bob's age in terms of *x*. However, the question gives information about their respective ages in 6 years times. Therefore, **you need an expression for their ages in 6 years' time.**

$\therefore$ In 6 years' time: Bob's age $= x + 6$

Jane's age $= x - 8 + 6 = x - 2$

$\therefore x + 6 = 2(x - 2) \rightarrow$ in 6 years' time Bob is twice as old as Jane

$\therefore x + 6 = 2x - 4$

$\therefore -x = -10$

$\therefore x = 10$

$\therefore$ Bob is 10 years old

If Bob is 10 years old, Jane must be 2 years old.

Example 2: **The sum of two consecutive even integers is equal to twice the second number less the first number. Determine what the numbers are.**

Let the first number $= x$

$\therefore$ the second number $= x + 2 \rightarrow$ the numbers are consecutive and even

$\therefore x + (x + 2) = 2(x + 2) - x \rightarrow$ sum of the integers is equal to twice the second number less the first number

$\therefore 2x + 2 = 2x + 4 - x$

$\therefore x = 2$

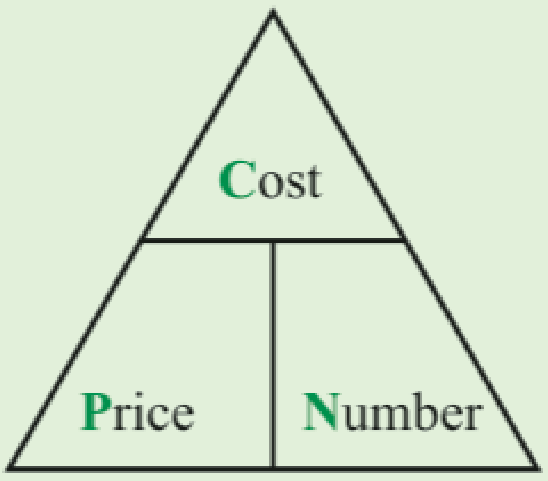
$\therefore$ The first integer is equal to 2 and the second integer is 4.

Complete Exercise 8 on page 267.

SHOPPING PROBLEMS

Example 1:

A man buys fillet steak and mince for a party. Fillet steak costs R96 per kilogram whilst mince costs R56 per kilogram. In total, the man pays R476 for 6 kilograms of meat. Determine the quantity (in kilograms) of each type of meat that was bought.



$\text{Cost} = \text{Price} \times \text{Number of items}$ $\quad \text{Number of items} = \dfrac{\text{Cost}}{\text{Price}}$ $\quad \text{Price} = \dfrac{\text{Cost}}{\text{Number of items}}$

Cost → Total amount that is paid
Price → Price per item (price of a single item)
Number of items → Number of items bought or sold

In order to draw a table, you need a **variable column**, **constant column** and **formula column**. You then use the extra information from the question to form an equation and solve for the unknown variable.

Situation	Number of kg	Price per kg	Cost (total)
Fillet	x	96	$96x$
Mince	$6-x$	56	$56(6-x)$

Always use the variable column and constant column in order to generate the formula column.

You can now use the extra information in the question (the total cost is R476) to create an equation:

$96x + 56(6-x) = 476$

$\therefore 96x + 336 - 56x = 476$

$\therefore 40x = 140$

$\therefore x = 3{,}5 \rightarrow x$ is the amount of fillet that was purchased

$\therefore$ The amount of mince bought $= 6 - x = 6 - 3{,}5 = 2{,}5$

$\therefore 3{,}5$ kilograms of fillet and $2{,}5$ $(6-3{,}5)$ kilograms of mince was bought.

Example 2:

Tom buys ice cream lollies to sell at his school's Fun Day. Each lolly costs him R3, which he then sells for R8. Tom sells all of his lollies, except for 4 which had melted during the day. If he makes a profit of R68, how many lollies did he originally purchase?

Situation	Number of lollies	Price per lolly	Cost (total)
Bought	x	3	$3x$
Sold	$x-4$	8	$8(x-4)$

You can now use the extra information in the question (R68 profit was made) to create an equation:

$8(x-4)-3x=68 \rightarrow \text{selling price} - \text{cost price} = \text{profit}$

$\therefore 8x-32-3x=68$

$\therefore 5x=100$

$\therefore x=20 \rightarrow x \text{ is the number of lollies originally purchased}$

$\therefore$ 20 lollies were originally purchased.

Profit = Selling Price – Cost Price

Example 3:

Nancy buys T-shirts and shorts. A T-shirt costs R40 more than a pair of shorts. If Nancy buys 8 T-shirts and 4 pairs of shorts at a cost of R800, determine the price of a T-shirt and the price of a pair of shorts.

Situation	Number of each item of clothing	Price per item of clothing	Cost (total)
Shorts	4	x	$4x$
T-Shirts	8	$x+40$	$8(x+40)$

You can now use the extra information in the question (the total cost is R800) to create an equation:

$4x+8(x+40)=800$

$\therefore 4x+8x+320=800$

$\therefore 12x=480$

$\therefore x=40 \rightarrow x \text{ is the price of a pair of shorts}$

$$\begin{aligned}\therefore \text{the price of a T-shirt} &= x+40\\ &= 40+40\\ &= 80\end{aligned}$$

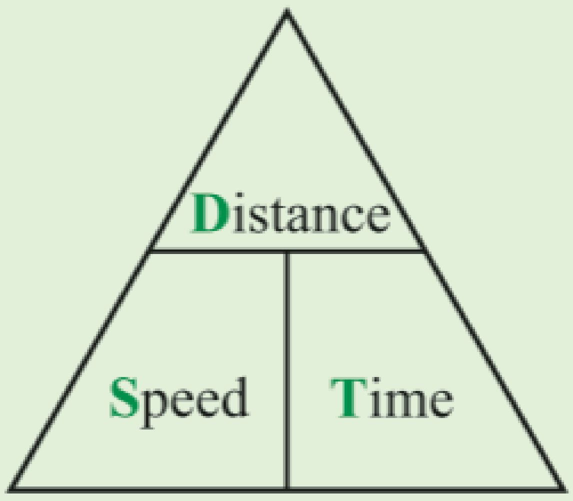
$\therefore$ a T-shirt costs R80 and a pair of shorts costs R40.

Complete Exercise 9 on page 268.

SPEED, DISTANCE AND TIME PROBLEMS

Example 1:

Two planes take off at 8:00am from Lanseria Airport in opposite directions. Plane A flies at a speed of 450 km/h whilst plane B flies at 500 km/h. At what time will the planes be 2375 km apart?



$\text{Distance} = \text{Speed} \times \text{Time}$ $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$ $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

Situation	Time	Speed	Distance
Plane A	x	**450**	$450x$
Plane B	x	**500**	$500x$

You can now use the extra information in the question (the planes are 2375 km apart) to create an equation:

$450x + 500x = 2375 \rightarrow$ the total distance of both planes must sum (add) to 2375 km

$\therefore 950x = 2375$

$\therefore x = 2{,}5$

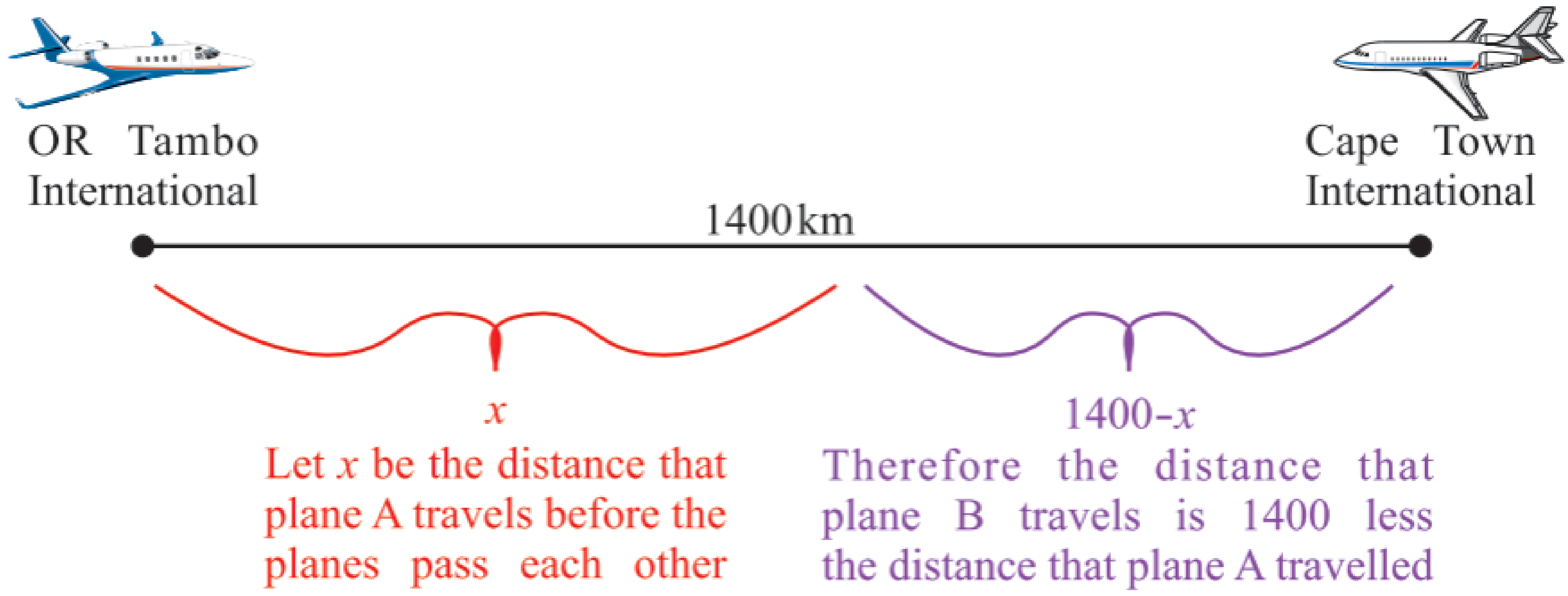
$\therefore$ after 2,5 hours the planes will be 2375 km apart

$\therefore$ The planes will be 2375 km apart at 10:30am. → always answer the question

Example 2:

Plane A takes off from OR Tambo International Airport heading towards Cape Town International Airport. At the same time plane B takes off from Cape Town International Airport, heading for OR Tambo International flying 20 km/h slower than Plane A. The airports are 1400 km apart and the planes meet after 54 minutes. Calculate the distance from OR Tambo International at which they meet and the speed of the planes.

When objects are moving towards each other a diagram can help in drawing up a table.



Let x be the distance that plane A travels before the planes pass each other

Therefore the distance that plane B travels is 1400 less the distance that plane A travelled

You cannot work in minutes as the speed is in kilometres per **hour**. Therefore, you must convert minutes to hours: $\frac{54 \text{ minutes}}{60} = 0{,}9 \text{ hours}$

Situation	Distance	Speed	Time
Plane A	x	$\frac{x}{0{,}9}$	0,9
Plane B	$1400 - x$	$\frac{1400 - x}{0{,}9}$	0,9

You can now use the extra information in the question (Plane A travels 20 km/h faster than Plane B) to make an equation.

$$\frac{x}{0{,}9} = \frac{1400 - x}{0{,}9} + 20 \rightarrow \text{Plane A flies 20 km/h faster than Plane B}$$

$$\therefore x = 1400 - x + 18 \rightarrow \text{both sides were mulitplied by } 0{,}9$$

$$\therefore 2x = 1418$$

$$\therefore x = 709$$

$\therefore$ The planes will pass each other 709 km from OR Tambo International

$$\therefore \text{Speed of plane A} = \frac{x}{0{,}9} = \frac{709}{0{,}9} = 787{,}78$$

$$\therefore \text{Speed of plane B} = 787{,}78 - 20 = 767{,}78$$

$\therefore$ Plane A is flying at 787,78 km/h and plane B is flying at a speed of 767,78 km/h

To convert minutes to hours you divide by 60.

Example 3:

In a race, Mohammed gives Donna a 30 second head start. Donna can run at 4 m/s (metres per second) and Mohammed can run at 6 m/s. How long will it take Mohammed to catch Donna?

When dealing with catch up problems, remember that the distance which people or objects travel is the same. In most cases you can let the distance travelled be *x*. In addition, one person or object will travel for longer than the other. You have to add this extra time in when making an equation.

When Mohammed catches Donna they have both travelled the same distance.
Let the distance they have travelled be x.

Situation	Distance	Speed	Time
Donna	x	**4**	$\frac{x}{4}$
Mohammed	x	**6**	$\frac{x}{6}$

As Mohammed started 30 seconds after Donna, Donna has been running for 30 seconds more than Mohammed (Donna's time is equal to Mohammed's time plus 30 seconds).

$$\therefore \frac{x}{6}+30=\frac{x}{4} \rightarrow \text{LCD}=12$$

$$\therefore 2x+30(12)=3x$$

$$\therefore x=360 \text{ metres}$$

$$\begin{aligned}\therefore \text{time} &= \frac{x}{6} \\ &= \frac{360}{60} \\ &= 60 \text{ seconds}\end{aligned}$$

$\therefore$ It will take Mohammed 60 seconds to catch Donna.

Complete Exercise 10 on page 268.

MATHEMATICAL MODELLING WITH SIMULTANEOUS EQUATIONS

Example 1: The sum of two numbers is 165. The difference of the same numbers is 53. Determine the value of the two numbers.

Let the numbers be x and y:

$\therefore x + y = 165 \rightarrow$ equation ①

and

$\therefore x - y = 53 \rightarrow$ equation ②

Using elimination:

equation ① + equation ②:

$\therefore x + y + (x - y) = 165 + 53$

$\therefore x + y + x - y = 218$

$\therefore 2x = 218$

$\therefore x = 109 \rightarrow$ equation ③

substitute ③ into ①:

$\therefore 109 + y = 165 \rightarrow$ substitute $x = 109$, for all the x's in equation 1

$\therefore y = 56$

$\therefore$ The numbers are 56 and 109

Example 2: The difference of two numbers is 12. If the larger number is divided by the smaller number the answer is 2. What are the numbers?

Let the larger number be x and the smaller number be y:

$\therefore x - y = 12 \rightarrow$ equation ① and $\frac{x}{y} = 2 \rightarrow$ equation ②

Using substitution:

from equation ②:

$x = 2y \rightarrow$ equation ③

substitute ③ into ①:

$\therefore 2y - y = 12 \rightarrow$ substitute $x = 2y$, for all the x's in equation 1

$\therefore y = 12 \rightarrow$ equation ④

substitute ④ into ③:

$\therefore x = 2(12)$

$\therefore x = 24$

$\therefore$ The numbers are 24 and 12

Example 3: **When the digits of a two digit number are added together their sum is equal to 6. If the same number is divided by the units digit the answer is 3. Determine what the number is.**

Any two digit number is made up of a tens digit and a units digit. A two digit number can be expressed as the sum of the units digit and ten times the tens digit. For example the number 52 can be written as $\mathbf{52} = \mathbf{50} + \mathbf{2} = (10 \times \mathbf{5}) + \mathbf{2}$. In order to answer this question you have to use this principle.

Let the tens digit $= x$

Let the units digit $= y$

$\therefore$ the number can be written as $10x + y$

You can now use the information in the question:

$\therefore x + y = 6 \rightarrow$ equation ①

$\therefore \frac{(10x + y)}{y} = 3 \rightarrow$ the number divided by the units digit is equal to 3

$\therefore 10x + y = 3y$

$\therefore 10x - 2y = 0 \rightarrow$ equation ②

Using substitution:

From equation ①:

$y = 6 - x \rightarrow$ equation ③

substitute ③ into ②:

$\therefore 10x - 2(6 - x) = 0 \rightarrow$ substitute $y = 6 - x,$ for all the x's in equation 2

$\therefore 10x - 12 + 2x = 0$

$\therefore 12x = 12$

$\therefore x = 1 \rightarrow$ equation ④

substitute ④ into ③:

$$\begin{aligned} \therefore y &= 6 - 1 \\ &= 5 \end{aligned}$$

$\therefore$ the number is $10(1) + 5 = 15$

Example 4: **The local school is performing a play in order to raise money for charity. There are two types of tickets available.**
One type costs R100 each and the other costs R75 each.

a) If a total of R80 000 is made on the sale of a thousand tickets calculate the number of each type of ticket sold.

b) If the question was solved graphically by drawing two graphs, what point would your answer in (a) represent?

a) Let the total number of R100 tickets sold $= x$

Let the total number of R75 tickets sold $= y$

$\therefore x + y = 1000 \rightarrow$ equation ①, 1000 tickets were sold

and

$\therefore 100x + 75y = 80\,000 \rightarrow$ equation ②, R80 000 was made from the sale of the tickets

Using substitution:

from equation ①:

$\therefore x = 1000 - y \rightarrow$ equation ③

substitute ③ into ②:

$\therefore 100(1000 - y) + 75y = 80\,000 \rightarrow$ substitute $x = 1000 - y$, for all the x's in equation 2

$\therefore 100\,000 - 100y + 75y = 80\,000$

$\therefore -25y = -20000$

$\therefore y = 800 \rightarrow$ equation ④

substitute ④ into ③:

$\therefore x = 1000 - 800 \rightarrow$ substitute $y = 800$, for all the y's in equation 3

$\therefore x = 200$

$\therefore$ Eight hundred R75 tickets were sold and two hundred R100 tickets were sold.

b) The x and y-value of the point of intersection of the graphs. The solutions to simultaneous equations represent the point of intersection of two (linear) graphs.

Complete Exercise 11 on page 268 and
The Mixed Exercise on pages 268, 269 and 270.

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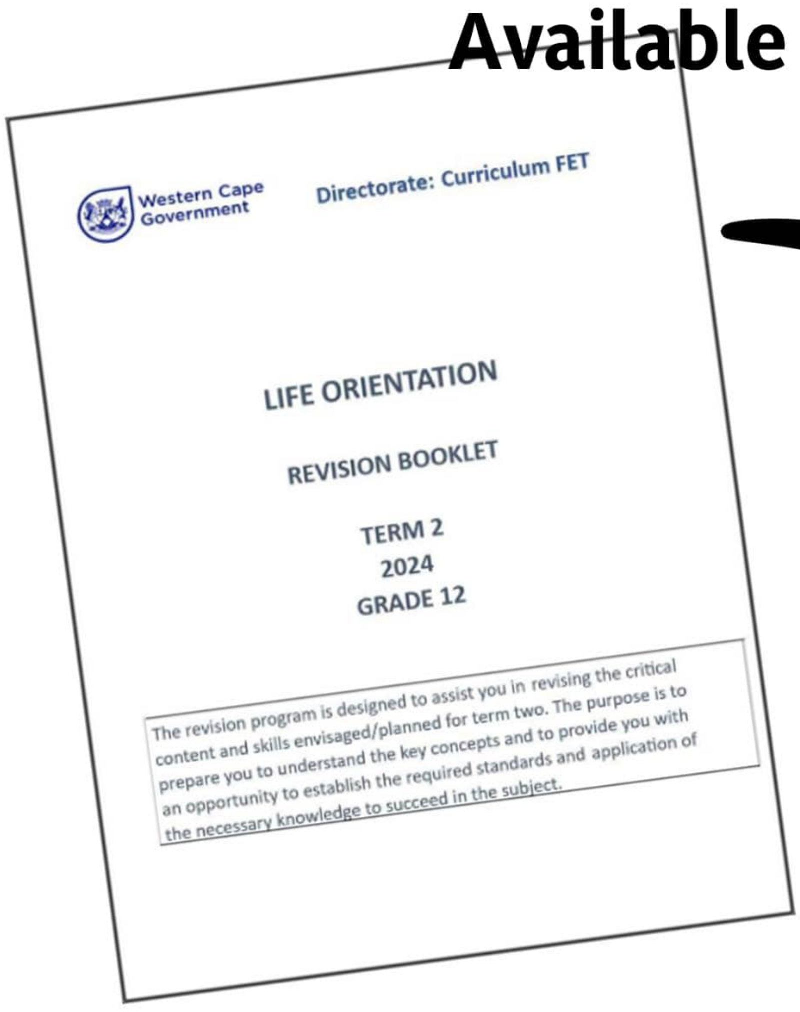

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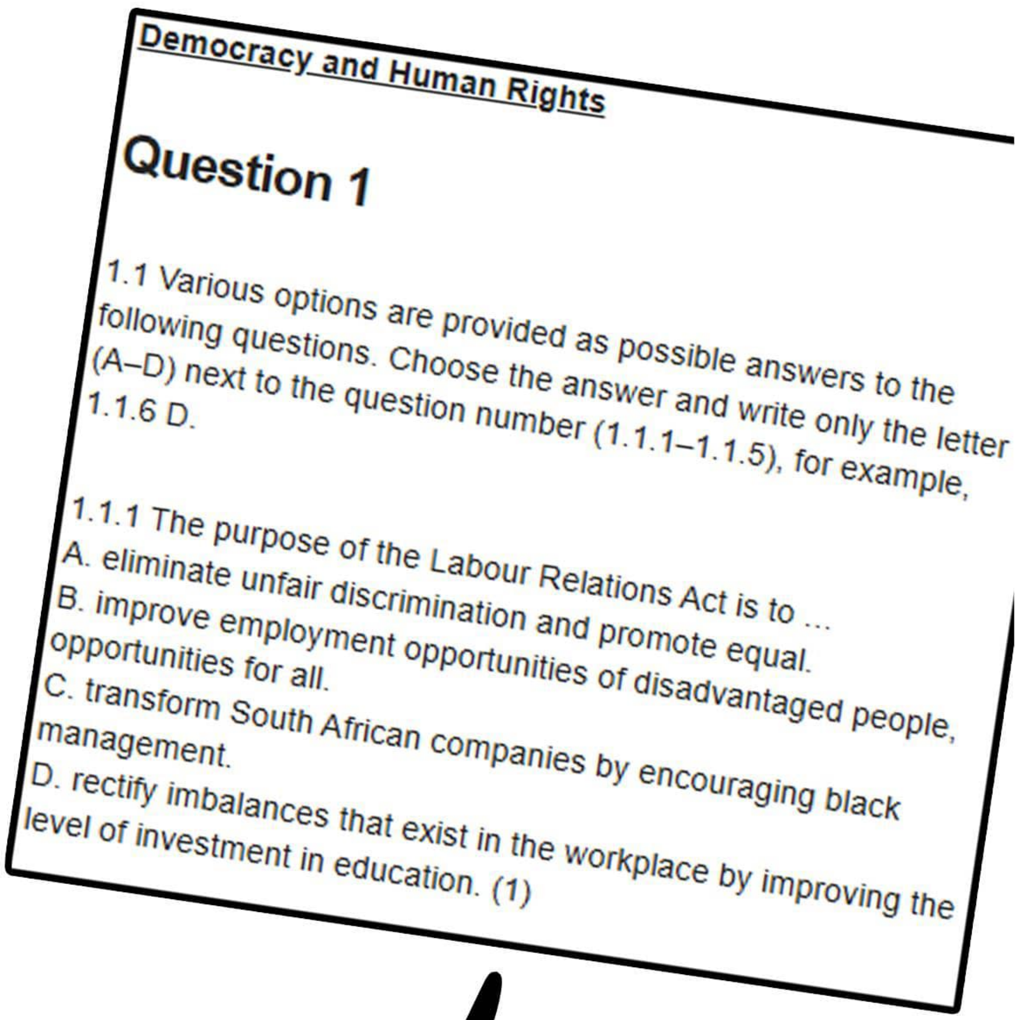
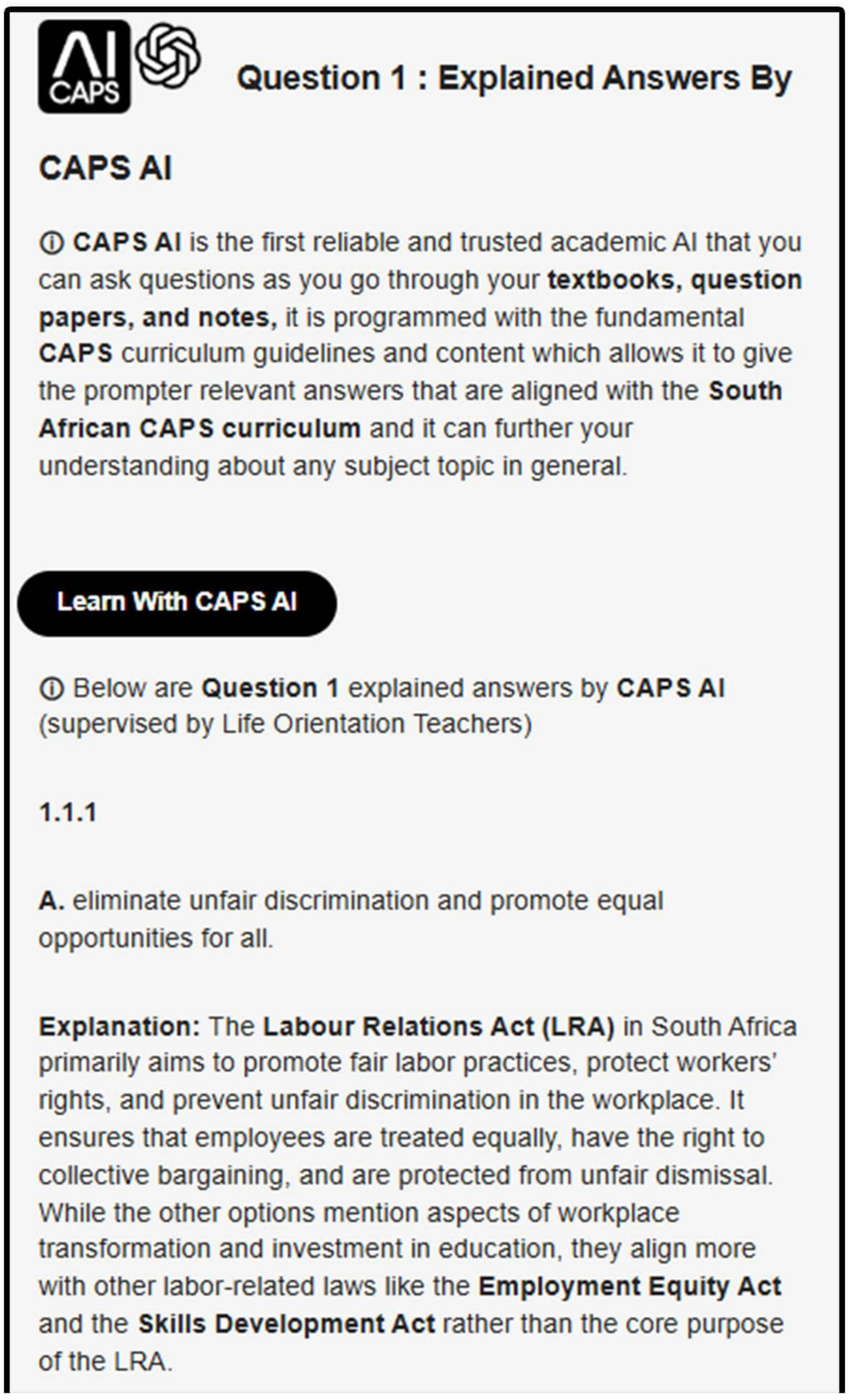


Democracy and Human Rights

Question 1

1.1 Various options are provided as possible answers to the following questions. Choose the answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5), for example, 1.1.6 D.

1.1.1 The purpose of the Labour Relations Act is to ...
A. eliminate unfair discrimination and promote equal opportunities for all.
B. improve employment opportunities of disadvantaged people.
C. transform South African companies by encouraging black management.
D. rectify imbalances that exist in the workplace by improving the level of investment in education. (1)



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1.1.1

A. eliminate unfair discrimination and promote equal opportunities for all.

Explanation: The **Labour Relations Act (LRA)** in South Africa primarily aims to promote fair labor practices, protect workers' rights, and prevent unfair discrimination in the workplace. It ensures that employees are treated equally, have the right to collective bargaining, and are protected from unfair dismissal. While the other options mention aspects of workplace transformation and investment in education, they align more with other labor-related laws like the **Employment Equity Act** and the **Skills Development Act** rather than the core purpose of the LRA.

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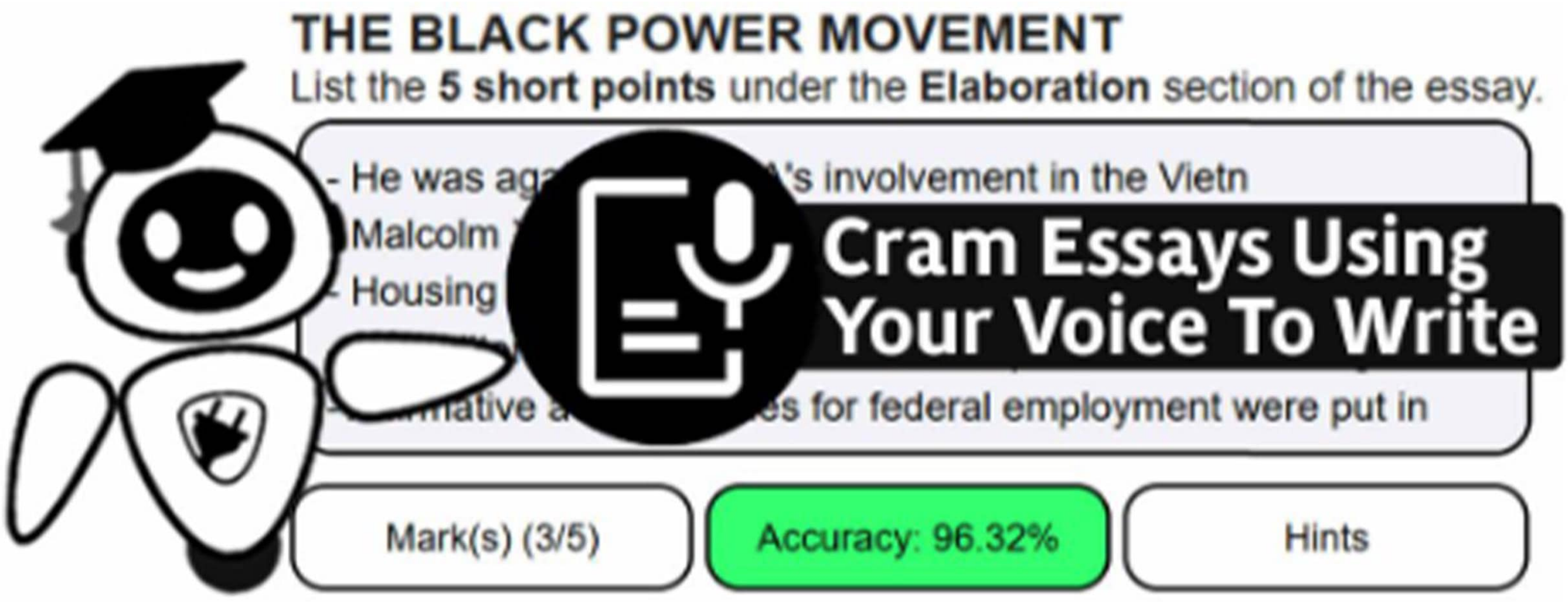

THE BLACK POWER MOVEMENT

List the **5 short points** under the **Elaboration** section of the essay.

- He was ag... 's involvement in the Vietn
- Malcolm
- Housing
- ...mative a... es for federal employment were put in

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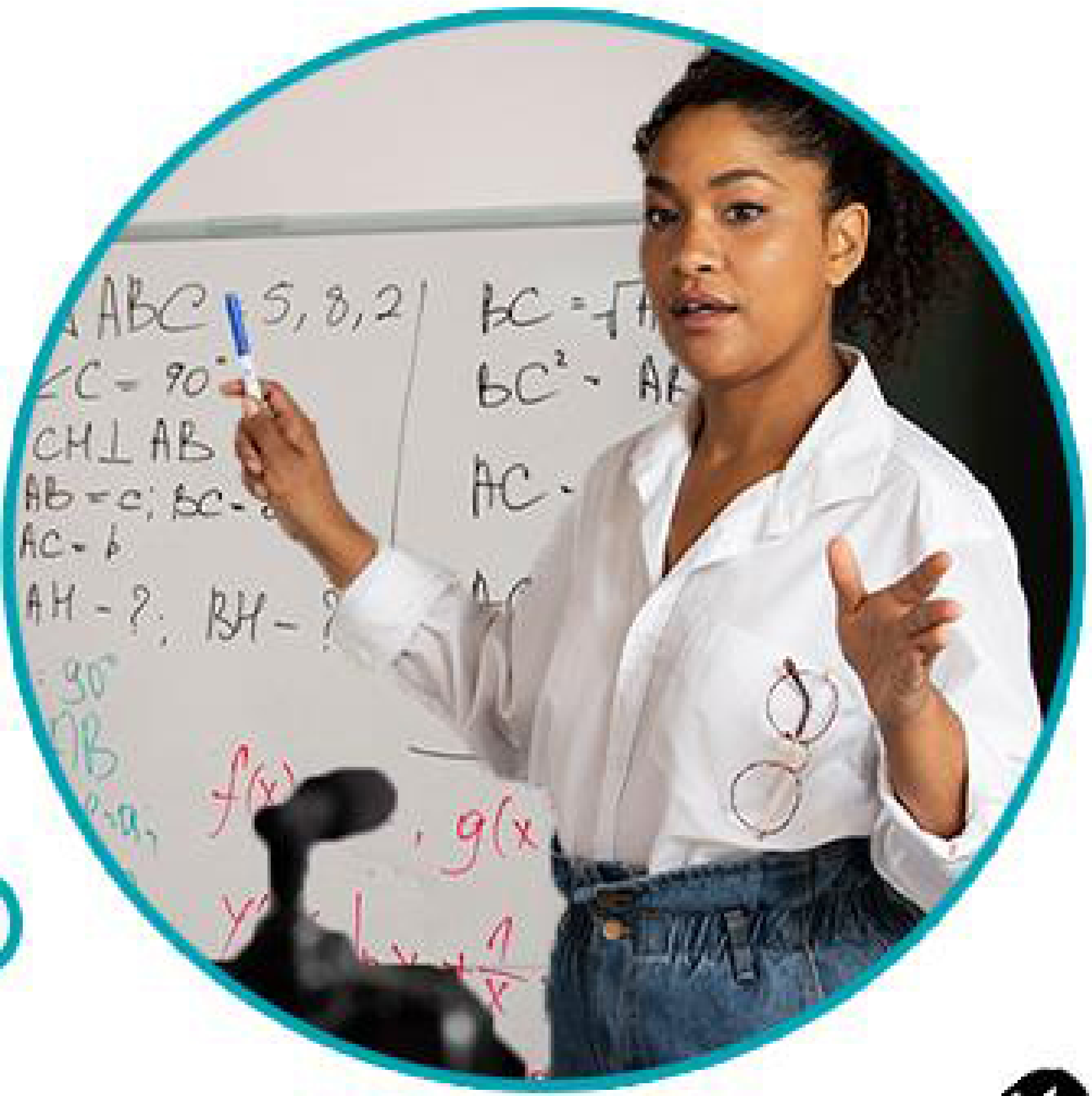

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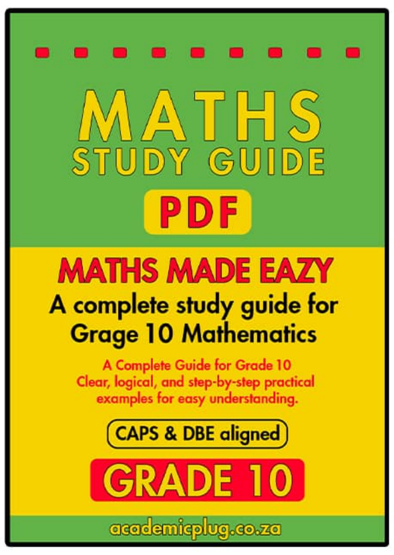
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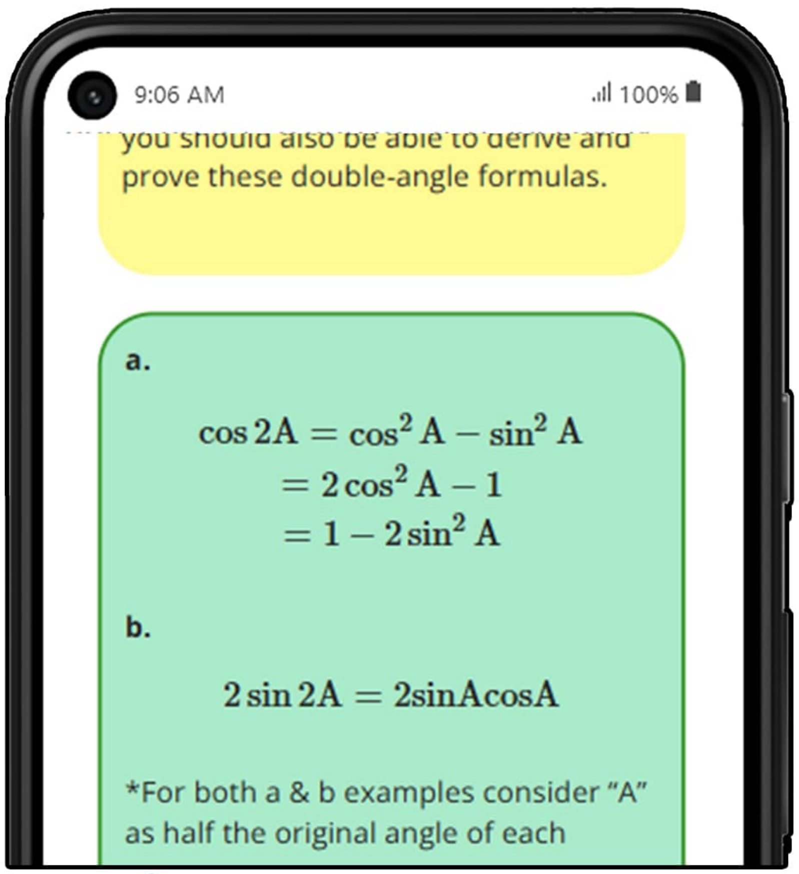

you should also be able to derive and prove these double-angle formulas.

a.

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

b.

$$2\sin 2A = 2\sin A\cos A$$

*For both a & b examples consider "A" as half the original angle of each

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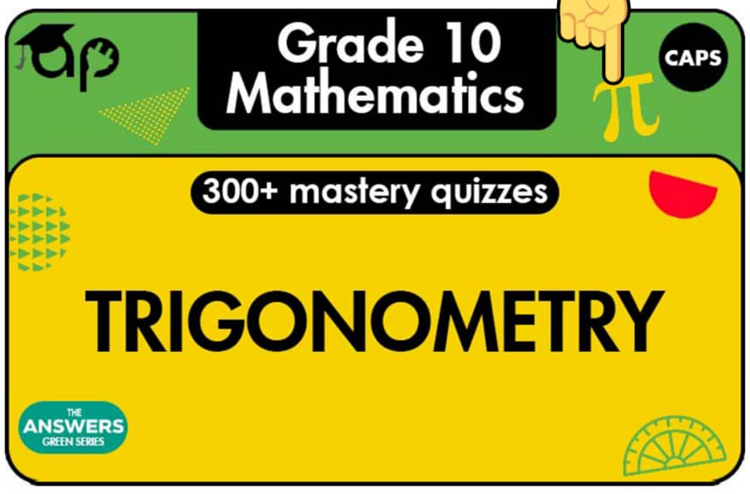
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CHAPTER 5

TRIGONOMETRY

WHAT YOU WILL LEARN IN THIS CHAPTER

PAGE	CONTENT	CAPS	ASSESSMENTS YOU WILL BE ABLE TO:
77	**A. Trigonometric Ratios**	8	• Define and use the functions sinθ, cosθ and tanθ in right-angled triangles. • Derive functions for the special cases: $\theta \in \{0°; 30°; 45°; 60°; 90°\}$ • Define the reciprocals of the above trigonometric functions. • Extend the definition of sinθ, cosθ and tanθ for $0° < \theta < 360°$ • Solve problems in two dimensions by using trigonometric functions and by constructing and interpreting geometric and trigonometric models
77	• Labelling of Triangles		
78	• Determining the Lengths of Sides		
79	• Determining the Size of Angles using a Calculator		
80	• Trigonometric Equations		
81	• Reciprocals of Trigonometric Functions		
81	• Trigonometric Functions Using Pythagoras		
84	• Trigonometric Functions in the Cartesian Plane		
87	**B. Special Angles**		
87	• The 30° - 60° - 90° and 45° - 45° - 90° Triangle		
88	• The Special Angles Fan Diagram		
90	**C. Angles of Elevation and Depression**		

A. TRIGONOMETRIC RATIOS

Trigonometric ratios can be used to determine an unknown length or angle in a right-angled triangle. In Grade 10 you will only have to solve trigonometric problems that deal with **right angled triangles**.

The trigonometric ratios are defined as follows:

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} \rightarrow \text{Oh Hell}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \rightarrow \text{Another Hour}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} \rightarrow \text{Of Algebra}$$

In order to remember the trigonometric ratios use the mnemonic:
Oh Hell Another Hour Of Algebra

LABELLING OF TRIANGLES

The sides of a right-angled triangle are labelled with reference to the angle that you are looking at. The side that is directly **opposite the 90° angle** is always the **hypotenuse.** The side that is **directly next to the** angle that is **not the hypotenuse** is the **adjacent side**. The side **directly opposite the angle** is the **opposite side**. Figures 1 and 2 below show how the labelling of the sides differs depending on the position of the relevant angle within the triangle.

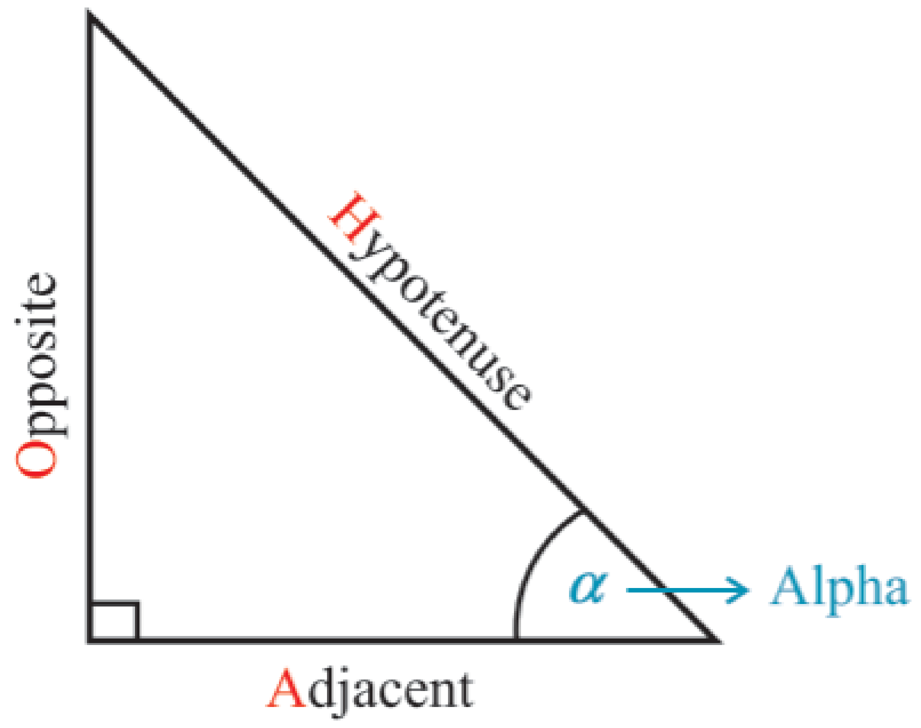


Figure 1

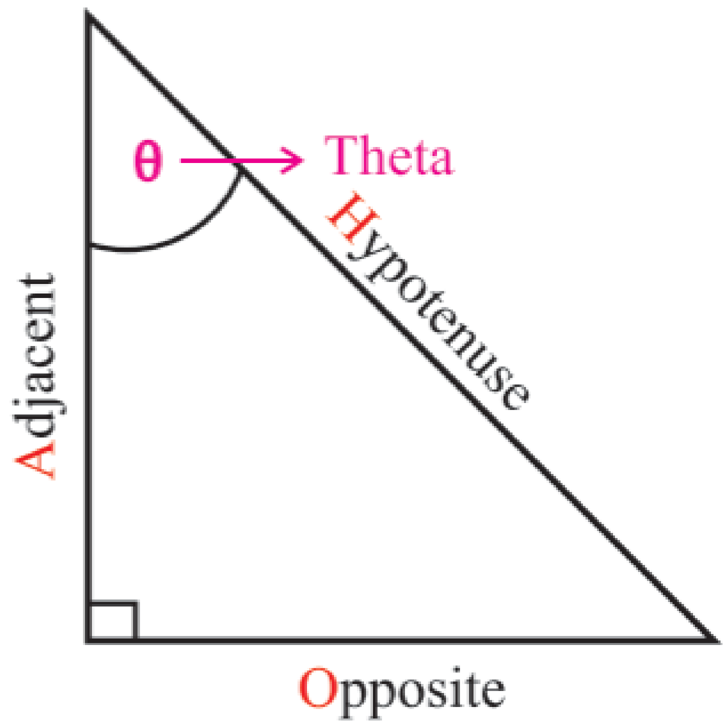
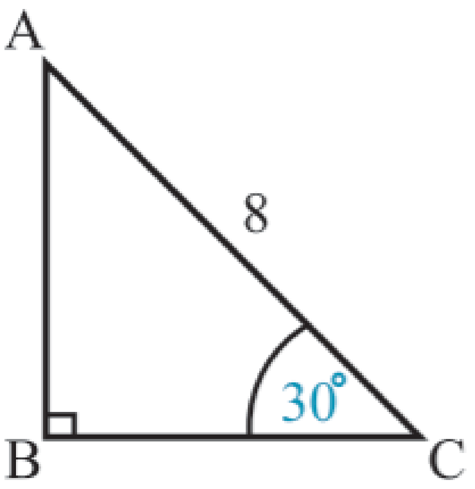


Figure 2

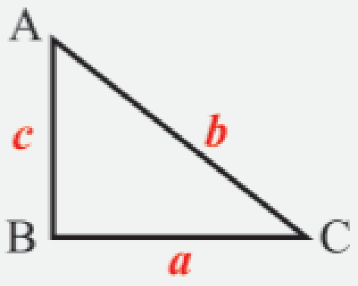
Using trigonometric ratios in a right angled triangle, you can calculate the length of an unknown side or the size of an unknown angle, as long as you have an angle (other than the ninety degree angle) and a side or two sides.

DETERMINING THE LENGTHS OF SIDES

Example 1: **Determine the length of line AB in the figure below.**



Recap: Labelling sides of Triangles:



The letter *b* can be used to label side AC opposite $\hat{B}$
The letter *a* can be used to label side BC opposite $\hat{A}$
The letter *c* can be used to label side AB opposite $\hat{C}$

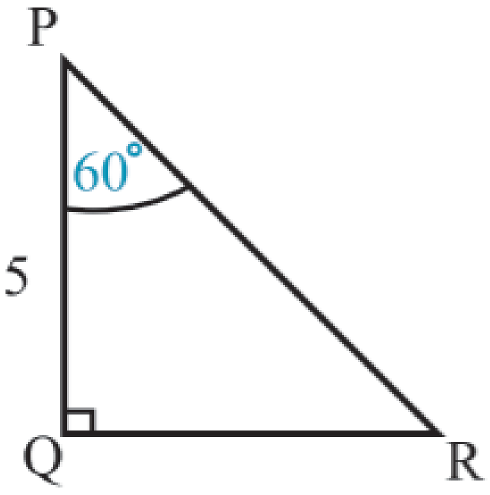
$$\sin 30^\circ = \frac{AB}{AC} \rightarrow \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\therefore \sin 30^\circ = \frac{AB}{8} \rightarrow \text{multiply both sides by } 8$$

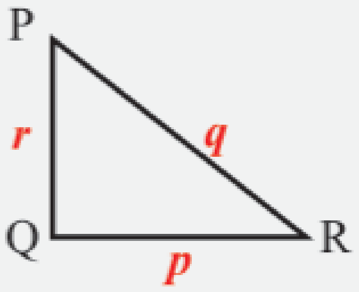
$$\therefore 8\sin 30^\circ = AB$$

$$\therefore AB = 4 \text{ units} \rightarrow \text{always put in units}$$

Example 2: **Determine the length of the hypotenuse in the figure below.**



Recap: Labelling sides of Triangles:



The letter *q* can be used to label side PR opposite $\hat{Q}$
The letter *p* can be used to label side QR opposite $\hat{P}$
The letter *r* can be used to label side QP opposite $\hat{R}$

$$\cos 60^\circ = \frac{PQ}{PR} \rightarrow \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\therefore \cos 60^\circ = \frac{5}{PR} \rightarrow \text{multiply both sides by PR}$$

$$\therefore PR\cos 60^\circ = 5 \rightarrow \text{divide both sides by } \cos 60^\circ$$

$$\therefore PR = \frac{5}{\cos 60^\circ} = 10 \text{ units} \rightarrow \text{always put in units}$$

When calculating the lengths of sides you use the sin, cos and tan buttons on the calculator without pressing the shift or 2ndF buttons.

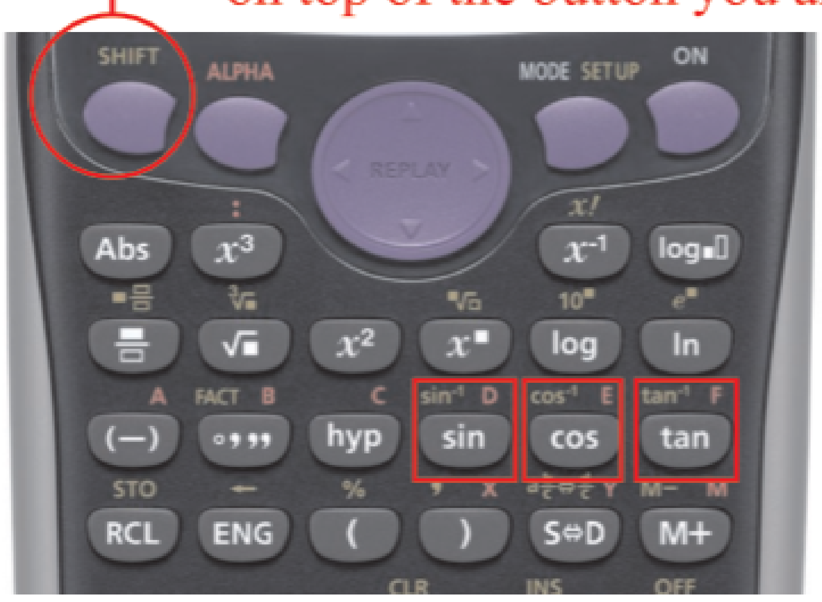
Complete Exercise 1 numbers 1 and 2 on page 287.

DETERMINING THE SIZE OF ANGLES USING A CALCULATOR

In order to determine the size of an angle you have to use the inverse trigonometric function **arcsine**, **arccos** and **arctan**. These functions are accessed on the calculator using the $\boxed{\textbf{shift}}$ button on *Casio* calculators or the $\boxed{\textbf{2ndF}}$ button on *Sharp* calculators.

Example 1: If $\cos\theta = \frac{\sqrt{2}}{2}$ determine the size of θ.

Shift is in yellow. When pressing shift the calculator uses the function in yellow on top of the button you are pressing.


SHIFT
ALPHA
REPLAY
MODE SETUP
ON
Abs
x^3
x^{-1}
$x!$
log
ln
x^2
$10^{\blacksquare}$
$e^{\blacksquare}$
hyp
$\sin^{-1}$ D
sin
$\cos^{-1}$ E
cos
$\tan^{-1}$ F
tan
(–)
FACT
STO
RCL
ENG
(
)
S⇔D
M+
CLR
INS
OFF

$$\cos\theta = \frac{\sqrt{2}}{2}$$

To calculate the angle press the following on your calculator :

$\boxed{\text{shift}}\ \boxed{\cos}\ \boxed{\left(\frac{\boxed{\sqrt{2}}}{\boxed{2}}\right)}\ \boxed{=}$ → pressing $\boxed{\text{shift}}$ $\boxed{\cos}$ uses the $\boxed{\cos^{-1}}$ function which is arccos

$\therefore \theta = 45^{\circ}$

Example 2: If $\sin x = \frac{4}{5}$ determine the size of x.

$$\sin x = \frac{4}{5}$$

to calculate the angle press the following on your calculator:

$\boxed{\text{shift}}\ \boxed{\sin}\ \boxed{\left(\frac{4}{5}\right)}\ \boxed{=}$ → pressing $\boxed{\text{shift}}$ $\boxed{\sin}$ uses the $\boxed{\sin^{-1}}$ function which is arcsin

$\therefore x = 53{,}13^{\circ}$

Complete Exercise 1 numbers 3 and 4 on page 287 and number 5 on page 288.

TRIGONOMETRIC EQUATIONS

When solving a trigonometric equation you are solving for an angle. Always isolate the trigonometric ratio and use the $\boxed{\text{shift}}$ or $\boxed{\text{2ndF}}$ button to solve for the angle.

Example 1: Determine the value of x if $\sin(2x) = \frac{1}{2}$.

$\sin(2x) = \frac{1}{2} \rightarrow \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$, use the shift button when solving for an angle

$\therefore 2x = 30^\circ \rightarrow$ divide both sides by 2

$\therefore x = 15^\circ$

Example 2: Determine the value of x if $4\cos\left(2x + 30^\circ\right) + 1 = 1$.

$4\cos\left(2x + 30^\circ\right) + 1 = 1$

$\therefore 4\cos\left(2x + 30^\circ\right) = 0$

$\therefore \cos\left(2x + 30^\circ\right) = 0 \rightarrow \cos^{-1}(0) = 90^\circ$, use the shift button when solving for an angle

$\therefore 2x + 30^\circ = 90^\circ \rightarrow$ subtract 30° from both sides

$\therefore 2x = 60^\circ \rightarrow$ divide both sides by 2

$\therefore x = 30^\circ$

Example 3: Determine the value of x if $\tan\left(\frac{x}{2}\right) - \frac{1}{\sqrt{3}} = 0$.

$\tan\left(\frac{x}{2}\right) - \frac{1}{\sqrt{3}} = 0$

$\therefore \tan\left(\frac{x}{2}\right) = \frac{1}{\sqrt{3}} \rightarrow \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$, use the shift button when solving for an angle

$\therefore \frac{x}{2} = 30^\circ$

$\therefore x = 60^\circ$

It is important to note that:

$\sin(2\theta) \neq 2\sin(\theta)$

$\cos(2\theta) \neq 2\cos(\theta)$

$\tan(2\theta) \neq 2\tan(\theta)$

Complete Exercise 1 number 6 on page 288.

RECIPROCALS OF TRIGONOMETRIC FUNCTIONS

There are special names which are given to the reciprocals of the trigonometric functions which you have learnt so far. The reciprocals are defined as follows:

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Example: Use a calculator to determine the value of:

a) $\sin 30^\circ \times \operatorname{cosec} 60^\circ$ **b)** $\cos 60^\circ \times \cot 60^\circ$ **c)** $\sec 30^\circ \times \sin 60^\circ \times \operatorname{cosec} 90^\circ$

a)
$$\sin 30^\circ \times \operatorname{cosec} 60^\circ$$
$$= \left(\frac{1}{2}\right)\left(\frac{1}{\sin 60^\circ}\right)$$
$$= \frac{\sqrt{3}}{3} \rightarrow \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

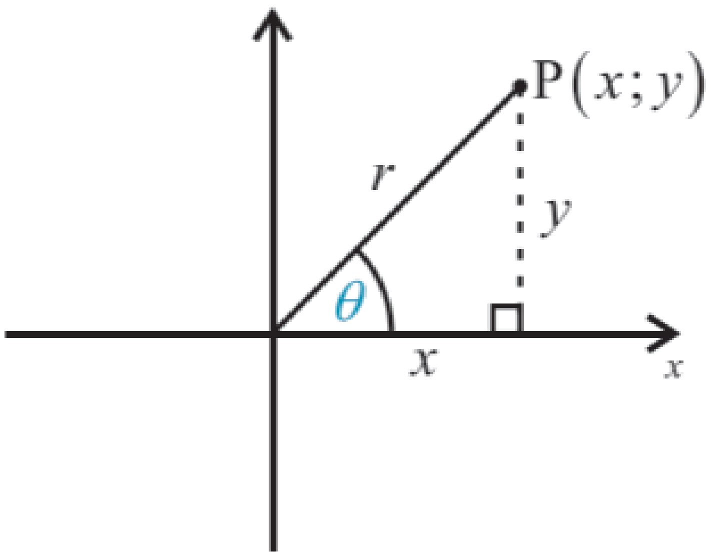
b)
$$\cos 60^\circ \times \cot 60^\circ$$
$$= \left(\frac{1}{2}\right)\left(\frac{1}{\tan 60^\circ}\right)$$
$$= \frac{\sqrt{3}}{6} \rightarrow \frac{\sqrt{3}}{6} = \frac{1}{2\sqrt{3}}$$

c)
$$\sec 30^\circ \times \sin 60^\circ \times \operatorname{cosec} 90^\circ$$
$$= \left(\frac{1}{\cos 30^\circ}\right)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sin 90^\circ}\right)$$
$$= 1$$

Complete Exercise 2 on page 288.

TRIGONOMETRIC FUNCTIONS USING PYTHAGORAS

When dealing with these type of problems it is important to note that for any point $P(x;y)$ in the Cartesian plane, a 90º triangle can be formed by joining the point to the *x*-axis with a vertical line. The distance from the origin to point P is called *r*. This is shown in the diagram below.

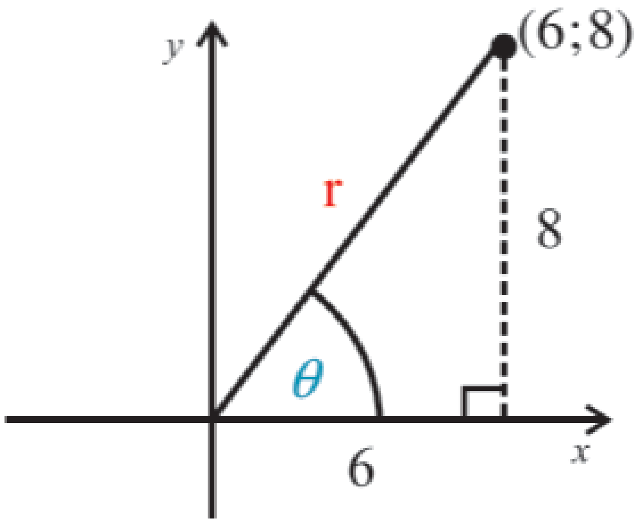


Using the trigonometric ratios which you have learnt so far, it can be seen that:

$\sin\theta = \frac{y}{r} \rightarrow \frac{\text{Opposite}}{\text{Hypotenuse}}$	$\operatorname{cosec}\theta = \frac{r}{y} \rightarrow 1 \div \frac{y}{r} = 1 \times \frac{r}{y} = \frac{r}{y}$
$\cos\theta = \frac{x}{r} \rightarrow \frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\sec\theta = \frac{r}{x} \rightarrow 1 \div \frac{x}{r} = 1 \times \frac{r}{x} = \frac{r}{x}$
$\tan\theta = \frac{y}{x} \rightarrow \frac{\text{Opposite}}{\text{Adjacent}}$	$\cot\theta = \frac{x}{y} \rightarrow 1 \div \frac{y}{x} = 1 \times \frac{x}{y} = \frac{x}{y}$

Always join the point to the *x*-axis to form a triangle, not the *y*-axis.

Example 1: **Using the diagram below and without the use of a calculator determine the value of:**



a) $\sin\theta$

b) $\cos\theta$

c) $\tan\theta$

d) $\dfrac{\sin\theta}{\cos\theta}$

e) $\dfrac{\sin\theta}{\tan\theta}$

f) $\sin^2\theta+\cos^2\theta$

The first step in an example of this type is to use the theorem of Pythagoras (pg. 142) in order to calculate *r*, the value of the unknown side of the triangle.

Using Pythagoras: $r^2 = x^2 + y^2$

$$\therefore r^2 = 6^2 + 8^2$$
$$= 100$$

$\therefore r = \pm 10 \rightarrow$ when square rooting a variable there is always a positive and negative answer however, the radius is a length which is always positive.

$$\therefore r = +10$$

a) $\sin\theta = \dfrac{8}{10} \rightarrow \dfrac{y}{r}$

$= \dfrac{4}{5}$

b) $\cos\theta = \dfrac{6}{10} \rightarrow \dfrac{x}{r}$

$= \dfrac{3}{5}$

c) $\tan\theta = \dfrac{8}{6} \rightarrow \dfrac{y}{x}$

$= \dfrac{4}{3}$

d) $\dfrac{\sin\theta}{\cos\theta} = \dfrac{4}{5} \div \dfrac{3}{5}$

$= \dfrac{4}{5} \times \dfrac{5}{3} \rightarrow$ 'tip and times'

$= \dfrac{4}{3}$

e) $\dfrac{\sin\theta}{\tan\theta} = \dfrac{4}{5} \div \dfrac{8}{6}$

$= \dfrac{4}{5} \times \dfrac{6}{8} \rightarrow$ 'tip and times'

$= \dfrac{3}{5}$

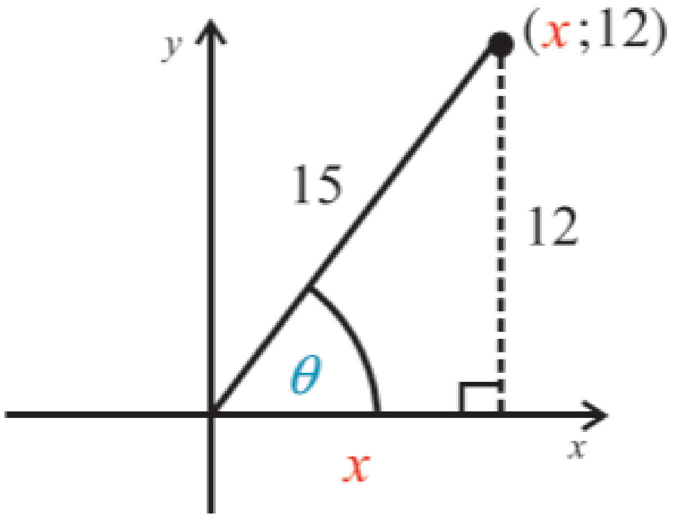
f) $\sin^2\theta + \cos^2\theta = \left(\dfrac{4}{5}\right)^2 + \left(\dfrac{3}{5}\right)^2$

$= \dfrac{16}{25} + \dfrac{9}{25}$

$= \dfrac{25}{25}$

$= 1$

Example 2: **Using the diagram below and without the use of a calculator determine the value of:**



a) **cosecθ**

b) **secθ**

c) **cotθ**

d) **tanθcotθ**

The first step in an example of this type is to use Pythagoras (pg. 142) in order to calculate *x*, the value of the unknown side of the triangle.

Using Pythagoras: $r^2 = x^2 + y^2$

$\therefore 15^2 = x^2 + 12^2$

$\therefore x^2 = 15^2 - 12^2$

$\therefore x^2 = 81$

$\therefore x = \pm 9 \rightarrow$ when square rooting a variable there is always a positive and negative answer

however, x is positive in the first quadrant.

$\therefore x = +9$

a) $\text{cosec}\,\theta = \frac{15}{12} \rightarrow \frac{r}{y}$

$= \frac{5}{4}$

b) $\sec\theta = \frac{15}{9} \rightarrow \frac{r}{x}$

$= \frac{5}{3}$

c) $\cot\theta = \frac{9}{12} \rightarrow \frac{x}{y}$

$= \frac{3}{4}$

d) $\tan\theta\cot\theta = \left(\frac{12}{9}\right)\left(\frac{9}{12}\right)$

$= 1$

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} \rightarrow \frac{y}{r}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \rightarrow \frac{x}{r}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} \rightarrow \frac{y}{x}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Opposite}} \rightarrow \frac{r}{y}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent}} \rightarrow \frac{r}{x}$$

$$\cot\theta = \frac{\text{Adjacent}}{\text{Opposite}} \rightarrow \frac{x}{y}$$

Complete Exercise 3 on pages 288 and 289.

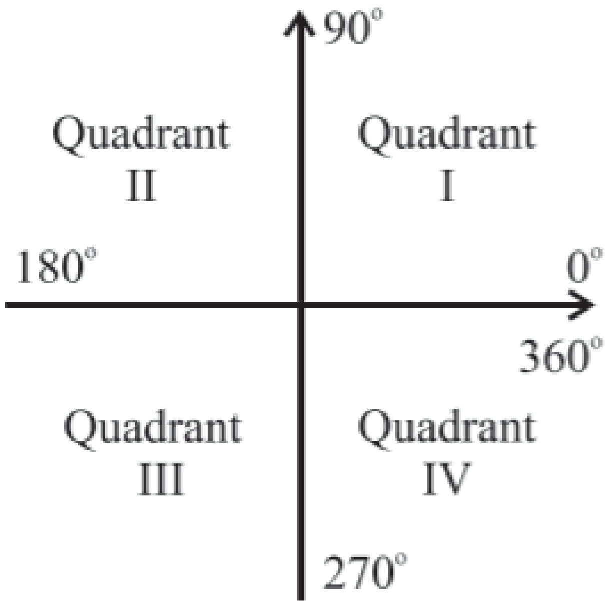
TRIGONOMETRIC FUNCTIONS IN THE CARTESIAN PLANE

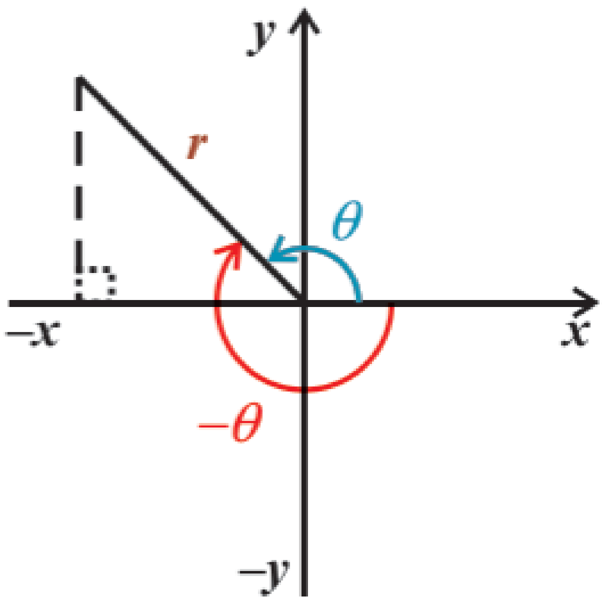
Up until now you have only dealt with angles between 0° and 90° in the 1st quadrant. In this section you will deal with angles from 0° to 360°.

The signs of the functions **sin** θ, **cos** θ and **tan** θ differ depending on the quadrant the angle is in. Note that the functions **sin** θ, **cos** θ and **tan** θ are also defined as:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

The Cartesian planes below show how angles are measured in the Cartesian plane and where the relative quadrants lie. It is important to note that if an angle is measured **anti-clockwise** it is considered to be **positive** and if measured **clockwise** it is considered **negative**.


90°
Quadrant II
Quadrant I
180°
0°
360°
Quadrant III
Quadrant IV
270°

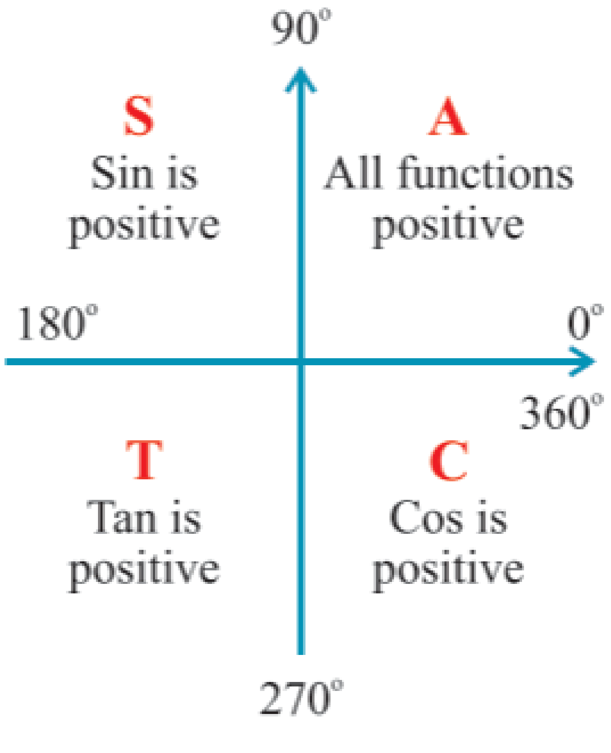


From the above diagrams it can be seen which trigonometric functions are positive or negative in the relevant quadrants.

Quadrant I	**Quadrant II**	**Quadrant III**	**Quadrant IV**
$\sin\theta = \frac{y}{r} = \frac{+}{+} = +$	$\sin\theta = \frac{y}{r} = \frac{+}{+} = +$	$\sin\theta = \frac{y}{r} = \frac{-}{+} = -$	$\sin\theta = \frac{y}{r} = \frac{-}{+} = -$
$\cos\theta = \frac{x}{r} = \frac{+}{+} = +$	$\cos\theta = \frac{x}{r} = \frac{-}{+} = -$	$\cos\theta = \frac{x}{r} = \frac{-}{+} = -$	$\cos\theta = \frac{x}{r} = \frac{+}{+} = +$
$\tan\theta = \frac{y}{x} = \frac{+}{+} = +$	$\tan\theta = \frac{y}{x} = \frac{+}{-} = -$	$\tan\theta = \frac{y}{x} = \frac{-}{-} = +$	$\tan\theta = \frac{y}{x} = \frac{-}{+} = -$

There are two mnemonics that may be used to remember which functions are positive in the different quadrants.

1. Moving **anti-clockwise** from the **first quadrant:** **A**ll **S**tudents **T**ake **C**are
2. Moving **anti-clockwise** from the **fourth quadrant** the word: **CAST**


90°
S
Sin is positive
A
All functions positive
180°
0°
360°
T
Tan is positive
C
Cos is positive
270°

Example 1: **If $\sin\theta = \frac{3}{5}$ and $\theta \in [90^\circ ; 270^\circ]$ determine the value of $\cos\theta \tan^2\theta$.**

With this type of question you have to draw a diagram in order to solve the problem.

Step 1: determine which quadrant you are in and draw a diagram:

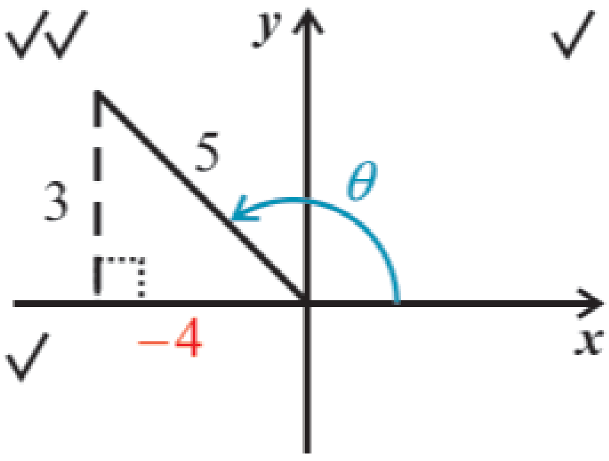
You are given that $\sin\theta = \frac{3}{5}$.

$\frac{3}{5}$ is positive, therefore θ lies in quadrant 1 or 2 as $\sin\theta$ is positive in these quadrants. Therefore place a tick in the 1st and 2nd quadrant. You now have to use the restriction in the question. You are told that θ is between 90° and 270°. This means that θ is in the 2nd or 3rd quadrant. Therefore place a tick in the 2nd and 3rd quadrant. The quadrant with two ticks is where both constraints are satisfied and this is the quadrant that the terminal arm lies in.

$\sin\theta = \frac{3}{5} \rightarrow \frac{y}{r}$ therefore the y-value is 3 and r is 5.

You can now draw the diagram as shown below:

The value of −4 in red is calculated in step 2 and then added to the diagram.



Step 2: Calculate the value of the unknown side using Pythagoras: $r^2 = x^2 + y^2$

$\therefore 5^2 = x^2 + 3^2$

$\therefore x^2 = 16$

$\therefore x = \pm 4$

$\therefore x = -4 \rightarrow$ negative as you are in the second quadrant

Step 3: Now solve the problem using your diagram.

$$\cos\theta \tan^2\theta = \left(\frac{-4}{5}\right)\left(\frac{3}{-4}\right)^2$$
$$= \left(\frac{-4}{5}\right)\left(\frac{9}{16}\right)$$
$$= -\frac{9}{20}$$

A common mistake is to write the above answer as:

$$\cos\left(\frac{-4}{5}\right)\tan^2\left(\frac{3}{-4}\right) = -\frac{9}{20}$$

This is incorrect! The sides of the triangle can never be written as the angle.

The line drawn in the Cartesian plane is known as the terminal arm. In this question the terminal arm has a length of 5 units and lies in the 2nd quadrant. Remember that the length of a line can never be negative. No matter which quadrant the terminal arm lies in, *r* is always positive.

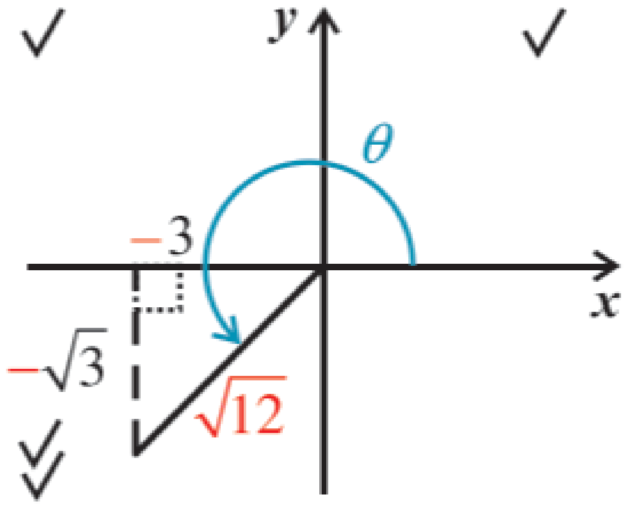
Example 2: If $\tan\theta = \frac{\sqrt{3}}{3}$ and $\cos\theta < 0$ determine the value of:

a) $\cos^2\theta + \sin^2\theta$ b) $\operatorname{cosec}\theta\cos\theta$ c) $\sqrt{3}\cot\theta + \cos^2\theta\sin^2\theta$

Step 1: determine which quadrant you are in and draw a diagram:

Tan θ is positive and cos θ is negative. Therefore the terminal arm lies in quadrant 3. Remember as you are in the 3rd quadrant the values of x and y are negative.

$\tan\theta = \frac{\sqrt{3}}{3} \rightarrow \frac{y}{x}$ $\therefore y$ is $-\sqrt{3}$ and x is -3



Step 2: Calculate the value of the unknown side using Pythagoras:

$$r^2 = x^2 + y^2$$

$$\therefore r^2 = (-3)^2 + \left(-\sqrt{3}\right)^2$$

$$\therefore r^2 = 12$$

$$\therefore r = \sqrt{12} \rightarrow r \text{ is always positive}$$

Step 3: Now solve the problem using your diagram.

a) $$\cos^2\theta + \sin^2\theta = \left(\frac{-3}{\sqrt{12}}\right)^2 + \left(\frac{-\sqrt{3}}{\sqrt{12}}\right)^2$$
$$= \frac{9}{12} + \frac{3}{12}$$
$$= 1$$

b) $$\operatorname{cosec}\theta\cos\theta = \left(\frac{\sqrt{12}}{-\sqrt{3}}\right)\left(\frac{-3}{\sqrt{12}}\right)$$
$$= \frac{3}{\sqrt{3}}$$

Note:
$\operatorname{cosec}\theta = \frac{r}{y}$
$\sec\theta = \frac{r}{x}$
$\cot\theta = \frac{x}{y}$

c) $$\sqrt{3}\cot\theta + \cos^2\theta\sin^2\theta = \sqrt{3}\left(\frac{-3}{-\sqrt{3}}\right) + \left(\frac{-3}{\sqrt{12}}\right)^2\left(\frac{-\sqrt{3}}{\sqrt{12}}\right)^2$$
$$= 3 + \left(\frac{9}{12}\right)\left(\frac{3}{12}\right)$$
$$= \frac{51}{16}$$

Complete Exercise 4 on page 289.

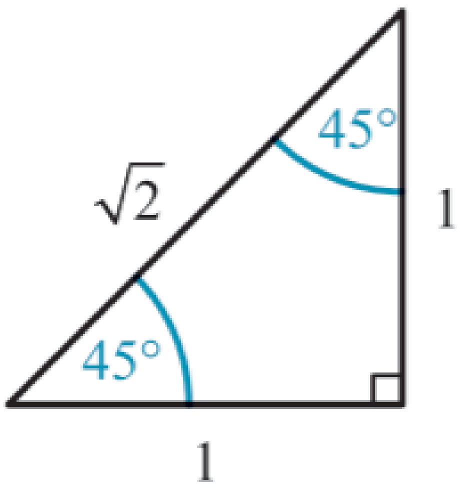
B. SPECIAL ANGLES

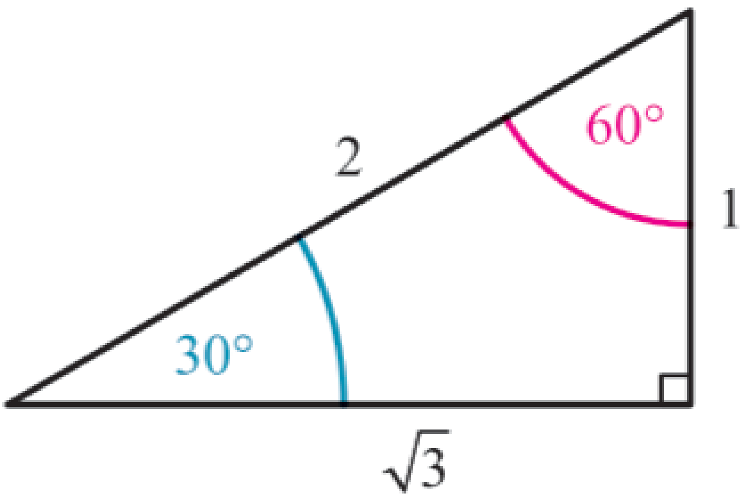
In Trigonometry there are certain special angles and ratios which can be found without using a calculator. These angles are 0º, 30º, 45º, 60º and 90º.

Trigonometric ratios are independent of the lengths of the sides of a triangle and only depend on the angles in the triangle.

THE 30° - 60° - 90° AND 45° - 45° - 90° TRIANGLE

The triangles shown below can be used to calculate the values of the trigonometric ratios of special angles.





It can be seen that for any 30º - 60º - 90º triangle and 45º - 45º - 90º triangle, the trigonometric ratios are as follows:

$$\sin 30^\circ = \frac{1}{2} \qquad \cos 30^\circ = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \qquad \cos 60^\circ = \frac{1}{2} \qquad \tan 60^\circ = \sqrt{3}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \qquad \cos 45^\circ = \frac{1}{\sqrt{2}} \qquad \tan 45^\circ = 1$$

It is useful to learn the following trigonometric ratios for the angles 0º and 90º.

$$\sin 0^\circ = 0 \qquad \cos 0^\circ = 1 \qquad \tan 0^\circ = 0$$

$$\sin 90^\circ = 1 \qquad \cos 90^\circ = 0 \qquad \tan 90^\circ = \text{undefined}$$

These ratios will be clear when you cover trigonometric functions later in the year on page 135.

Rationalising the denominator:

It is important to note that your calculator will not display values with irrational numbers in the denominator.

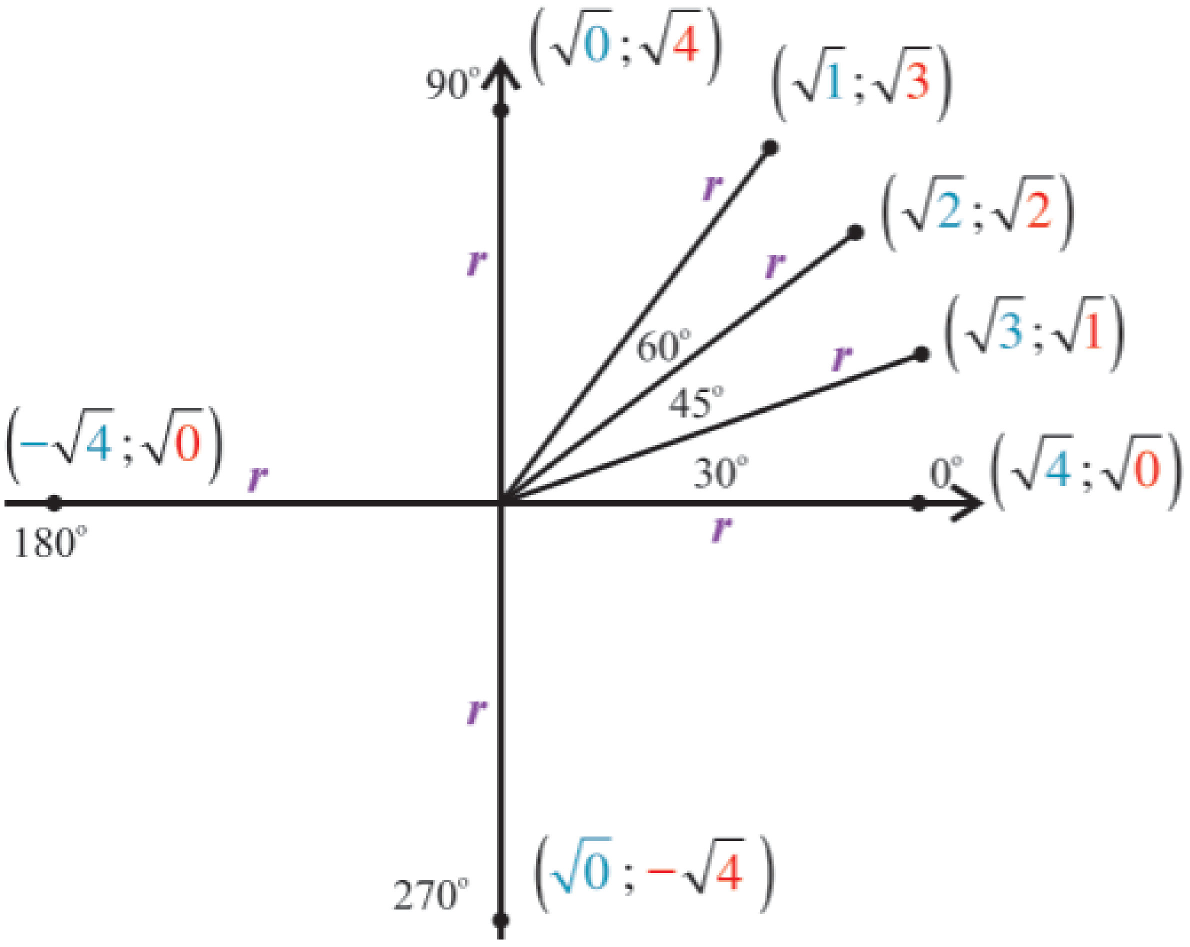
This means that your calculator will display $\frac{1}{\sqrt{2}}$ as $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{3}}$ as $\frac{\sqrt{3}}{3}$.

Theses answers are the same. This is known as rationalising the denominator.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2} \text{ and } \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{\sqrt{3}}{3}$$

THE SPECIAL ANGLES FAN DIAGRAM

The Fan diagram below is useful in remembering special angles. The fan is constructed by drawing a terminal arm ***r*** for each of the special angles 30°, 45° and 60°. The *x*-coordinates are then labelled 0, 1, 2, 3 and 4 starting at 90° moving clockwise to 0°. The *y*-coordinates are then labelled 0, 1, 2, 3 and 4 starting at 0° moving anti-clockwise to 90°. Once all the co-ordinates are in place **square root** each coordinate and use your diagram to solve for all the special angles as shown below.
Each terminal arm has a length ***r*** which is always equal to **2**.



Recap:

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

$\theta = 0°:\ (x;y) = (2;0)$

$\theta = 30°:\ (x;y) = (\sqrt{3};1)$

$\theta = 45°:\ (x;y) = (\sqrt{2};\sqrt{2})$

$\theta = 60°:\ (x;y) = (1;\sqrt{3})$

$\theta = 90°:\ (x;y) = (0;2)$

$\theta = 180°:\ (x;y) = (-2;0)$

$\theta = 270°:\ (x;y) = (0;-2)$

$\theta = 360°:\ (x;y) = (2;0)$

When dealing with the special angles fan diagram *r* is always equal to 2.

Example 1: Without the use of a calculator determine the value of:

a) $\sin 30° \cos 60°$ **b)** $\tan 45 \cos 30°$ **c)** $\sin 45° \cos 45°$

a) $\sin 30° \cos 60°$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{1}{4}$

b) $\tan 45 \cos 30°$

$= (1)\left(\frac{\sqrt{3}}{2}\right)$

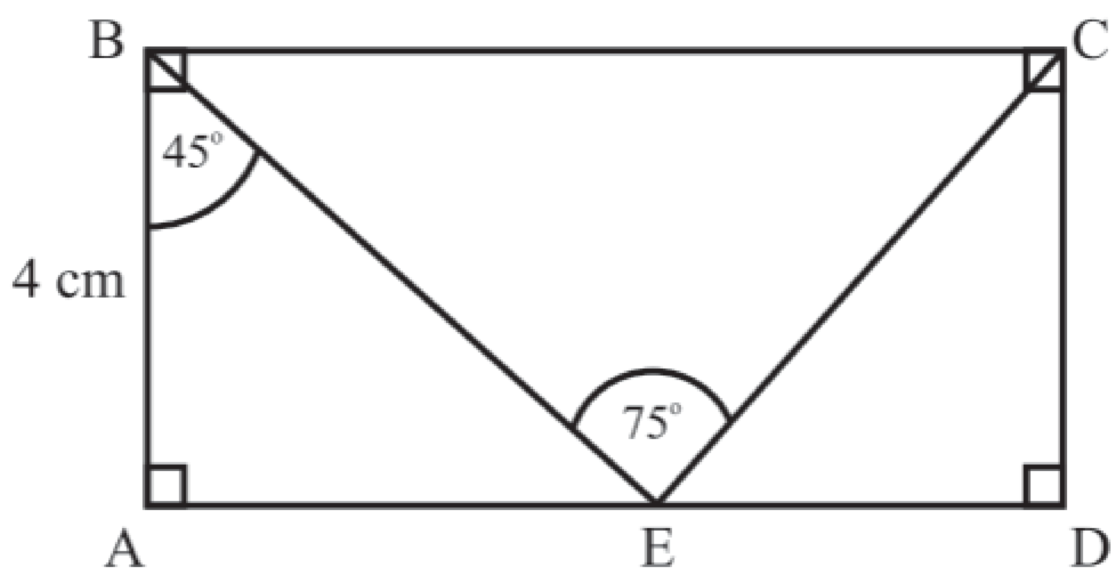
$= \frac{\sqrt{3}}{2}$

c) $\sin 45° \cos 45°$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{1}{2}$

Example 2: **In rectangle ABCD below, $AB = 4\,\text{cm}$, $A\hat{B}E = 45^\circ$ and $B\hat{E}C = 75^\circ$.**



Without the use of a calculator:

a) Determine angles $A\hat{E}B$ and $C\hat{E}D$.

b) Determine the length of AE and ED.

c) Hence, determine the area of the rectangle.

a) $A\hat{E}B = 180^\circ - 90^\circ - 45^\circ$ $\quad$ ($\angle$'s in a triangle are supplementary)

$= 45^\circ$

$C\hat{E}D = 180^\circ - 45^\circ - 75^\circ$ $\quad$ (adj. $\angle$'s on a straight line are supplementary)

$= 60^\circ$

b) $\tan 45^\circ = \dfrac{AE}{BA} \rightarrow \dfrac{\text{Opposite}}{\text{Adjacent}}$

$\therefore \tan 45^\circ = \dfrac{AE}{4}$

$\therefore AE = 4\tan 45^\circ$

$\therefore AE = 4(1) = 4\,\text{cm} \rightarrow$ special angles

$\tan 60^\circ = \dfrac{CD}{ED} \rightarrow \dfrac{\text{Opposite}}{\text{Adjacent}}$

$\therefore \tan 60^\circ = \dfrac{4}{ED} \rightarrow BA = CD$

$\therefore ED = \dfrac{4}{\tan 60^\circ}$

$\therefore ED = \dfrac{4}{\sqrt{3}} = 2{,}31\,\text{cm} \rightarrow$ special angles

c) Area = Length × Breadth

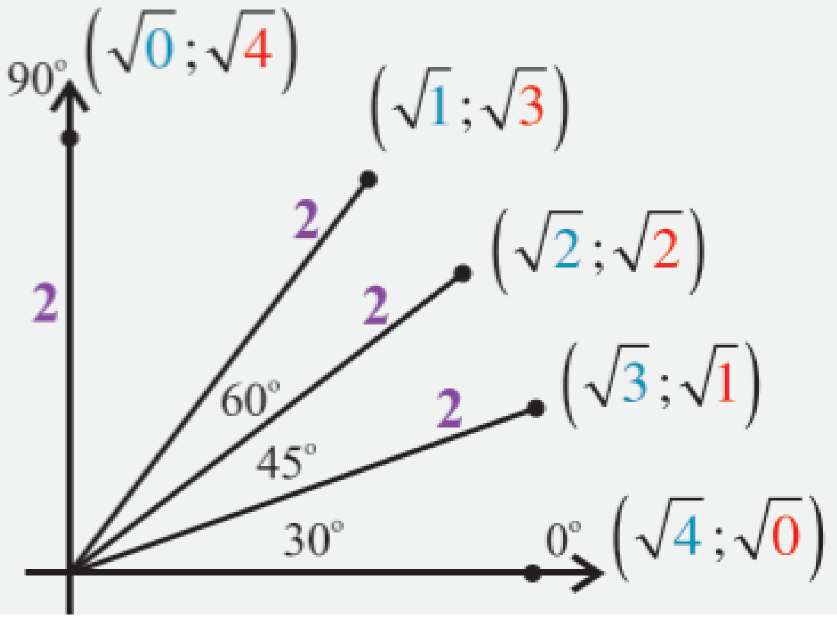
$= AB \times AD$

$= AB \times (AE + ED) \rightarrow AD = AE + ED$

$= 4(4 + 2{,}31)$

$= 25{,}24\,\text{cm}^2 \rightarrow$ area is measured in units^2

Recap:



Complete Exercise 5 on page 289.

C. ANGLES OF ELEVATION AND DEPRESSION

The **angle of elevation** is the angle between the horizontal and the line of sight **up** to an object. This is shown in figure 1 below.

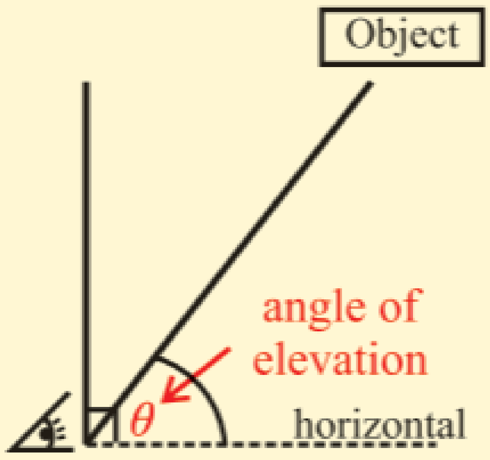

Object
angle of elevation
θ
horizontal

Figure 1: Angle of elevation

The **angle of depression** is the angle between the horizontal and the line of sight **down** to an object. This is shown in figure 2 below.

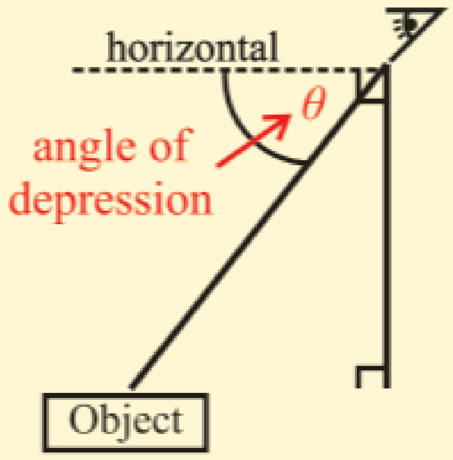
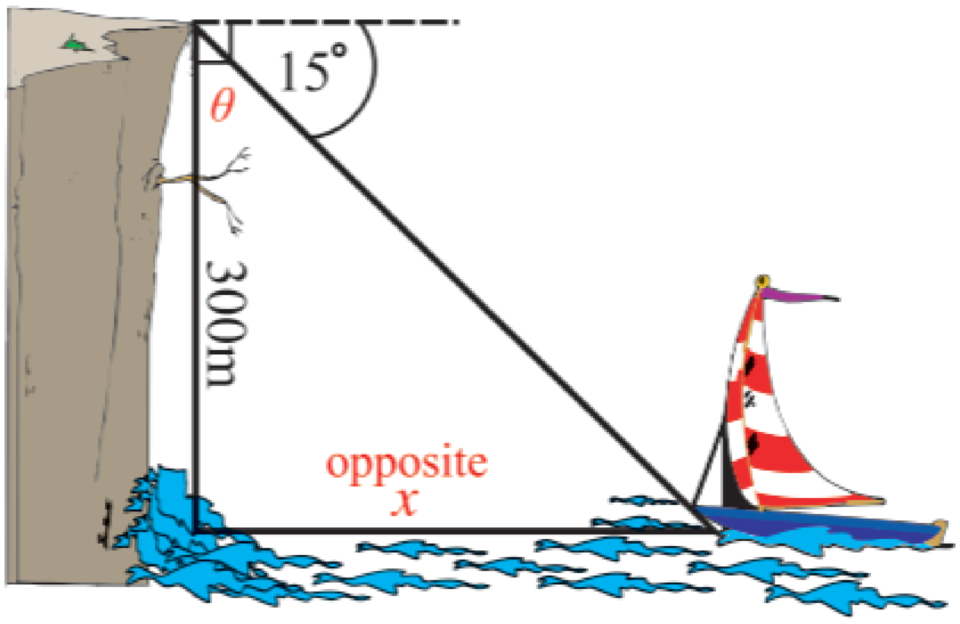

horizontal
θ
angle of depression
Object

Figure 2: Angle of depression

Example 1: A man is watching a boat on the ocean from the edge of a 300m high cliff. He calculates that the angle of depression from the cliff to the boat is 15°. Determine how far the boat is from the base of the cliff to the nearest metre.


15°
θ
300m
opposite
x

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \rightarrow \theta = 90^\circ - 15^\circ = 75^\circ$$

$$\therefore \tan 75^\circ = \frac{x}{300} \rightarrow \frac{\text{opposite}}{\text{adjacent}}$$

$$\therefore 300\tan 75^\circ = x$$

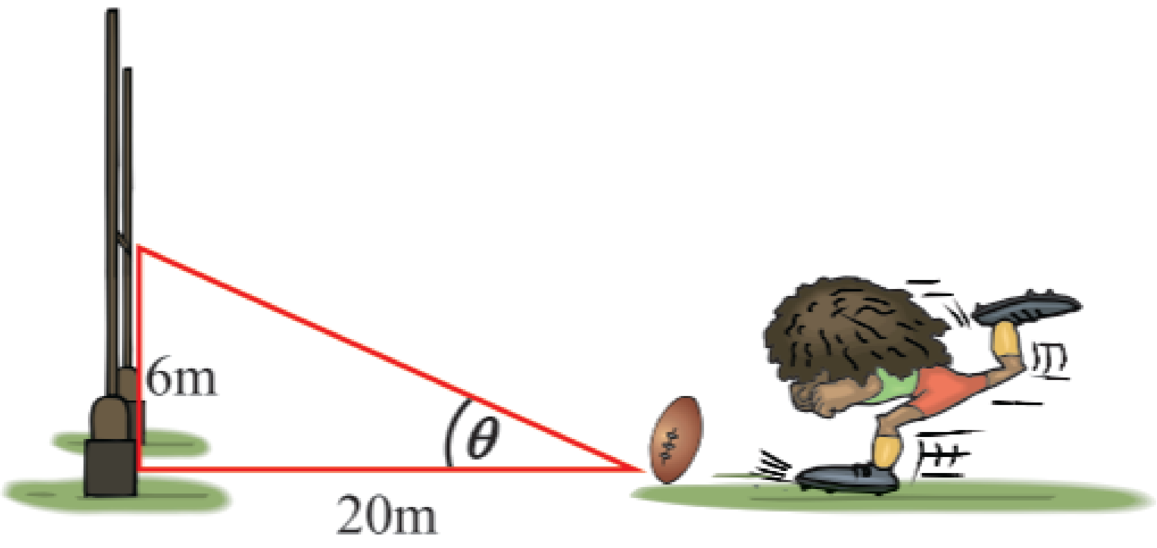
$$\therefore x = 1119{,}61$$

$\therefore$ the boat is 1120 m from the base of the cliff. → round to the nearest metre

Example 2: **In a rugby game Joe has been awarded a penalty kick. The cross beam of the rugby posts is 6m high and Joe is standing 20m from centre of the posts.**

a) Will Joe score if he kicks the ball at an angle of 15°?

b) What is the minimum angle (θ) at which Joe must kick the ball in order to score?



a) In order to solve the problem you have to calculate the height that the ball will reach after 20m if it is kicked at an angle of 15°.

$$\therefore \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\therefore \tan\left(15^\circ\right) = \frac{y}{20} \rightarrow \text{you are trying to find the height that the ball will reach}$$

$$\therefore y = 20\tan\left(15^\circ\right)$$

$$\therefore y = 5{,}36\,\text{m} \rightarrow \text{always put in units}$$

$$\therefore \text{Joe won't score as the ball will not reach a height of } 6\,\text{m.}$$

b) The minimum angle is the angle that would result in the ball being more than 6m high after 20m.

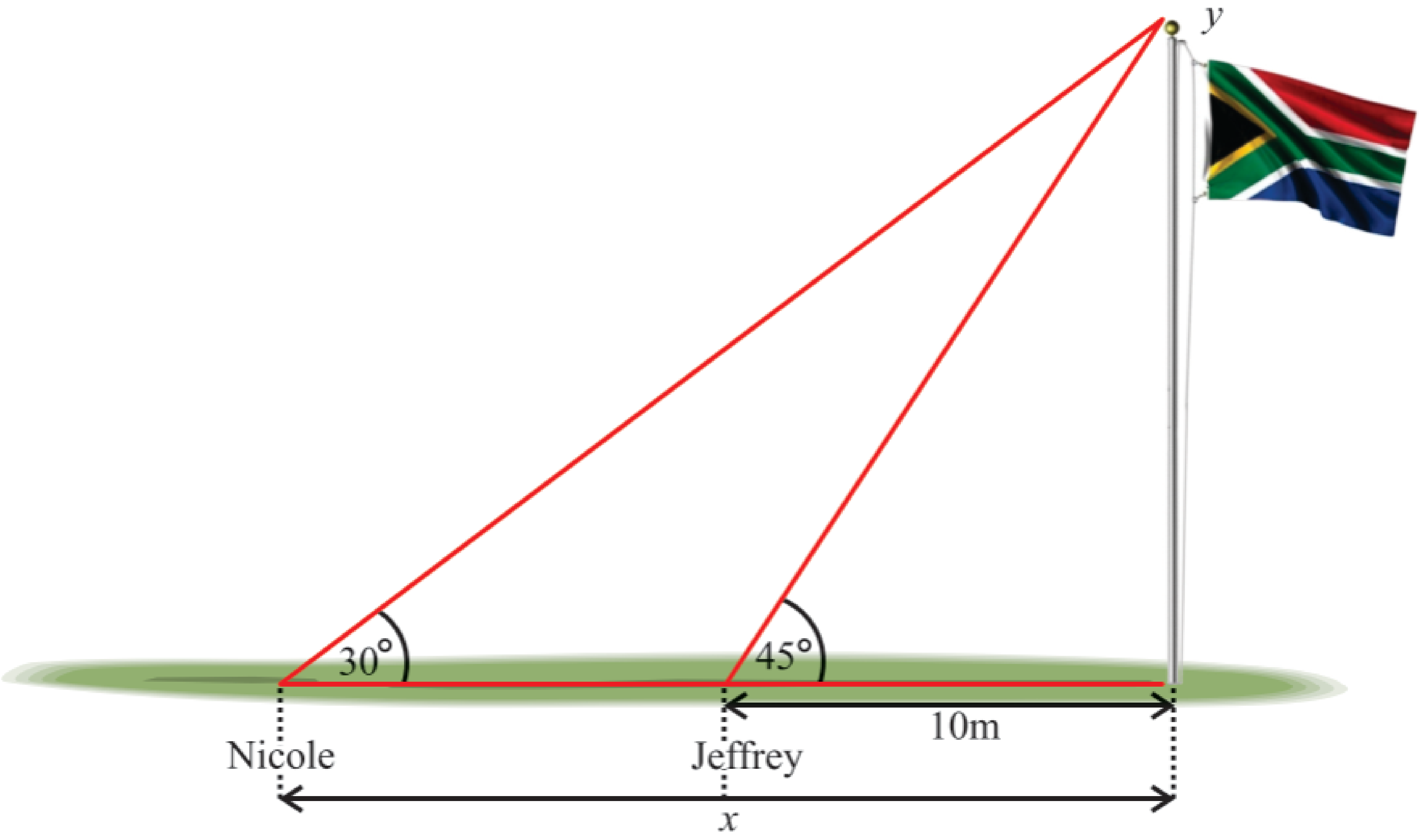
$$\therefore \tan\theta = \frac{6}{20} \rightarrow \frac{\text{opposite}}{\text{adjacent}}$$

$$\therefore \theta = 16{,}7^\circ \rightarrow \arctan\left(\frac{6}{20}\right) = 16{,}7^\circ,\ \text{note that arctan is shift/2ndF tan on the calculator}$$

$$\therefore \theta > 16{,}7^\circ \text{ in order for Joe to score.} \rightarrow \text{if } \theta = 16{,}7^\circ \text{ the ball will hit the cross beam}$$

Example 3: **Nicole and Jeffrey are standing behind each other in a straight line watching a flag blowing in the wind. The angle of elevation of the flag from Jeffrey is 45 degrees and Nicole's angle of elevation to the flag is 30 degrees.**

a) If Jeffrey is standing 10m from the base of the flagpole, calculate the height of the flag pole.
b) How far are Jeffrey and Nicole standing from each other?



a) Using the triangle that Jeffrey makes with the pole: let y be the height of the pole.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$\therefore \tan(45^\circ) = \frac{y}{10} \rightarrow$ y is the height of the pole

$\therefore y = 10\tan(45^\circ)$

$\therefore y = 10\,\text{m} \rightarrow$ always put in units

$\therefore$ The height of the flagpole is 10m

b) Using the triangle that Nicole makes with the pole:

$\therefore \tan(30^\circ) = \frac{y}{x} \rightarrow y$ is the height of the pole, x is the distance Nicole is from the pole

$\therefore \tan(30^\circ) = \frac{10}{x}$

$\therefore x = \frac{10}{\tan(30^\circ)}$

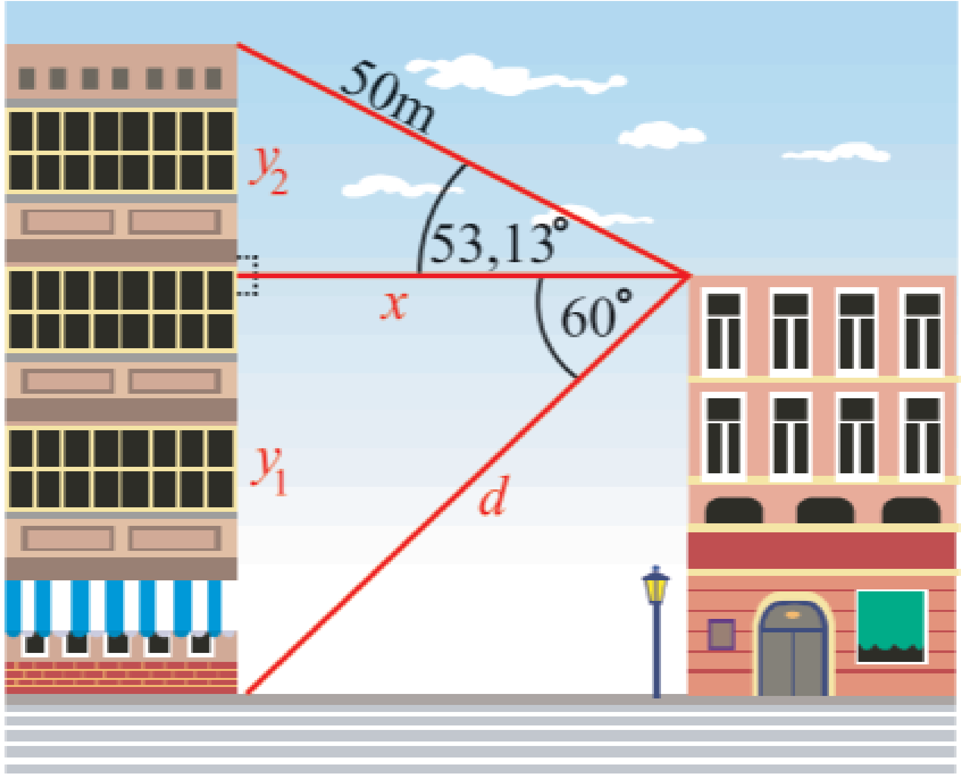
$\therefore x = 17{,}3\,\text{m} \rightarrow$ this is the distance Nicole is from the base of the pole

The question asked for the distance that Jeffrey and Nicole are from each other

$\therefore$ Nicole is standing $7{,}3\,\text{m}$ from Jeffrey $\rightarrow 17{,}3 - 10 = 7{,}3$

Example 4: **A foefie slide is set up between two buildings and the ground as an emergency escape route. The angle of elevation from the top of the shorter building to the taller building is 53,13° and the angle of depression to the ground from the shorter building is 60°. Determine:**

- **a) The distance between the buildings and hence the length of the second slide, if the length of the first slide is 50m.**
- **b) The height of each building.**



a) Using the triangle that the first slide makes: let the distance between the buildings be ***x***

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\therefore \cos\left(53{,}13^\circ\right) = \frac{x}{50} \rightarrow x \text{ is the distance between the buildings}$$

$$\therefore x = 50\cos\left(53{,}13^\circ\right)$$

$$\therefore x = 30\,\text{m} \rightarrow \text{always put in units}$$

Using the triangle that the second slide makes: let ***d*** be the length of the second slide

$$\therefore \cos\left(60^\circ\right) = \frac{x}{d} \rightarrow d \text{ is the length of the second slide (the hypotenuse)}$$

$$\therefore d = \frac{30}{\cos\left(60^\circ\right)} \rightarrow x = 30\text{m, this was calculated in the first part of the question}$$

$$\therefore d = 60\,\text{m} \rightarrow \text{always put in units}$$

$\therefore$ the buildings are 30 m apart and the second slide is 60 m long

b) $\sin\left(60^\circ\right) = \frac{y_1}{d} \rightarrow y_1$ is the height of the shorter building

$$\therefore y_1 = 60\sin\left(60^\circ\right) = 51{,}96\,\text{m} \rightarrow \text{put in units}$$

$$\sin\left(53{,}13^\circ\right) = \frac{y_2}{50}$$

$$\therefore y_2 = 50\sin\left(53{,}13^\circ\right) = 40\,\text{m} \rightarrow \text{put in units}$$

The height of the taller building $= y_1 + y_2 = 40 + 51{,}96 = 91{,}96\,\text{m} \rightarrow$ put in units

$\therefore$ the height of the shorter building is 51,96 m and the taller building is 91,96 m

Complete Exercise 6 on page 290 and
The Mixed Exercise on pages 291 and 292.

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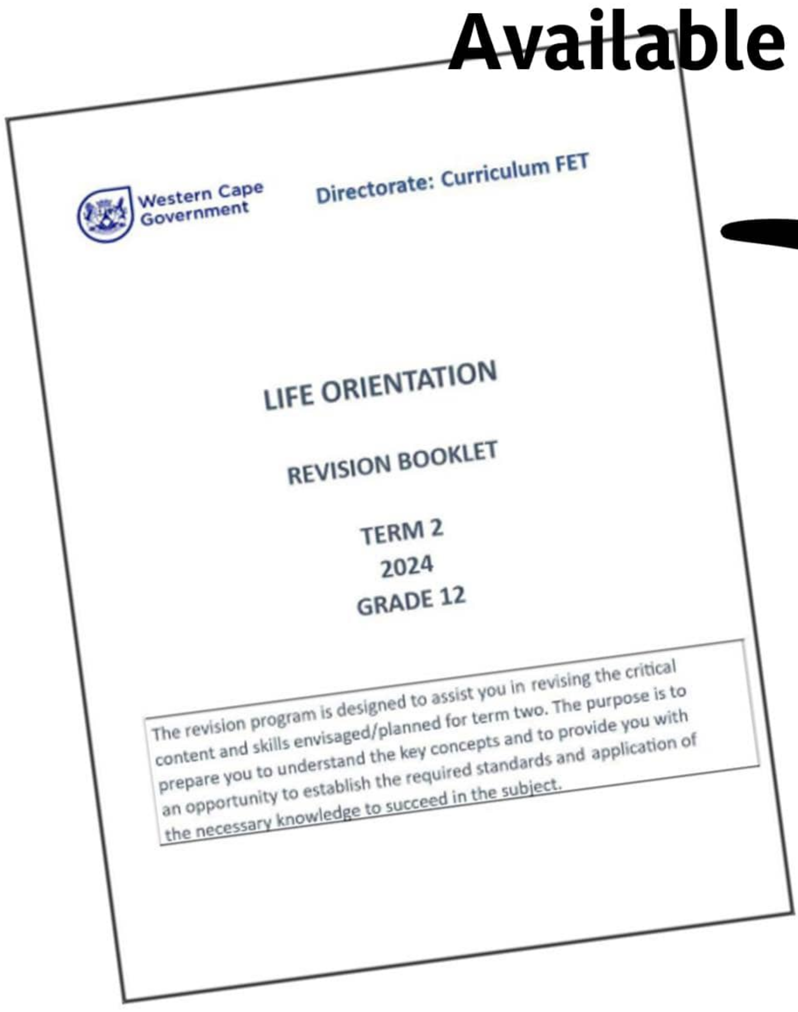

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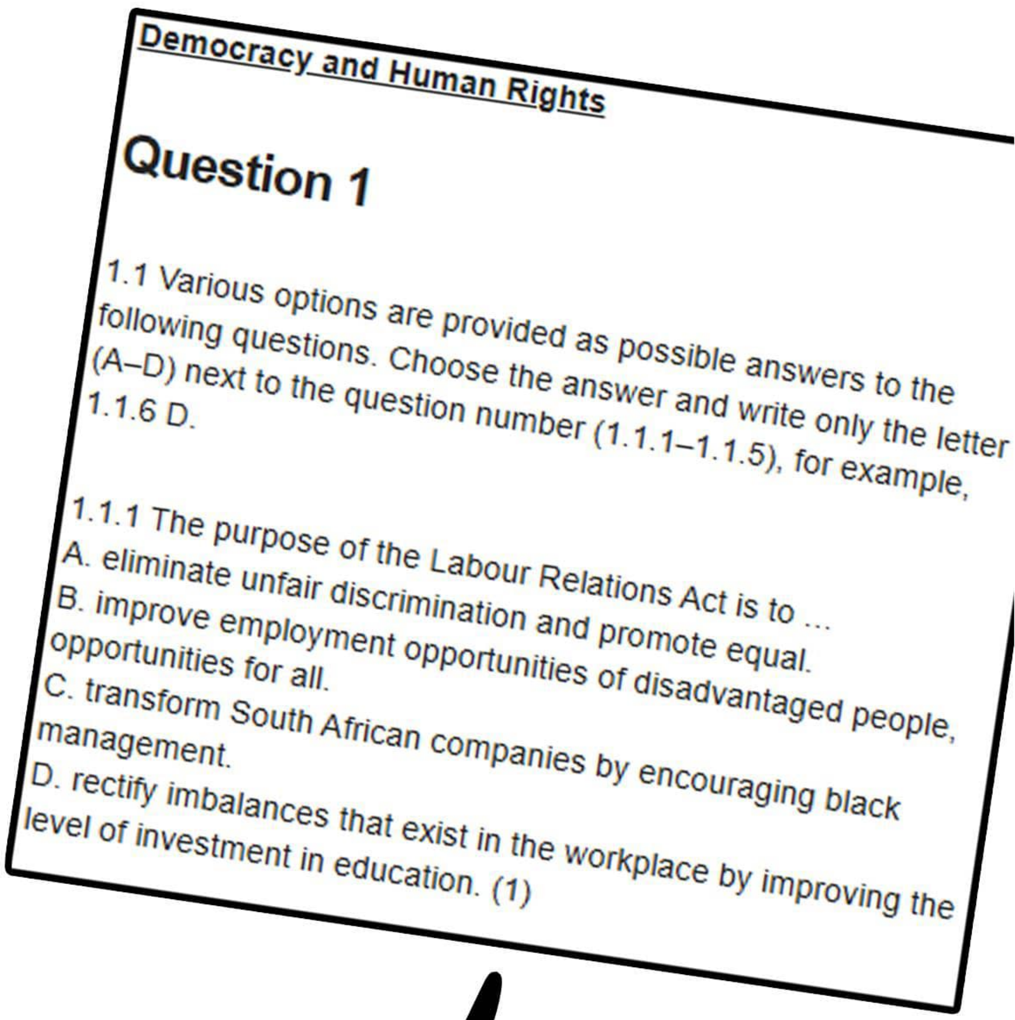
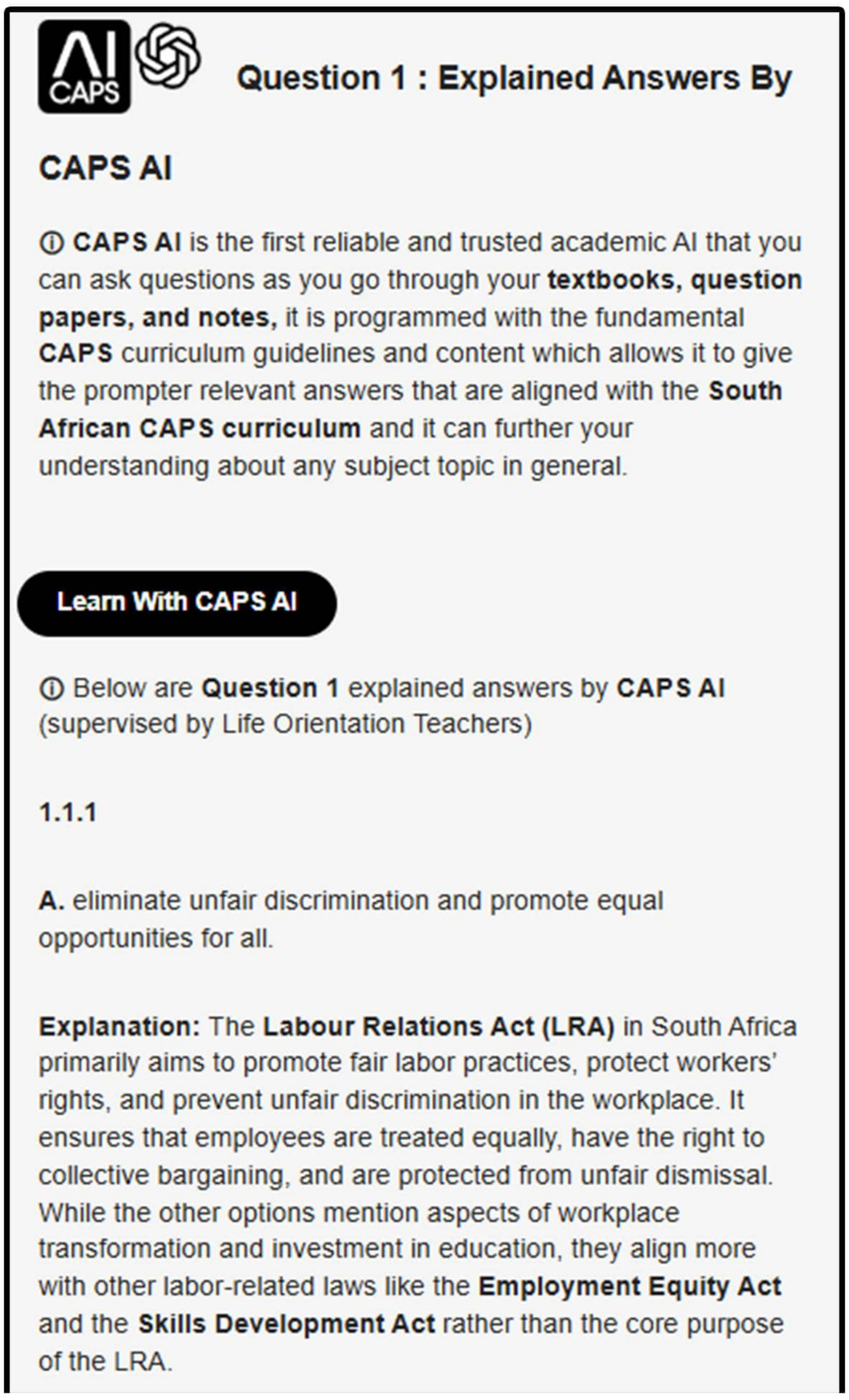


Democracy and Human Rights

Question 1

1.1 Various options are provided as possible answers to the following questions. Choose the answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5), for example, 1.1.6 D.

1.1.1 The purpose of the Labour Relations Act is to ...
A. eliminate unfair discrimination and promote equal opportunities for all.
B. improve employment opportunities of disadvantaged people.
C. transform South African companies by encouraging black management.
D. rectify imbalances that exist in the workplace by improving the level of investment in education. (1)



Question 1 : Explained Answers By CAPS AI

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ⓘ Below are **Question 1** explained answers by **CAPS AI** (supervised by Life Orientation Teachers)

1.1.1

A. eliminate unfair discrimination and promote equal opportunities for all.

Explanation: The **Labour Relations Act (LRA)** in South Africa primarily aims to promote fair labor practices, protect workers' rights, and prevent unfair discrimination in the workplace. It ensures that employees are treated equally, have the right to collective bargaining, and are protected from unfair dismissal. While the other options mention aspects of workplace transformation and investment in education, they align more with other labor-related laws like the **Employment Equity Act** and the **Skills Development Act** rather than the core purpose of the LRA.

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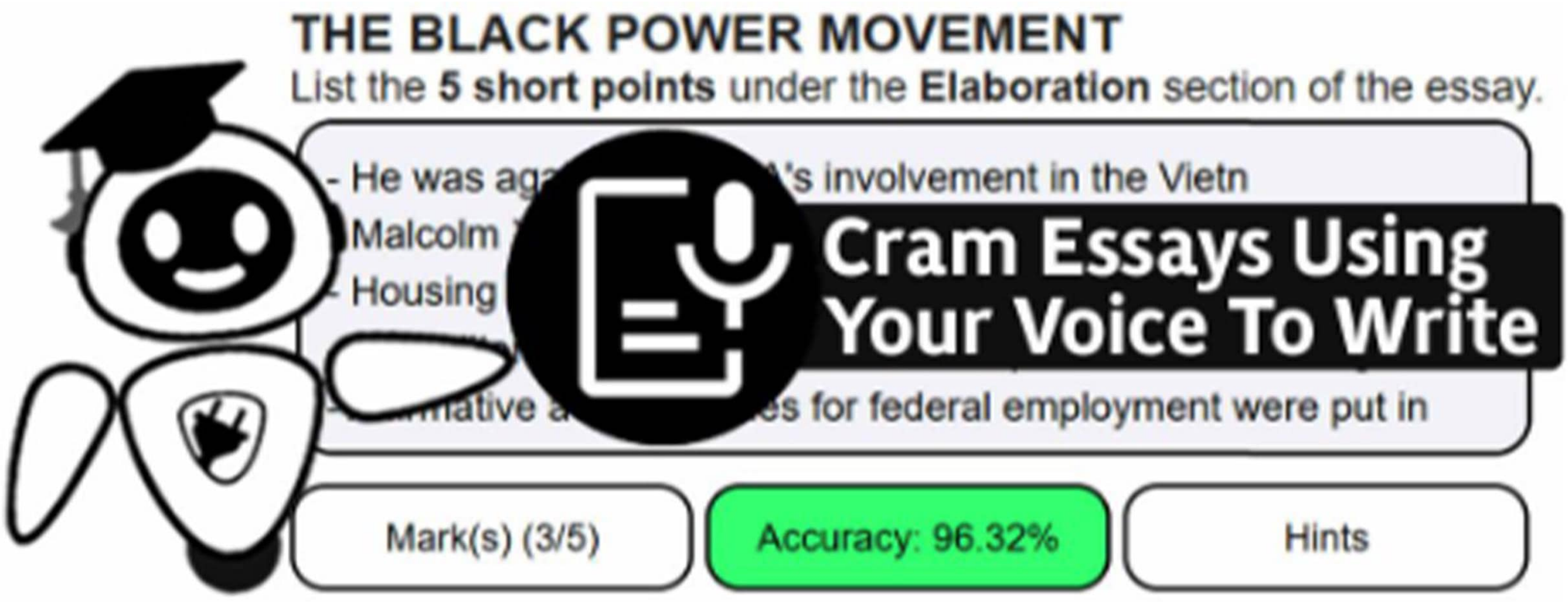

THE BLACK POWER MOVEMENT

List the **5 short points** under the **Elaboration** section of the essay.

- He was ag 's involvement in the Vietn
- Malcolm
- Housing
- mative a es for federal employment were put in

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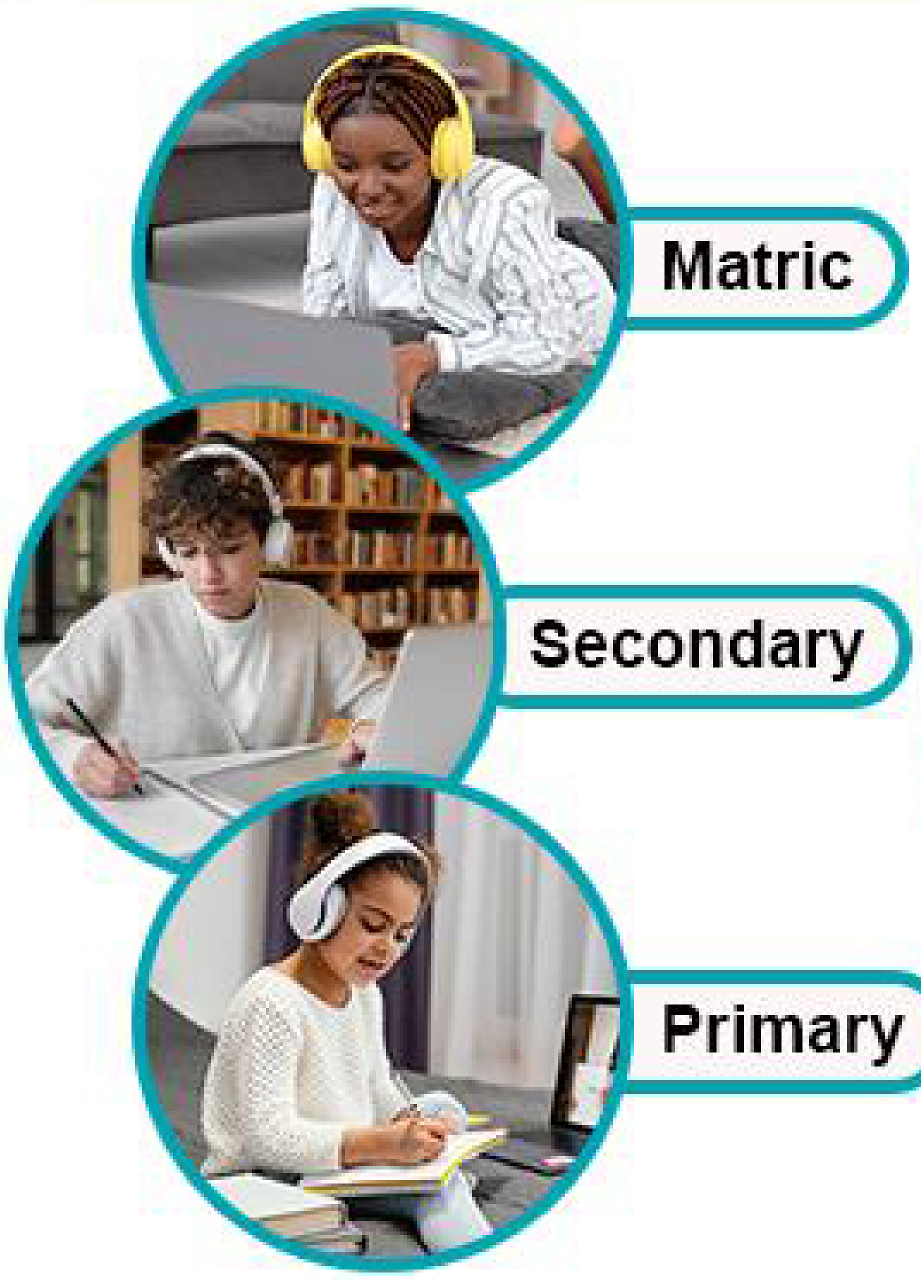

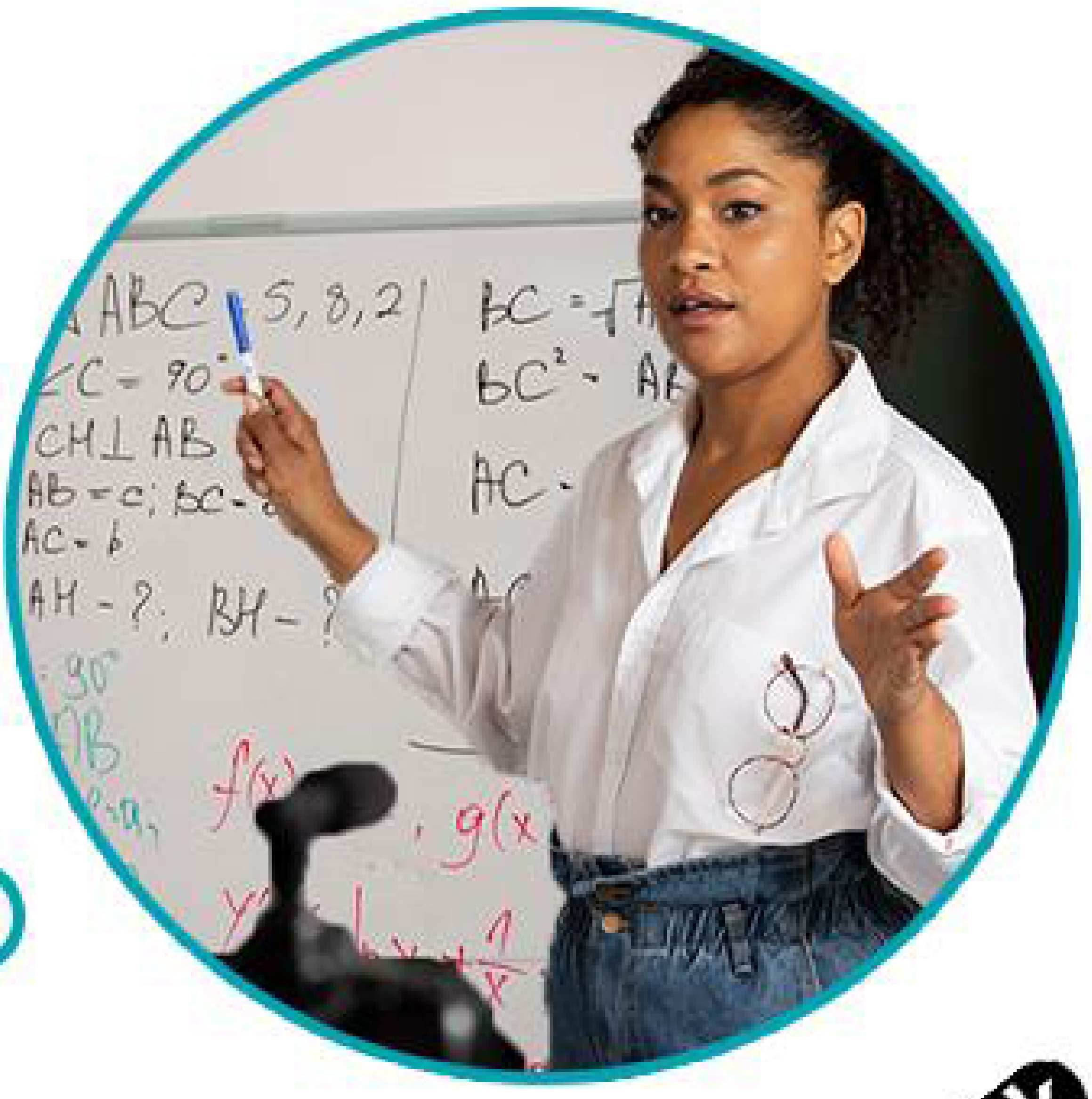

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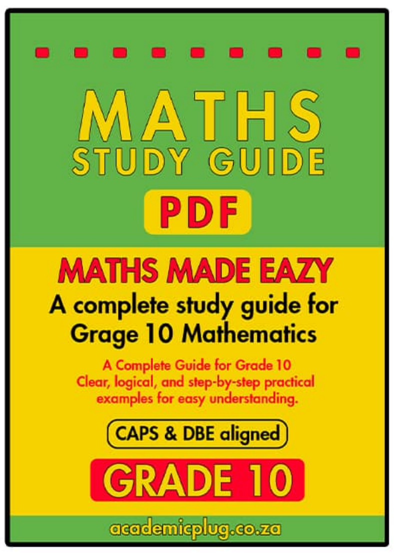
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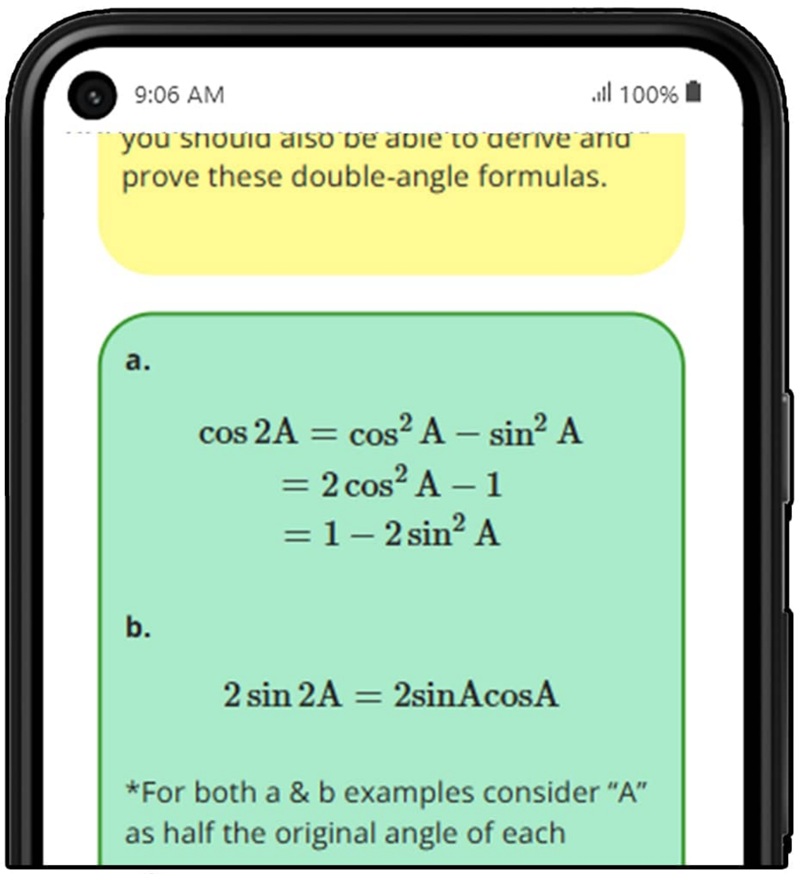

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you should also be able to derive and prove these double-angle formulas.

a.

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

b.

$$2\sin 2A = 2\sin A\cos A$$

*For both a & b examples consider "A" as half the original angle of each

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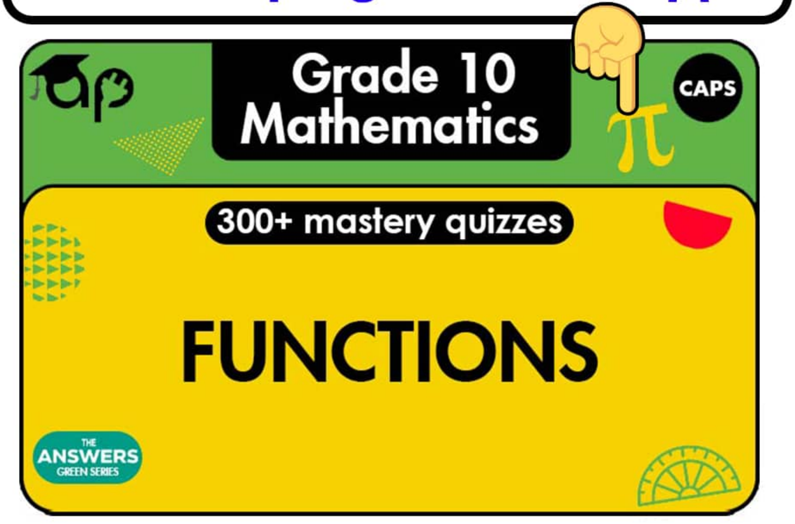
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CHAPTER 6 – FUNCTIONS

WHAT YOU WILL LEARN IN THIS CHAPTER

PAGE	CONTENT	CAPS	ASSESSMENTS YOU WILL BE ABLE TO:
96	**A. Graphs**	2	• Work with relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations. • Generate as many graphs as necessary, initially by point by point plotting, supported by available technology, to make and test conjectures and hence generalise the effect of the parameter which results in a vertical shift and that which results in a vertical stretch and / or reflection about the x-axis. • Problem solving and graph work involving the prescribed functions. • Investigate the average rate of change of a function between two values of the independent variable demonstrating an understanding of average rate of change over different intervals. • Understand the effects of *a* and *q* on the above graphs.
96	• Notation		
97	**B. The Linear Function (Straight Line Graph)**		
97	• Sketching Straight Line Graphs		
100	• Determining the Equation of a Straight Line		
100	• Case 1: Given the *y* - intercept and one other point		
100	• Case 2: Given Any Two Points		
101	**C. The Quadratic Function (The Parabola)**		
101	• Steps to Sketching the Quadratic Function		
104	• Determining the Equation of a Parabola		
104	• Case 1: Given the Turning Point and One other Point		
105	• Case 2: Given any Two Points		
106	**D. The Hyperbolic Function (The Hyperbola)**		
107	• Steps to sketching the Hyperbolic Function		
109	• Determining the Equation of a Hyperbola		
109	• Case 1: Given the Horizontal Asymptote		
110	• Case 2: Given Any Two Points on the Graph		
111	**E. The Exponential Function**		
112	• Steps to Sketching the Exponential Function		
116	• Determining the Equation of an Exponential Function		
116	• Case 1: Given the Horizontal Asymptote		
117	• Case 2: Given Any Two Points on the Graph		

CHAPTER 6 - CONTINUED

WHAT YOU WILL LEARN IN THIS CHAPTER

PAGE	CONTENT	CAPS	ASSESSMENTS YOU WILL BE ABLE TO:
119	**F. Characteristics of Functions**	2	
119	• Domain and Range		
120	• Axes Of Symmetry		
121	• Asymptotes		
121	• Continuity		
121	• Increasing and Decreasing Functions		
122	• The effects of *a* and *q* on Functions		
124	• Reflections and Translations		
125	• Summary of Functions and their Characteristics		
127	**G. Graph Interpretation**		
127	• Lengths of Lines		
129	• Mixed Examples		
133	**H. Mathematical Modelling with Functions**		
135	**I. Trigonometric Functions**		
136	• Change in Amplitude		
137	• Shifting Trigonometric Functions Up and Down		
138	**J. Interpretation of Trigonometric Functions**		

A. GRAPHS

NOTATION

Examine the function: $y = 3x + 4$

The value of y is **dependent** on the value of x.
By substituting different values for x you can obtain a set of **ordered pairs** $(x\,;y)$.
These ordered pairs represent points on the graph of the function.

This function can also be represented using functional and mapping notation.
In grade 10 you can assume that all of the notations mean the same thing.

The table below shows how functional and mapping notations are used to represent the above function.

Functional Notation	$f(x) = 3x + 4$	This reads as ***f*** of ***x*** is equal to $3x + 4$. In this notation $f(x)$ is the same as the y-value. By substituting values of x into the function, a corresponding y-value can be found. This results in a set of **ordered pairs** $(x; f(x))$.
Mapping Notation	$f : x \rightarrow 3x + 4$	This reads as ***f*** maps ***x*** onto $3x + 4$. As in functional Notation, a set of **ordered pairs** can be obtained by substituting values of x into the function in order to find a corresponding y-value.

A function is a set of ordered pairs $(x\,;y)$ in which there are no two ordered pairs that have the same x value.

Example 1: Given the function $y = 2x + 8$, determine the value of y and the ordered pairs if $\{x : -3 \leq x \leq 2; x \in \mathbb{Z}\}$.

Draw a table of x and y-values by substituting the possible values of x into the function.

x	-3	-2	-1	0	1	2
y	**2**	**4**	**6**	**8**	**10**	**12**

$\therefore$ The ordered pairs are: $(-3;2)$ $(-2;4)$ $(-1;6)$ $(0;8)$ $(1;10)$ $(2;12)$

Example 2: Given the function $f(x) = 3x^2 + 2$, determine the values of: $f(-2)$ and $f(p)$.

$f(-2)$ is saying: What is the y value when $x = -2$

$\therefore f(-2) = 3(-2)^2 + 2 \rightarrow$ substitute -2 for all of the x's

$= 3(4) + 2 = 14$

$f(p)$ is saying: What is the y value when $x = p$

$\therefore f(p) = 3(p)^2 + 2 \rightarrow$ substitute p for all of the x's

$= 3p^2 + 2$

Any function $f(x) = \ldots$ can also be written as $y = \ldots$

Complete Exercise 1 on page 300.

B. THE LINEAR FUNCTION (STRAIGHT LINE GRAPH)

SKETCHING STRAIGHT LINE GRAPHS

Straight line graphs have the form: $y = mx + c$ **or** $y = ax + q$

Here m **or** a **is the gradient of the graph and** c or q **is the** y**-intercept.**

If the gradient is positive the graph will have a positive slope →

If the gradient is negative the graph will have a negative slope →

Note that the x and y values are variables that can change, while the **gradient** and the y-intercept are constant values. In order to sketch a straight line graph, the values of m and c have to be known.

There are three ways to sketch a straight line graph:

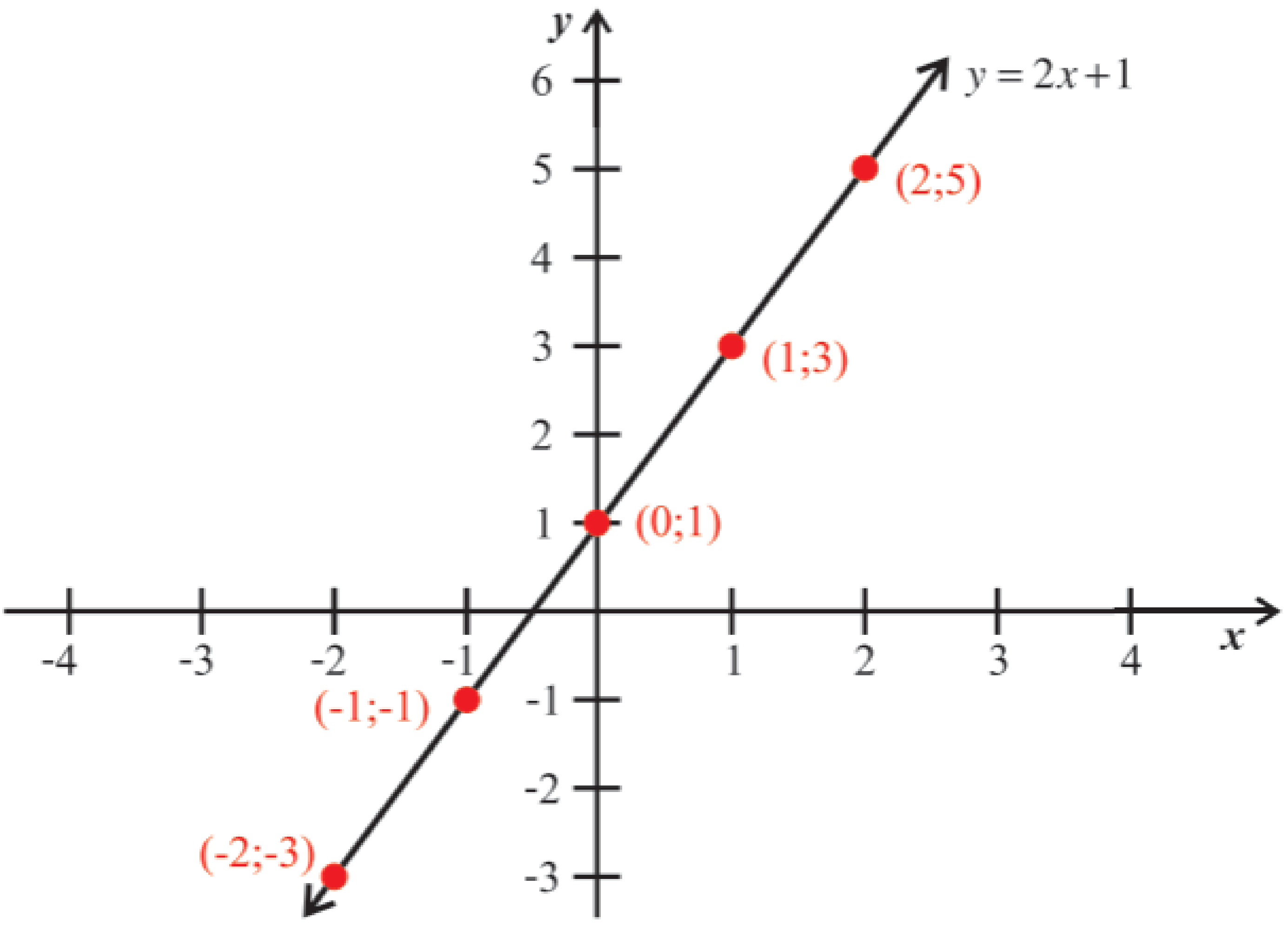
1. **Table method**
2. **Dual intercept method**
3. **Gradient-intercept method ('rise over run method')**

1. The Table Method

In this method you use a table and substitute values for x into the equation in order to calculate what the corresponding y-value would be at that specific point.
You can choose any values to substitute for x. However, two to three numbers on either side of zero are usually sufficient.

Example: Sketch the graph of $y = 2x + 1$ **using the table method.**

x	-2	-1	0	1	2
y	-3	-1	1	3	5



Complete Exercise 2 number 1 on page 300.

2. The Dual Intercept Method

In this method you have to **determine the x and y-intercepts**. The **x-intercept** is the point where the **graph cuts the x-axis** and is found by setting the **y-value equal to zero.**

The **y-intercept** is the point where the **graph cuts the y-axis** and is found by setting the **x-value equal to zero.** You then draw a line through the two points.

Example: Sketch the graph of $y=-3x+6$ using the dual intercept method.

$x-\text{int}:\text{ let } y=0$

$\therefore 0=-3x+6$

$\therefore 3x=6$

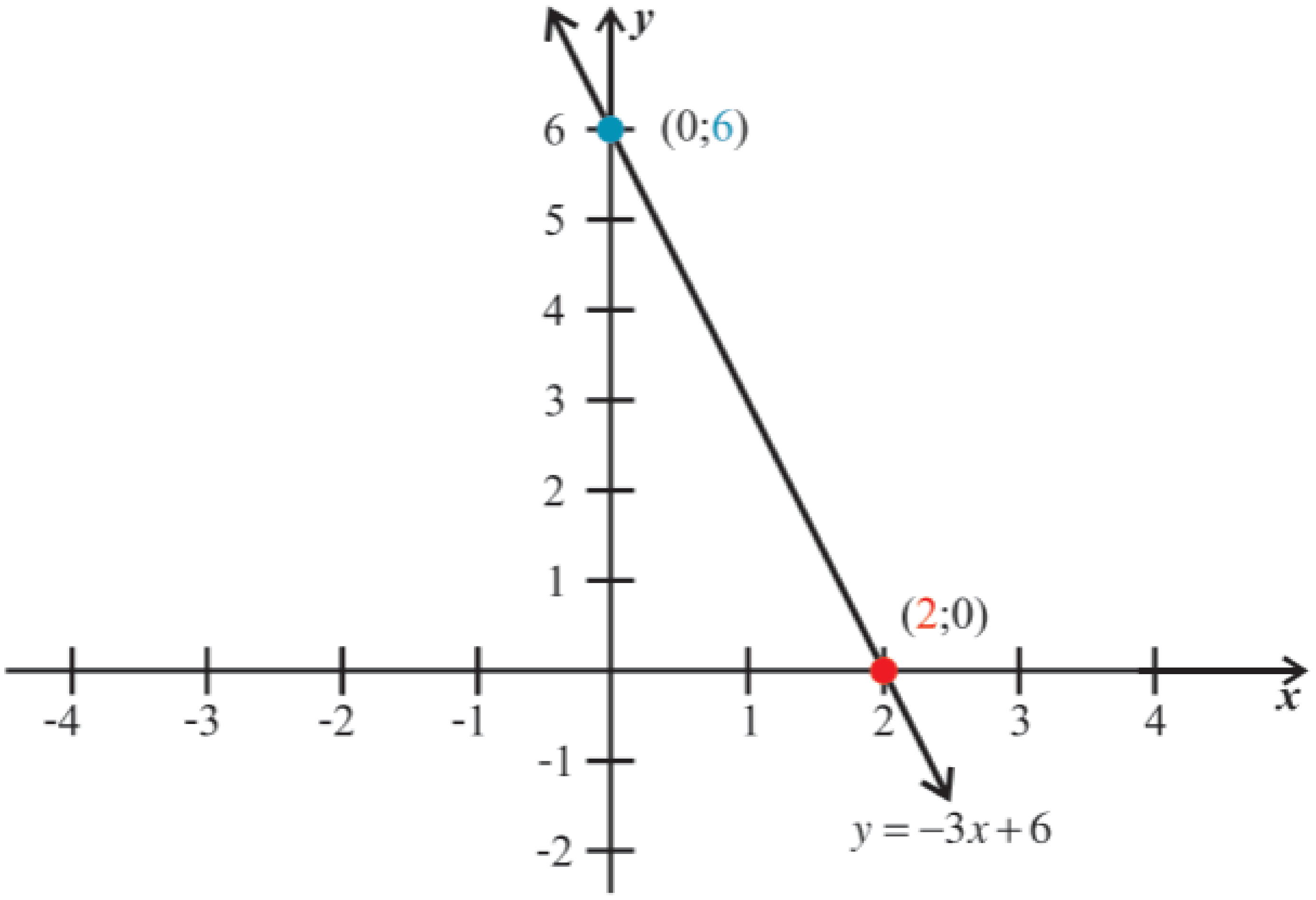
$\therefore x=2$

$y-\text{int}:\text{ let } x=0$

$\therefore y=-3(0)+6$

$\therefore y=6 \rightarrow$ you also know this from the equation

$\therefore$ The x-intercept is at the point $(2;0)$ and the y-intercept is at the point $(0;6)$



The easiest way to plot a linear function is to use the dual intercept method. However, it cannot be used if the gradient or y-intercept is equal to zero.

Complete Exercise 2 number 2 on page 300.

3. The Gradient Intercept Method

In this method you use the gradient to calculate the points that the graph passes through.

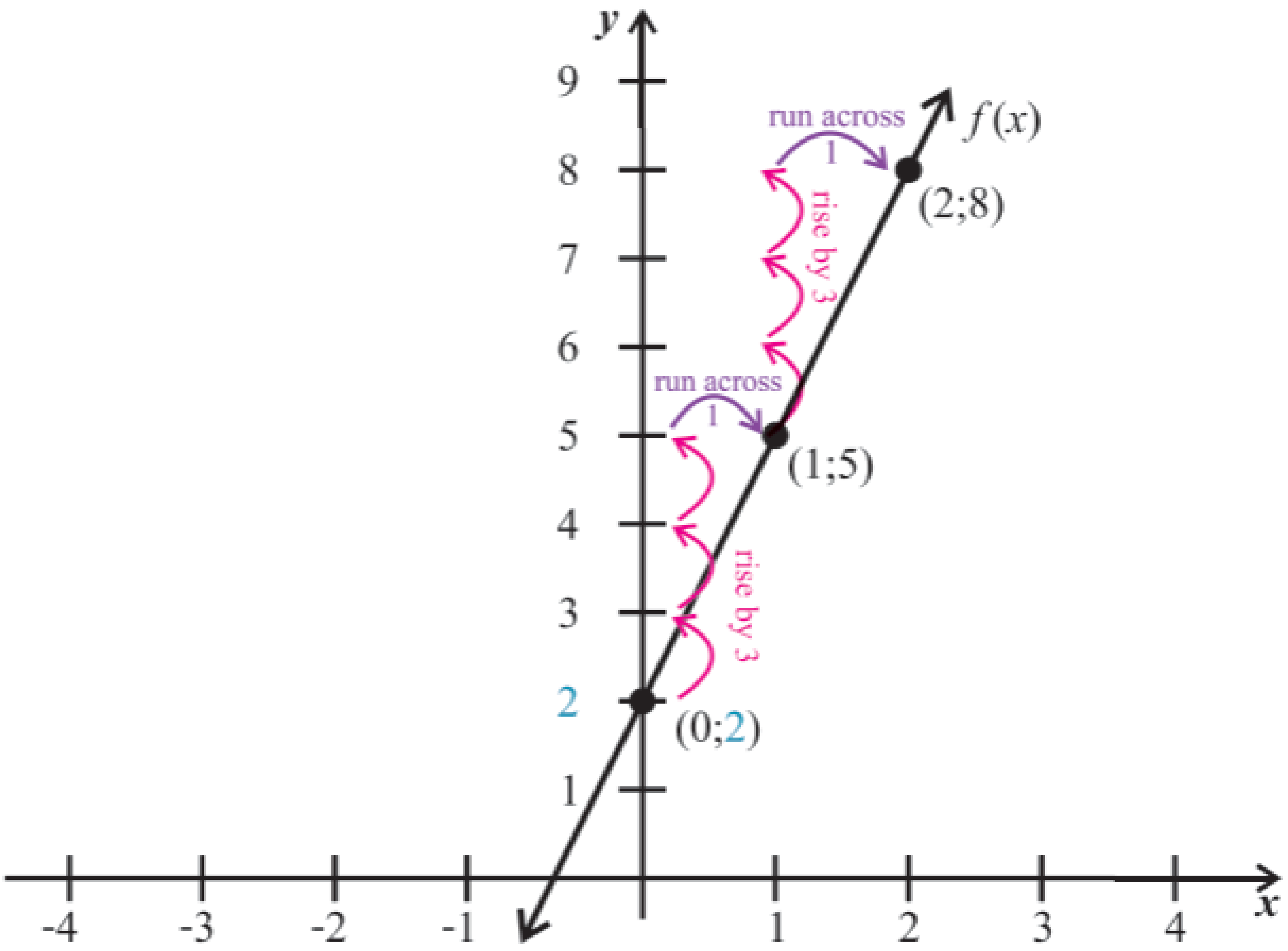
$$\text{Gradient} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: **Sketch the graph of $f(x) = 3x + 2$ using the gradient intercept method.**

Using this method you have to substitute a value in for x in order to obtain a starting point. It is easiest to start with the y-intercept. Therefore the starting point is $(0;2)$

From the equation of the function the gradient equals to 3. Therefore $m = \frac{3}{1} = \frac{\text{rise}}{\text{run}}$.

This tells you that from each point on the graph, the next point can be found by increasing the y-value (rise) by 3 and increasing the x-value (run across) by 1.



Recap:

This method is also known as the 'rise over run' method as it uses the gradient of the straight line. The gradient is the ratio of the change in y-values (rise) against the change in x-values (run).

If the gradient is negative you decrease the y-value (move down) instead of rising.

Complete Exercise 2 numbers 3, 4, 5 and 6 on page 300.

DETERMINING THE EQUATION OF A STRAIGHT LINE

There are two ways in which you will be expected to determine the equation of a linear function:

1. **You will be given the y-intercept and one other point on the line.**
2. **You will be given the coordinates of two random points on the line.**

CASE 1: GIVEN THE y - INTERCEPT AND ONE OTHER POINT

In this case, immediately substitute the y-intercept into c in the equation: $y = mx + c$

Then determine the gradient of the line using the equation: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Example: Determine the equation of the straight line passing through the points (0;9) and (3;6)

The point $(0;9)$ is the $y-$intercept

$\therefore y = mx + 9$

The gradient of the line is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-6}{0-3} = -1$$

$\therefore$ The equation of the line is: $y = -x + 9$

CASE 2: GIVEN ANY TWO POINTS

In this case, find the gradient of the line using the equation: $m = \frac{y_2 - y_1}{x_2 - x_1}$

You then substitute one of the given points into the equation $y = mx + c$ in order to solve for c.

Example: Determine the equation of the straight line passing through the points (4;4) and (6;8)

The gradient of the line is: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{6-4} = 2$

$\therefore y = 2x + c$

Now substitute either $(4;4)$ or $(6;8)$ into the equation:

Substituting $(4;4)$

$\therefore 4 = 2(4) + c$

$\therefore 4 = 8 + c$

$\therefore c = -4$

$\therefore y = 2x - 4$

or

Substituting $(6;8)$

$\therefore 8 = 2(6) + c$

$\therefore 8 = 12 + c$

$\therefore c = -4$

$\therefore y = 2x - 4$

You only have to substitute one point in order to solve for c.

Complete Exercise 3 on pages 300 and 301.

C. THE QUADRATIC FUNCTION (THE PARABOLA)

A parabola is a graph that will always take the shape of a sad or happy face. In Grade 10 all parabolas will have a turning point on the y-axis.

The general form of a parabola is:

$$y = ax^2 + c \qquad \text{or} \qquad y = ax^2 + q$$

Where: $a \rightarrow$ **shape**

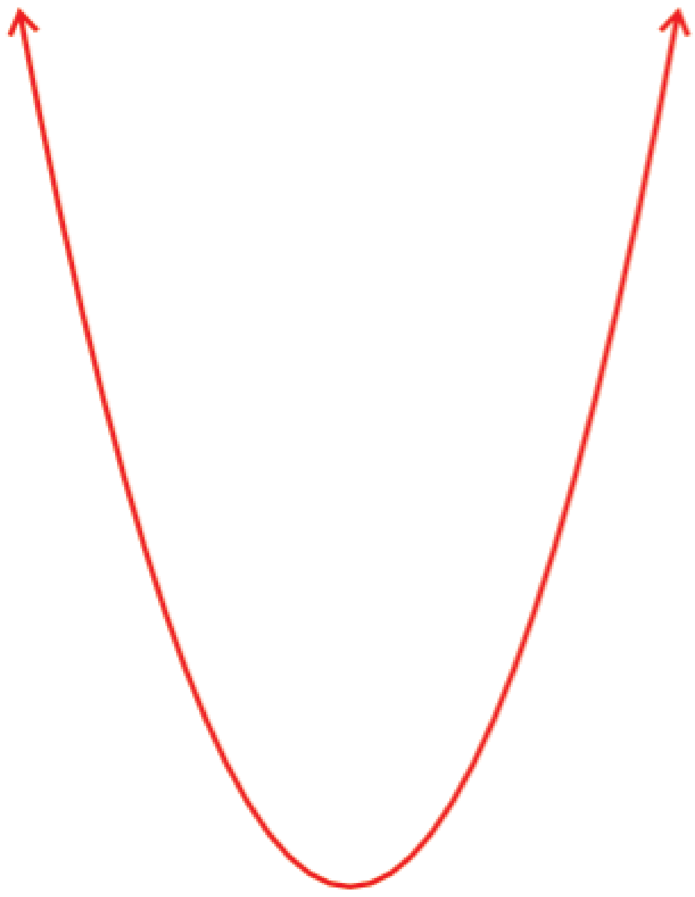
c **or** $q \rightarrow y$**-intercept**

STEPS TO SKETCHING THE QUADRATIC FUNCTION

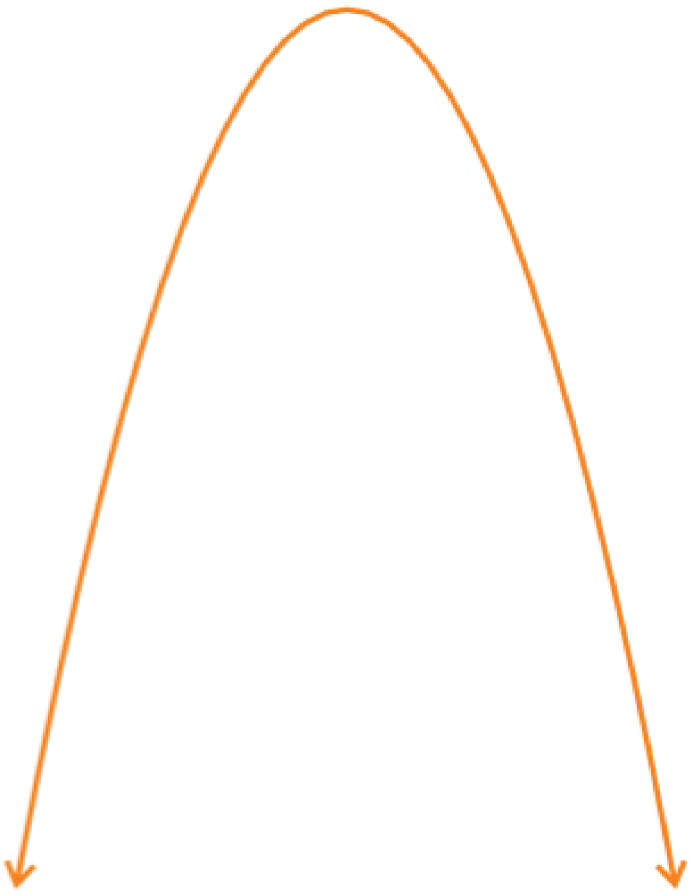
1. **Test for shape:** if $a > 0 \rightarrow$ **happy face parabola (concave up)** ☺
   if $a < 0 \rightarrow$ **sad face parabola (concave down)** ☹
2. **Find the y-intercept by setting x equal to 0.**
3. **Find the x-intercept/s by setting y equal to 0.**

If $a > 0$ then the turning point is a minimum

if $a < 0$ then the turning point is a maximum



$a > 0$: Happy Face
(Concave up)
☺



$a < 0$: Sad Face
(Concave down)
☹

You can also use the Table Method to sketch the graph of a parabola.

Example 1: **Sketch the graph of $f(x) = 2x^2 - 8$.**

1) $a > 0 \therefore$ happy face

2) $y - \text{int}: \text{set } x = 0$

$\therefore y = 2(0)^2 - 8$

$= -8$

$\therefore$ The y-intercept (turning point) is the point $(0; -8)$

3) $x - \text{int}: \text{set } y = 0$

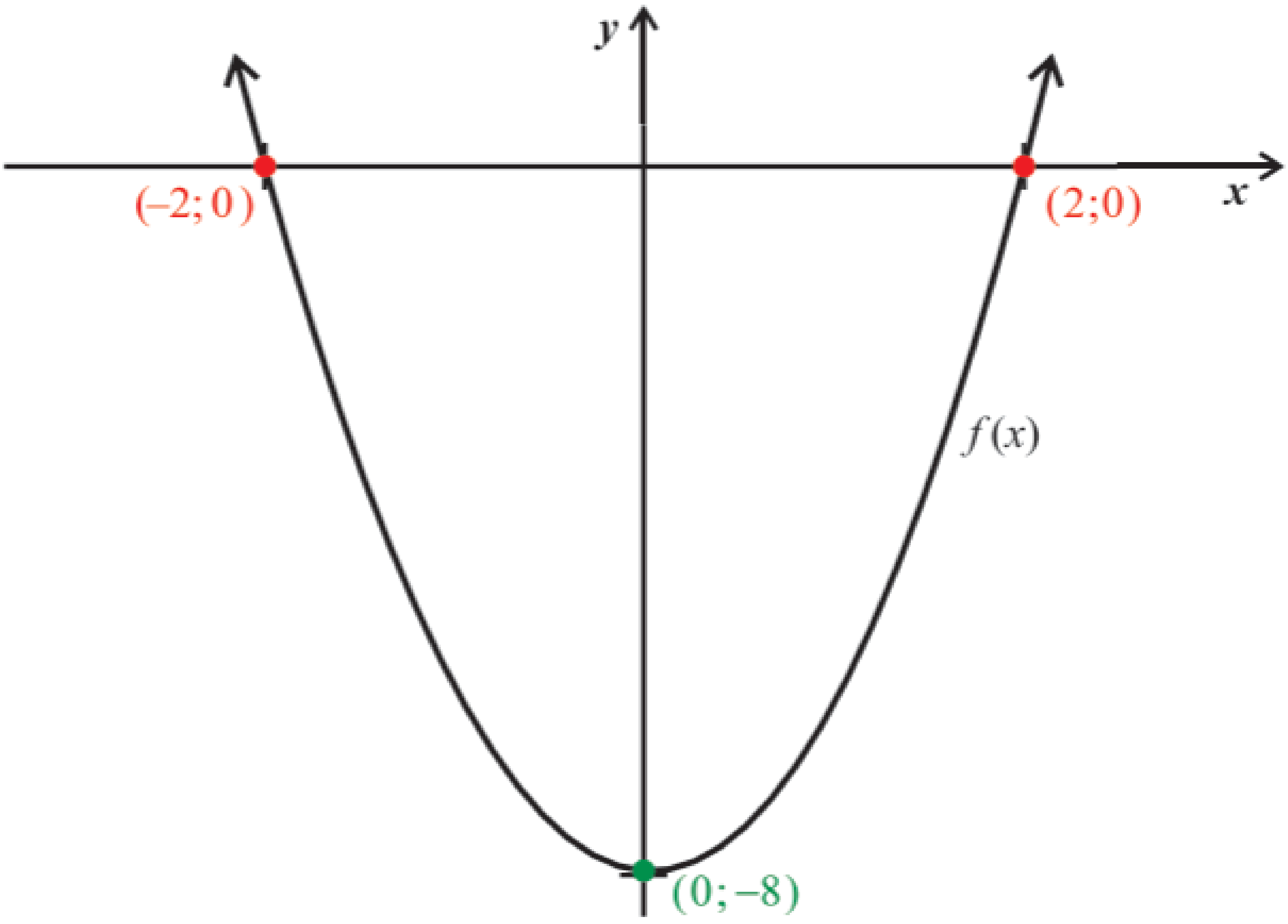
$\therefore 2x^2 - 8 = 0$

$\therefore 2(x^2 - 4) = 0$

$\therefore (x-2)(x+2) = 0$

$\therefore x = 2 \quad \text{or} \quad x = -2$

$\therefore$ x-intercepts are: $(-2; 0)$ and $(2; 0)$



In Grade 10 the parabola will always be symmetrical about the *y*-axis. For further details on symmetry refer to pg. 120.

Example 2: **Sketch the graph of $f(x) = -2x^2 - 2$.**

1) $a < 0 \therefore$ sad face

2) $y-\text{int}: \text{set } x = 0$

$\therefore y = -2(0)^2 - 2 = -2$

3) $x-\text{int}: \text{set } y = 0$

$\therefore -2x^2 - 2 = 0$

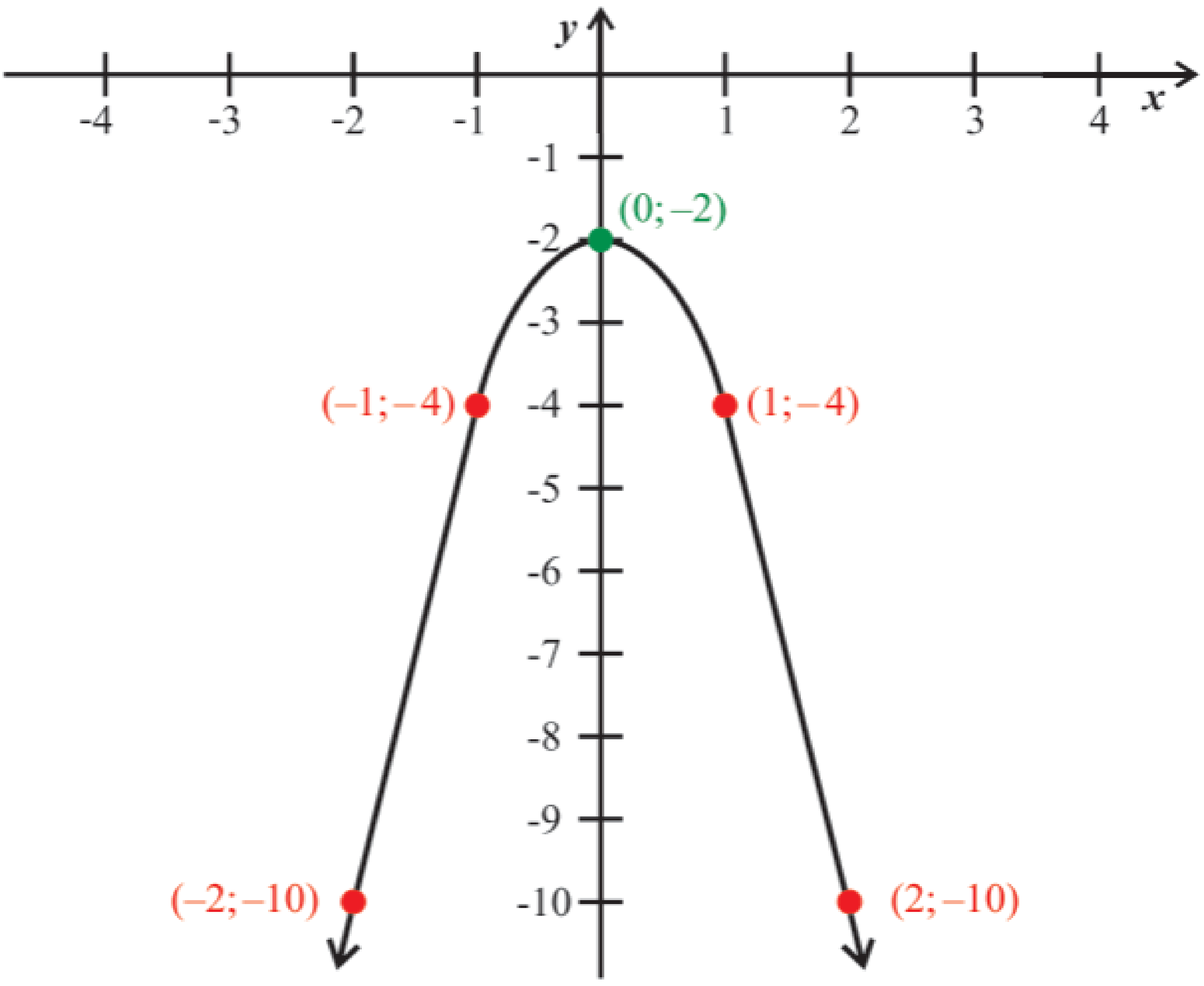
$\therefore -2(x^2 + 1) = 0$

$\therefore x^2 + 1 = 0 \rightarrow$ cannot factorise, therefore the graph has no x-intercepts

The turning point is at -2 and the shape of the graph is a sad face. From this, it can also be seen that there are no x-intercepts and the graph has a maximum value of -2.

In a case where there are no x-intercepts, it is useful to draw a table to obtain the correct shape of the graph.

x	-2	-1	0	1	2
y	-10	-4	-2	-4	-10



If the question says sketch the graph, it does not have to be accurate. However, the general shape and intercepts have to be correct.

Complete Exercise 4 on page 301.

DETERMINING THE EQUATION OF A PARABOLA

There are two ways in which you will be expected to determine the equation of a parabola (quadratic function):

1. **You will be given the turning point and one other point on the function.**
2. **You will be given two points on the function, where neither point is the turning point. In this case you will have to use simultaneous equations.**

Recap (pg. 101): The parabola has the form: $y = ax^2 + c$

CASE 1: GIVEN THE TURNING POINT AND ONE OTHER POINT

Example 1: Determine the equation of the parabola passing through the points (0;4) and (–1;6).

$y = ax^2 + c$

You know that the point $(0;4)$ is the turning point and y-intercept

$\therefore y = ax^2 + 4$

Now substitute the remaining point $(-1;6)$ in order to solve for a:

$\therefore 6 = a(-1)^2 + 4$

$\therefore 2 = a$

$\therefore y = 2x^2 + 4 \rightarrow$ always write down the final equation

Example 2: Determine the equation of the parabola passing through the point (0;3) with an x-intercept of negative one.

$y = ax^2 + c$

You know that the point $(0;3)$ is the turning point and y-intercept

$\therefore y = ax^2 + 3$

Now substitute the remaining point $(-1;0)$ in order to solve for a: $\rightarrow -1$ is an x-intercept

$\therefore 0 = a(-1)^2 + 3$

$\therefore -3 = a$

$\therefore y = -3x^2 + 3 \rightarrow$ always write down the final equation

Always check to see if you have been given the y-intercept (turning point) even if the question doesn't explicitly tell you that the given point is an intercept or turning point. If the x-value of a point is equal to 0 the point is a y-intercept and turning point of the parabola.

CASE 2: GIVEN ANY TWO POINTS

If you are given **two points** of which **neither is the *y*-intercept**, you have to use **simultaneous equations** in order to determine the equation of a parabola.

Example: **Determine the equation of the parabola passing through the points (–2;–36) and (1;–12).**

In this case you have to substitute both points into the equation of the parabola in order to obtain two equations. You can then **solve simultaneously** for ***a*** and ***c***.

$y = ax^2 + c$

substitute the point $(-2;-36)$ and substitute the point $(1;-12)$

$\therefore -36 = a(-2)^2 + c$

$\therefore -36 = 4a + c \rightarrow$ ①

$\therefore -12 = a(1)^2 + c$

$\therefore -12 = a + c \rightarrow$ ②

Using elimination:

② – ①:

$\therefore -12 - (-36) = a + c - (4a + c)$

$\therefore -12 + 36 = a + c - 4a - c$

$\therefore 24 = -3a$

$\therefore \mathbf{a = -8} \rightarrow$ ③

Substitute ③ into ②

$\therefore -12 = -8 + c$

$\therefore \mathbf{c = -4}$

$\therefore y = -8x^2 - 4$

The method of elimination or substitution can be used to determine the equation of a parabola. Use whichever method you prefer and if necessary revise the section on simultaneous equations (pg. 63).

Complete Exercise 5 on pages 301 and 302.

D. THE HYPERBOLIC FUNCTION (THE HYPERBOLA)

The hyperbola always lies on either side of two asymptotes and has the form:

$$y = \frac{a}{x} + q$$

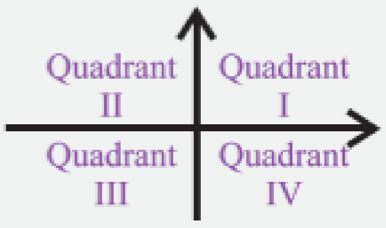
$a \rightarrow$ a is a constant which determines the quadrants that the hyperbola will be in.

If $a > 0$ the hyperbola will be in the 1st and 3rd quadrants.

If $a < 0$ the hyperbola will be in the 2nd and 4th quadrants.

$q \rightarrow$ q is the horizontal asymptote (the line $y = q$)

Recap: Quadrants



An asymptote is an imaginary line that the graph can never touch.
In Grade 10 the y-axis will always be the vertical asymptote for hyperbolas.

Axis of Symmetry

The hyperbola is always symmetrical about the straight lines:

$$y = x + q \qquad \text{and} \qquad y = -x + q$$

If $a > 0$:

the line of symmetry will cut the hyperbola at $\left(+\sqrt{a}; +\sqrt{a} + q\right)$ and $\left(-\sqrt{a}; -\sqrt{a} + q\right)$

If $a < 0$:

the line of symmetry will cut the hyperbola at $\left(-\sqrt{a}; +\sqrt{a} + q\right)$ and $\left(+\sqrt{a}; -\sqrt{a} + q\right)$

Ignore the sign of a (+/−) when calculating the points of intersection.

Figure 1 and figure 2 below, show hyperbolas with q equal to zero:

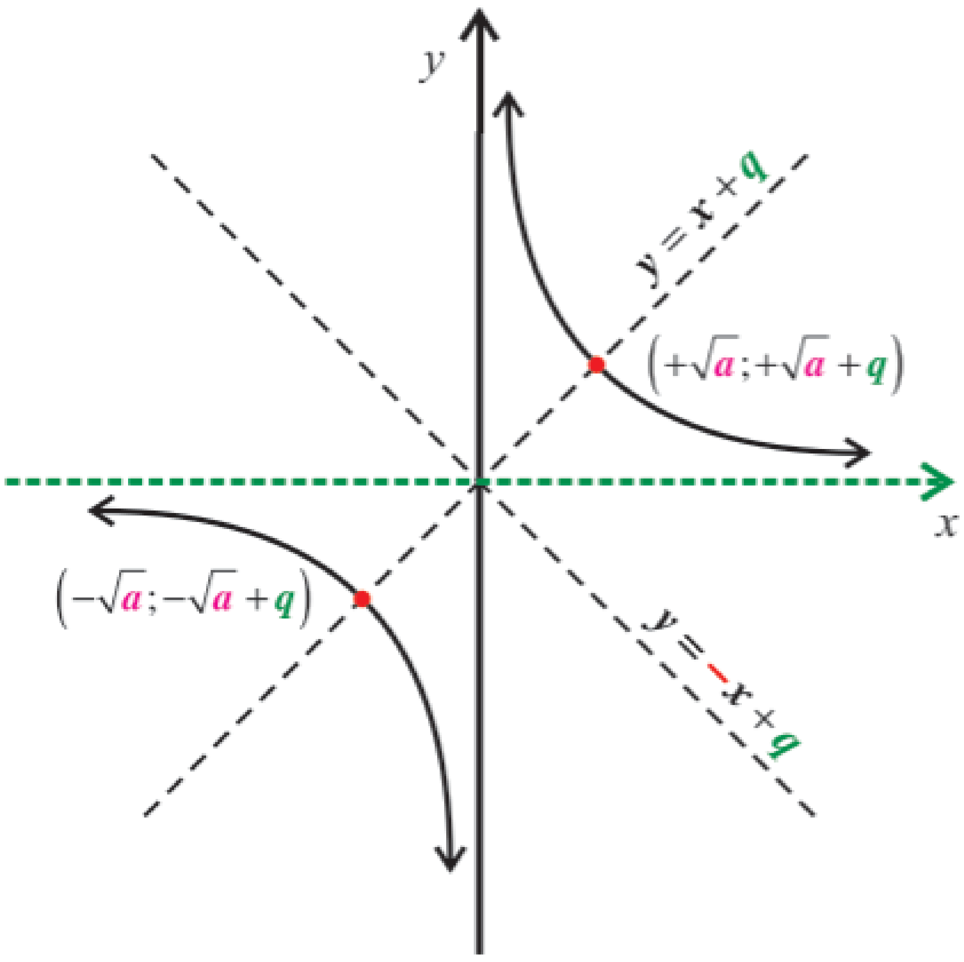


Figure 1: $a > 0$

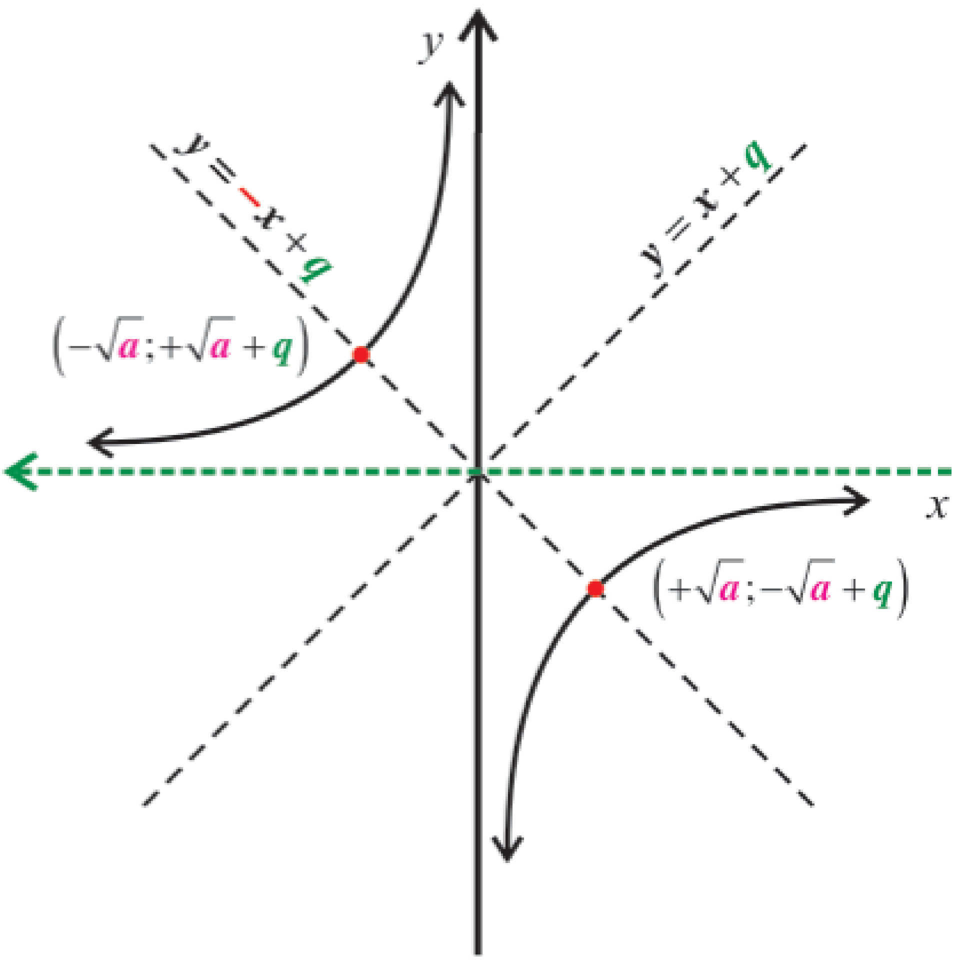


Figure 2: $a < 0$

Increasing the value of q shifts the graph up.
Decreasing the value of q shifts the graph down.

STEPS TO SKETCHING THE HYPERBOLIC FUNCTION

1. **Determine the equation of the asymptote.**
2. **Determine the shape by examining the sign of a.**
3. **Find the x-intercept by setting y equal to 0 (there is no intercept if $q = 0$).**
4. **If asked, draw the axis of symmetry and label the points of intersection.**

Example 1: Sketch the graph of $f(x) = \frac{4}{x} + 2$ and determine the equation of an axis of symmetry and its points of intersection with the graph.

Steps 1 and 2: Sketch the asymptotes and determine the shape.

$a = +4$ Asymptote: $y = 2$

$\therefore a > 0 \ \therefore$ 1st and 3rd quadrants

Step 3: Find the x-intercept.

$x - \text{int} : y = 0$

$\therefore 0 = \frac{4}{x} + 2$

$\therefore -2 = \frac{4}{x}$

$\therefore -2x = 4$

$\therefore x = -2$

Step 4: Find the equation of the axis of symmetry and the points of intersection.

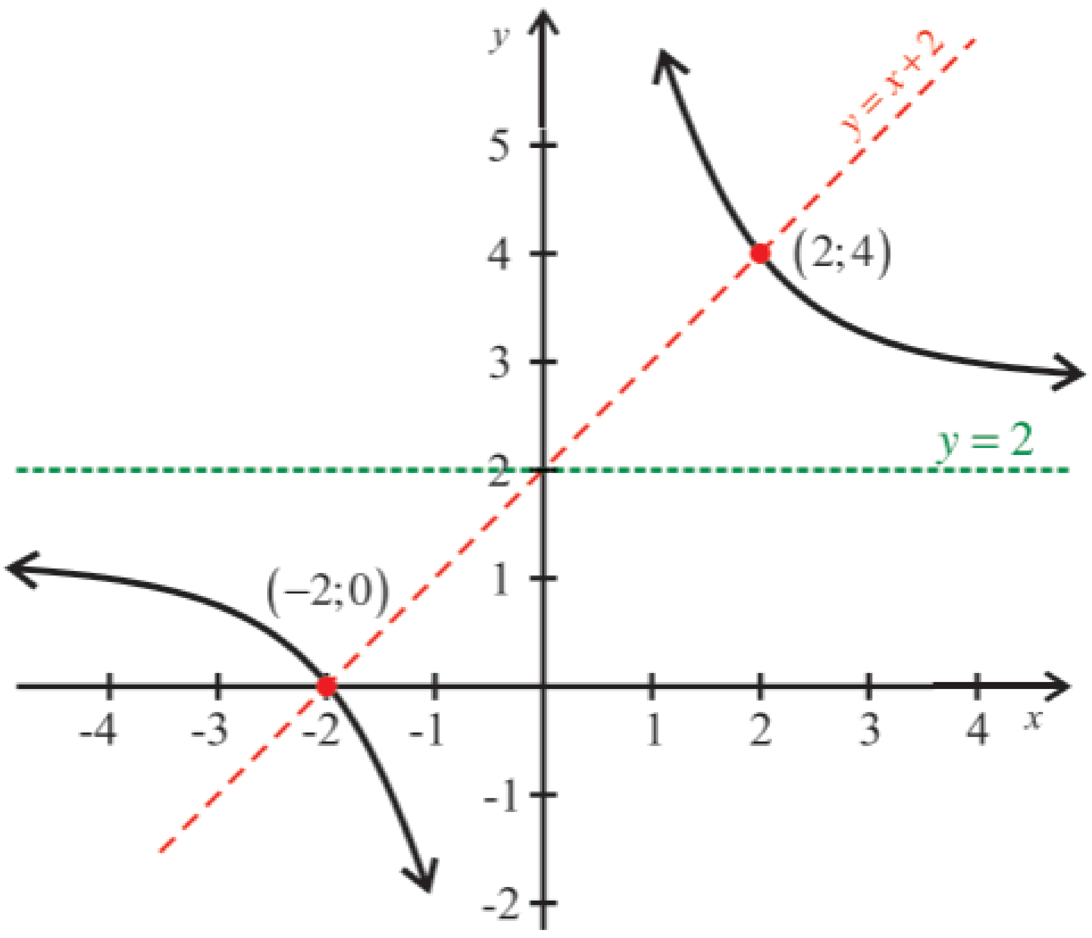
$a > 0$:

$\therefore y = x + q$

$\therefore y = x + 2$

$\therefore$ Points of intersection are $\left(+\sqrt{4}; +\sqrt{4} + 2\right)$ and $\left(-\sqrt{4}; -\sqrt{4} + 2\right)$

$\therefore$ Points of intersection are $(2; 4)$ and $(-2; 0) \rightarrow$ this is also an x-intercept



The table method can also be used to sketch hyperbolas.

Example 2: **Sketch the graph of $f(x)=\frac{-9}{x}+3$ and determine the equation of an axis of symmetry and its points of intersection with the graph.**

Steps 1 and 2: Sketch the asymptotes and determine the shape.

$a=-9$ Asymptote: $y=3$

$\therefore a<0 \ \therefore$ 2nd and 4th quadrants

Step 3: Find the x-intercept.

$x-\text{int}: y=0$

$\therefore 0=\frac{-9}{x}+3$

$\therefore -3=\frac{-9}{x}$

$\therefore -3x=-9$

$\therefore x=3$

Step 4: Find the equation of the axis of symmetry and the points of intersection.

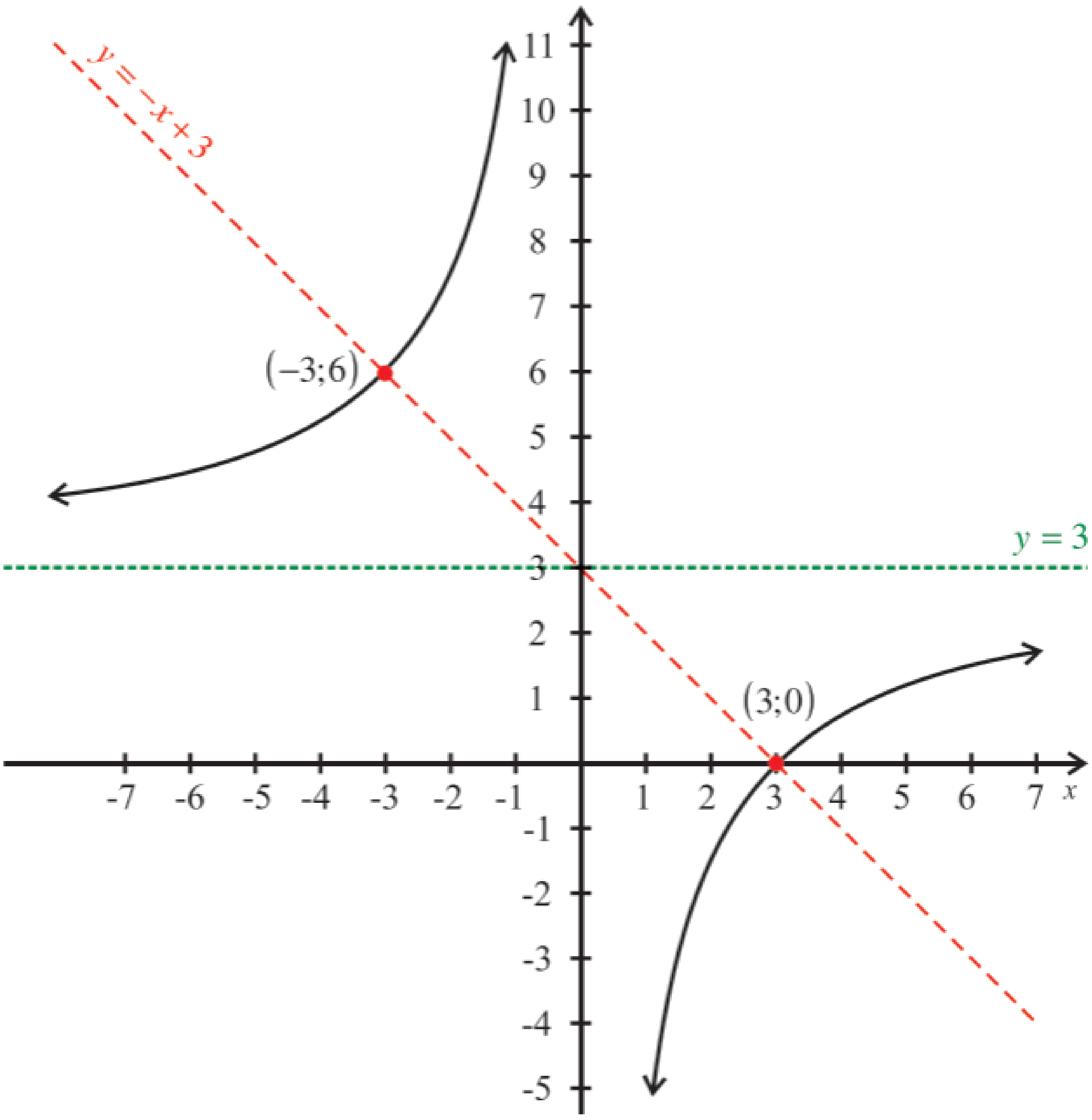
$a<0$:

$\therefore y=-x+q$

$\therefore y=-x+3$

$\therefore$ Points of intersection are $\left(-\sqrt{9};+\sqrt{9}+3\right)$ and $\left(+\sqrt{9};-\sqrt{9}+3\right)$

$\therefore$ Points of intersection are $(-3;6)$ and $(3;0) \rightarrow$ this is also an x-intercept



Complete Exercise 6 on page 302.

DETERMINING THE EQUATION OF A HYPERBOLA

Recap (pg. 106): The hyperbola has the form: $y = \frac{a}{x} + q$

In general, there are two cases that you will be given when determining the equation of a hyperbola.

1. **You will be given the horizontal asymptote and a point on the function.**
2. **You will be given two points which lie on the function. In this case you will have to use simultaneous equations.**

CASE 1: GIVEN THE HORIZONTAL ASYMPTOTE

In this case you will be **given the horizontal asymptote**. You will either be given it in words or be required to read it off of a graph. Once you have **this value it can be substituted into q. The value of a can then be found by substituting one other point $(x;y)$ into the equation of the hyperbola**.

Example: **Determine the equation of a hyperbola with a horizontal asymptote at negative five and passes through the point (1;4). In addition, determine the equation of an axis of symmetry and its points of intersection with the graph.**

Equation of hyperbola:

$y = \frac{a}{x} + q$

$\therefore y = \frac{a}{x} - 5 \rightarrow$ the graph has a horizontal asymptote at $y = -5$

Substitute the given point $(1;4)$

$\therefore 4 = \frac{a}{1} - 5$

$\therefore a = 9$

$\therefore y = \frac{9}{x} - 5$

Points of intersection (axis of symmetry):

$a > 0 \therefore$ the equation of the axis of symmetry which cuts the graph is $y = x + q$

$\therefore y = x - 5$

$a > 0 \therefore$ the points of intersection are $\left(+\sqrt{a};+\sqrt{a}+q\right)$ and $\left(-\sqrt{a};-\sqrt{a}+q\right)$

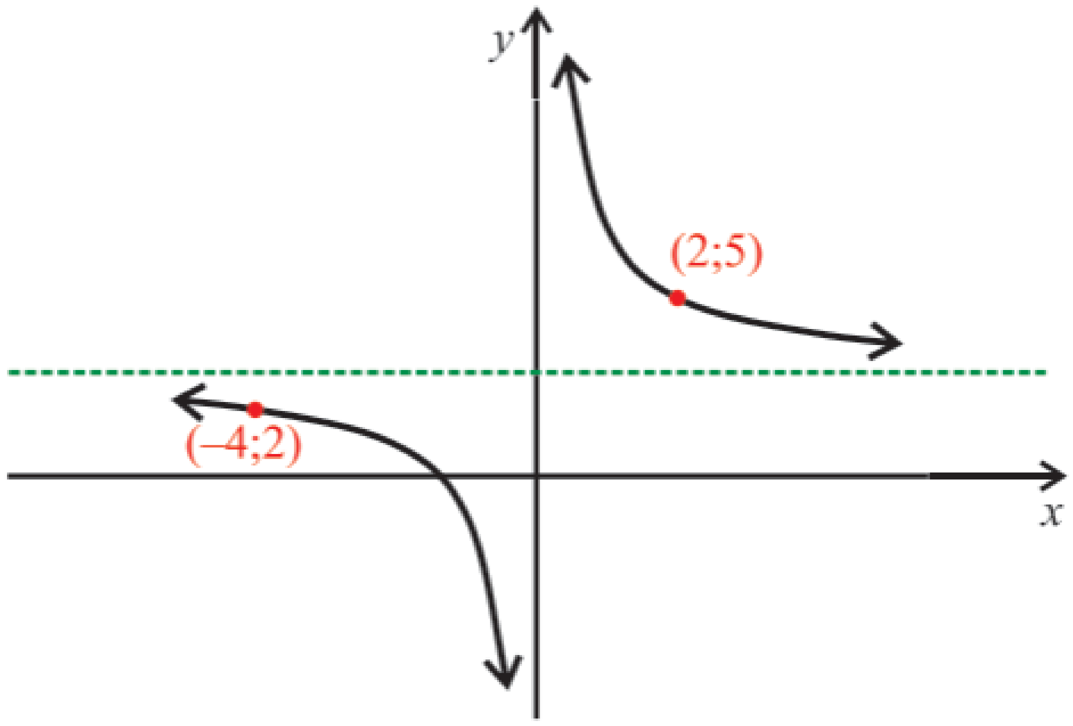
$\therefore$ the points of intersection are $\left(+\sqrt{9};+\sqrt{9}-5\right)$ and $\left(-\sqrt{9};-\sqrt{9}-5\right)$

$\therefore$ the points of intersection are $(3;-2)$ and $(-3;-8)$

CASE 2: GIVEN ANY TWO POINTS ON THE GRAPH

In this case you will substitute both points into the equation of the hyperbola. You can then use simultaneous equations to solve for the values of a and q.

Example: **The graph of $y = \frac{a}{x} + q$ is shown below. Determine the values of a and q.**



There are two unknowns and two points. Therefore, you have to substitute both points into the general equation of the hyperbola and solve simultaneously for a and q.

$$y = \frac{a}{x} + q$$

substitute the point $(-4;2)$	and	substitute the point $(2;5)$
$\therefore 2 = \frac{a}{-4} + q$		$\therefore 5 = \frac{a}{2} + q$
$\therefore -8 = a - 4q$		$\therefore 10 = a + 2q \rightarrow$ ②
$\therefore a = 4q - 8 \rightarrow$ ①		

Using substitution:

Substitute ① into ②:

$\therefore 10 = (4q - 8) + 2q$

$\therefore 10 = 6q - 8$

$\therefore 18 = 6q$

$\therefore \mathbf{q = 3} \rightarrow$ ③

Substitute ③ into ①

$\therefore a = 4(3) - 8$

$\therefore \mathbf{a = 4}$

$\therefore a = 4$ and $q = 3$

$\therefore$ the equation of the function is: $y = \frac{4}{x} + 3$

Complete Exercise 7 on page 302.

E. THE EXPONENTIAL FUNCTION

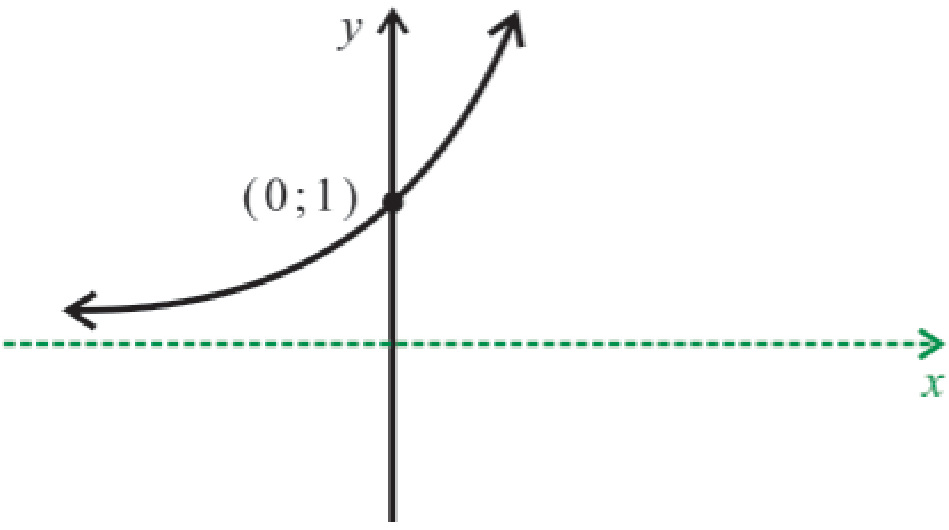
The exponential function always lies above or below a horizontal asymptote.
It is an **increasing function** if the **y-value increases as x increases**.
If the **y-value decreases as x increases** then it is a **decreasing function**.
The exponential function has the form: $f(x) = ab^x + q$ **and** $b > 0$ **and** $b \neq 1$

In order to demonstrate the increasing and decreasing nature of the exponential graph the graphs that are sketched below assume that $a = 1$ and $q = 0$.

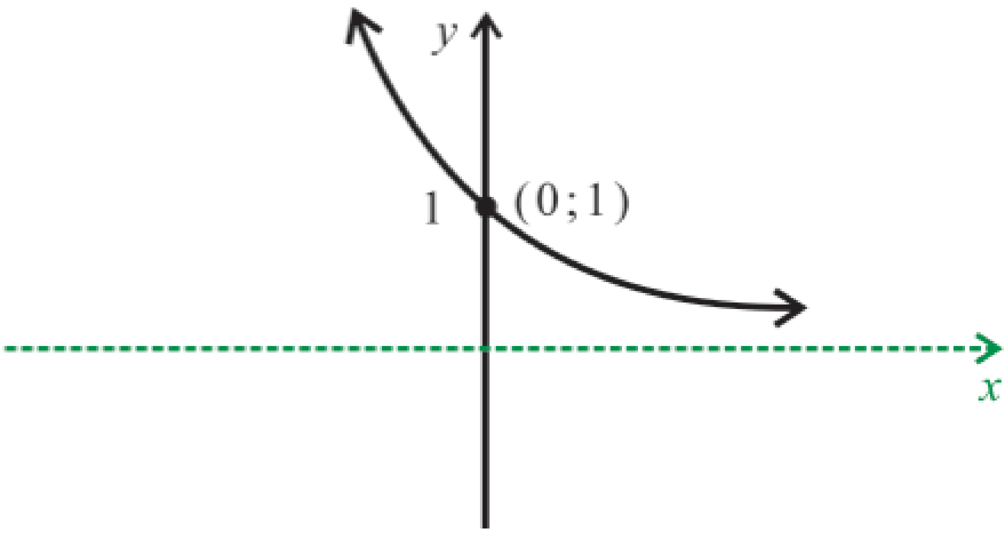
This results in the standard exponential graph of: $f(x) = b^x$

If:

$b > 1 \rightarrow$ increasing function



$0 < b < 1 \rightarrow$ decreasing function



An increasing function means that as x increases so does the value of y.
A decreasing function means that as x increases the value of y decreases.

The y-intercept of the exponential graph:

Determine the y-intercept by letting x equal 0.

$y - \text{int} : x = 0$

$\therefore y = b^0$

$\therefore y = 1 \rightarrow$ anything raised to the power of 0 is equal to 1

$\therefore$ in the case where $a = 1$ and $q = 0$, the y-intercept will always be at the point $(0;1)$ with a horizontal asymptote of $y = 0$ (the x-axis).

The values of a and q shift the standard exponential graph as follows:

$a \rightarrow$ The sign of a determines on which side of the asymptote the graph will lie (shape).
If $a > 0$ then the graph will lie above the asymptote.
If $a < 0$ then the graph will lie below the asymptote.

$q \rightarrow$ Shifts the graph up or down. Note that the line of $y = q$ is the **horizontal asymptote**.

$f(x) = ab^x + q$ can also be written as $y = ab^x + q$.

STEPS TO SKETCHING THE EXPONENTIAL FUNCTION

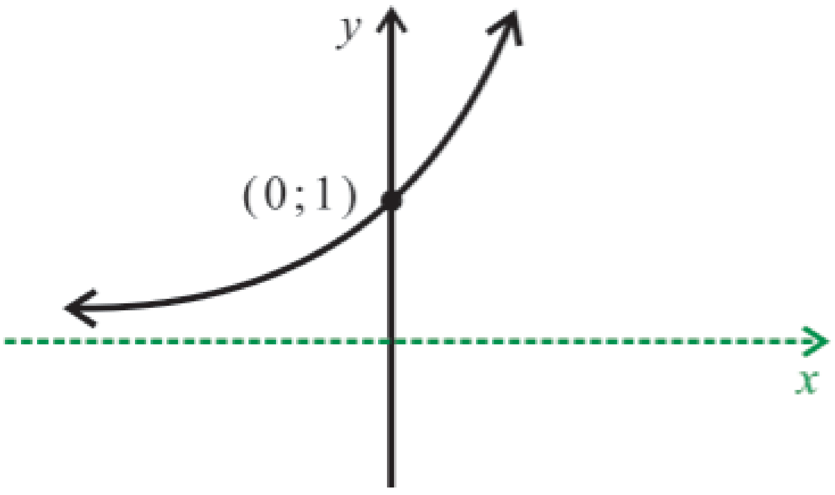
1. **Check if the graph is above or below the asymptote:**

   If $a > 0$ → the graph is above the asymptote
   If $a < 0$ → the graph is below the asymptote

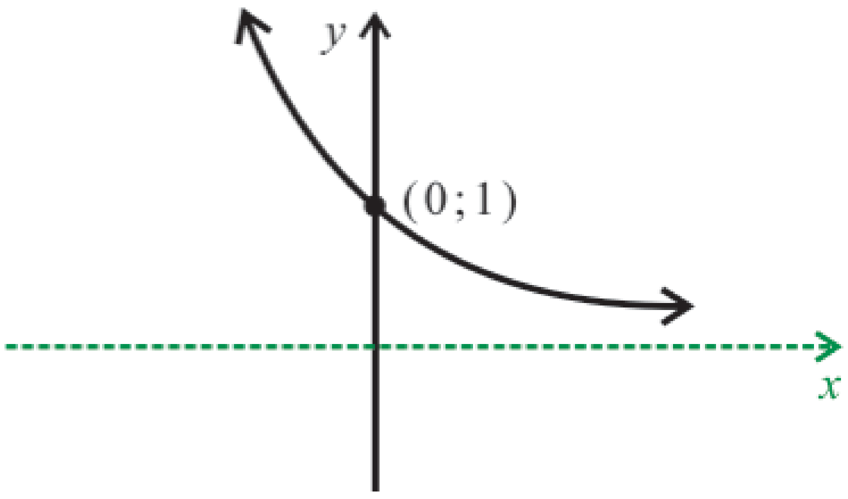
2. **Check if the function is increasing or decreasing:**

   If $b > 1$ and $a > 0$ → the graph is **increasing** (as x increases so does the value of y)
   If $0 < b < 1$ and $a > 0$ → the graph is **decreasing** (as x increases the value of y decreases)

$b > 1$ and $a > 0$

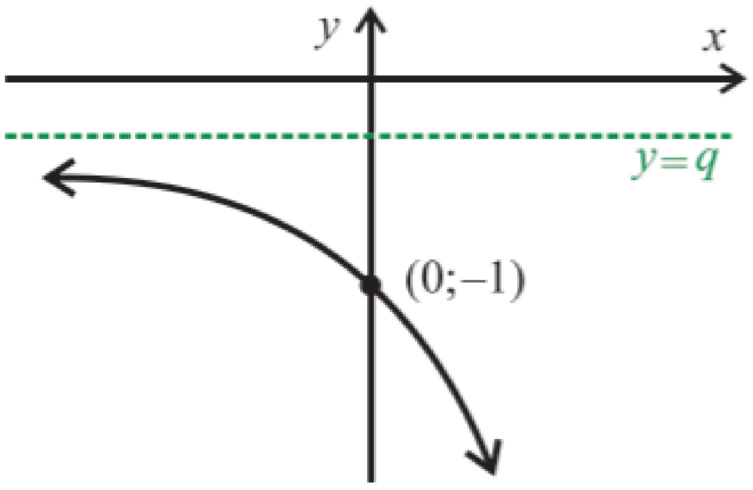


$0 < b < 1$ and $a > 0$

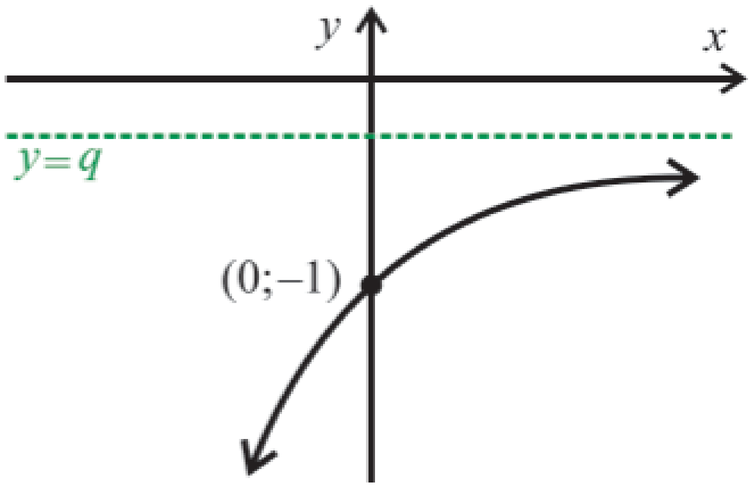


   If $b > 1$ and $a < 0$ → the graph is **decreasing** (as x increases the value of y decreases)
   If $0 < b < 1$ and $a < 0$ → the graph is **increasing** (as x increases so does the value of y)

$b > 1$ and $a < 0$



$0 < b < 1$ and $a < 0$



Note : if b is raised to a negative it is the same as $\frac{1}{b}$

3. **Determine the y intercept by letting x equal to 0.**
4. **Find the horizontal asymptote $(y = q)$.**
5. **Substitute a value for x in order to obtain another point on the graph so that you have the correct shape.**
6. **If there is an x-intercept determine the x-intercept by letting y equal to 0.**

The Table Method can also be used to draw the exponential graph. However, it is important to understand the effects that a, b and q have on the graph.

Example 1: **Sketch the graph of** $y = 2(2)^x + 2$.

Step 1: Determine the shape of the graph.

$a = 2$

$\therefore a > 0 \therefore$ the graph is **above** the asymptote

Step 2: Determine if the graph is increasing or decreasing.

$a = 2$ and $b = 2$

$\therefore a > 0$ and $b > 1$

$\therefore$ the graph is **increasing**

Step 3: Determine the y-intercept.

$y - \text{int} : x = 0$

$\therefore y = 2(2)^0 + 2$

$\therefore y = 2(1) + 2$

$= 4$

Step 4: Determine the horizontal asymptote.

$y = 2 \rightarrow$ the horizontal asymptote is the line of $y = q$.

Step 5: Find another point on the graph to obtain the correct shape.

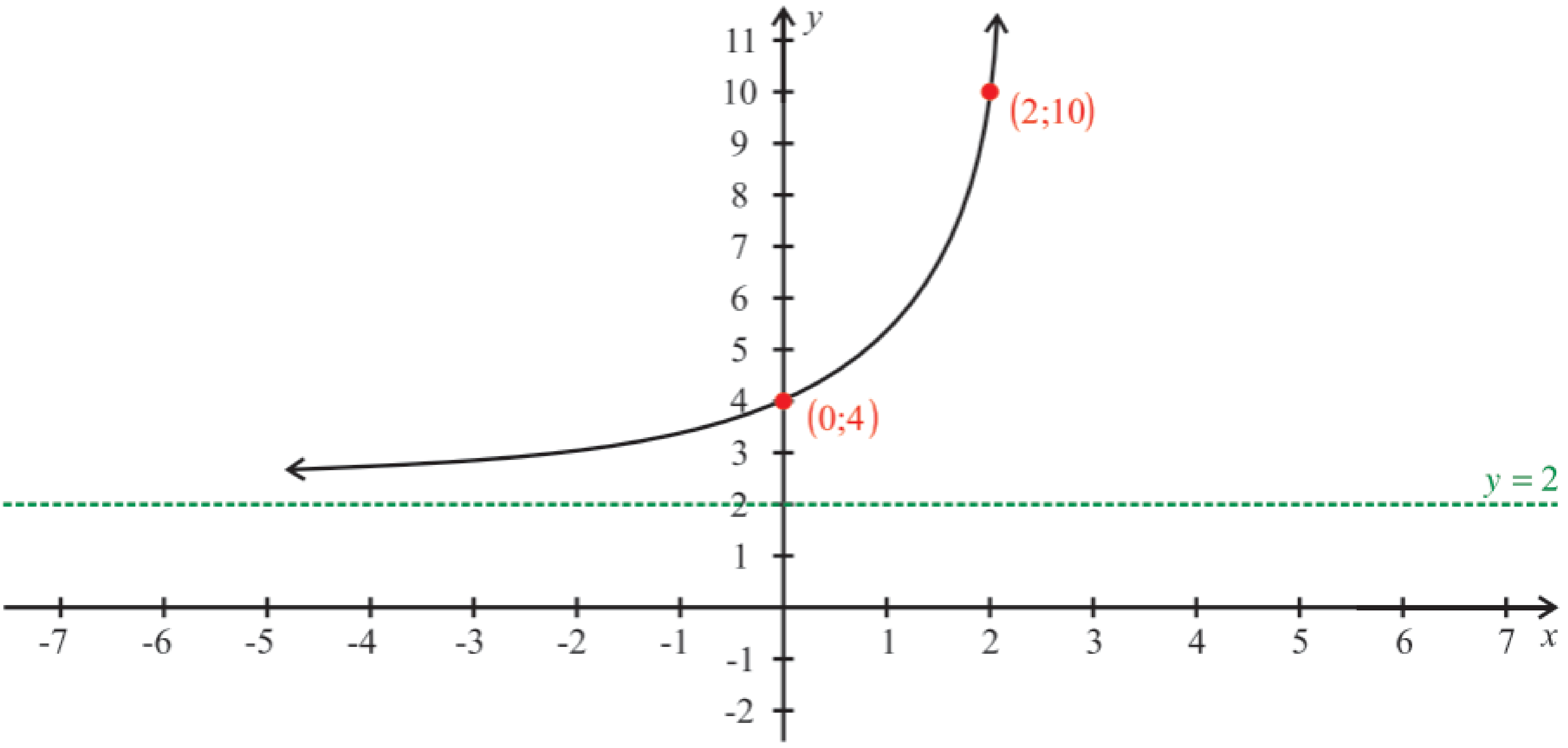
Substitute $x = 2 \rightarrow$ substitute any value for x to obtain another point on the graph

$\therefore y = 2(2)^2 + 2$

$= 2(4) + 2$

$= 10$

$\therefore$ another point on the graph is $(2;10)$



Do not choose a large a value for x when substituting to determine an additional point.

Example 2: Sketch the graph of $y = -(2)^x - 3$.

Step 1: Determine the shape of the graph.

$a = -1$

$\therefore a < 0$ $\therefore$ the graph is **below** the asymptote

Step 2: Determine if the graph is increasing or decreasing.

$a = -1$ and $b = 2$

$\therefore a < 0$ and $b > 1$

$\therefore$ the graph is **decreasing**

Step 3: Determine the y-intercept.

$y - \text{int} : x = 0$

$\therefore y = -(2)^0 - 3$

$\therefore y = -(1) - 3$

$= -4$

Step 4: Determine the horizontal asymptote.

$y = -3 \rightarrow$ the horizontal asymptote is the line of $y = q$

Step 5: Find another point on the graph to obtain the correct shape.

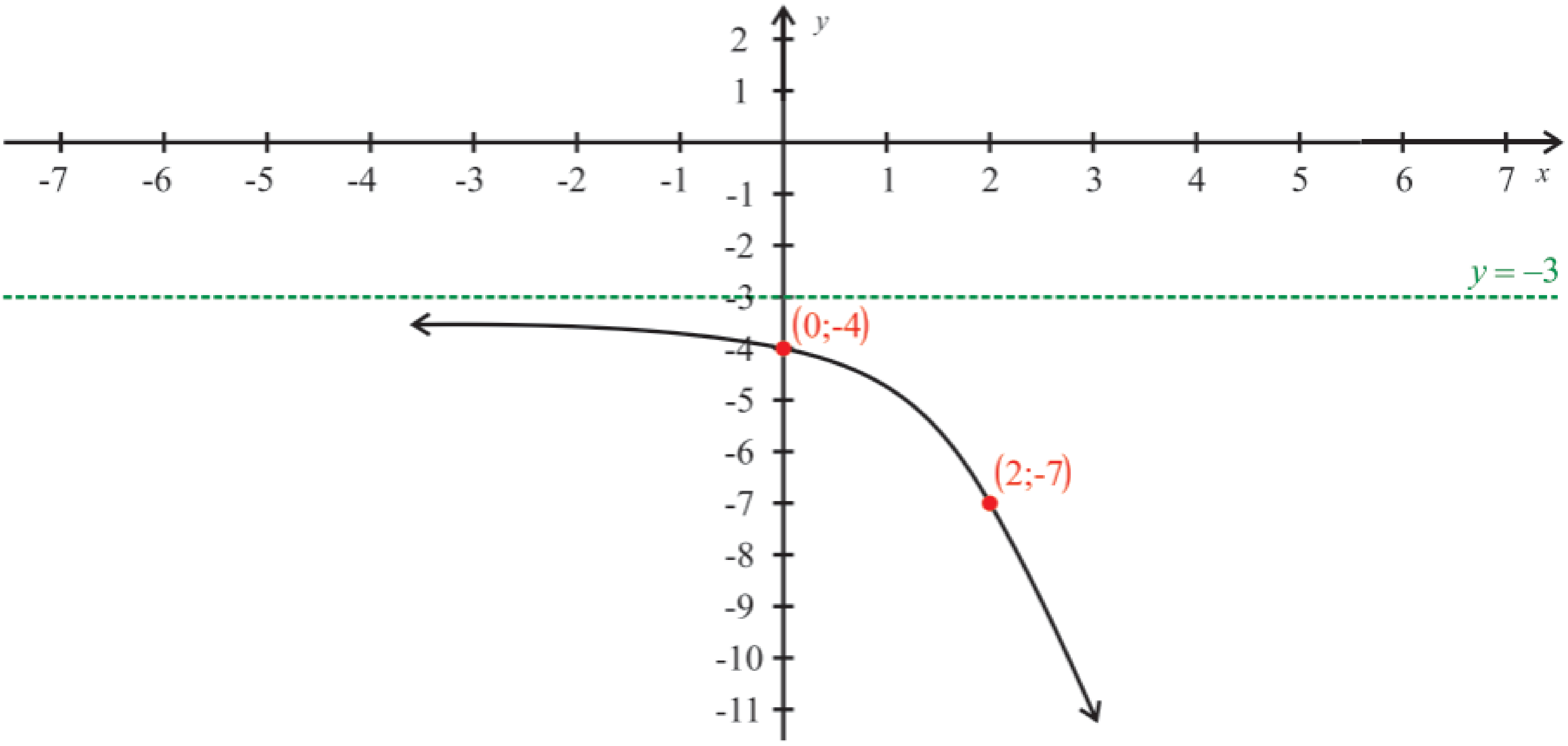
Substitute $x = 2 \rightarrow$ substitute any value for x to obtain another point on the graph

$\therefore y = -(2)^2 - 3$

$= -(4) - 3$

$= -7$

$\therefore$ another point on the graph is $(2;-7)$



Example 3: **Sketch the graph of $y = 4(2)^{-x} + 1$.**

If x is negative, always rewrite the equation so that x is positive.

$\therefore y = 4(2)^{-x} + 1 = 4\left(\frac{1}{2}\right)^{x} + 1$

Step 1: Determine the shape of the graph.

$a = 4$

$\therefore a > 0$ $\therefore$ the graph is **above** the asymptote

Step 2: Determine if the graph is increasing or decreasing.

$a = 4$ and $b = \frac{1}{2}$

$\therefore a > 0$ and $0 < b < 1$

$\therefore$ the graph is **decreasing**

Step 3: Determine the y-intercept.

$y - \text{int} : x = 0$

$\therefore y = 4\left(\frac{1}{2}\right)^{0} + 1$

$\therefore y = 4(1) + 1 = 5$

Step 4: Determine the horizontal asymptote.

$y = 1 \rightarrow$ the horizontal asymptote is the line of $y = q$

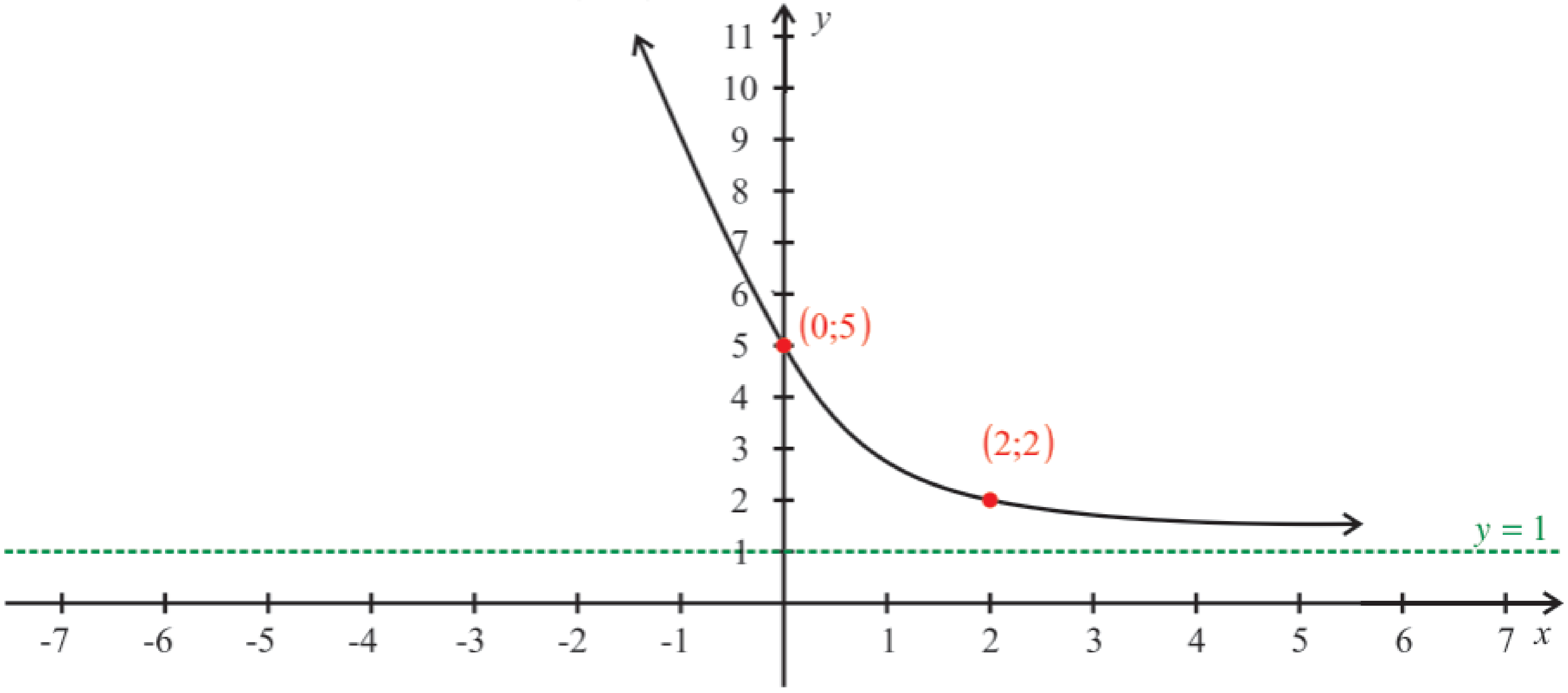
Step 5: Find another point on the graph to obtain the correct shape.

Substitute $x = 2 \rightarrow$ substitute any value for x to obtain another point on the graph

$\therefore y = 4(2)^{-2} + 1$

$\therefore y = 4\left(\frac{1}{4}\right) + 1 = 2$

$\therefore$ another point on the graph is $(2;2)$



Complete Exercise 8 on page 303.

DETERMINING THE EQUATION OF AN EXPONENTIAL FUNCTION

Recap (pg. 111): The Exponential function has the form: $y = ab^x + q$

In general, there are two cases that you will be given when determining the equation of an Exponential Function.

1. **You will be given the horizontal asymptote and up to 2 points on the function.**
2. **You will be given two points which lie on the function. In this case you will have to use simultaneous equations.**

CASE 1: GIVEN THE HORIZONTAL ASYMPTOTE

In this case you will be **given the horizontal asymptote**. You will either be given it in words or be required to read it off of a graph. Once you have **this value it can be substituted into q. The value of b can then be found by substituting one other point $(x;y)$ into the equation of the exponential function.**

Example 1: Determine the equation of the graph of g in the form $g(x) = b^x + q$, if the horizontal asymptote is the line $y = -3$ and $A(2;1)$ is a point which lies on the graph.

$g(x) = b^x - 3 \rightarrow -3$ is the horizontal asymptote

Substitute $A(2;1)$

$\therefore 1 = b^2 - 3$

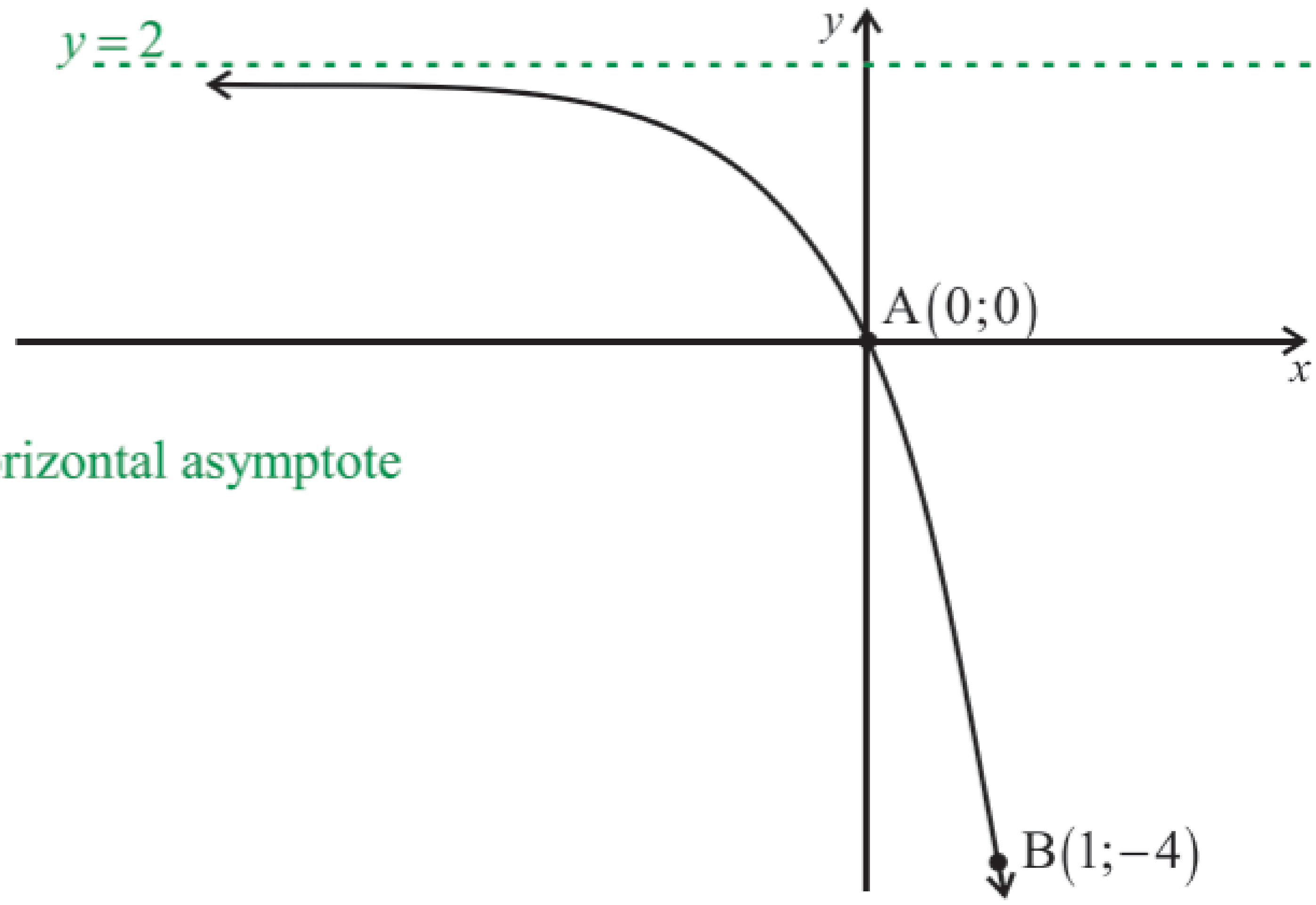
$\therefore 4 = b^2$

$\therefore b = \pm 2 \rightarrow$ but $b > 0$

$\therefore b = 2$

$\therefore g(x) = 2^x - 3$

Example 2: The graph of f is shown below. The graph has the equation $f(x) = -2b^x + q$. Determine the values of b and q.



$f(x) = -2b^x + 2 \rightarrow 2$ is the horizontal asymptote

Substitute $B(1;-4)$

$\therefore -4 = -2b^1 + 2$

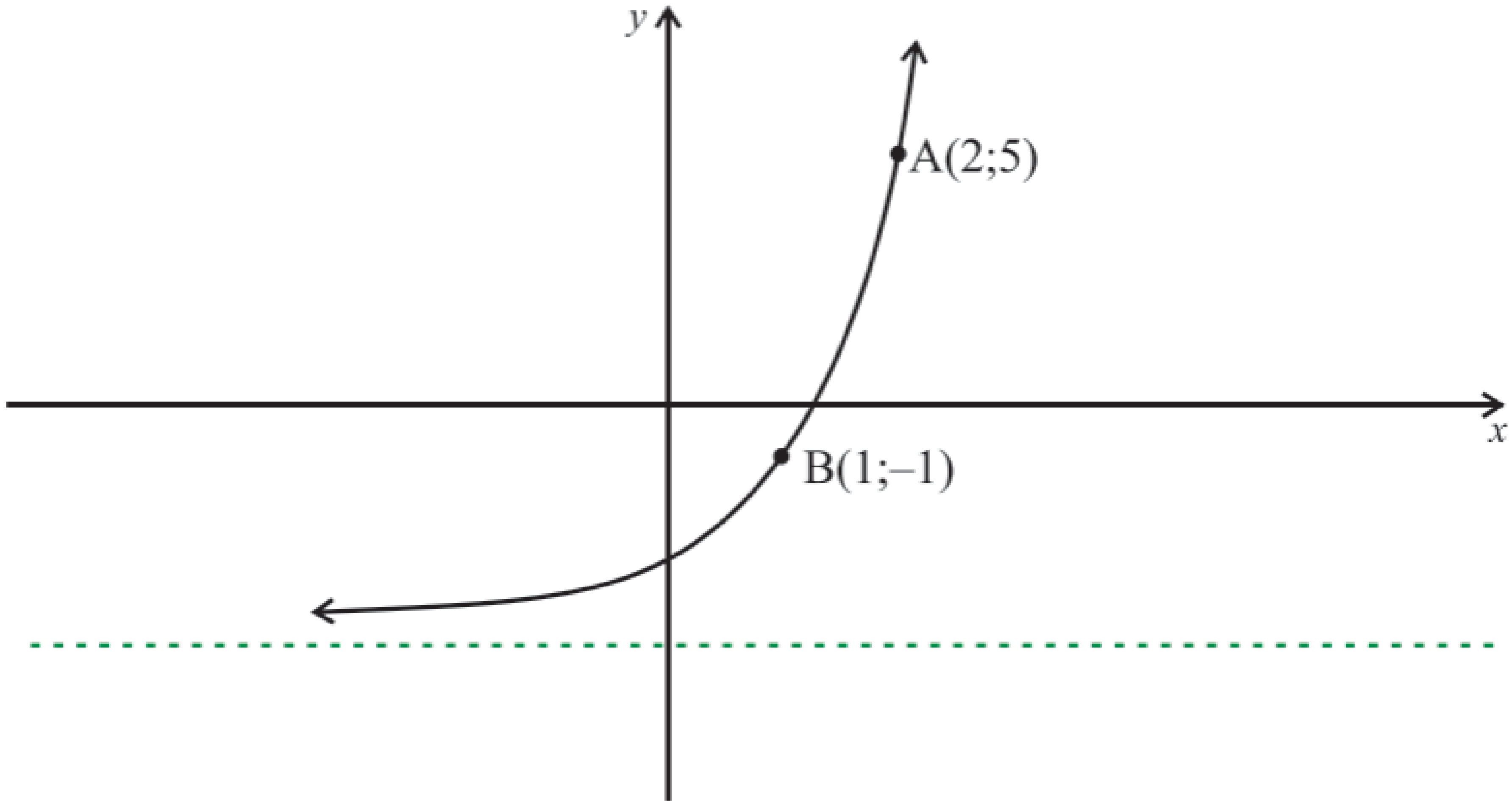
$\therefore -6 = -2b$

$\therefore b = 3$ and $q = 2$

CASE 2: GIVEN ANY TWO POINTS ON THE GRAPH

In this case you will substitute both points into the equation of the exponential. You can then use simultaneous equations to solve for the values of a, b and q. It is important to note that a question can combine case 1 and 2 together.

Example 1: Determine the equation of the graph shown below in the form $y = b^x + q$.



Substitute $\mathrm{A}(2;5)$

$5 = b^2 + q \rightarrow$ ①

Substitute $\mathrm{B}(1;-1)$

$-1 = b^1 + q$

$\therefore q = -b - 1 \rightarrow$ ②

Substitute ② into ①:

$\therefore 5 = b^2 - b - 1$

$\therefore 0 = b^2 - b - 6$

$\therefore 0 = (b-3)(b+2)$

$\therefore b = 3$ or $b = -2 \rightarrow$ N/A as $b > 0$

$\therefore b = 3 \rightarrow$ ③

Substitute ③ into ②:

$\therefore q = -3 - 1$

$\therefore q = -4$

$\therefore y = 3^x - 4$

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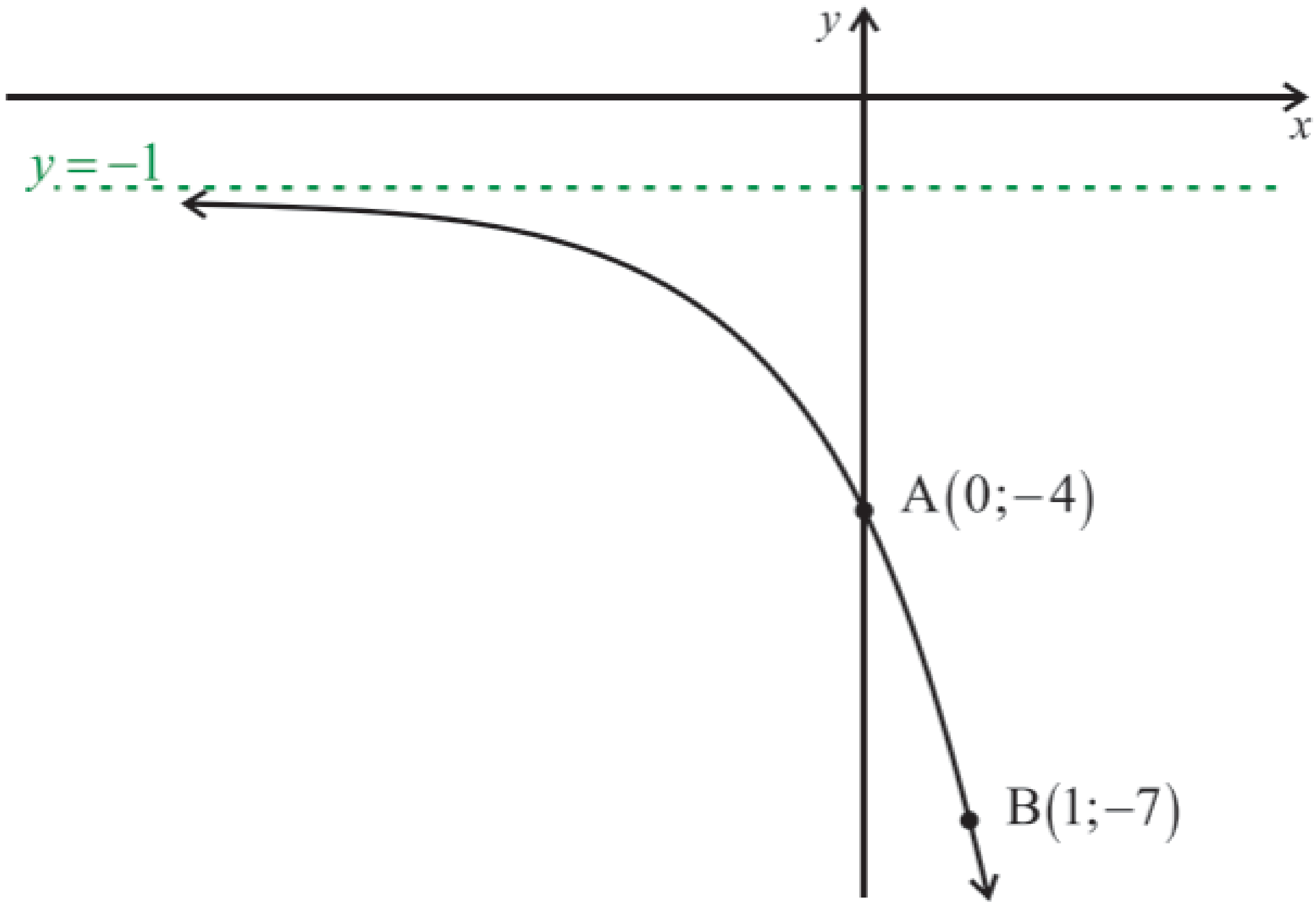
Example 2: **The graph shown below has the form $y = ab^x + q$. Determine the equation of the graph.**



Before substituting any points in it is important to note that the horizontal asymptote has been given to you as –1. Start by substituting this value for *q*.

$y = ab^x - 1 \rightarrow -1$ is the horizontal asymptote

Substitute A$(0;-4)$

$-4 = ab^0 - 1 \rightarrow b^0 = 1$

$\therefore -4 = a - 1$

$\therefore -3 = a \rightarrow$ ①

Substitute B$(1;-7)$

$-7 = ab^1 - 1 \rightarrow$ ②

Substitute ① into ②:

$\therefore -7 = -3(b)^1 - 1$

$\therefore -6 = -3b$

$b = 2$

$\therefore y = -3(2)^x - 1$

Complete Exercise 9 on page 303.

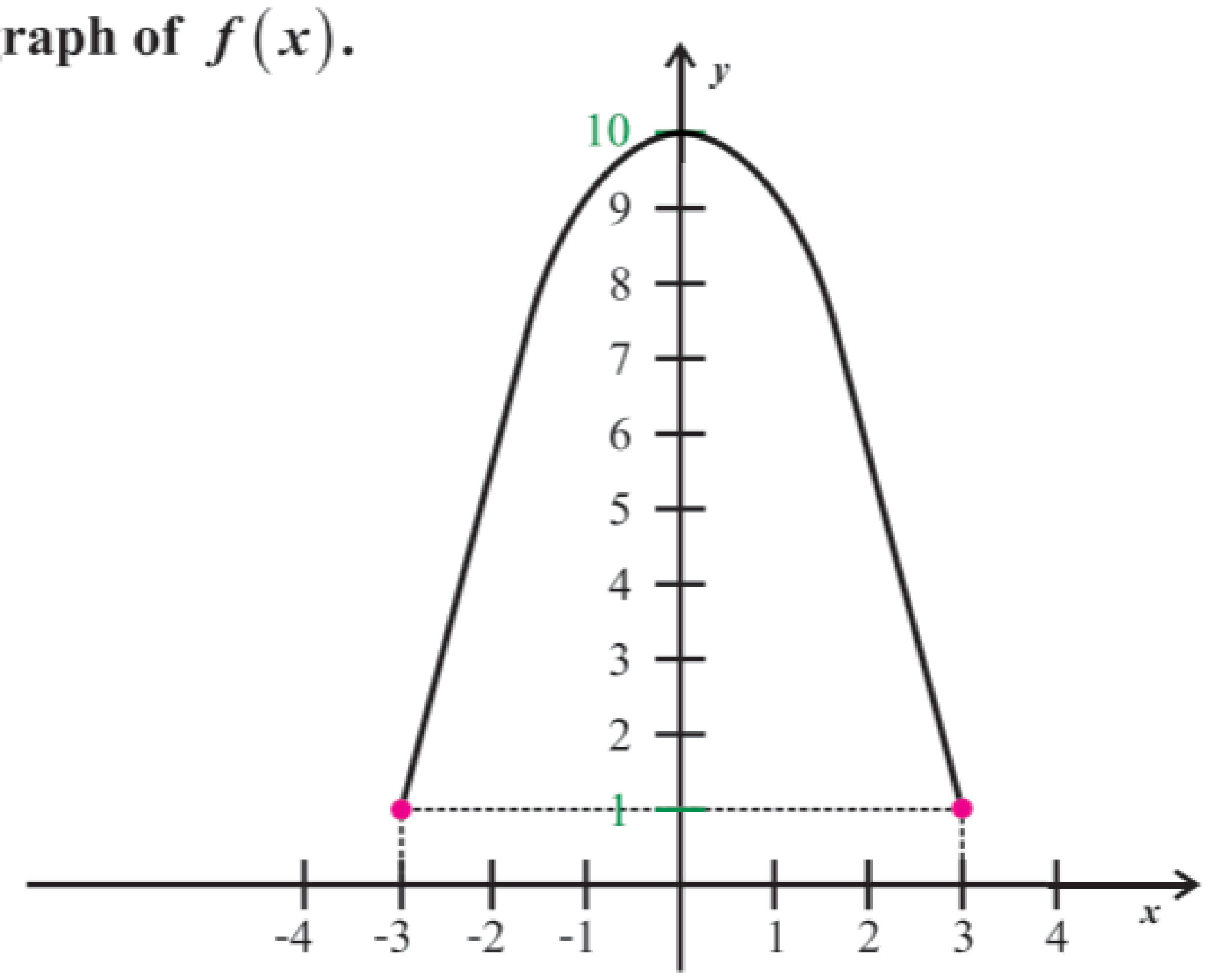
F. CHARACTERISTICS OF FUNCTIONS

DOMAIN AND RANGE

Domain: The domain refers to the possible ***x*-values** that a function can have.

Range: The range refers to the possible ***y*-values** that a function can have.

Example 1: The graph of $f(x) = -x^2 + 10$ is sketched below.
From the graph, clearly state the domain and range of $f(x)$.
In addition, state if there is a minimum or maximum value for the graph of $f(x)$.



Domain: $x \in [-3;3]$
Range: $y \in [1;10]$

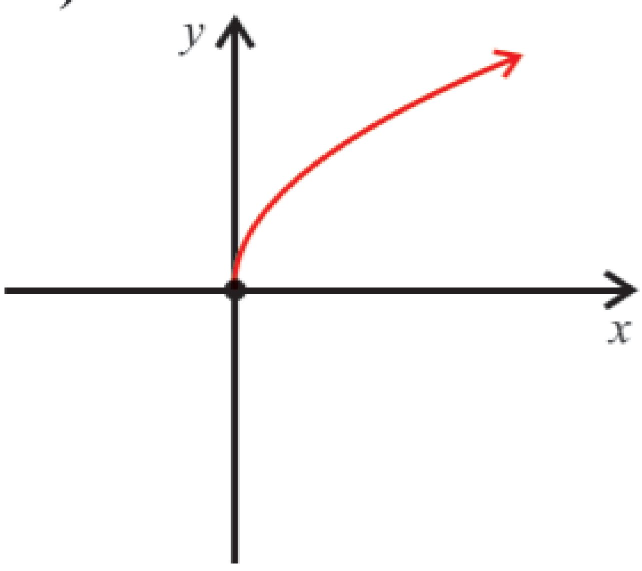
or

Domain: $-3 \leq x \leq 3$
Range: $1 \leq y \leq 10$

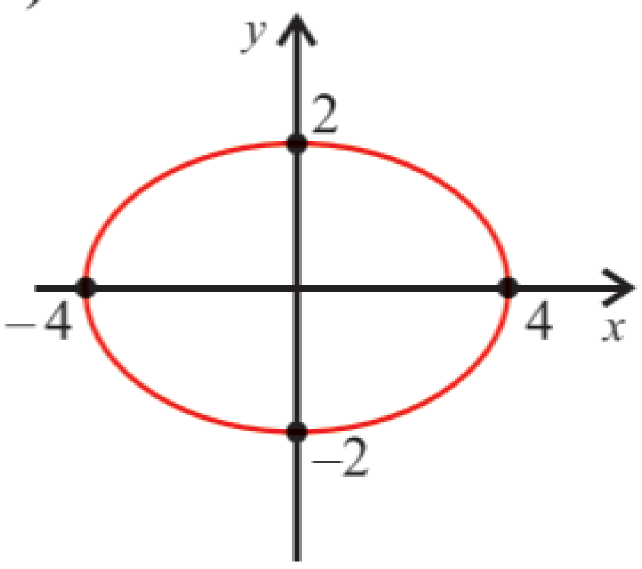
The maximum value of the function is: 10
The minimum value of the function is : 1

Example 2: State the domain and range for each of the graphs shown below.

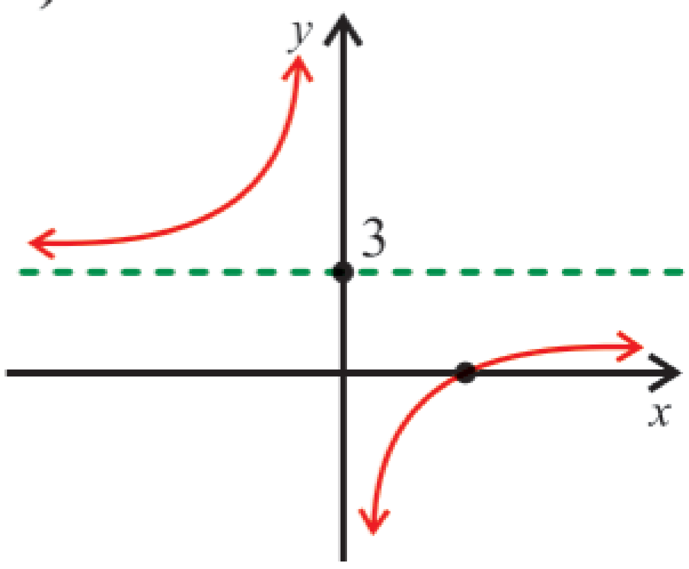
a)



b)



c)



a) Domain: $x \geq 0$
This is also written as:
$x \in [0;\infty)$
Range: $y \geq 0$
This is also written as:
$y \in [0;\infty)$

b) Domain: $-4 \leq x \leq 4$
This is also written as:
$x \in [-4;4]$
Range: $-2 \leq y \leq 2$
This is also written as:
$y \in [-2;2]$

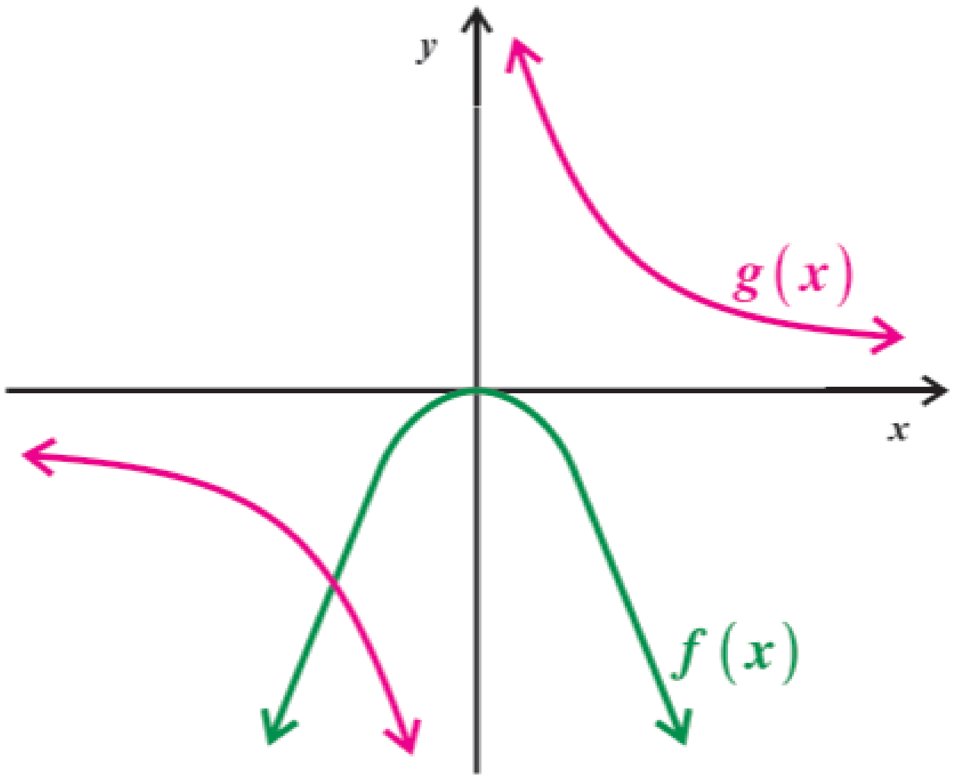
c) Domain: $x \in \mathbb{R}; x \neq 0$
This is also written as:
$x \in (-\infty;\infty); x \neq 0$
Range: $y \in \mathbb{R}; y \neq 3$
This is also written as:
$y \in (-\infty;\infty); y \neq 3$

AXES OF SYMMETRY

As you have already seen, some graphs have lines of symmetry. If a function has a line of symmetry, it means that the function is a mirror image of itself about the line of symmetry. In other words, if the function were folded along the line of symmetry it would duplicate the graph on the other side of the line.

Example: **The graphs of $f(x) = -x^2$ and $g(x) = \frac{3}{x}$ are shown below.**

Write down the equations of the axes of symmetry for $f(x)$ and $g(x)$.



$f(x)$ is a parabola. It can be seen from the graph that the parabola is symmetrical about the y-axis (the line $x = 0$).

$\therefore$ the axis of symmetry of $f(x)$ is: $x = 0$

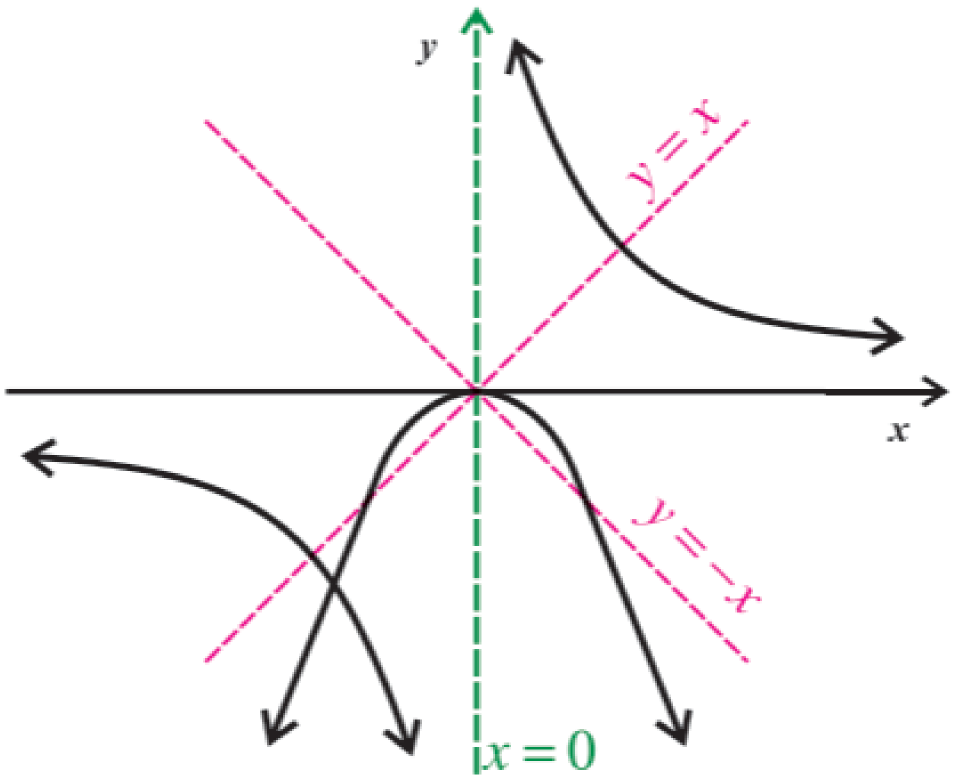
It was seen earlier that hyperbolas are symmetrical about two lines (pg. 106).

These lines are: $\boldsymbol{y = x + q}$ **and** $\boldsymbol{y = -x + q}$

In this case $q = 0$.

$\therefore$ the axis of symmetry of $g(x)$ is: $y = x$ and $y = -x$

Below are the graphs showing the axes of symmetry for $f(x)$ and $g(x)$ respectively.



In Grade 10 the parabola will always be symmetrical about the y-axis.

ASYMPTOTES

Asymptotes are lines that a function approaches, but never touches or cuts.
These were encountered earlier when sketching graphs of the hyperbolic function (pg. 106) and exponential function (pg. 111).

CONTINUITY

A function is said to be continuous if it can be drawn as a single line
(You can draw the function without lifting your pen from the paper).

A function that is not continuous is said to be discrete. The **hyperbola** is an example of a **discrete function** as it cannot be drawn as one line. It has to be drawn in two separate motions.

INCREASING AND DECREASING FUNCTIONS

A function is **increasing** if the **y-value increases** as the **x-value increases**.
A function is **decreasing** if the **y-value decreases** as the **x-value increases.**

Example: Figure 1 and figure 2 below show sketch graphs of an exponential and a quadratic function respectively. Give the interval on which each function is increasing or decreasing.

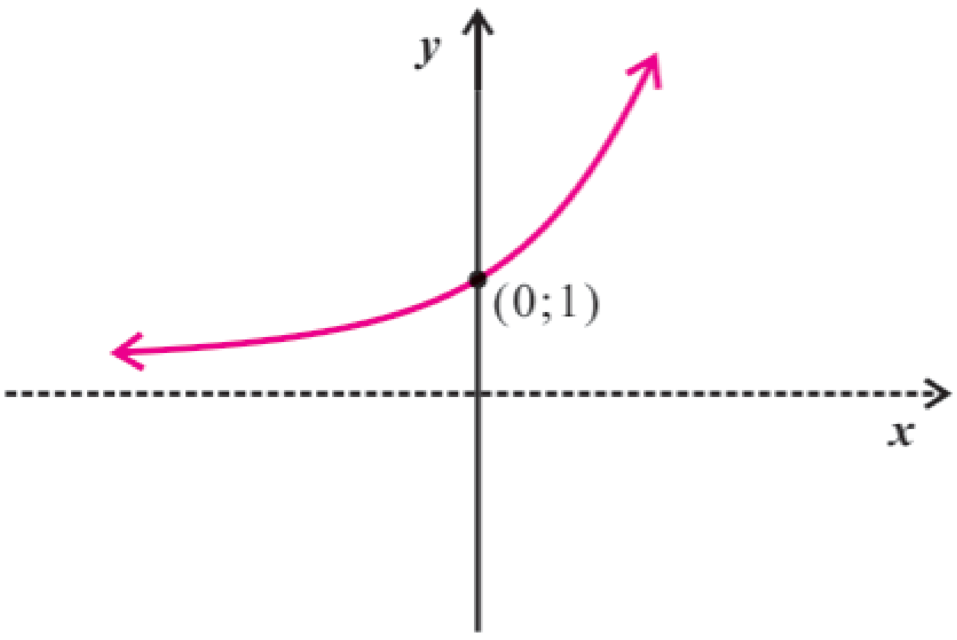


Figure 1

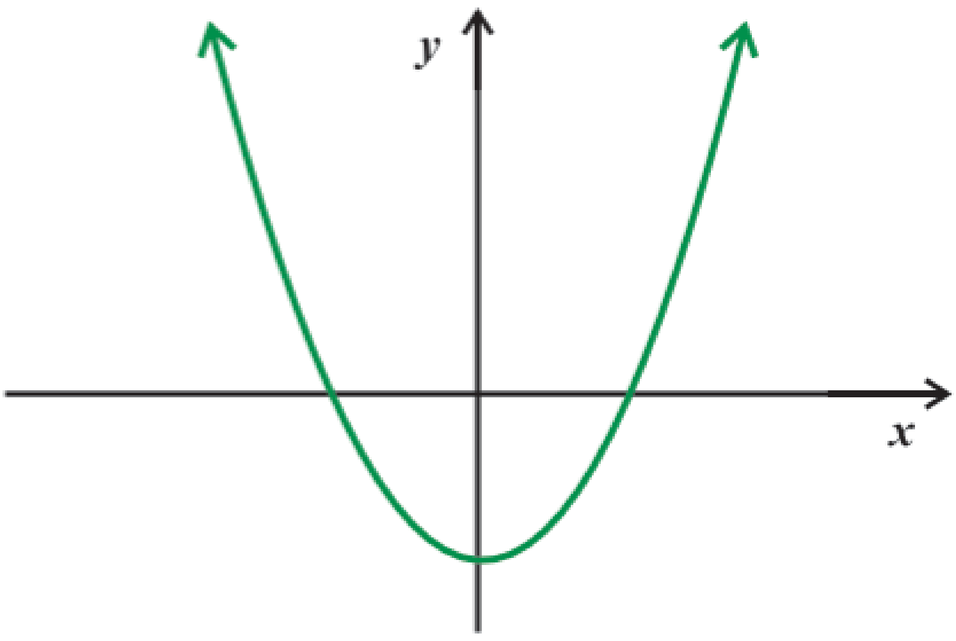


Figure 2

Exponential Function:
For all values of x, the y-value increases as the x-value increases.
$\therefore$ The exponential function increases for $x \in \mathbb{R}$

Quadratic Function:
Looking at the graph from left to right, the y-values decrease as the x-values increase for all values of x until zero. After zero, as the x-values increase so do the y-values.
$\therefore$ The quadratic function decreases for $x \in (-\infty;0)$ and increases for $x \in (0;\infty)$.

Complete Exercise 10 on page 304.

THE EFFECTS OF a AND q ON FUNCTIONS

It is important to understand the effects that a and q have on the shape and position of the graphs that you have learnt thus far.

Example 1: Using the table method sketch the graphs of:

a) $f(x)=x^2$

b) $h(x)=2x^2$

c) $s(x)=\frac{1}{2}x^2$

d) $p(x)=-x^2$

e) $r(x)=-2x^2$

f) $z(x)=-\frac{1}{2}x^2$

a)

x	–4	–2	0	2	4
y	16	4	0	4	16

b)

x	–4	–2	0	2	4
y	32	8	0	8	32

c)

x	–4	–2	0	2	4
y	8	2	0	2	8

d)

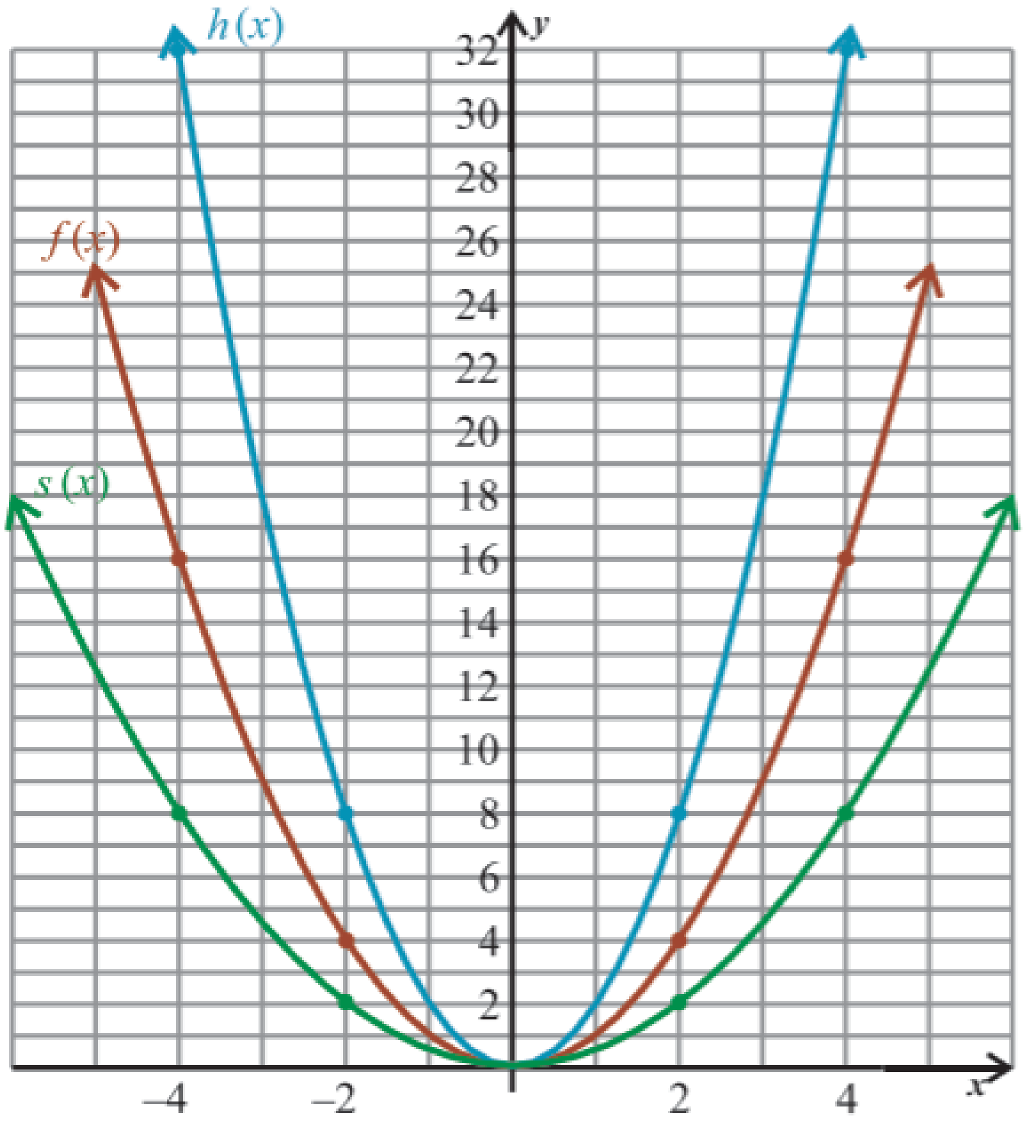
x	–4	–2	0	2	4
y	–16	–4	0	–4	–16

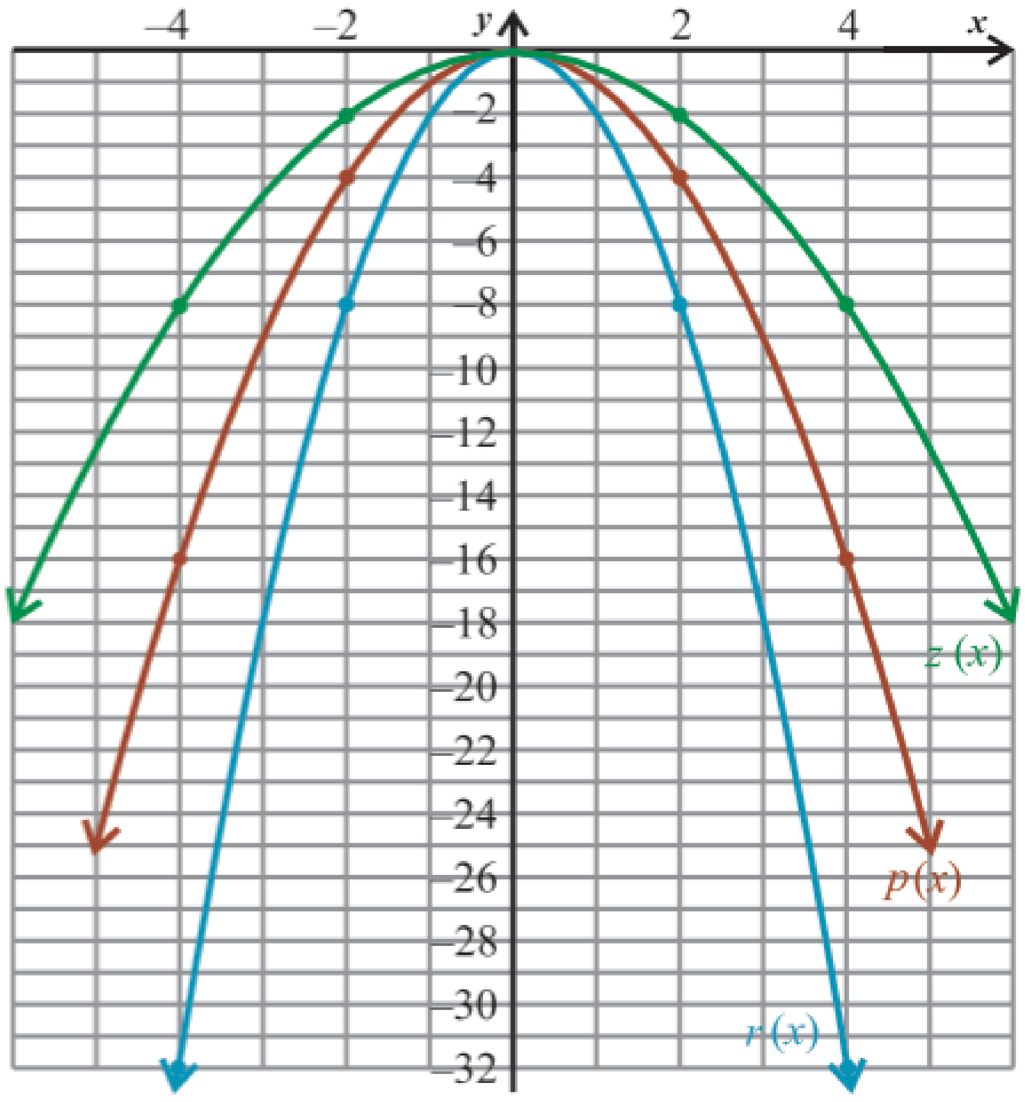
e)

x	–4	–2	0	2	4
y	–32	–8	0	–8	–32

f)

x	–4	–2	0	2	4
y	–8	–2	0	–2	–8





From the above example it should be clear that the value of a effects the graph as follows:

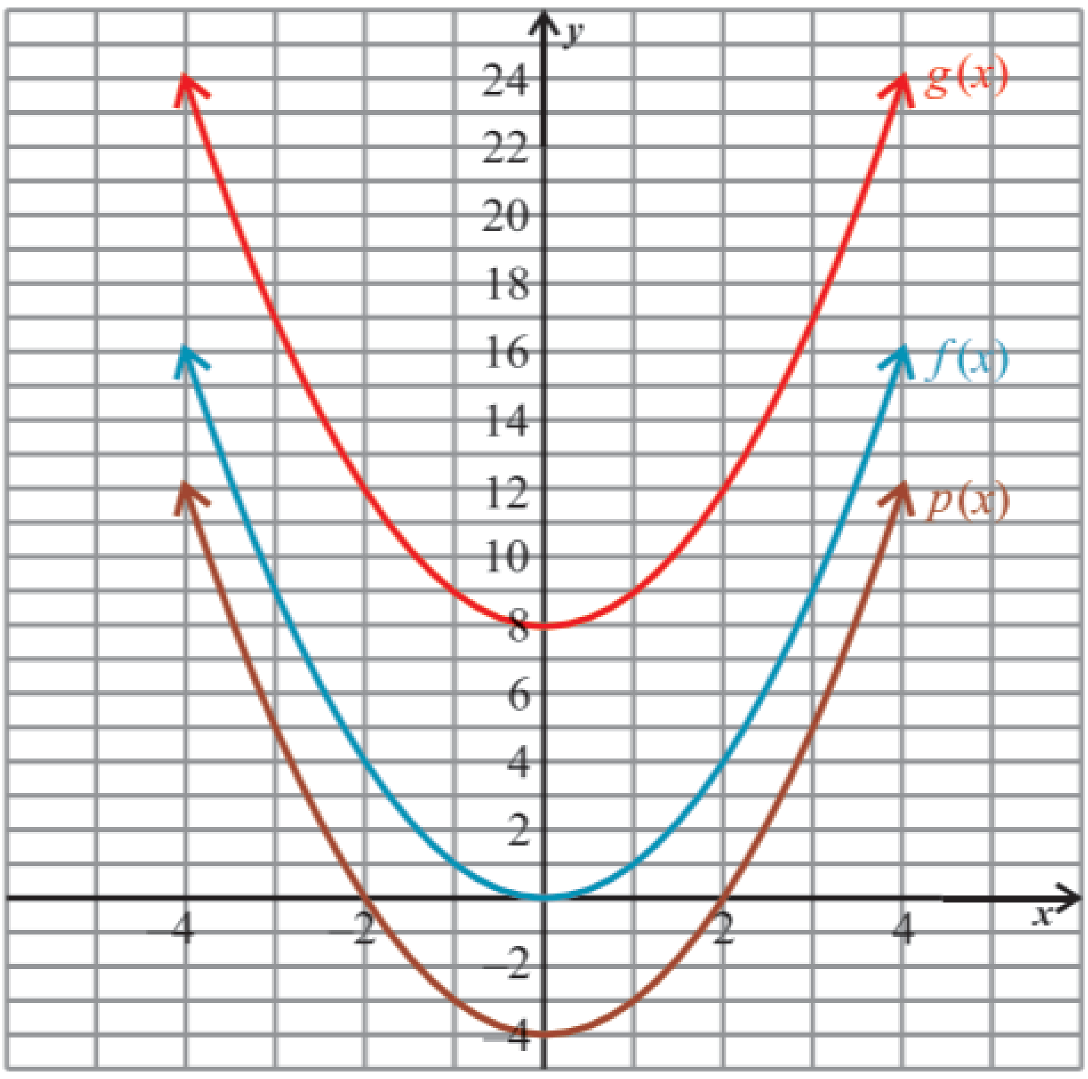
The sign of a determines the shape of the graph (happy or sad) and whether the graph has a maximum value or minimum value.

If $-1<a<1$ (a fraction between –1 and 1) the graph is stretched and gets wider. (note that: $a \neq 0$)

The graph is compressed as the value of a increases (or decreases) and the graph gets narrower.

Example 2: **The graphs of $f(x)=x^2$, $g(x)=x^2+8$ and $p(x)=x^2-4$ are sketched below given that the standard form of a parabola is $f(x)=ax^2+q$:**

a) State how the value of q effects the graph of $f(x)$.
b) Determine the domain and range of $f(x)$, $g(x)$ and $p(x)$.
c) Determine the minimum value of $f(x)$, $g(x)$ and $p(x)$.
d) Hence, determine the maximum value of $h(x)$ if $h(x)=-g(x)$.



a) Changing the value of q results in a vertical translation (shift) of the graph. If $q>0$ the graph will shift up by q units. If $q<0$ the graph will shift down by q units.

b) Domain:

$f(x): x\in\mathbb{R}$ $\quad g(x): x\in\mathbb{R}$ $\quad p(x): x\in\mathbb{R}$

Range:

$f(x): y\in[0;\infty)$ $\quad g(x): y\in[8;\infty)$ $\quad p(x): y\in[-4;\infty)$

c) $f(x)$: minimum value $=0$ $\quad g(x)$: minimum value $=8$ $\quad p(x)$: minimum value $=-4$.

d) $h(x)=-g(x)=-(x^2+8)=-x^2-8$

In order to obtain the graph of h, the graph of g has been reflected about the x-axis. This means that the graph of h has a turning point of -8 and the shape is that of a 'sad face'. Therefore, the graph will have a maximum value of -8.

From the above example it can be seen that the domain, range and maximum/minimum values of a parabola are as follows:

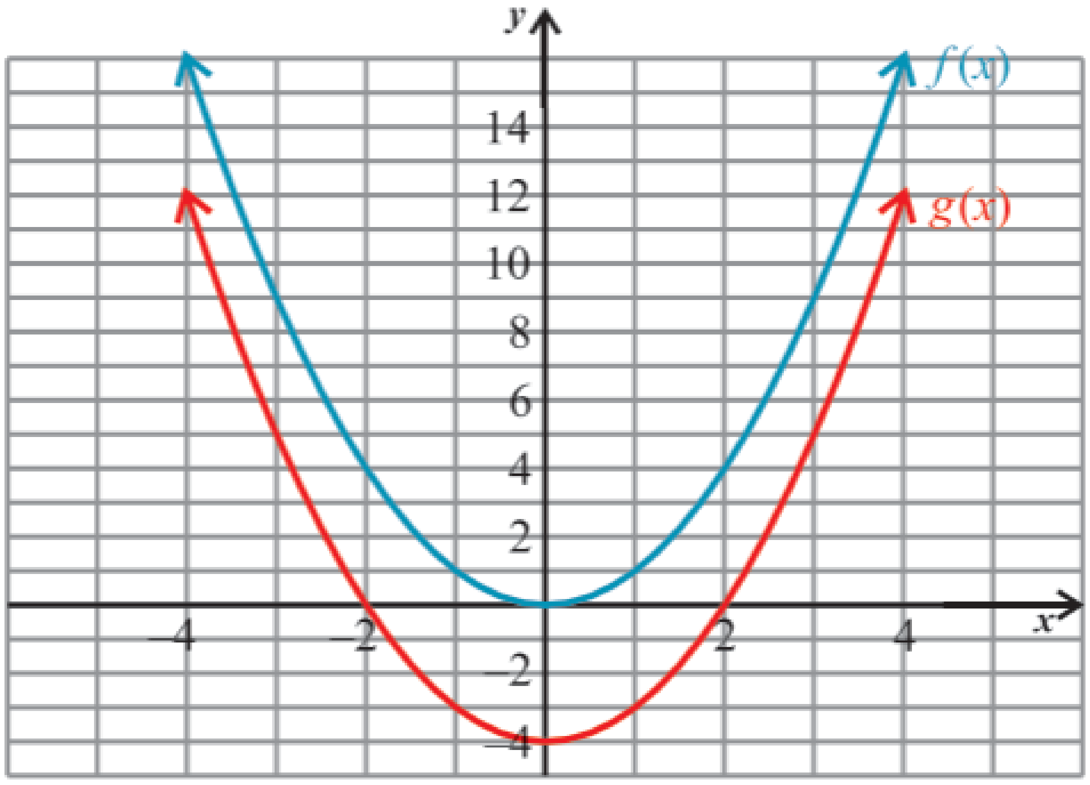
If $a>0$:

The graph will have a minimum value of q and the range will be: $y\in[q;\infty)$.
If $a<0$:

The graph will have a maximum value of q and the range will be: $y\in[-\infty;q)$.
Provided that there are no restrictions on the domain, the domain of any parabola is $x\in\mathbb{R}$. This can also be written as $x\in(-\infty;\infty)$.

REFLECTIONS AND TRANSLATIONS

Illustrative Example: **The graphs of $f(x)=x^2$ and $g(x)=x^2-4$ are sketched below:**



a) Use the table method to sketch the graphs of $h(x)$ and $q(x)$ if $h(x)=-f(x)$ and $q(x)=-g(x)$.

b) Describe the effect $q(x)-6$ would have on the graph of $q(x)$. In your answer discuss how the turning point changes.

a) $h(x)=-f(x)=-x^2$

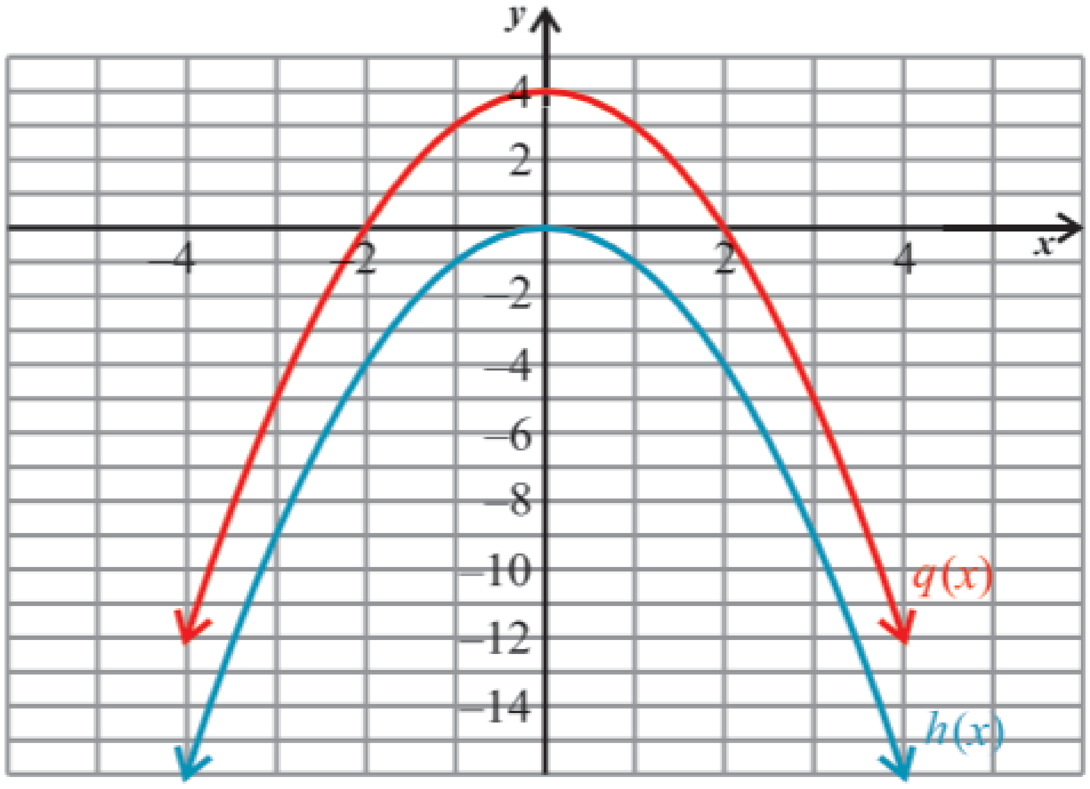
Table for $h(x)$:

x	−2	−1	0	1	2
y	−4	−1	0	−1	−4

$q(x)=-g(x)=-\left(x^2-4\right)=-x^2+4$

Table for $q(x)$:

x	−2	−1	0	1	2
y	0	3	4	3	0



b) $q(x)-6=-x^2+4-6$

$=-x^2-2$

This results in a vertical shift down by 6 units. The graph of $q(x)-6$ will have a turning point (and y-intercept) of –2. In addition, the entire graph will lie below the x-axis. This means that there will also be no x-intercepts.

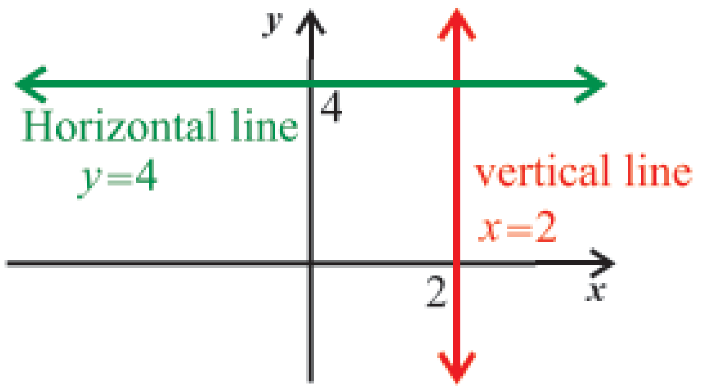
From the above example it should be clear that the effect of:
$-f(x)+q$ reflects the graph of $f(x)$ about the x-axis and shifts every point on the graph vertically by q units. This is the case for any function.

SUMMARY OF FUNCTIONS AND THEIR CHARACTERISTICS

Horizontal and vertical lines:

Form of a horizontal line: $y = k \rightarrow k$ is a number

Form of a vertical line: $x = k \rightarrow k$ is a number

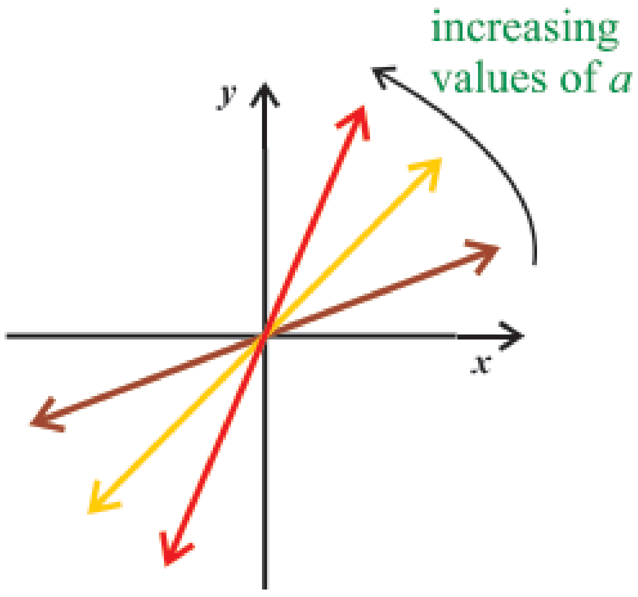


The Straight line graph (The Linear Function):

Forms: $y = mx + c$ *or* $y = ax + q$

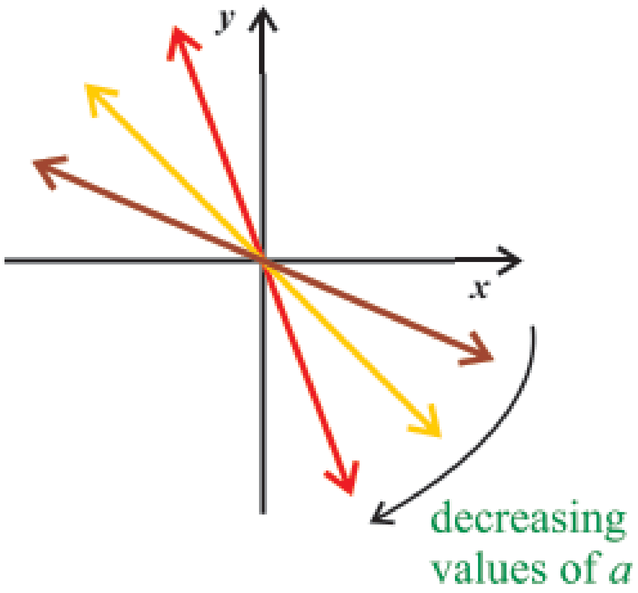
For $a > 0$:

- If the value of m or a is greater than 0 the graph has a positive slope.
- As the value of m or a increases the steepness of the line (slope) increases from left to right.
- c or q is the **y-intercept** and shifts the graph vertically up (for $q > 0$) or down ($q < 0$).



For $a < 0$:

- If value of m or a is less than 0 the graph has a negative slope.
- As the value of m or a decreases the steepness of the line (slope) increases from left to right.
- c or q is the **y-intercept** and shifts the graph vertically up (for $q > 0$) or down (for $q < 0$).



The Parabola (The Quadratic Function):

Form: $y = ax^2 + q$

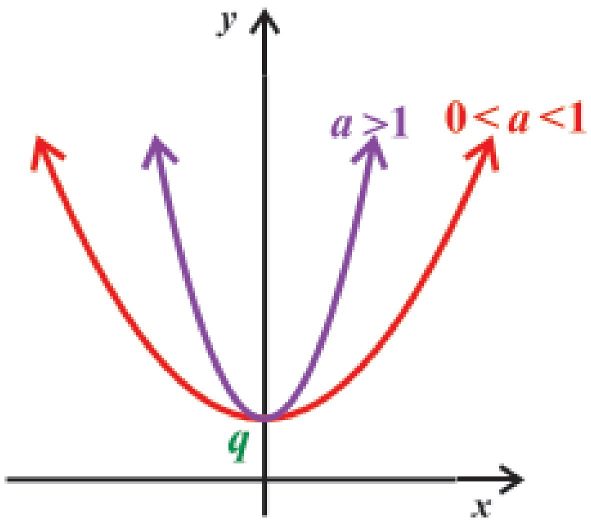
The domain of an unrestricted parabola is: $x \in \mathbb{R} \rightarrow$ also written as $x \in (-\infty; \infty)$

The turning point of parabolas in Grade 10 is $(0; q)$

The axis of symmetry of parabolas in Grade 10 is the y-axis (line of $x = 0$).

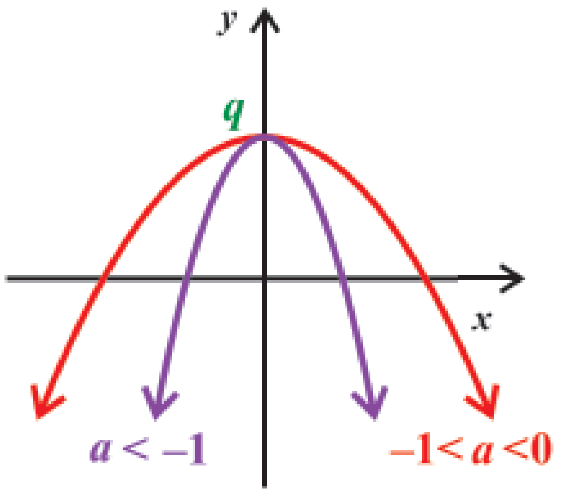
For $a > 0$:

- If $a > 0$ the graph is a 'happy shaped' parabola.
- If $0 < a < 1$ the graph is stretched and gets wider.
- If $a > 1$ the graph is compressed and gets narrower.
- If $a > 0$ the graph has a **minimum** value of q.
- If $a > 0$ the range of a parabola is: $y \in [q; \infty)$.



For $a < 0$:

- If $a < 0$ the graph is a 'sad shaped' parabola.
- If $-1 < a < 0$ the graph is stretched and gets wider.
- If $a < -1$ the graph is compressed and gets narrower.
- If $a < 0$ the graph has a **maximum** value of q.
- If $a < 0$ the range of a parabola is: $y \in (-\infty; q]$.



The Hyperbola (The Hyperbolic Function):

Form: $y = \frac{a}{x} + q$

The domain of the hyperbola is: $x \in \mathbb{R},\ x \neq 0 \rightarrow$ you have to put in the restriction

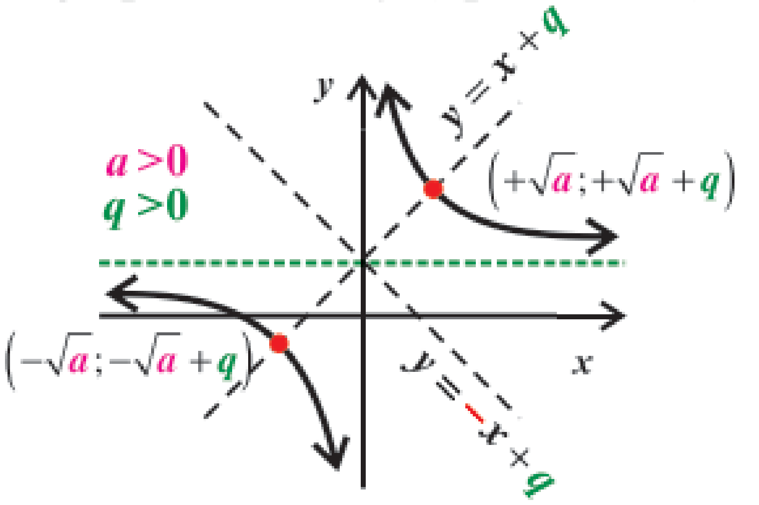
The range of a hyperbola is: $y \in \mathbb{R},\ y \neq q \rightarrow$ you have to put in the restriction

The axes of symmetry of hyperbolas in Grade 10 are $y = x + q$ **and** $y = -x + q$.

The value of q is the **horizontal asymptote** and shifts the graph vertically (up or down).

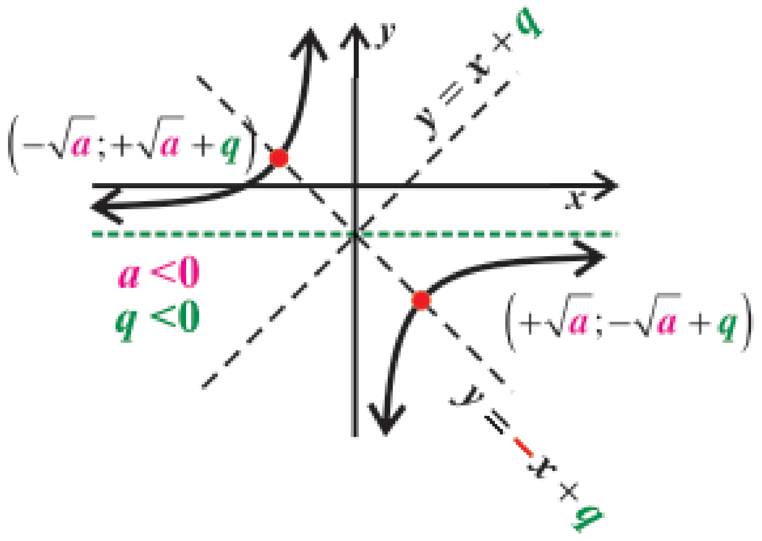
For $a > 0$:

- If $a > 0$ the graph lies in the 1st and 3rd quadrants.
- The axis of symmetry $y = x + q$ intersects the graph at $\left(+\sqrt{a}; +\sqrt{a} + q\right)$ and $\left(-\sqrt{a}; -\sqrt{a} + q\right)$.


$y = x + q$
y
$a > 0$
$q > 0$
$\left(+\sqrt{a}; +\sqrt{a} + q\right)$
$\left(-\sqrt{a}; -\sqrt{a} + q\right)$
x
$y = -x + q$

For $a < 0$:

- If $a < 0$ the graph lies in the 2nd and 4th quadrants.
- The axis of symmetry $y = -x + q$ intersects the graph at $\left(-\sqrt{a}; +\sqrt{a} + q\right)$ and $\left(+\sqrt{a}; -\sqrt{a} + q\right)$.


y
$y = x + q$
$\left(-\sqrt{a}; +\sqrt{a} + q\right)$
x
$a < 0$
$q < 0$
$\left(+\sqrt{a}; -\sqrt{a} + q\right)$
$y = -x + q$

The Exponential Function:

Form: $y = ab^x + q$ **where** $b > 0$ **and** $b \neq 0$

The **domain** of an exponential function is: $x \in \mathbb{R} \rightarrow$ also be written as $x \in (-\infty; \infty)$

The **range** of an exponential function is:

- $y > q$ if $a > 0 \rightarrow$ also be written as $y \in (q; \infty)$
- $y < q$ if $a < 0 \rightarrow$ also be written as $y \in (-\infty; q)$

The value of q is the **horizontal asymptote** and shifts the graph vertically (up or down).

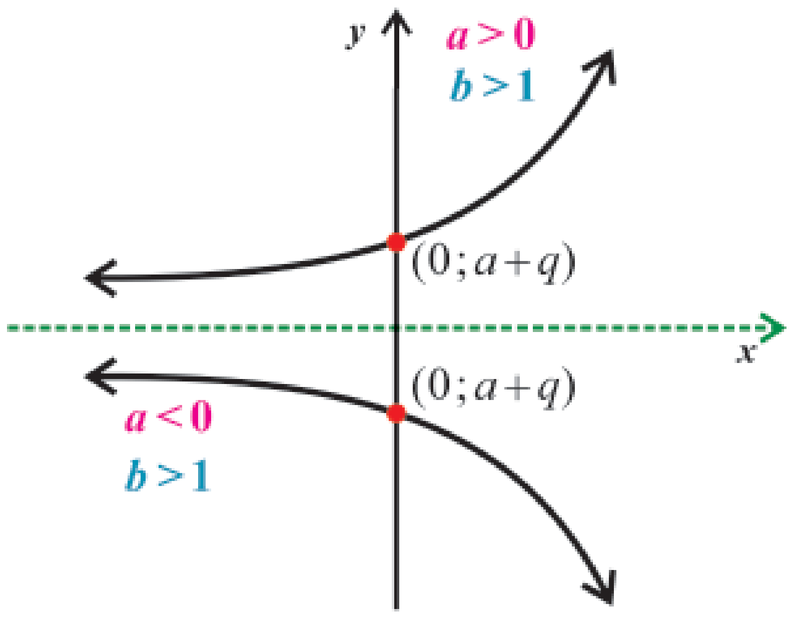
The **y-intercept** is always at the point $(0; a + q)$.

For $a > 0$ and $b > 1$:

- The function is **increasing** and is above the horizontal asymptote.

For $a < 0$ and $b > 1$:

- The function is **decreasing** and is below the horizontal asymptote.

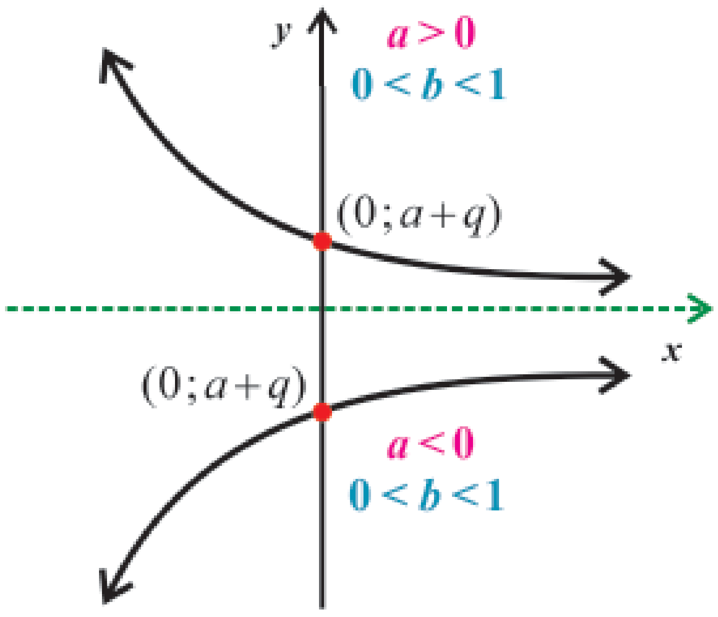

y
$a > 0$
$b > 1$
$(0; a + q)$
x
$(0; a + q)$
$a < 0$
$b > 1$

For $a > 0$ and $0 < b < 1$:

- The function is **decreasing** and is above the horizontal asymptote.

For $a < 0$ and $0 < b < 1$:

- The function is **increasing** and is below the horizontal asymptote.


y
$a > 0$
$0 < b < 1$
$(0; a + q)$
x
$(0; a + q)$
$a < 0$
$0 < b < 1$

G. GRAPH INTERPRETATION

LENGTHS OF LINES

There are three types of lines that you will be required to find the lengths of:

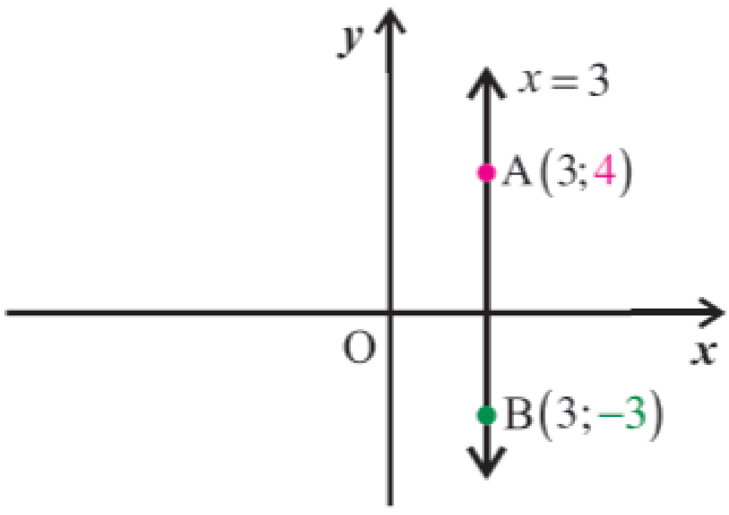
1. **Vertical Lines**
2. **Horizontal Lines**
3. **Slanted Lines (Oblique Lines)**

1. VERTICAL LINES

The length of any vertical line AB, is the y-value at the top of the line minus the y-value at the bottom of the line:

$$AB = y_{top} - y_{bottom}$$

Example 1: The graph of $x = 3$ is sketched below. Determine the length of AB.

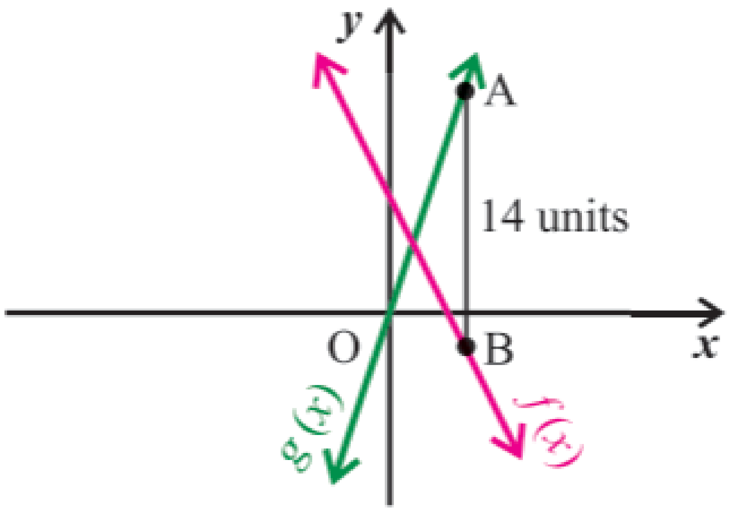


$AB = y_{top} - y_{bottom}$

$= 4 - (-3)$

$= 7$ units → always put in units

Example 2: The graphs of $f(x) = -2x + 6$ and $g(x) = 3x$ are sketched below. Determine the coordinates of A and B if the length of AB is 14 units.



$AB = y_{top} - y_{bottom}$

$\therefore 14 = g(x) - f(x) \rightarrow$ $g(x)$ and $f(x)$ are the 'y' values of each of the graphs

$\therefore 14 = 3x - (-2x + 6)$

$\therefore 14 = 3x + 2x - 6$

$\therefore 20 = 5x$

$\therefore x = 4 \rightarrow$ this is the x-value at points A and B

In order to find the y-values at A and B, the x-value is substituted into $f(x)$ and $g(x)$

$\therefore f(4) = -2(4) + 6 = \mathbf{-2}$ and $g(4) = 3(4) = \mathbf{12}$

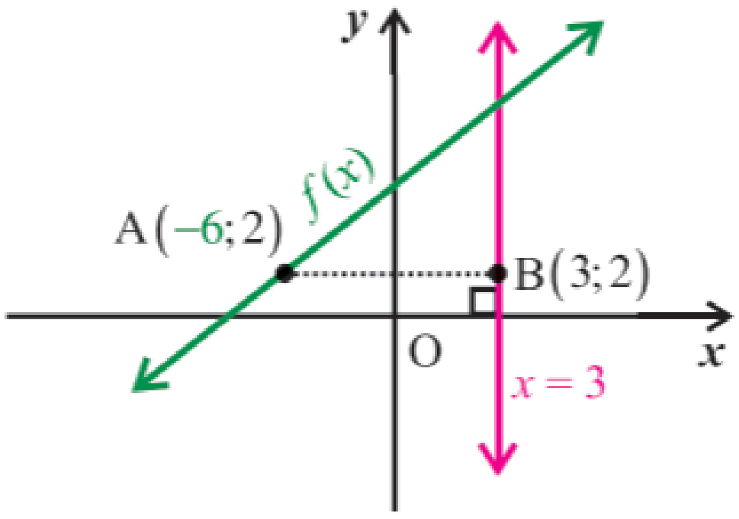
$\therefore$ A is the point $(4;12)$ and B is the point $(4;-2)$

2. HORIZONTAL LINES

The length of any horizontal line AB, is the x-value at the right of the line minus the x-value at the left of the line:

$$\mathbf{AB} = x_{right} - x_{left}$$

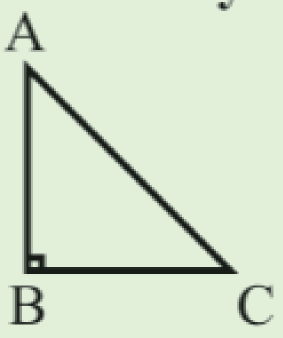
Example: **The graphs of $f(x) = x + 8$ and $x = 3$ are sketched below. Determine the length of AB.**



$$\text{AB} = x_{right} - x_{left}$$
$$= 3 - (-6) = 9 \text{ units} \rightarrow \text{always put in units}$$

3. SLANTED LINES

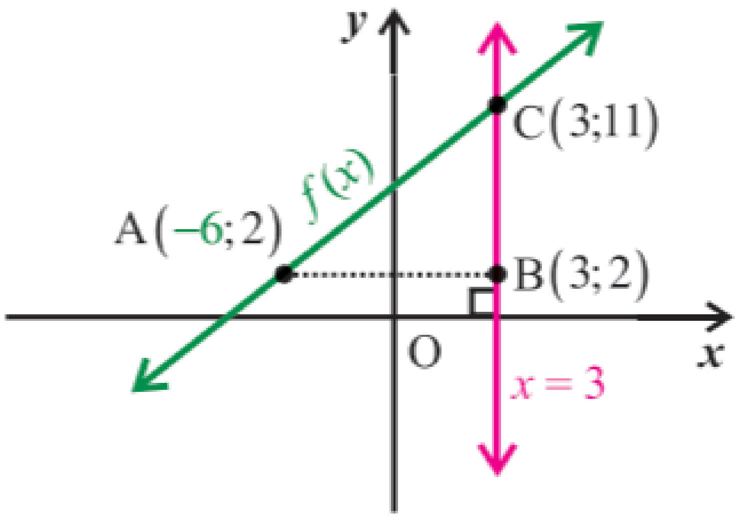
To calculate the length of any slanted line you use the theorem of Pythagoras:



$$\text{AC}^2 = \text{AB}^2 + \text{BC}^2 \quad \textbf{or} \quad \text{AC} = \sqrt{\text{AB}^2 + \text{BC}^2}$$

The distance formula (pg. 156) can also be used to calculate the lengths of slanted lines.

Example: **The graphs of $f(x) = x + 8$ and $x = 3$ are sketched below. Determine the length of AB, BC and AC.**



$$\text{AB} = x_{right} - x_{left}$$
$$= 3 - (-6)$$
$$= 9 \text{ units} \rightarrow \text{always put in units}$$

$$\text{BC} = y_{top} - y_{bottom}$$
$$= 11 - 2$$
$$= 9 \text{ units} \rightarrow \text{always put in units}$$

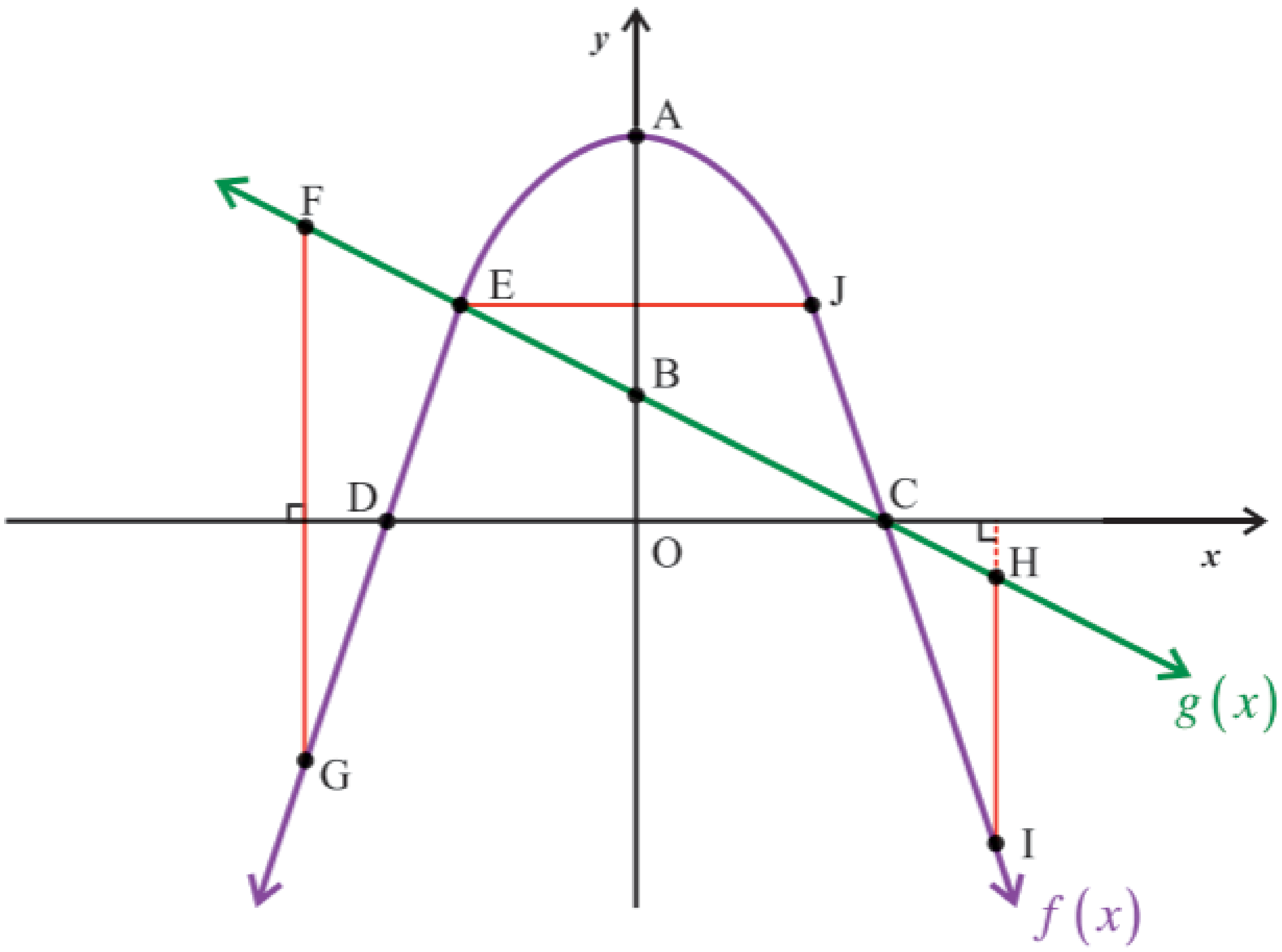
$$\text{AC} = \sqrt{\text{AB}^2 + \text{BC}^2} = \sqrt{9^2 + 9^2} = \sqrt{162} = 9\sqrt{2} \text{ units} \rightarrow \text{always put in units}$$

Complete Exercise 11 on page 304.

MIXED EXAMPLES

Example 1: **The graphs of $f(x)=-2x^2+8$ and $g(x)=-3x+6$ are sketched below. Determine:**

a) The coordinates of A, B, C and D.
b) The coordinates of E.
c) The lengths of AB and CD.
d) The length of HI, if the x-value of H is 10.
e) The coordinates of F and G if FG is 25 units long.
f) The y-coordinate of J if EJ is parallel to the x-axis.
g) The co-ordinates of J.
h) The values of x for which $f(x)>g(x)$.
i) The values of x for which $\frac{f(x)}{g(x)}\geq 0$.



a) A is the y-intercept of the parabola. This can be read from the equation.
$\therefore$ A is the point $(0;8)$

B is the y-intercept of the straight line which can be read from the equation
$\therefore$ B is the point $(0;6)$

C and D are the x-intercepts of the parabola. Therefore let $f(x)=0$

$\therefore -2x^2+8=0 \rightarrow$ take out a HCF of -2

$\therefore -2(x^2-4)=0 \rightarrow$ difference of two squares, you can divide both sides by -2

$\therefore (x-2)(x+2)=0$

$\therefore x=2$ or $x=-2$

$\therefore$ C is the point $(2;0)$ and D is the point $(-2;0) \rightarrow$ D is on the negative x-axis

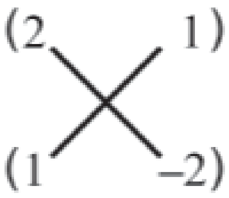
b) E is a point of intersection of the two graphs. At this point the y-values are the same.

$\therefore f(x) = g(x) \rightarrow$ at E the y-values are equal

$\therefore -2x^2 + 8 = -3x + 6$

$\therefore -2x^2 + 3x + 2 = 0 \rightarrow$ divide both sides by -1

$\therefore 2x^2 - 3x - 2 = 0 \rightarrow$ cross method



$\therefore (2x+1)(x-2) = 0$

$\therefore x = -\frac{1}{2}$ or $x = 2 \rightarrow$ the graphs intersect at two points, E is on the negative x-axis

You now have to substitute the x-value back into the equation of the parabola or straight line graph in order to obtain the y-value at point E.

$\therefore y = -3\left(-\frac{1}{2}\right) + 6 = \frac{15}{2} \rightarrow$ always substitute into the easier equation (linear function)

$\therefore$ E is the point $\left(-\frac{1}{2}; \frac{15}{2}\right)$

c) AB is a vertical line.

$\therefore AB = y_{top} - y_{bottom} = 8 - 6 = 2$ units $\rightarrow$ always give the units

CD is a horizontal line.

$\therefore CD = x_{right} - x_{left} = 2 - (-2) = 4$ units $\rightarrow$ always give the units

d) HI is a vertical line

$\therefore HI = y_{top} - y_{bottom}$

The top y-value is the value of $g(x)$ when $x = 10$ ie. $g(10)$

$\therefore g(10) = -3(10) + 6 \;= \mathbf{-24}$

The bottom y-value is the value of $f(x)$ when $x = 10$ ie. $f(10)$

$\therefore f(10) = -2(10)^2 + 8 \;= \mathbf{-192}$

$\therefore HI = \mathbf{-24} - (\mathbf{-192})$

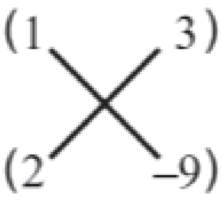
$\quad = 168$ units $\rightarrow$ always give the units

e) $FG = g(x) - f(x) \rightarrow y_{top} - y_{bottom}$

$\therefore 25 = -3x + 6 - (-2x^2 + 8)$

$\therefore 25 = -3x + 6 + 2x^2 - 8$

$\therefore 0 = 2x^2 - 3x - 27 \rightarrow$ cross method



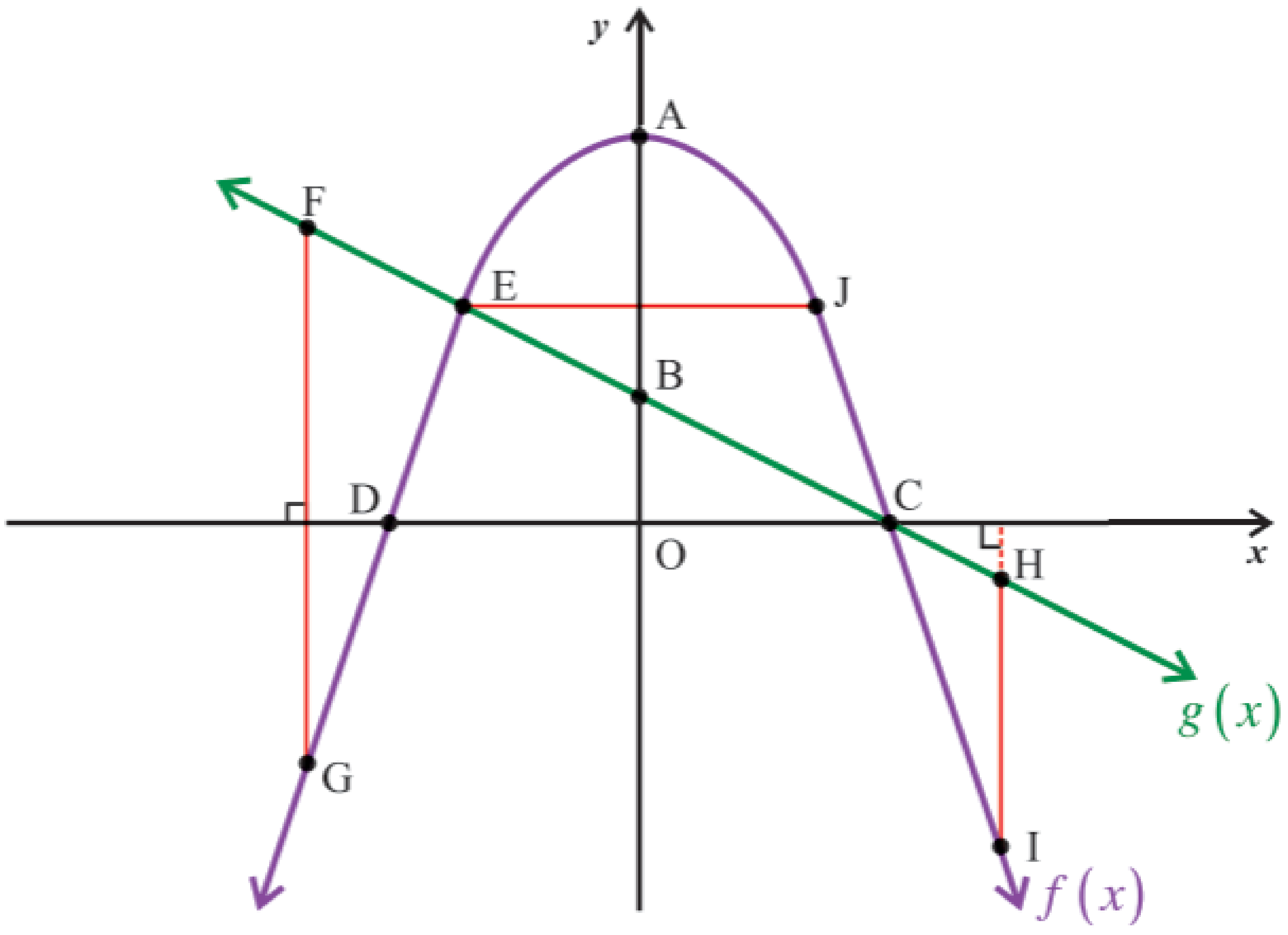
$\therefore (x+3)(2x-9) = 0$

$\therefore x = -3$ or $x = \frac{9}{2} \rightarrow -3$ is on the negative side of the x-axis

You now have the x-value at F and G. In order to get the y-values you have to substitute the x-value into $f(x)$ and $g(x)$.

$g(-3) = -3(-3) + 6 = 15$ and $f(-3) = -2(-3)^2 + 8 = -10$

$\therefore$ F is the point $(-3;15)$ and G is the point $(-3;-10)$



f) AS EJ is parallel to the x-axis, the y-value of E is the same as the y-value at J.

$\therefore$ The y-value at J is $\frac{15}{2}$

g) The parabola is symmetrical about the y-axis. This means that J is a reflection of E about the y-axis.

$\therefore$ J is the point $\left(\frac{1}{2};\frac{15}{2}\right) \rightarrow$ the x-value at E is $-\frac{1}{2}$

h) The question is asking for what values of x are the y-values of $f(x) > g(x)$.

This occurs from point E up until point C.

$\therefore f(x) > g(x)$ where $x \in \left(-\frac{1}{2};2\right) \rightarrow$ this could also be written as $-\frac{1}{2} < x < 2$

i) The question is asking for what values of x are the y-values of $\frac{f(x)}{g(x)} \geq 0$.

For this to happen $f(x)$ and $g(x)$ both have to be positive or both have to be negative, as $\frac{'+'}{'+'} \rightarrow '+'$ or $\frac{'-'}{'-'} \rightarrow '+'$

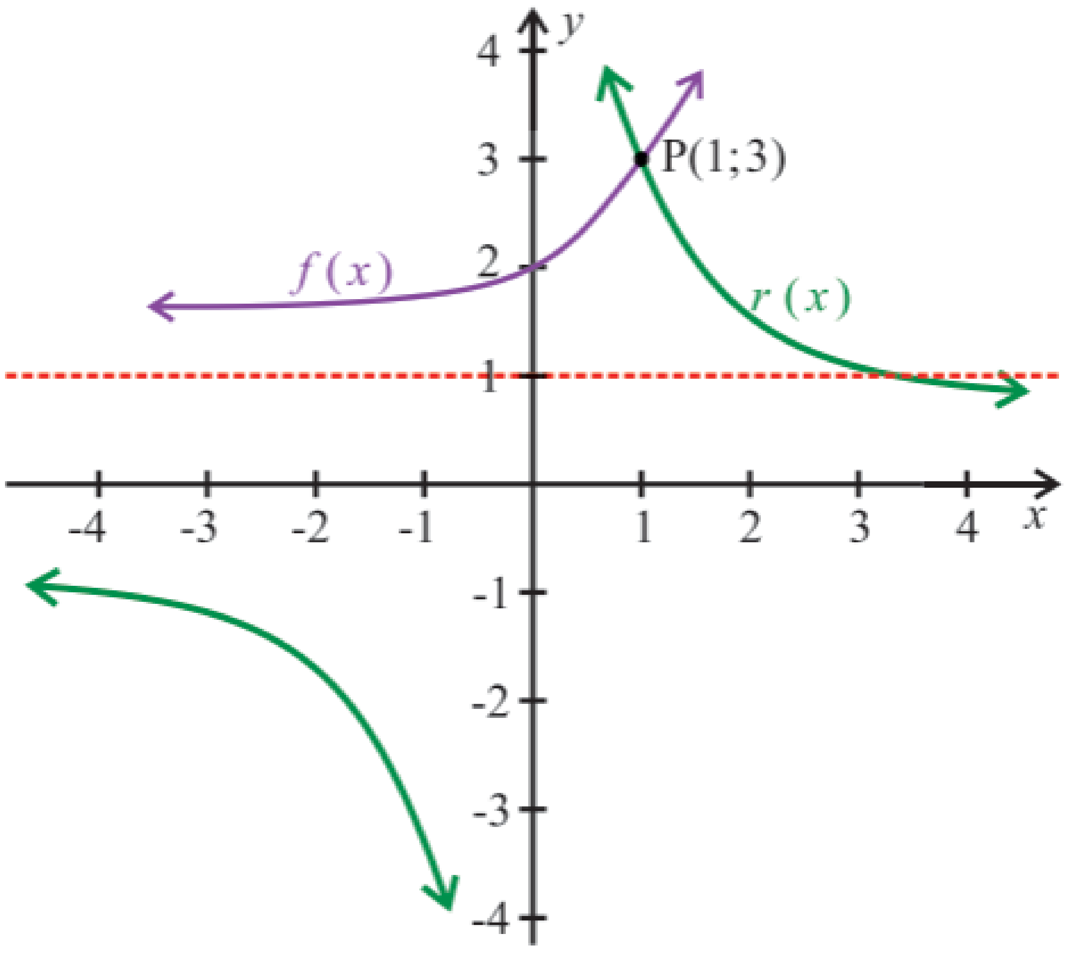
- From points D to C $f(x)$ and $g(x)$ are both positive.
- From point C onwards $f(x)$ and $g(x)$ are both negative.
- It is important to note that at the point where $x = 2$, $g(x) = 0$. Therefore $x \neq 2$ as division by 0 would occur.

$\therefore \frac{f(x)}{g(x)} \geq 0$ where $x \in [-2;\infty)$; $x \neq 2 \rightarrow$ this could also be written as $x \geq -2$; $x \neq 2$

Remember that where a graph lies above the x-axis it is positive and where it lies below the x-axis the graph is negative.

Example 2: **The graphs of $f(x)=b^x+q$ and $r(x)=\frac{a}{x}$ intersect at the point P(1;3) as shown below. Determine:**

a) The equations of $f(x)$ and $r(x)$.
b) The range of $f(x)$.
c) The domain of $r(x)$.



a) $r(x)=\frac{a}{x} \rightarrow y=\frac{a}{x}$

Substitute $P(1;3)$

$\therefore 3=\frac{a}{1}$

$\therefore a=3$

$f(x)=b^x+1 \rightarrow q$ is the asymptote of $f(x)$ which can be seen from the graph

Substitute $P(1;3)$

$\therefore 3=b^1+1$

$\therefore 2=b$

$\therefore f(x)=(2)^x+1$ and $r(x)=\frac{3}{x} \rightarrow$ write down the equations of the graphs

b) $y \in (1;\infty) \rightarrow$ this could also be written as $y>1$

c) $x \in \mathbb{R}$; $x \neq 0 \rightarrow$ make sure that you include the restriction

Complete Exercise 12 on pages 305 and 306.

H. MATHEMATICAL MODELLING WITH FUNCTIONS

Example 1: **A student sets up an experiment to measure the resistance (R) of a resistor in a circuit. He does this by changing the current (I) in the circuit and measuring the voltage (V) over the resistor. The table below shows some of the student's results.**

The student uses the formula $R = \frac{V}{I}$ to calculate resistance.

I	**2**	**4**			**10**
V	**1**	**2**	**3**	**4**	

a) Fill in the missing values in the table.

b) Which is the independent and which is the dependent variable in this experiment?

c) Write down a formula that connects the variables V and I.

d) Draw a graph of current against voltage.

e) If the trend continues, use your graph to determine the value of the current when the voltage reads eight volts.

f) What does the gradient of the graph represent?

a)

I	2	4	6	8	10
V	**1**	**2**	**3**	**4**	**5**

b) Voltage is changed by varying the current. Therefore voltage is dependent on current. Therefore, voltage is the dependent variable and current is the independent variable.

c) If current increases by 2 the voltage increases by 1. This is a linear relationship.

$\therefore y = mx + c$

$\therefore m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{4-2} = \frac{1}{2} \rightarrow$ any set of values from the table can be used

Substitute another value into the equation to solve for c: $(6;3)$

$\therefore 3 = \frac{1}{2}(6) + c$

$\therefore c = 0$

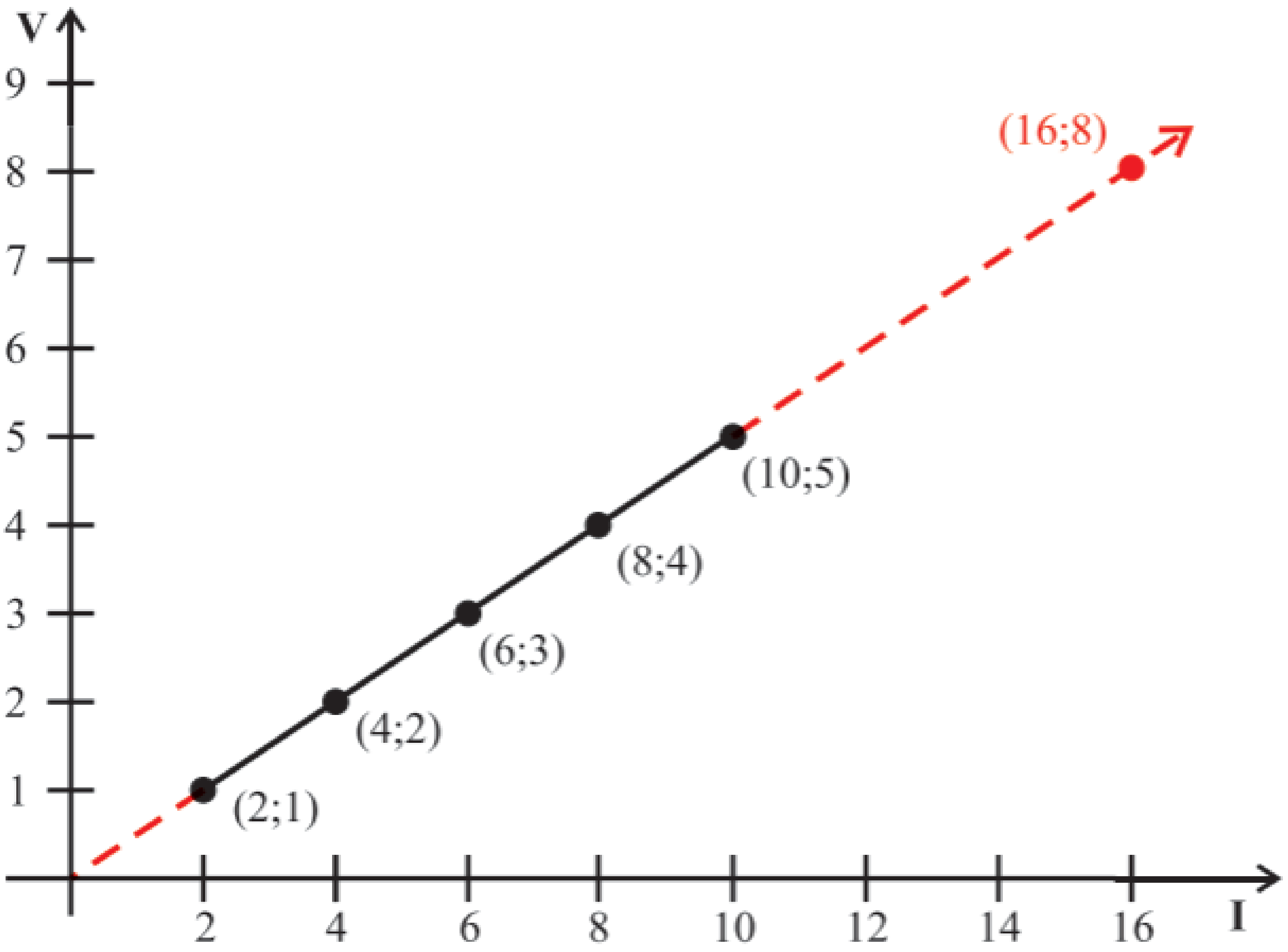
$\therefore y = \frac{1}{2}x$

$\therefore V = \frac{1}{2}I$

The dependent variable is always plotted on the *y*-axis and the independent variable is always plotted on the *x*-axis.

I	2	4	6	8	10
V	1	2	3	4	5

d)



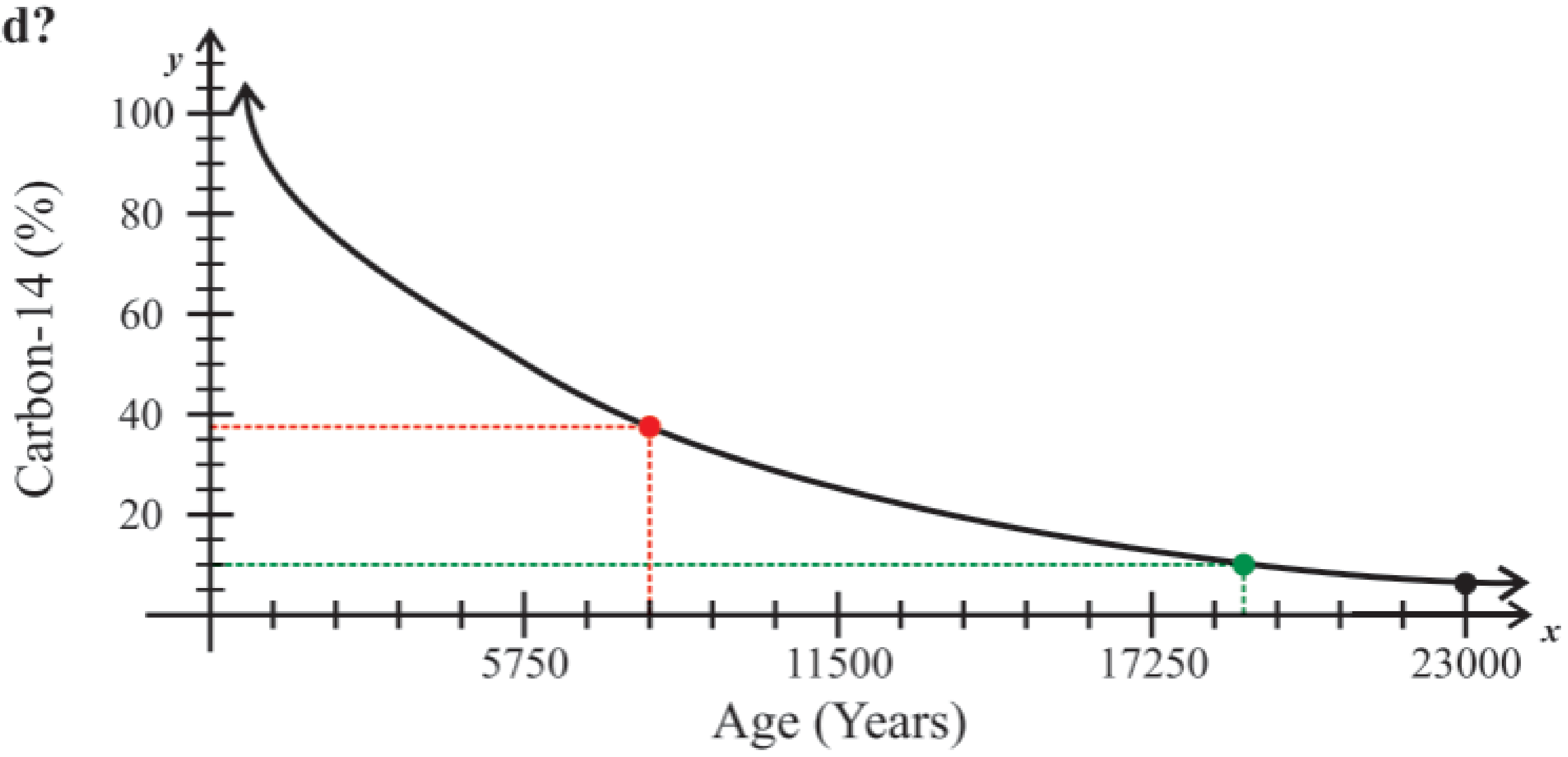
e) Draw a horizontal line from 8 on the y-axis (Voltage axis) until you reach the graph. Then draw a vertical line down to the x-axis (Current axis).
$\therefore$ At 8 volts the current will be 16 A → current is measured in Amperes

f) The gradient of the graph represents Resistance. → gradient $= \dfrac{\text{change in } y}{\text{change in } x}$ → $R = \dfrac{V}{I}$

Example 2:

Scientists use a technique called carbon dating to determine the age of ancient artefacts. To do this they measure the amount of Carbon-14 that is present in the artefact. The rate of decay of Carbon-14 fits an exponential function which is shown below. Using the graph below determine:

a) The age of an artefact that has 37,5% Carbon-14 present.
b) What percentage of Carbon-14 would be present in a sample that is 18 975 years old?

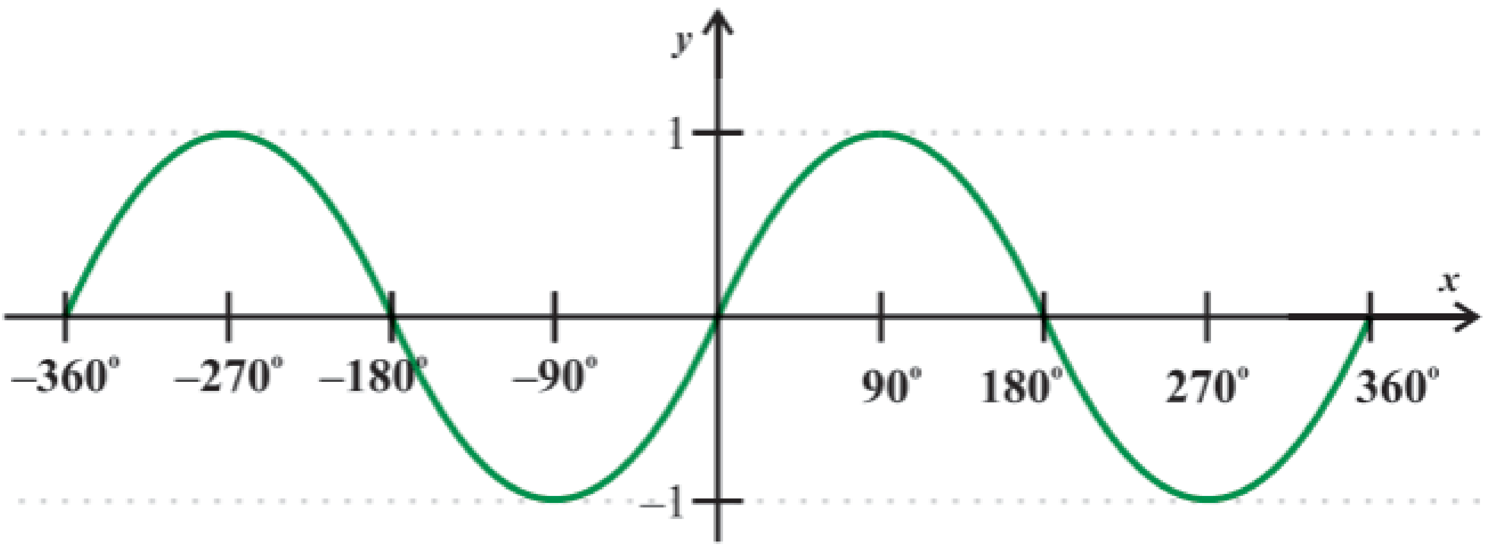


a) In order to answer this question, draw a horizontal line from the y-axis at 37,5% until it reaches the graph. At this point draw a vertical line down to the x-axis. The value on the x-axis will be the artefact's age.
$\therefore$ the artefact would be 8 050 years old → the x-axis increases in increments of 1150

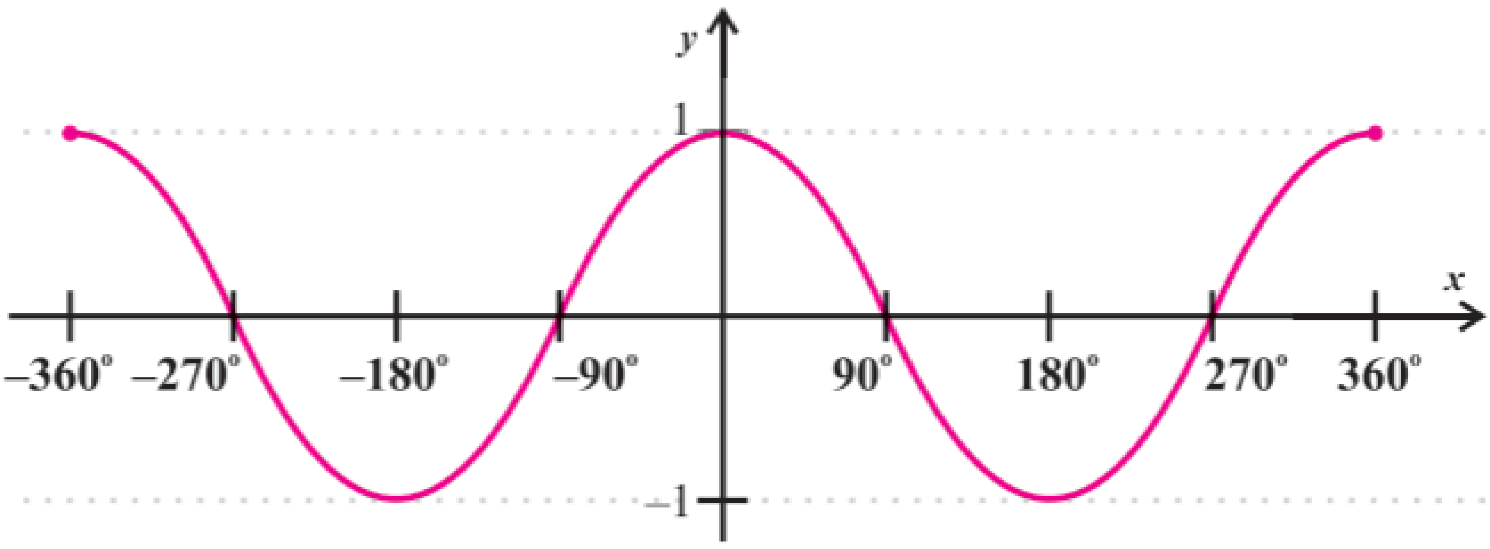
b) In order to answer this question, draw a vertical line up from the x-axis at 18 975 years until it reaches the graph. At this point draw a horizontal line across to the y-axis. The value on the y-axis will be the percentage of carbon-14 remaining.
$\therefore$ 10% of carbon-14 will be left in the sample. → the y-axis increases in increments of 5

I. TRIGONOMETRIC FUNCTIONS

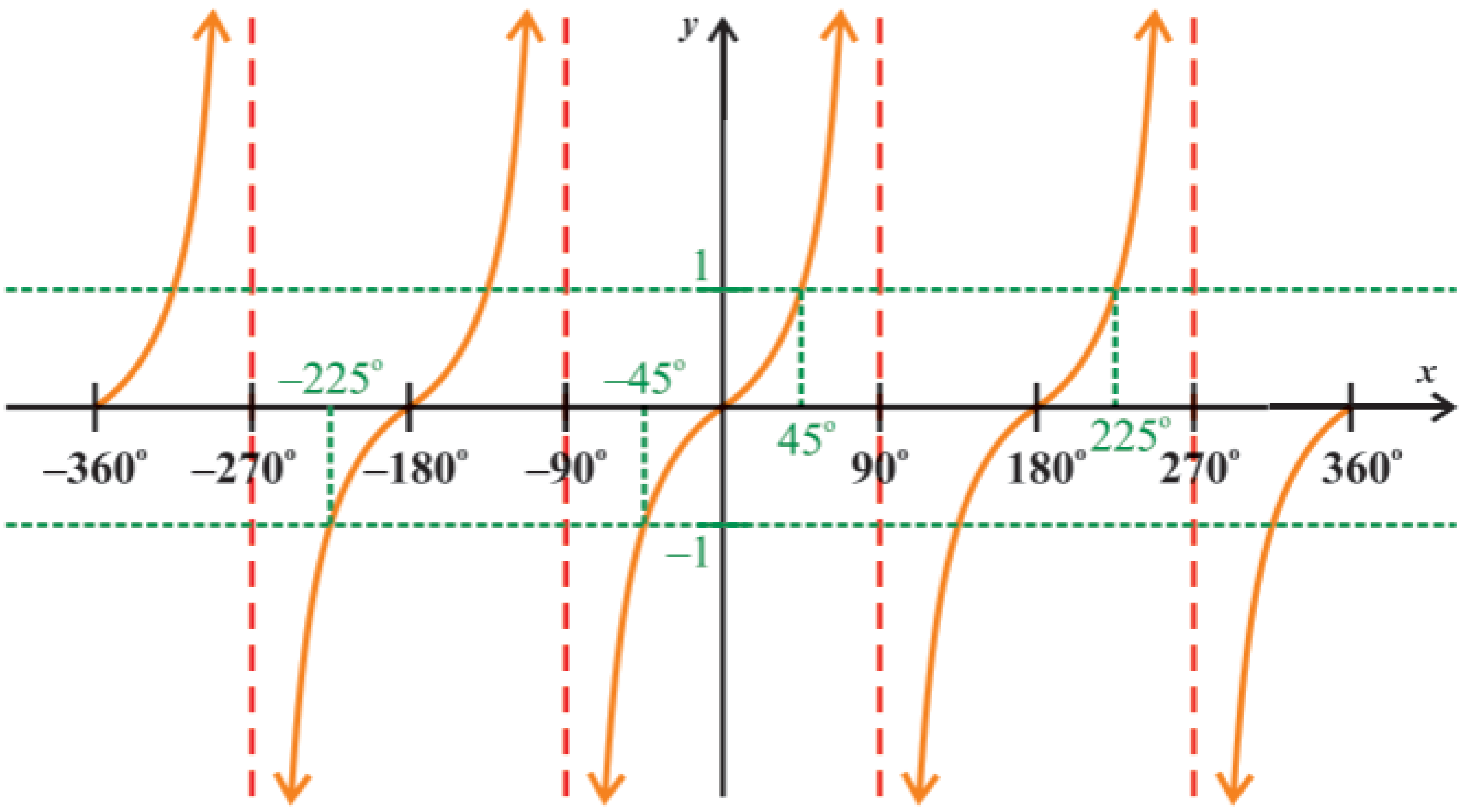
In the diagram below the graph of $y = \sin x$ is sketched for $x \in [-360^\circ ; 360^\circ]$. If the domain of the graph is not restricted the domain is $x \in \mathbb{R}$. The Range of the graph below is $y \in [-1;1]$. The period of a Sine graph is 360°.



In the diagram below the graph of $y = \cos x$ is sketched for $x \in [-360^\circ ; 360^\circ]$. If the domain of the graph is not restricted the domain is $x \in \mathbb{R}$. The Range of the graph below is $y \in [-1;1]$. The period of a Cosine graph is 360°.



In the diagram below the graph of $y = \tan x$ is sketched for $x \in [-360^\circ ; 360^\circ]$. Take note that in the graph there are asymptotes at $x = \pm 90^\circ$ and $x = \pm 270^\circ$. The range of a Tan graph is $y \in \mathbb{R}$. The period of a Tan graph is 180°.



The period of a trigonometric function is how often the graph repeats itself. In Grade 10 the period of the above trigonometric functions will not change.

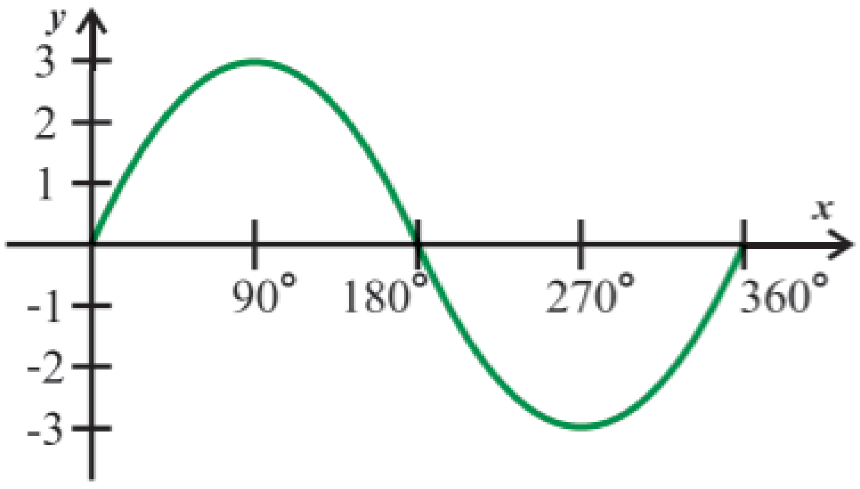
CHANGE IN AMPLITUDE

In the case of a change in amplitude the graphs will be multiplied by a. This causes a change in the amplitude by a factor of a. The graphs will be in the forms of:

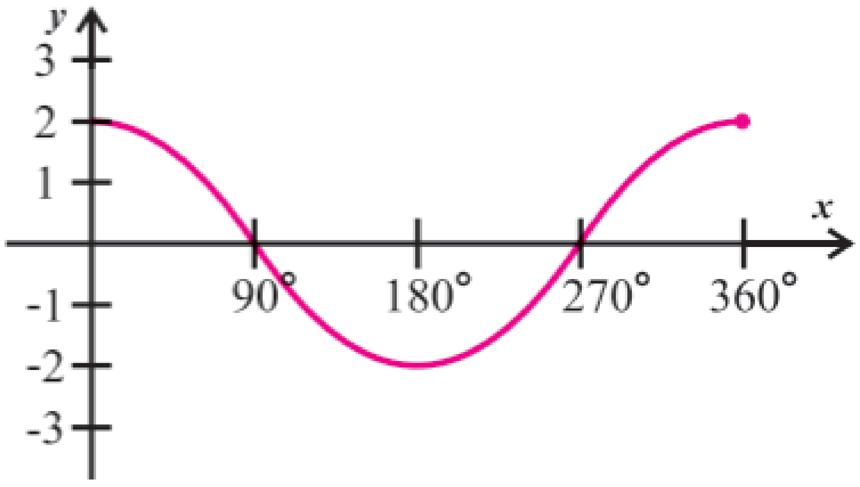
$$y = a\sin(x) \qquad y = a\cos(x) \qquad y = a\tan(x)$$

Example 1: **On separate axes sketch the graphs of:**

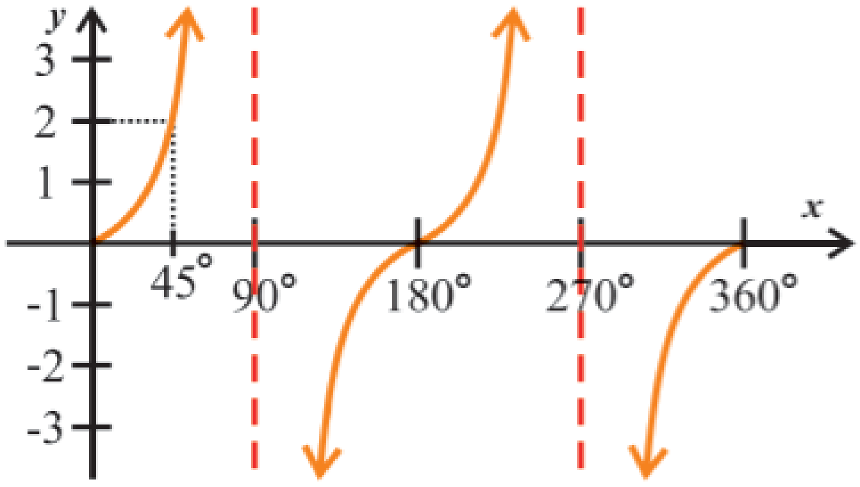
$$y = 3\sin x,\ y = 2\cos x \text{ and } y = 2\tan x \text{ where } x \in [0^\circ;360^\circ]$$



$y = 3\sin x$



$y = 2\cos x$



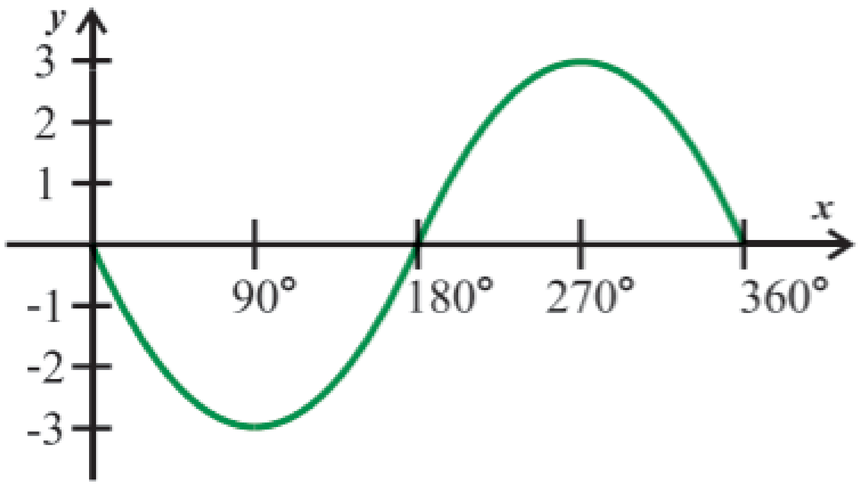
$y = 2\tan x$

The standard tan graph (where a is equal to 1) has a y-value of 1 at 45 degrees. In this case, as a is equal to 2, the y-value at 45 degrees is equal to 2.

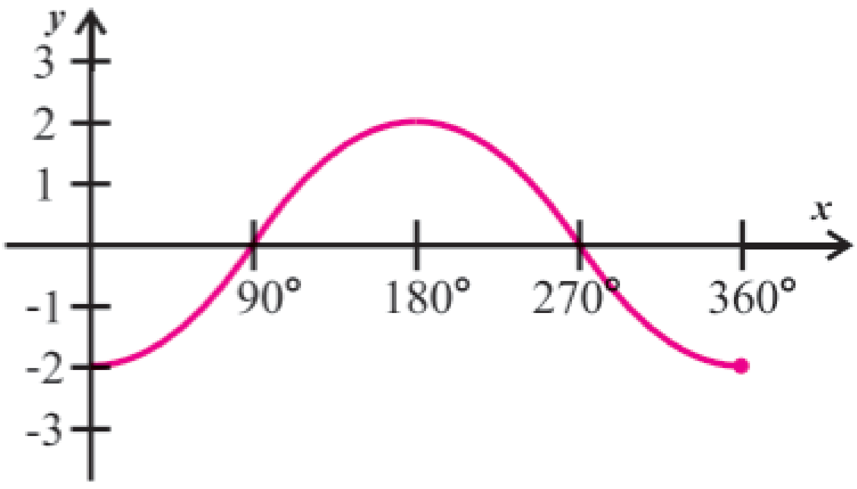
The domain is restricted in the question. Therefore, the graph is drawn from 0 to 360 degrees.

Example 2: **On separate axes sketch the graphs of:**

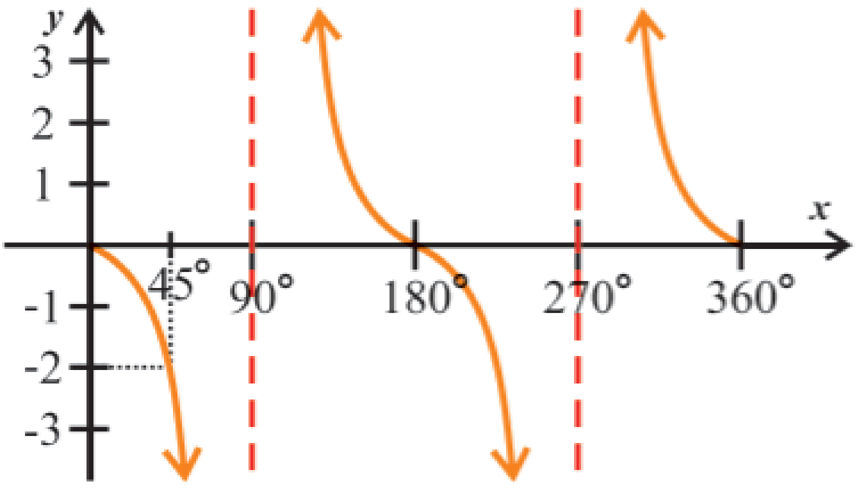
$$y = -3\sin x,\ y = -2\cos x \text{ and } y = -2\tan x \text{ where } x \in [0^\circ;360^\circ]$$



$y = -3\sin x$



$y = -2\cos x$



$y = -2\tan x$

Multiplying by a negative reflects the graph about the x-axis. In other words, all of the y-values become negative

Complete Exercise 13 numbers 1 and 2 on page 306.

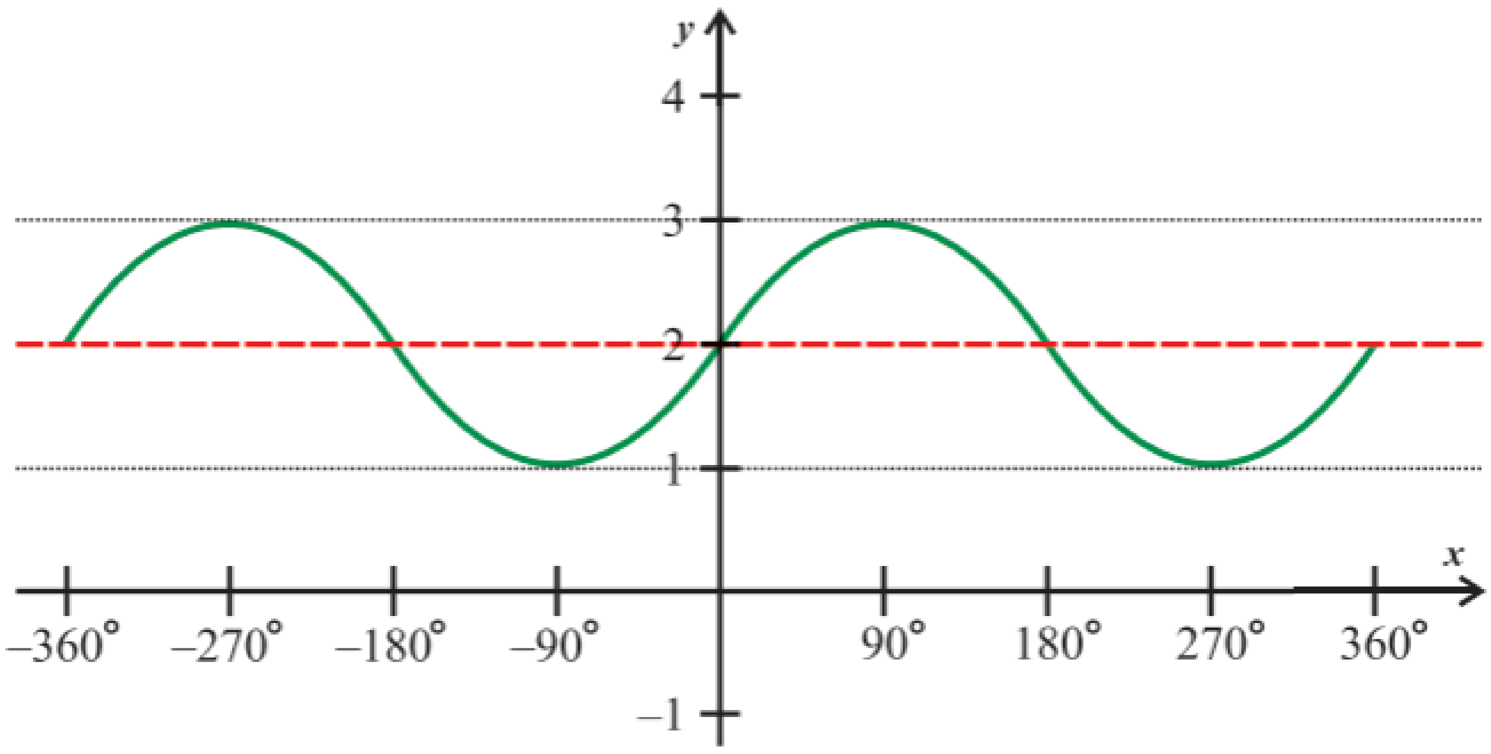
SHIFTING TRIGONOMETRIC FUNCTIONS UP AND DOWN

In the case of a vertical shift the graphs will be in the form of:

$y = a\sin(x) \pm q$ $\qquad$ $y = a\cos(x) \pm q$ $\qquad$ $y = a\tan(x) \pm q$

If q is **added** to the **graph** it **will shift up** by q units.
If q is **subtracted** from the **graph** it **will shift down** by q units.

Example 1: Sketch the graph of $y = \sin x + 2$ where $x \in [-360°; 360°]$

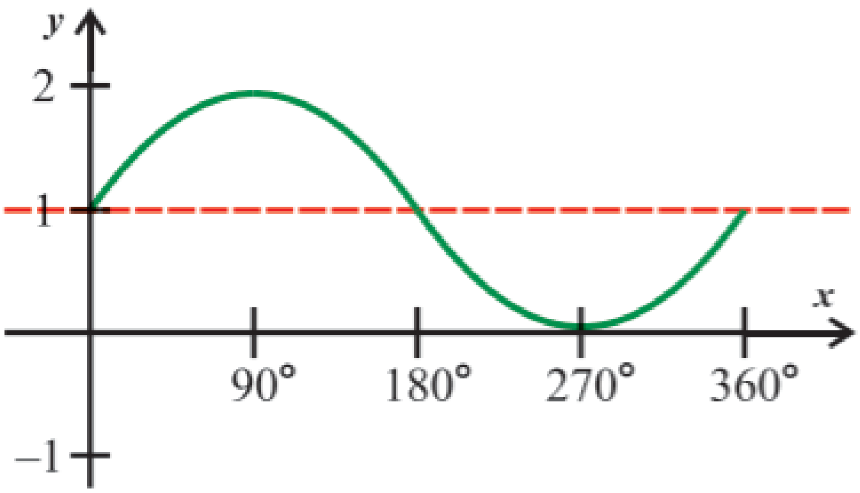


$y = \sin x + 2$

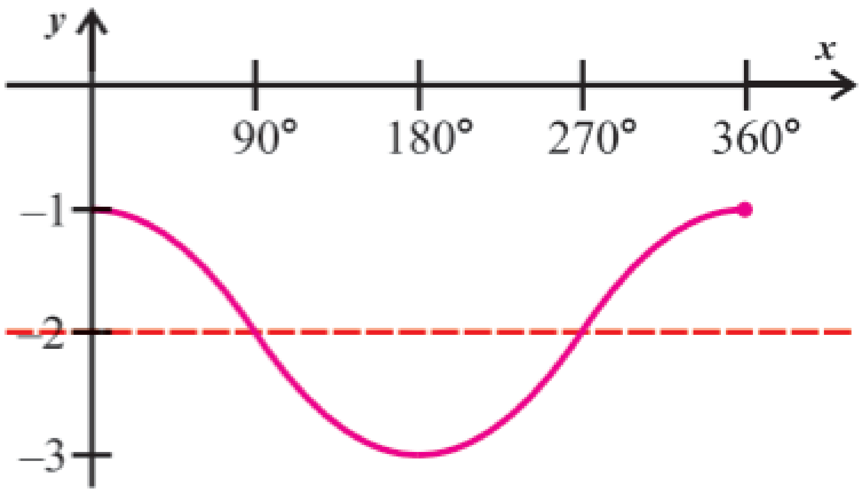
You can think of q as the new x-axis.

Example 2: On separate axes sketch the graphs of:

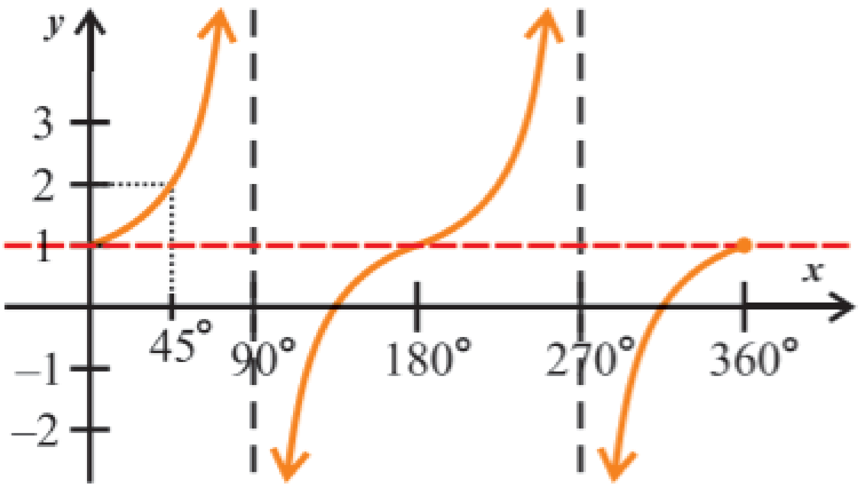
$y = \sin x + 1$, $y = \cos x - 2$ and $y = \tan x + 1$ where $x \in [0°; 360°]$



$y = \sin x + 1$



$y = \cos x - 2$



$y = \tan x + 1$

The standard tan graph (where q is equal to 0) has a y-value of 1 at 45 degrees. In this case, as q is equal to 1, the y-value at 45 degrees is equal to 2. This is because one is added to all of the y-values.

Complete Exercise 13 number 3 on page 306.

J. INTERPRETATION OF TRIGONOMETRIC FUNCTIONS

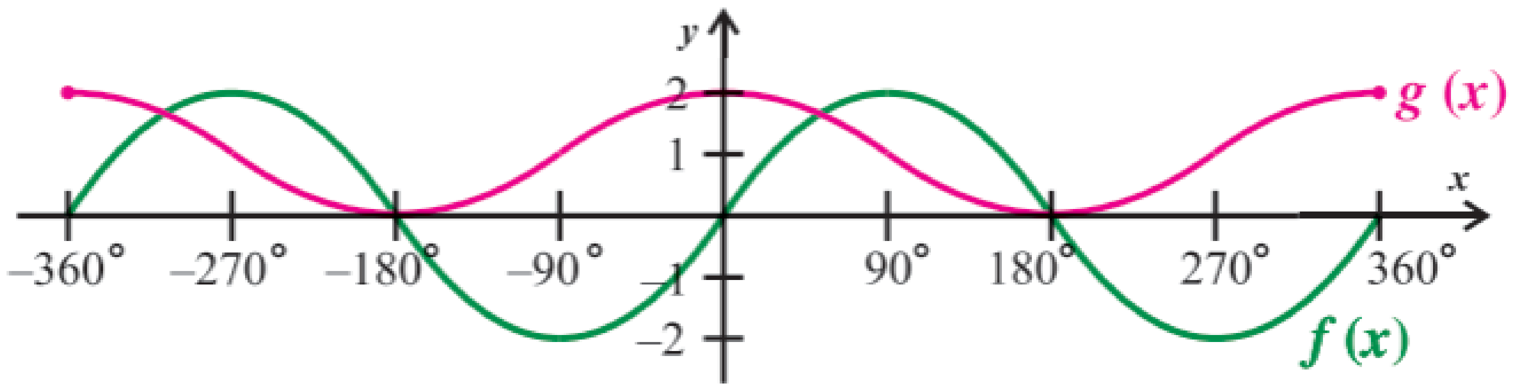
In grade 10 you will be expected to identify trigonometric functions and derive their equations. You will also have to identify key characteristics such as the functions period, amplitude, domain and range.

The period of a trigonometric function is how often the graph repeats itself. In Grade 10 the period of a trigonometric function does not change.

The graph of $y = \sin\theta$ has a period of 360°
The graph of $y = \cos\theta$ has a period of 360°
The graph of $y = \tan\theta$ has a period of 180°

Example 1: **The graphs of $f(x)$ and $g(x)$ are sketched below. Determine:**

- **a) The amplitude of $f(x)$ and $g(x)$.**
- **b) The equations of $f(x)$ and $g(x)$.**
- **c) The domain and range of $f(x)$ and $g(x)$.**
- **d) The period of $f(x)$ and $g(x)$.**



a) The amplitude of the Sine and Cosine graphs are: $\dfrac{y_{\max} - y_{\min}}{2}$

It can be seen that $f(x)$ is a sin graph and $g(x)$ is a cos graph.

$$f(x): \text{amplitude} = \frac{y_{\max} - y_{\min}}{2} = \frac{2-(-2)}{2} = \frac{4}{2} = 2$$

$$g(x): \text{amplitude} = \frac{y_{\max} - y_{\min}}{2} = \frac{2-0}{2} = 1$$

b) $f(x) = a\sin(x) + q$

$a = 2 \rightarrow$ this was calculated in (a)

As in the standard sine graph, the graph of $f(x)$ starts at 0. Therefore, there is no up or down shift.

$\therefore q = 0$

$\therefore f(x) = 2\sin(x)$

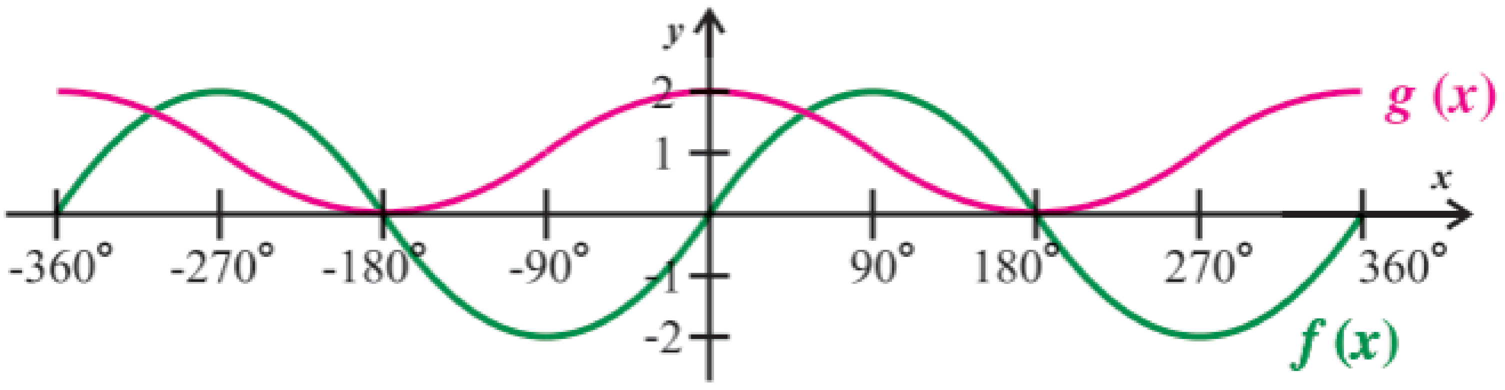
$g(x) = a\cos(x) + q$

$a = 1 \rightarrow$ this was calculated in (a)

The standard cosine graph starts at 1 and has an amplitude of 1. The graph of $g(x)$ starts at 2 and has an amplitude of 1. Therefore, there is a shift up by one unit.

$\therefore q = 1$

$\therefore g(x) = \cos(x) + 1$



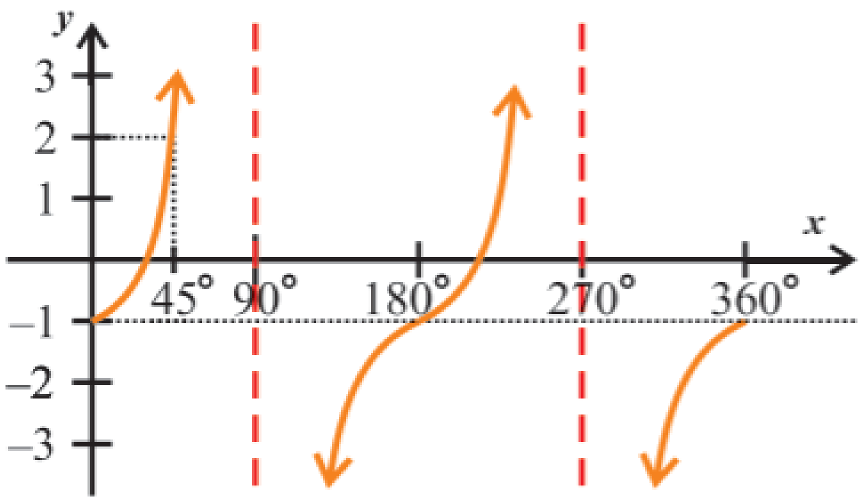
c) Domain of $f(x): x \in [-360^\circ; 360^\circ]$ Range of $f(x): y \in [-2; 2]$

Domain of $g(x): x \in [-360^\circ; 360^\circ]$ Range of $g(x): y \in [0; 2]$

d) $f(x)$ and $g(x)$ have a period of 360°.

Example 2: The graph of $y = a\tan x + q$ is sketched below. Determine:

- **a) The values of *a* and *q*. Hence, determine the equation of the graph.**
- **b) The domain and range of the graph.**
- **c) The period of the graph.**



When finding the equation of a tan graph, calculate the value of *q* first and then solve for *a* using algebra.

a) The value of q can be read from the graph by looking at where the graph starts. The standard tan graph should start at $(0;0)$. However this graph starts at $(0;-1)$.

$\therefore q = -1$

$\therefore y = a\tan x - 1$

Now substitute the point $(45^\circ; 2)$ into the above equation to solve for a.

$\therefore 2 = a\tan(45^\circ) - 1 \rightarrow$ the value of $\tan(45^\circ)$ is always equal to 1

$\therefore 2 = a(1) - 1$

$\therefore a = 3$

$\therefore$ The equation of the graph is: $y = 3\tan x - 1$

b) Domain: $x \in [0^\circ; 360^\circ],\ x \neq 90^\circ$ or 270°

Range: $y \in \mathbb{R}$

c) The graph repeats itself every 180°, therefore it has a period of 180°.

Complete Exercise 14 on pages 306 and 307
and The Mixed Exercise on pages 307, 308 and 309.

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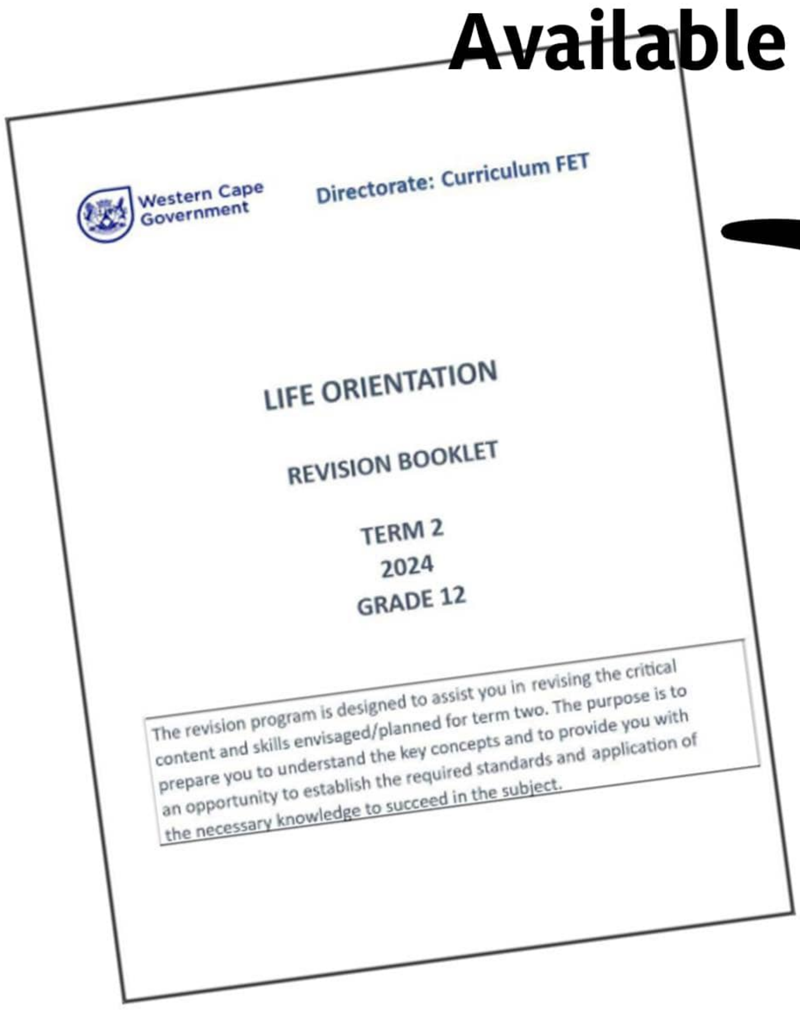

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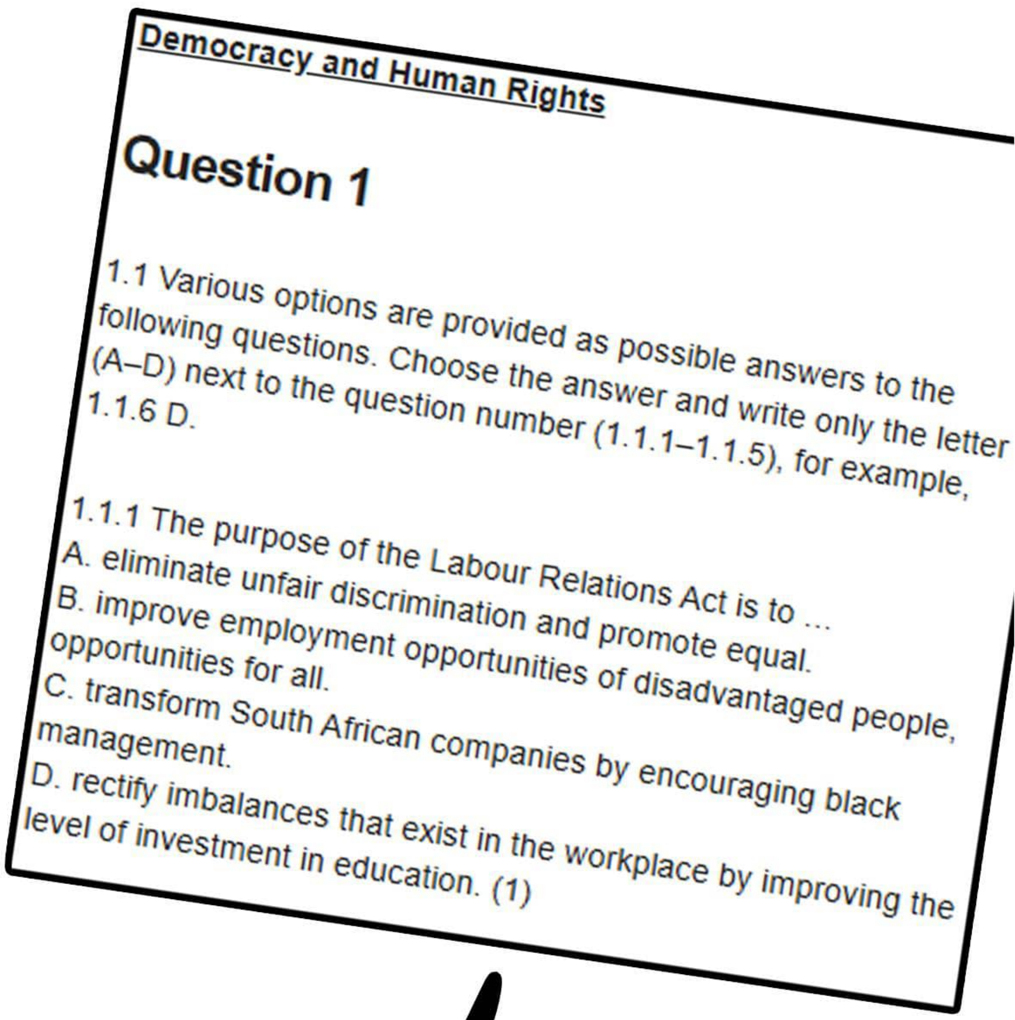
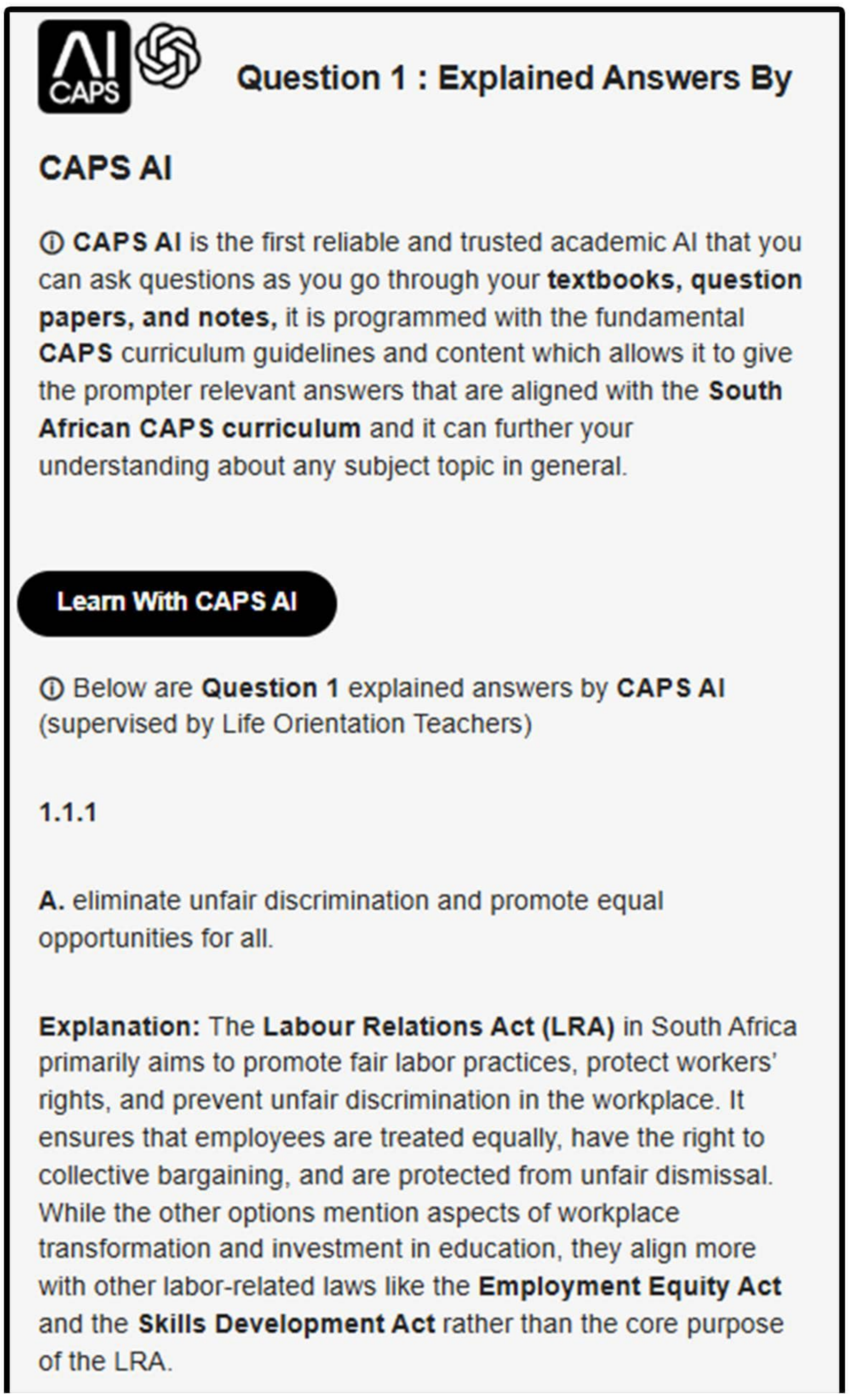


Democracy and Human Rights

Question 1

1.1 Various options are provided as possible answers to the following questions. Choose the answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5), for example, 1.1.6 D.

1.1.1 The purpose of the Labour Relations Act is to ...
A. eliminate unfair discrimination and promote equal opportunities for all.
B. improve employment opportunities of disadvantaged people.
C. transform South African companies by encouraging black management.
D. rectify imbalances that exist in the workplace by improving the level of investment in education. (1)



Question 1 : Explained Answers By CAPS AI

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ⓘ Below are **Question 1** explained answers by **CAPS AI** (supervised by Life Orientation Teachers)

1.1.1

A. eliminate unfair discrimination and promote equal opportunities for all.

Explanation: The **Labour Relations Act (LRA)** in South Africa primarily aims to promote fair labor practices, protect workers' rights, and prevent unfair discrimination in the workplace. It ensures that employees are treated equally, have the right to collective bargaining, and are protected from unfair dismissal. While the other options mention aspects of workplace transformation and investment in education, they align more with other labor-related laws like the **Employment Equity Act** and the **Skills Development Act** rather than the core purpose of the LRA.

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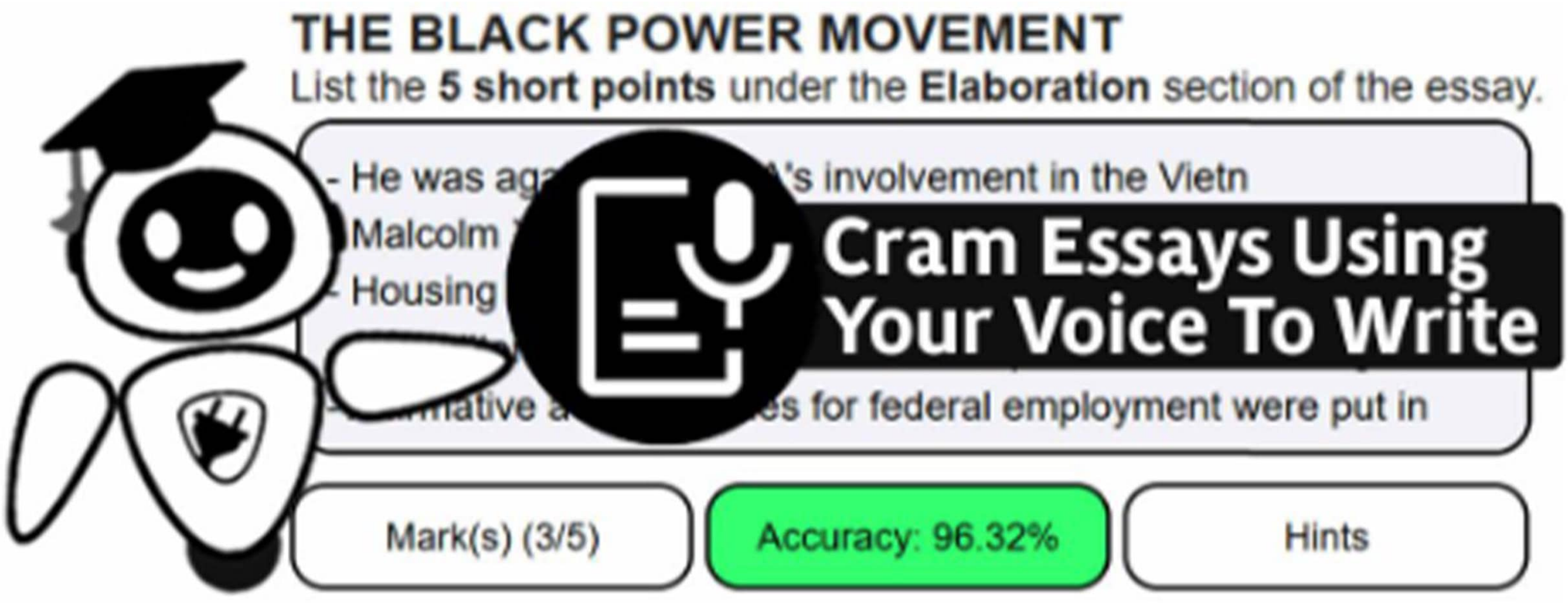

THE BLACK POWER MOVEMENT
List the **5 short points** under the **Elaboration** section of the essay.

- He was ag 's involvement in the Vietn
Malcolm
- Housing
- mative a es for federal employment were put in

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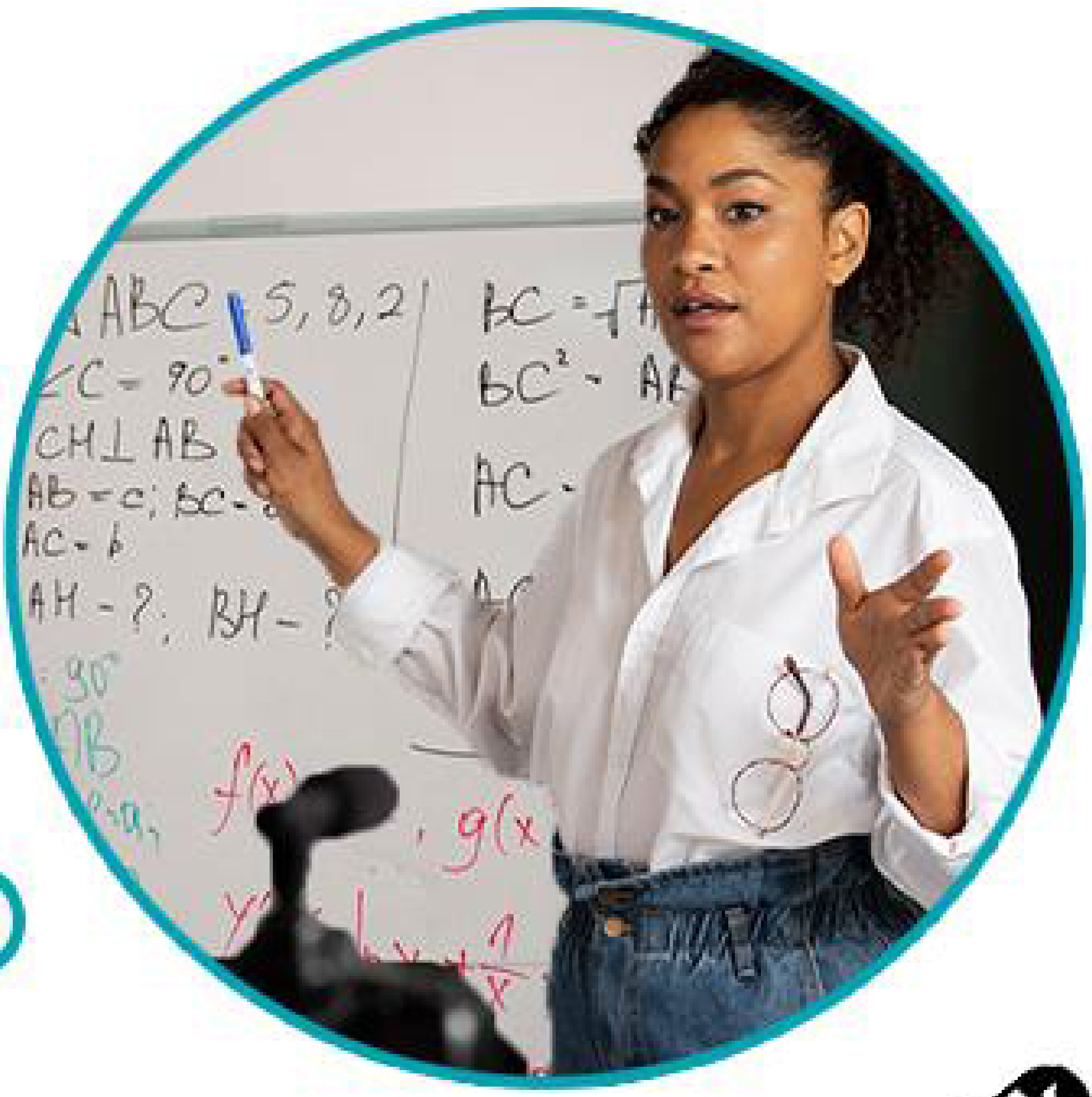

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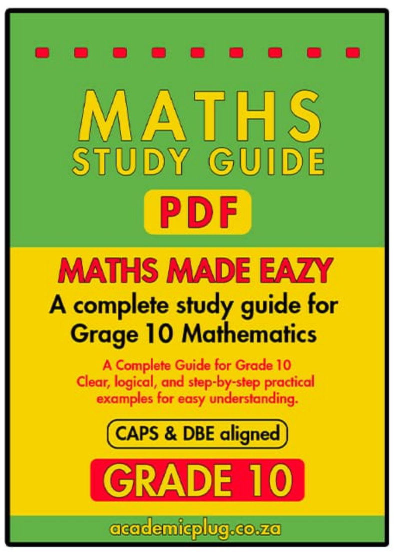
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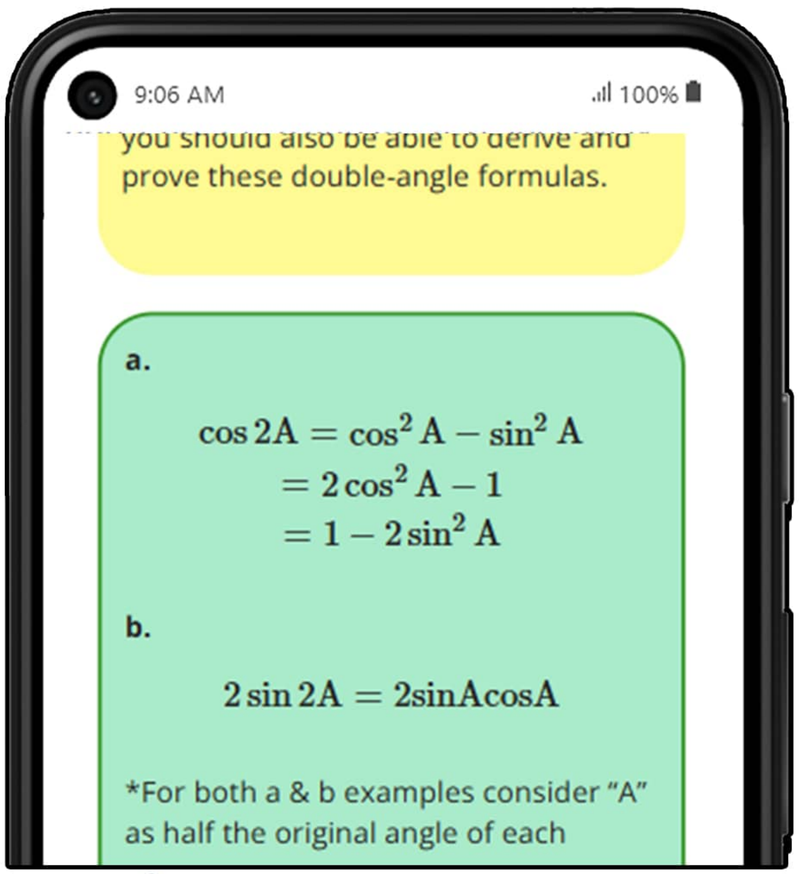

you should also be able to derive and prove these double-angle formulas.

a.

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

b.

$$2\sin 2A = 2\sin A\cos A$$

*For both a & b examples consider "A" as half the original angle of each

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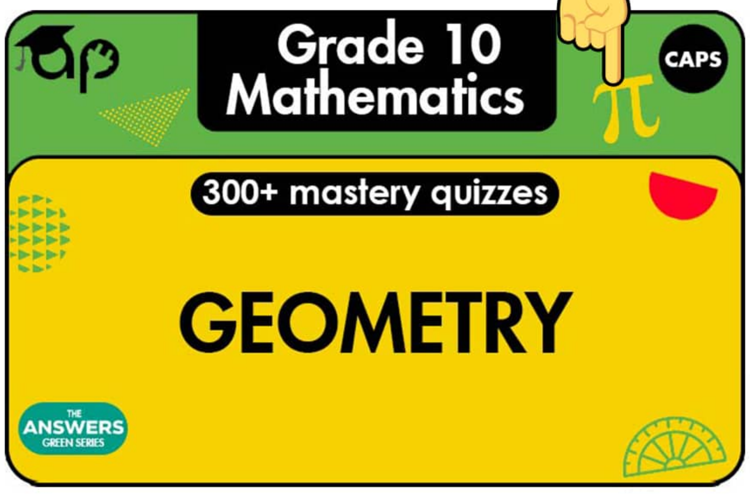
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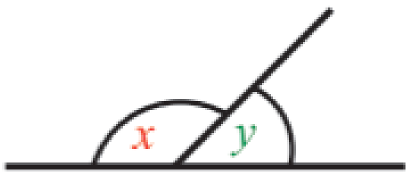
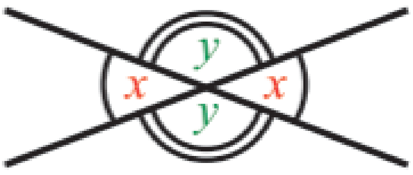
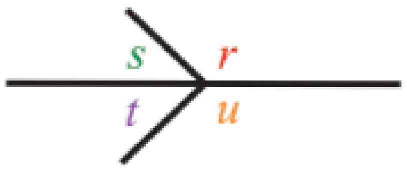
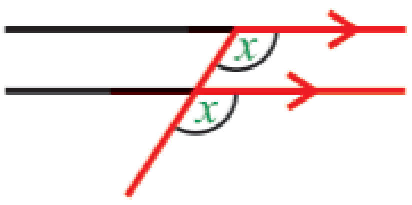
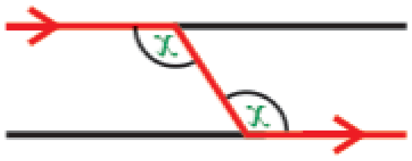
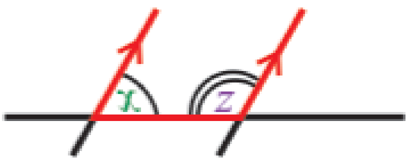
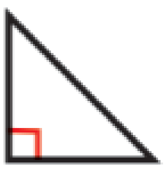
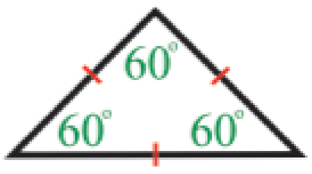
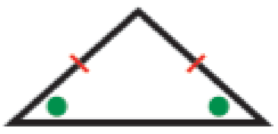
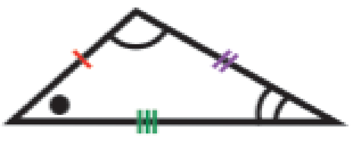
CHAPTER 7- GEOMETRY

WHAT YOU WILL LEARN IN THIS CHAPTER

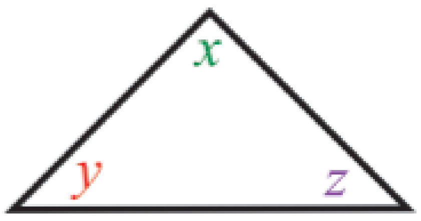
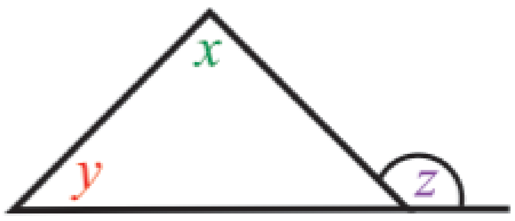
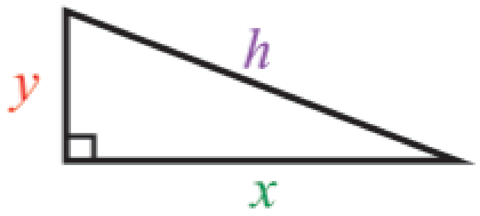
PAGE	CONTENT	CAPS	ASSESSMENTS YOU WILL BE ABLE TO:
141	**A. Euclidean Geometry**	7	- Investigate and form conjectures about the properties of special triangles, quadrilaterals and other polygons. Try to validate or prove conjectures using any logical method.
141	- Revision of Lines, Angles and Triangles		
142	- Rules and Theorems of Triangles		
142	- Similar Triangles		
144	- Congruency		- Investigate line segments joining the midpoints of two sides of a triangle.
147	- Quadrilaterals		
151	- Polygons		
152	- Angles in a Polygon		- Represent geometric figures on a Cartesian coordinate system and derive and apply a formula for calculating the distance between two points, the gradient of a line segment and the midpoint of a line segment joining two points.
153	- Midpoint Theorem		
155	**B. Analytical Geometry**	9	
155	- Formulas		
156	- The Distance Formula		
158	- The Midpoint Formula		
159	- Average Gradient		
160	- Mixed Examples		
165	- Determining the Equation of Straight Line		

A. EUCLIDEAN GEOMETRY

REVISION OF LINES, ANGLES AND TRIANGLES

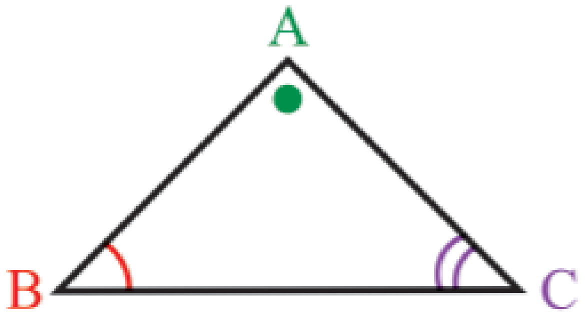
Definition	Example
Adjacent angles on a straight line are supplementary (add up to 180°).	 $x + y = 180^{\circ}$
Vertically opposite angles are equal.	
Angles around a point add up to 360°.	 $r + s + t + u = 360^{\circ}$
Corresponding angles of parallel lines are equal. (F-Shape)	
Alternate angles of parallel lines are equal. (Z-Shape)	
Co-interior angles of parallel lines are supplementary. (U-Shape)	 $x + z = 180^{\circ}$
A right-angled triangle has an angle that is equal to 90°.	
Equilateral triangles have equal sides and equal angles. All angles in an equilateral triangle are equal to 60°.	
An isosceles triangle has two sides which are equal. The angles opposite the equal sides are also equal.	
Scalene triangles have no sides or angles that are equal.	

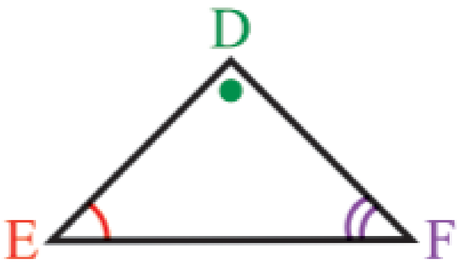
RULES AND THEOREMS OF TRIANGLES

Definition	Example
The sum of the angles inside a triangle add up to 180°.	 $x + y + z = 180^{\circ}$
The exterior angle of a triangle is equal to the sum of the opposite interior angles.	 $z = x + y$
The Theorem of Pythagoras states that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	 $h^2 = x^2 + y^2$ or $x^2 = h^2 - y^2$ or $y^2 = h^2 - x^2$

SIMILAR TRIANGLES

Similar triangles are triangles that have the **same angles** and whose **corresponding sides are in proportion**. For triangles to be similar, **either** all of the angles in the triangles have to be equal (equiangular) **or** the corresponding sides have to be in proportion. When proving triangles are similar the reason for similarity is always angle, angle, angle (AAA).





In ΔABC and ΔDEF

$\hat{A} = \hat{D}$

$\hat{B} = \hat{E}$

$\hat{C} = \hat{F}$

$\therefore \Delta ABC /// \Delta DEF \rightarrow$ The reason for similarity is AAA, '///' is the **symbol for similarity**

$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

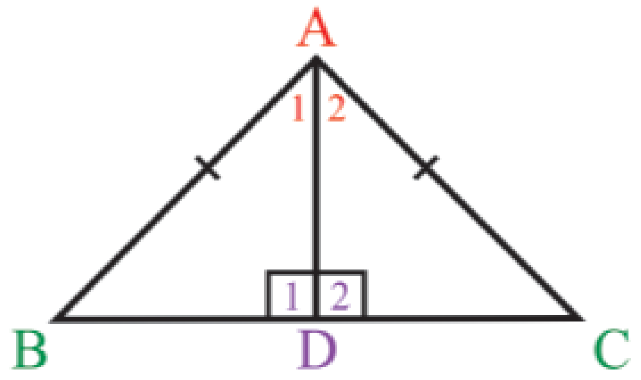
If the sides are in proportion it means that the ratios of the sides are equal.

The order in which you write the triangles is important. The angles that are equal can be read from $\Delta ABC /// \Delta DEF$. This tells you that $\hat{A} = \hat{D}$ and $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

The sides that are in proportion can be read in the same way.

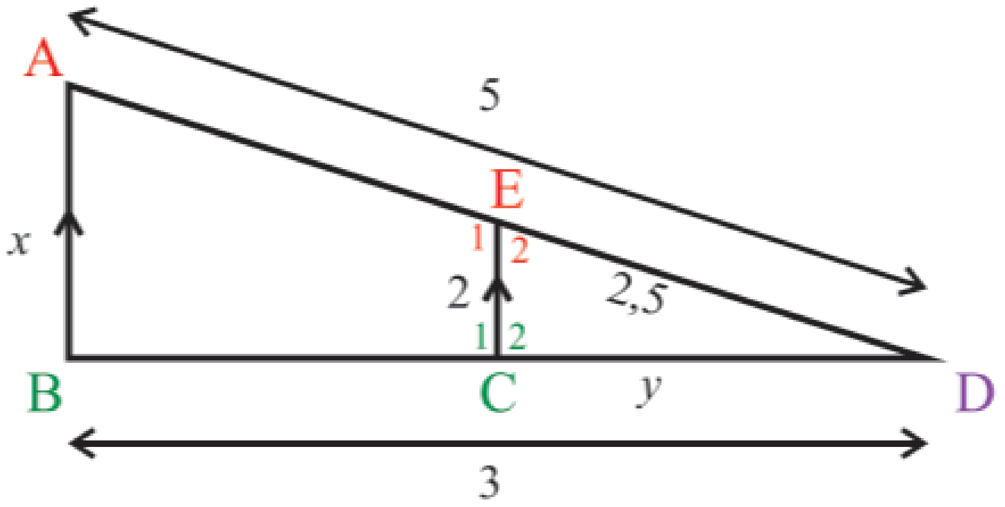
Complete Exercise 1 on page 326.

Example 1: **Prove that triangles ABD and ACD are similar.**



Statement	Reason
In ΔABD and ΔACD	
$\hat{B} = \hat{C}$	$\angle$'s opp. equal sides, $AB = AC$
$\hat{D}_1 = \hat{D}_2 = 90^\circ$	given
$\hat{A}_1 = \hat{A}_2$	3rd angle of a Δ
$\therefore \Delta ABD /// \Delta ACD$	AAA

Example 2: **Determine the lengths of the unknown sides in the triangles below.**



Statement	Reason
In ΔABD and ΔECD	
$\hat{D} = \hat{D}$	Common $\angle$
$\hat{B} = \hat{C}_2$	corresponding $\angle$'s, $AB // EC$
$\hat{A} = \hat{E}_2$	3rd angle of a Δ
$\therefore \Delta ABD /// \Delta ECD$	AAA

$$\therefore \frac{AB}{EC} = \frac{BD}{CD} = \frac{AD}{ED}$$

$$\therefore \frac{x}{2} = \frac{5}{2,5} \rightarrow \frac{AB}{EC} = \frac{AD}{ED}$$

$\therefore x = 4$ units $\rightarrow$ always put in units

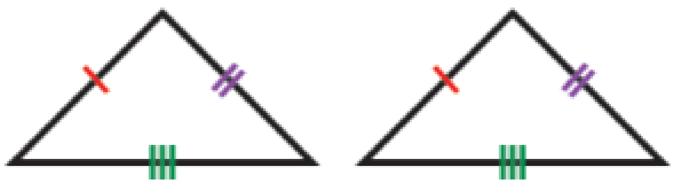
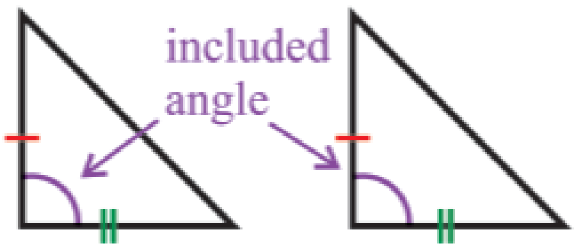
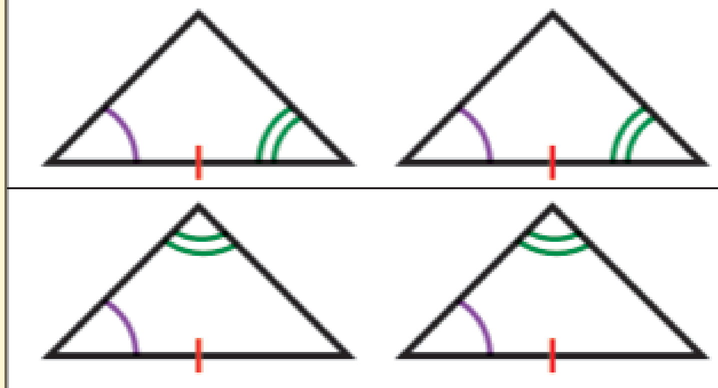
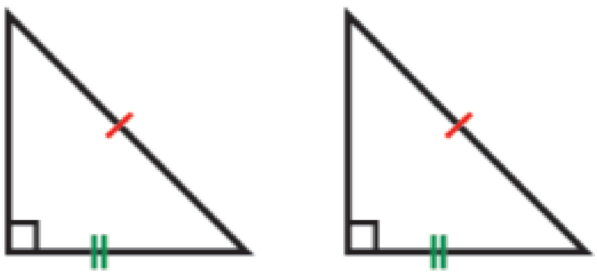
$$\therefore \frac{y}{3} = \frac{2,5}{5} \rightarrow \frac{CD}{BD} = \frac{ED}{AD}$$

$\therefore y = 1,5$ units

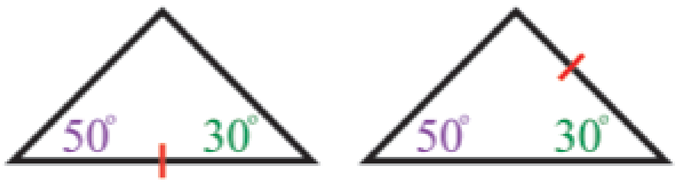
Complete Exercise 2 on pages 326 and 327.

CONGRUENCY

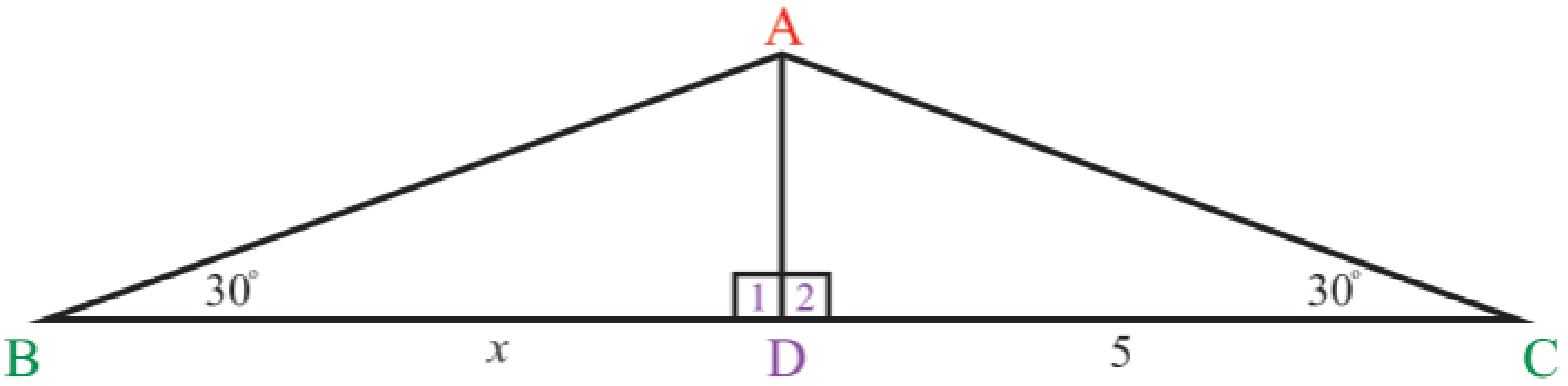
If triangles are congruent it means that they are exactly the same, i.e. every corresponding angle and side in each triangle is equal.
There are 4 conditions for triangles to be congruent.
If any of the conditions listed below are true then the triangles are congruent.
The symbol ≡ is used for congruency.

Condition/ Reason	Example	Explanation
Side Side Side (SSS)		**If triangles have three sides that are equal the triangles are congruent**
Side Angle Side (SAS)		**If triangles have two sides and an included angle that are equal, the triangles are congruent.**
Angle Angle Side Side Angle Angle Angle Side Angle (AAS) / (SAA) / (ASA)		**If triangles have two angles and a single corresponding side that are equal, the triangles are congruent.**
Right angle Hypotenuse Side (RHS)		**If the hypotenuse and one other side are equal in right angled triangles, the triangles are congruent.**

Using AAS, the sides that are equal have to be the corresponding sides.
The triangles below are not congruent as the equal sides are not the same relative to the angles.



Example 1: **Determine the length of x in the triangle below:**

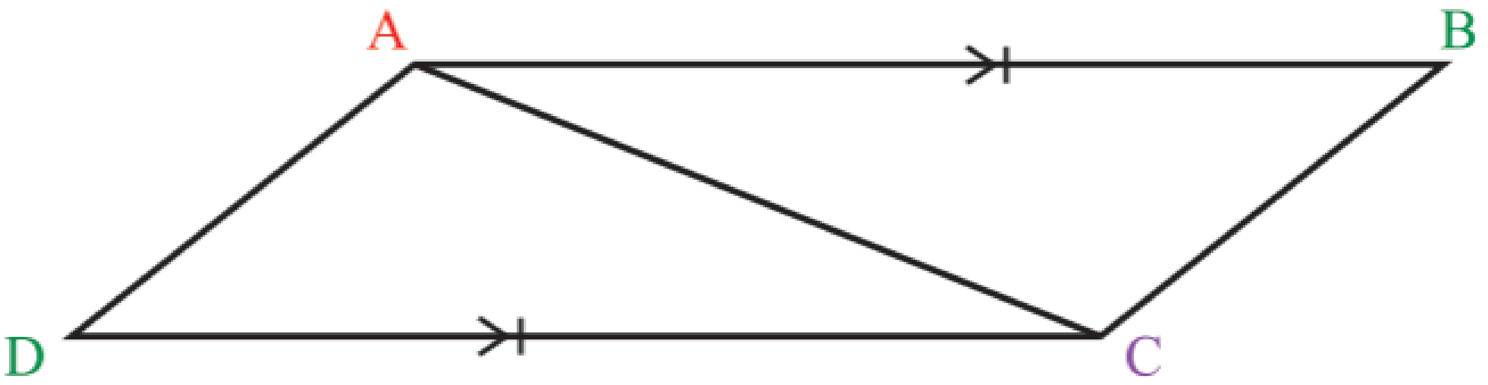


Although it looks like BD = DC, you cannot assume that it is true unless you prove it.

$\therefore$ Required To Prove (RTP): $\Delta ABD \equiv \Delta ACD \rightarrow$ congruent

Statement	Reason
In ΔABD and ΔACD	
$AB = AC$	sides opp. $=$ $\angle$'s, $\hat{B} = \hat{C}$
$AD = AD$	common side
$\hat{D}_1 = \hat{D}_2 = 90^\circ$	given
$\therefore \Delta ABD \equiv \Delta ACD$	RHS (AAS could also be the reason used)
$\therefore BD = CD$ $\therefore x = 5$ units $\rightarrow$ always put in units	

Example 2: **Given that AB is parallel and equal to DC, prove that ΔABC is congruent to ΔCDA.**



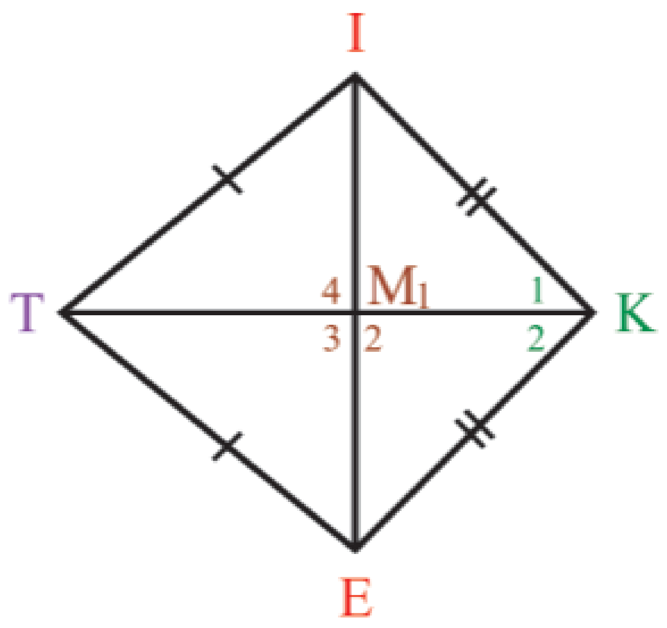
Statement	Reason
In ΔABC and ΔCDA	
$B\hat{A}C = A\hat{C}D$	alt $\angle$'s, $AB // DC$
$AB = DC$	given
$AC = AC$	common side
$\therefore \Delta ABC \equiv \Delta CDA$	SAS

$A\hat{C}D$ refers to the angle at $\hat{C}$ which is between the lines AC and DC.
$B\hat{A}C$ refers to the angle at $\hat{A}$ which is between the lines AB and AC.

Example 3: **In quadrilateral KITE below, KI = KE and IT = ET. If the diagonals intersect at M, prove that:**

a) MI = ME.

b) The diagonals intersect at 90º.



a) In order to prove that MI = ME, you have to prove that $\Delta KIT \equiv \Delta KET$.

Statement	**Reason**
In ΔKIT and ΔKET	
$KI = KE$	given
$IT = ET$	given
$KT = KT$	common side
$\therefore \Delta KIT \equiv \Delta KET$	SSS

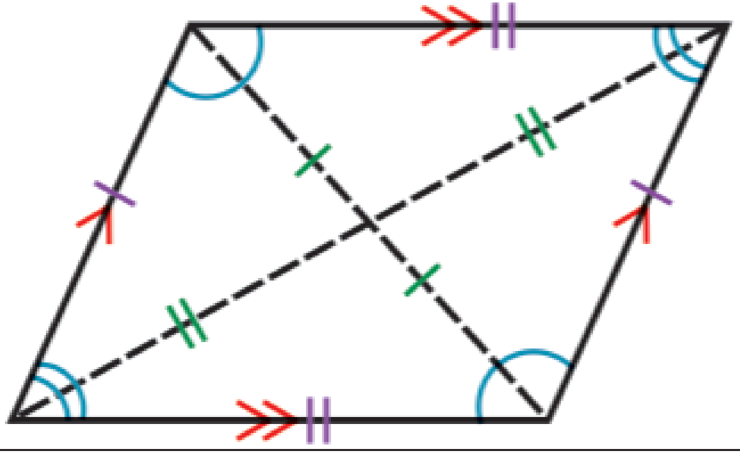
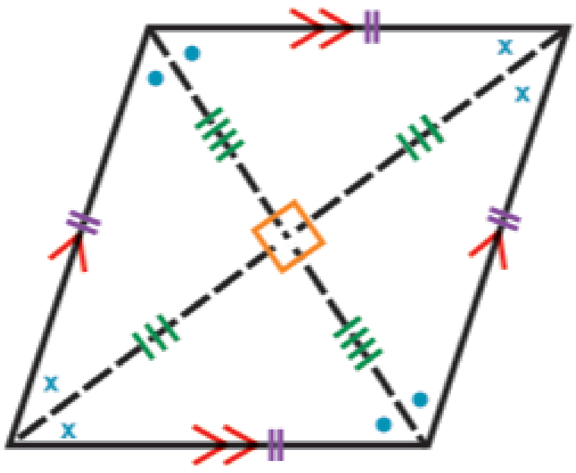
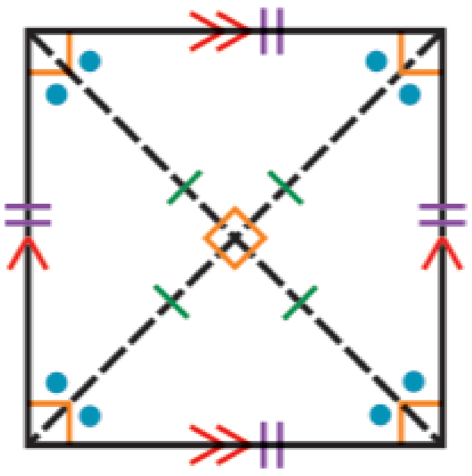
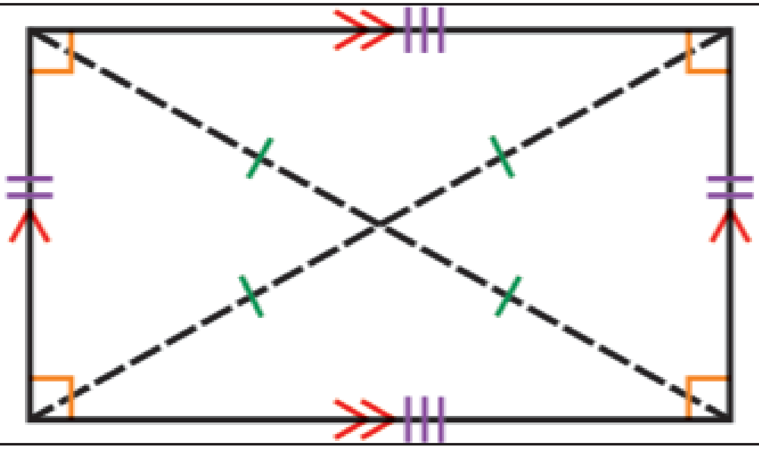
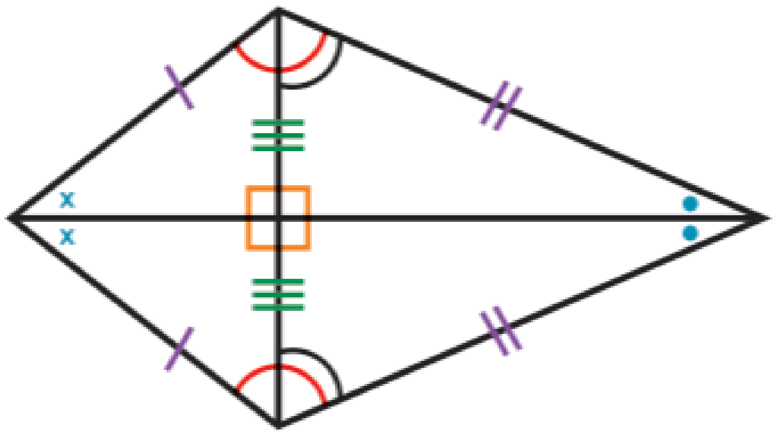
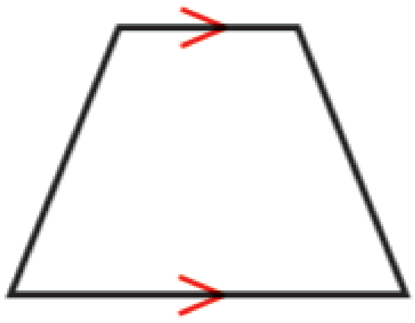
b) In order to prove that the diagonals intersect at 90º you have to prove that $\Delta KIM \equiv \Delta KEM$.

Statement	**Reason**
In ΔKIM and ΔKEM	
$KI = KE$	given
$\hat{K}_1 = \hat{K}_2$	congruent Δ's $(\Delta KIT \equiv \Delta KET)$
$KM = KM$	common side
$\therefore \Delta KIM \equiv \Delta KEM$	SAS
$\therefore IM = ME$ and $\hat{M}_1 = \hat{M}_2$	
$\hat{M}_1 + \hat{M}_2 = 180^\circ$	adj. $\angle$'s on a str. line are supp.
$\hat{M}_1 = \hat{M}_2 = 90^\circ$	
$\therefore$ The diagonals intersect at 90°.	

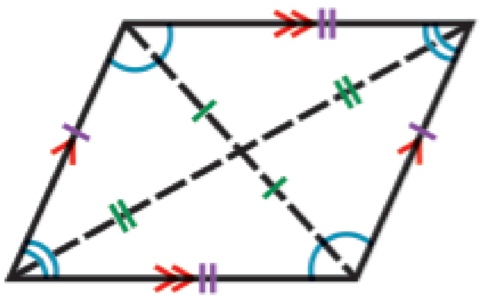
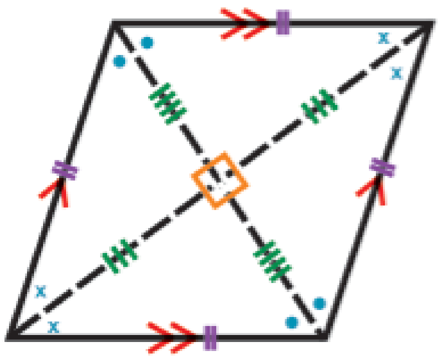
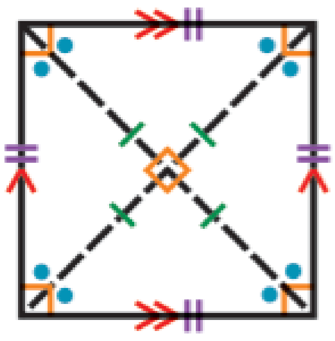
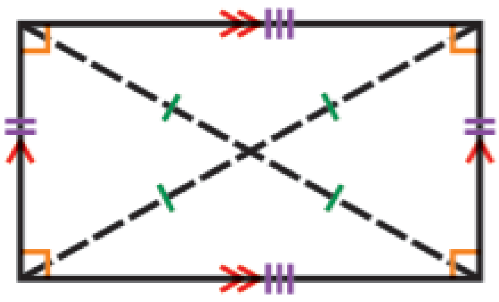
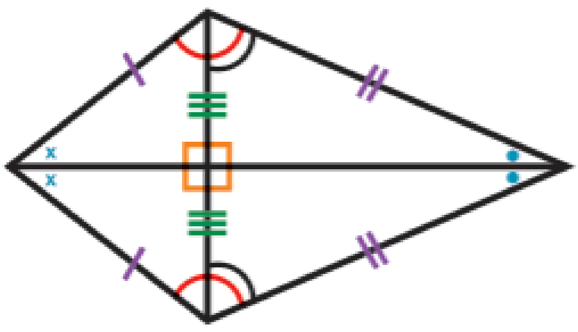
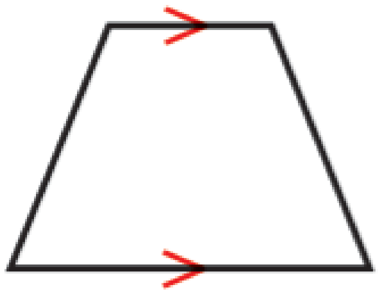
Complete Exercise 3 on page 327.

QUADRILATERALS

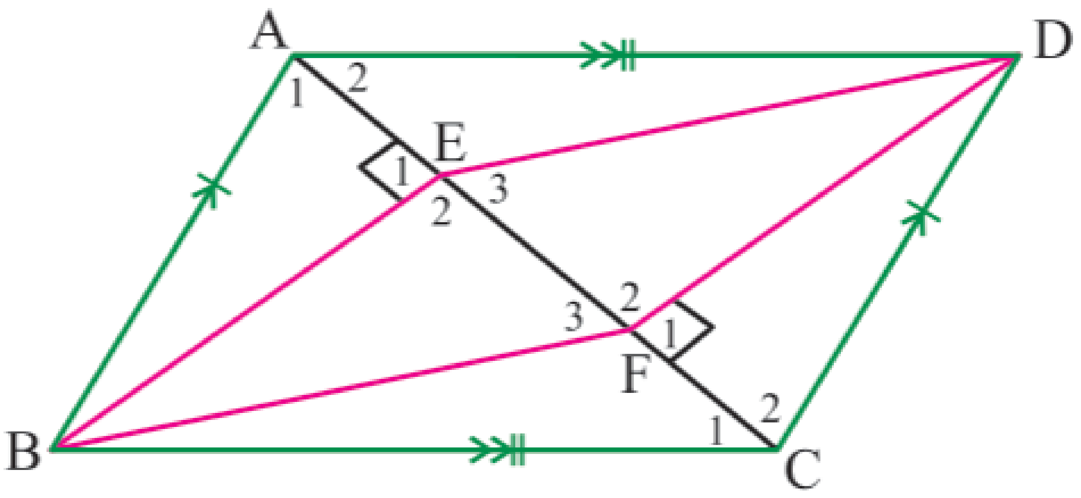
A quadrilateral is defined as a closed four-sided figure whose interior angles add up to 360°. The table below shows some common quadrilaterals and their properties.

Quadrilateral	Figure	Properties
Parallelogram (parm)		• Opposite sides are **parallel** and **equal** in length • Opposite angles are **equal** • **Diagonals bisect** each other
Rhombus		• All **sides** are **equal** in length • Opposite sides are **parallel** • Opposite angles are **equal** • **Diagonals bisect** each other at **90°**. • Diagonals bisect the corner **angles**
Square		• All **sides** are **equal** in length • Opposite sides are **parallel** • Corner **angles** equal **90°** • Diagonals are **equal** and **bisect** each other at **90°** • Diagonals **bisect** the corner **angles**
Rectangle		• Opposite sides are **parallel** and **equal** in length • Corner **angles equal 90°** • **Diagonals** are **equal** and **bisect** each other
Kite		• Two pairs of **adjacent sides** are **equal** • A single pair of **opposite angles are equal** • Diagonals intersect each other at **90°** • One diagonal **bisects** the corner **angles** • Shorter diagonal is **bisected** by the longer diagonal
Trapezium		• One pair of **opposite sides** are **parallel**

The table below shows the properties needed to prove that a quadrilateral is a specific shape.

Quadrilateral	Properties to prove
Parallelogram (parm) 	Prove any **one** of the following: 1) **Both** pairs of **opposite sides parallel** 2) **Both** pairs of **opposite sides equal** 3) **Both** pairs of **opposite angles equal** 4) **One** pair of **opposite sides equal** and **parallel** 5) **Diagonals bisect** each other (They have the same midpoint)
Rhombus 	Prove the quadrilateral is a parallelogram **and one** of the following: 1) **One** pair of **adjacent sides equal** 2) **Diagonals bisect at 90º**
Square 	Prove any **one** of the following: 1) The quadrilateral is a **parm** with a **corner angle equal to 90º** and a **pair of adjacent sides equal** 2) The quadrilateral is a **rhombus** with a **corner angle equal to 90º** 3) The quadrilateral is a **rhombus** with equal diagonals
Rectangle 	Prove that the quadrilateral is a parallelogram **and one** of the following: 1) The **parm has a corner angle equal to 90º** 2) The **parm has diagonals that are equal in length**
Kite 	Prove that: 1) **Each pair** of **adjacent sides are equal** in length
Trapezium 	Prove that: 1) One pair of **opposite sides** are **parallel**

Example 1: **In the diagram below ABCD is a parallelogram.**



a) Prove that $\Delta ABE \equiv \Delta CDF$.

b) Hence, prove that BEDF is also a parallelogram.

a)

Statement	Reason
In ΔABE and ΔCDF	
$\hat{A}_1 = \hat{C}_2$	alt $\angle$'s, AB// DC
$\hat{E}_1 = \hat{F}_1 = 90^\circ$	given
$AB = CD$	opp sides of a parm are =
$\therefore \Delta ABE \equiv \Delta CDF$	AAS

b)

Statement	Reason
$BE = DF$	Congruent Δ's $(\Delta ABE \equiv \Delta CDF)$
$\hat{E}_1 + \hat{E}_2 = 180^\circ$	adj. $\angle$'s on a str. line are supp.
$\therefore \hat{E}_2 = 90^\circ$	
$\hat{F}_1 + \hat{F}_2 = 180^\circ$	adj. $\angle$'s on a str. line are supp.
$\therefore \hat{F}_2 = 90^\circ$	
$\therefore$ BE // DF	alt $\angle$'s, $\hat{E}_2 = \hat{F}_2 = 90^\circ$
$\therefore$ BE // DF and BE = DF	
$\therefore$ BEDF is a parm	One pair of opp. sides = and //

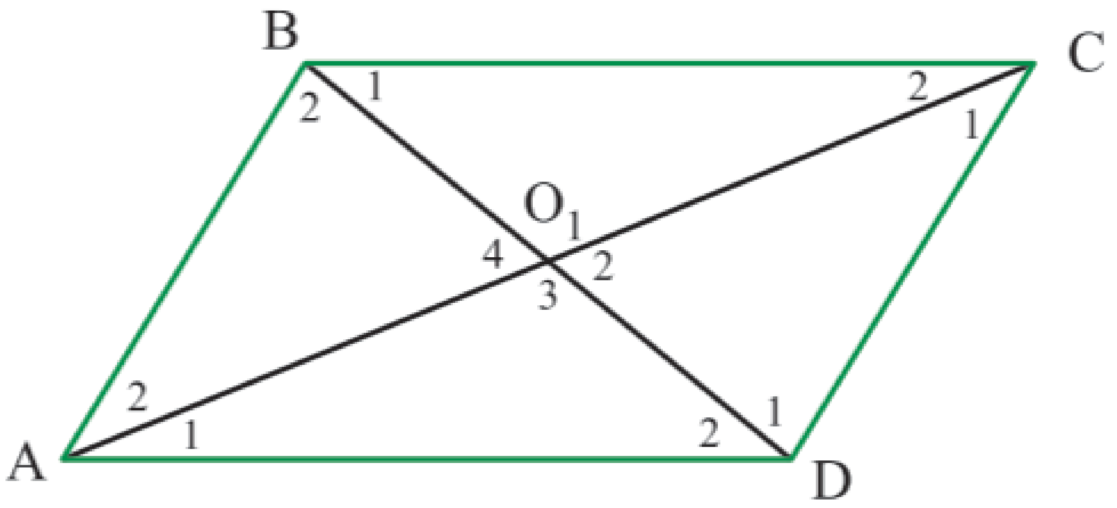
In order to prove that a quadrilateral is a parallelogram you have to prove any one of the following:

1) **Both pairs of opposite sides parallel**
2) **Both pairs of opposite sides equal**
3) **Both pairs of opposite angles equal**
4) **One pair of opposite sides equal and parallel**
5) **Diagonals bisect each other (have the same midpoint)**

Example 2: **Given that ABCD is a parallelogram:**

a) Prove that $\Delta BOC \equiv \Delta DOA$.

b) If BO=3cm, OC=4cm and BC=5cm, prove that ABCD is a rhombus?



a)

Statement	Reason
In ΔBOC and ΔDOA	
$BO = DO$	diags of a parm bisect each other
$OC = OA$	diags of a parm bisect each other
$\hat{O}_1 = \hat{O}_3$	vert. opp. $\angle$'s
$\therefore \Delta BOC \equiv \Delta DOA$	SAS

Alternate angles could also have been used to prove that $\Delta BOC \equiv \Delta DOA$. In this case SAS would have been the reason used to prove congruency.

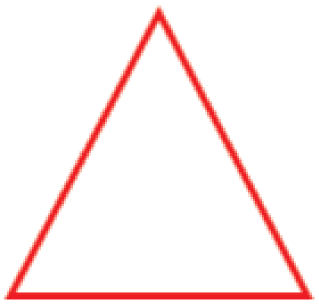
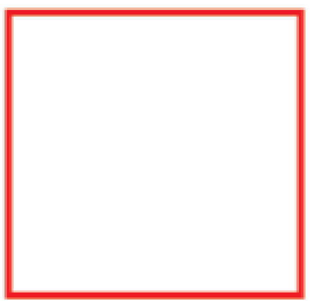
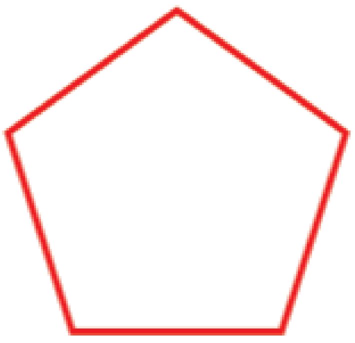
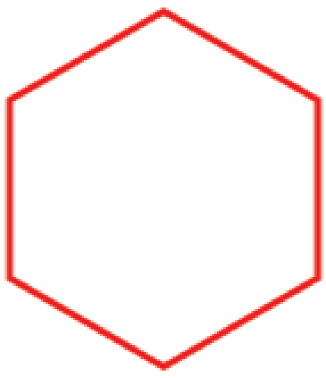
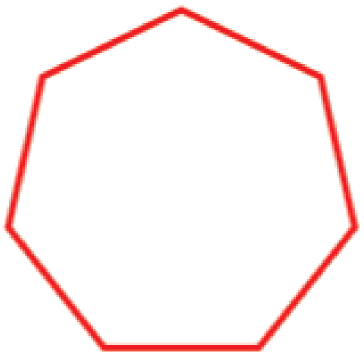
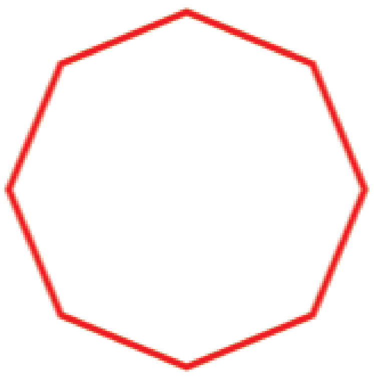
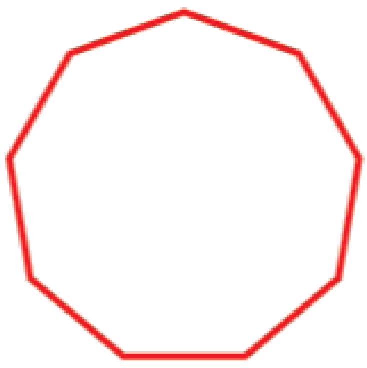
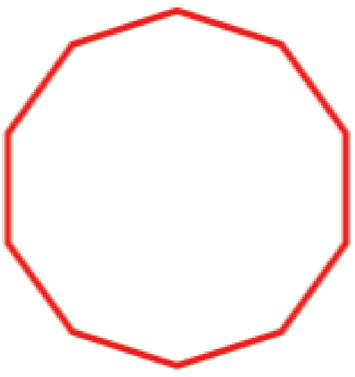
b) **You have now proved that the diagonals of ABCD bisect each other. In order to prove that ABCD is a rhombus, you have to prove that the diagonals bisect each other at 90º. You can do this by proving that the theorem of Pythagoras is true for the given triangle.**

Statement	Reason
In ΔBOC and ΔDOA	
$BO^2 + CO^2 = 3^2 + 4^2 = 25$	
$BC^2 = 25$	
$\therefore BC^2 = BO^2 + CO^2$	
$\therefore \hat{O}_1 = 90^\circ$	Pythag.
$\therefore$ ABCD is a rhombus	diagonals bisect at 90°

Complete Exercise 4 on pages 327 and 328.

POLYGONS

A polygon is defined as a closed shape with three or more straight sides.
The table below shows some common polygons.

Polygon	Figure	Number of Sides
Triangle		**3**
Quadrilateral		**4**
Pentagon		**5**
Hexagon		**6**
Heptagon (Septagon)		**7**
Octagon		**8**
Nonagon		**9**
Decagon		**10**

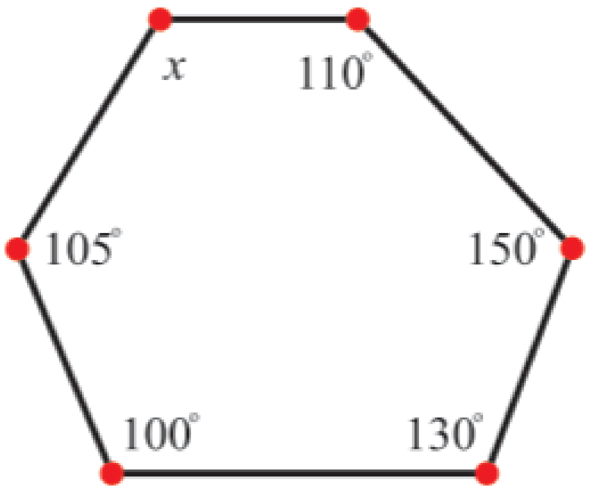
ANGLES IN A POLYGON

To calculate the sum of the angles in any polygon, use the formula:

$$\boldsymbol{S = 180^\circ (n-2)}$$

where: $S \rightarrow$ the sum of the angles in any polygon
$n \rightarrow$ the number of sides of the polygon

Example 1: Calculate the unknown angle (x) in the figure below:



The figure is a 6 sided polygon.

$\therefore S = 180^\circ (n-2)$

$\therefore S = 180^\circ (6-2) = 720^\circ$

$\therefore$ The sum of the angles in the figure is equal to 720°

$\therefore x + 110^\circ + 150^\circ + 130^\circ + 100^\circ + 105^\circ = 720^\circ$

$\therefore x + 595^\circ = 720^\circ$

$\therefore x = 125^\circ$

Example 2: If the sum of the angles in a polygon is equal to 2880°, calculate the number of sides that the polygon has.

$S = 180^\circ (n-2)$

$\therefore 2880^\circ = 180^\circ (n-2)$

$\therefore \dfrac{2880^\circ}{180^\circ} = n-2$

$\therefore 16 = n-2$

$\therefore n = 18$

$\therefore$ the polygon has 18 sides

In a regular polygon the lengths of each side of the polygon are equal. All angles in a regular polygon are also equal.

Complete Exercise 5 on page 328.

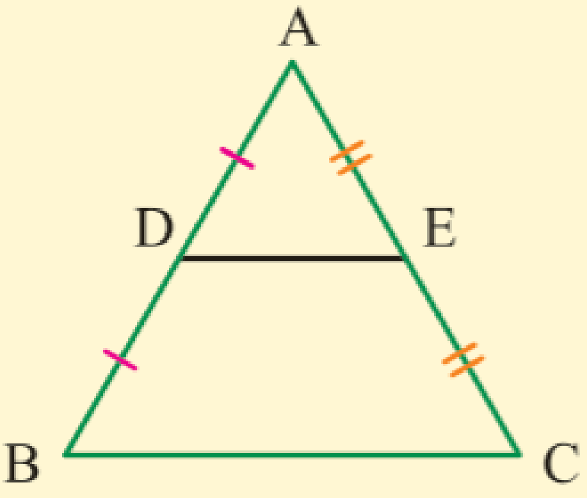
MIDPOINT THEOREM

The midpoint theorem states that if a line joins the midpoints of two sides of a triangle it is parallel to the third side and equal to half the length of the third side.

There are two cases in which this theorem can be interpreted:

Case 1:

Given $AD = DB$ and $AE = EC$

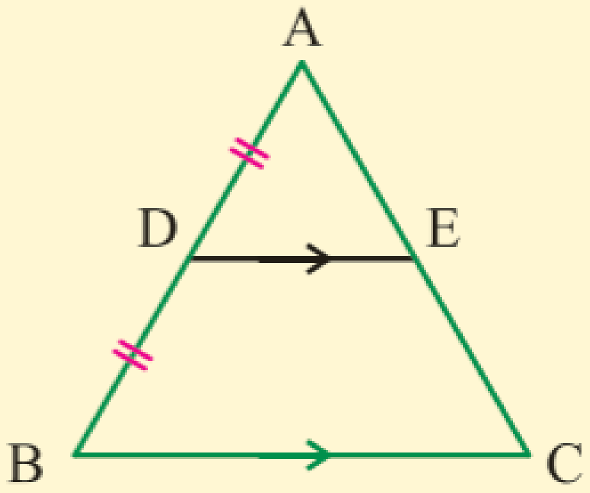


Conclusion:

$DE // BC$ and $BC = 2DE$

Case 2: (converse)

Given $AD = DB$ and $DE // BC$



Conclusion:

$AE = EC$ and $BC = 2DE$

Proof: **Given AD=DB and AE=EC extend DE to F so that DE=EF and join F to C as shown in figure 2.**

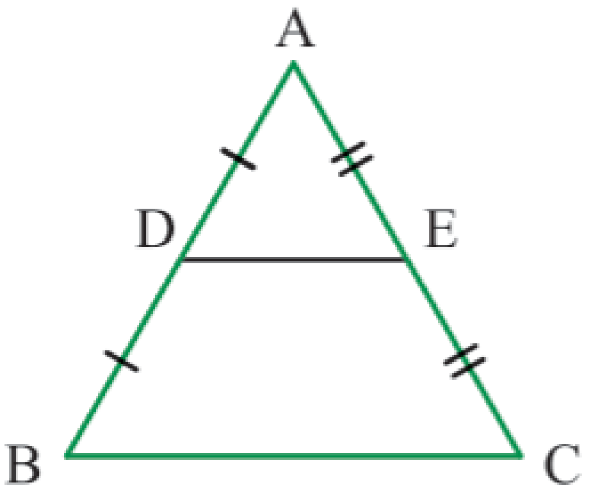


Figure 1

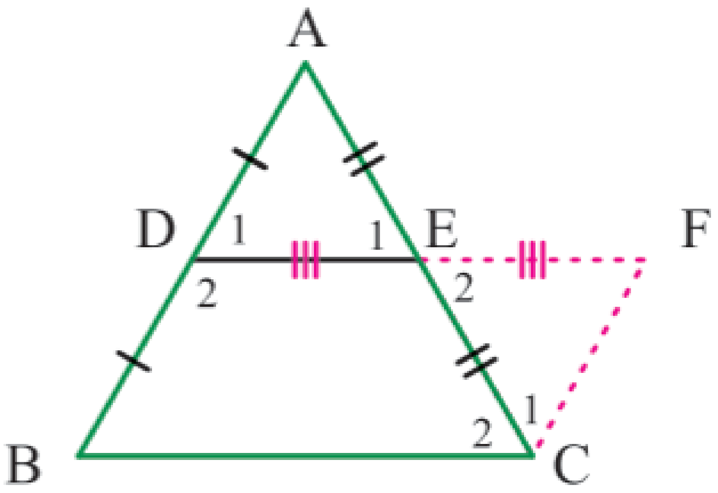
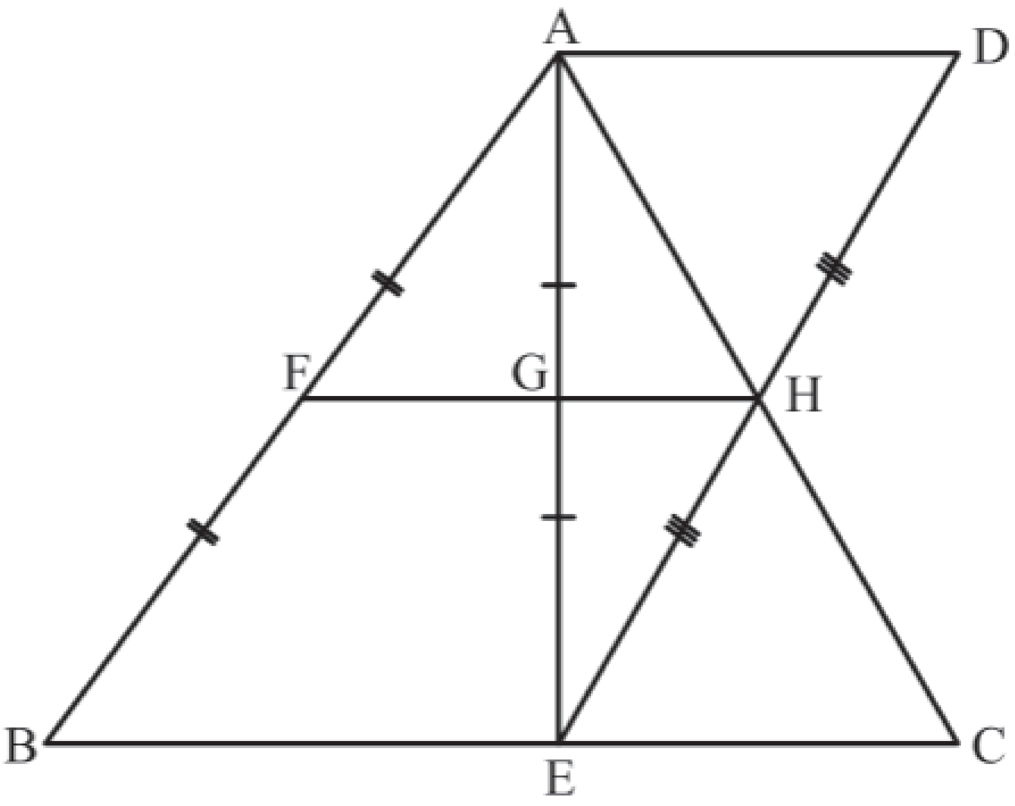


Figure 2

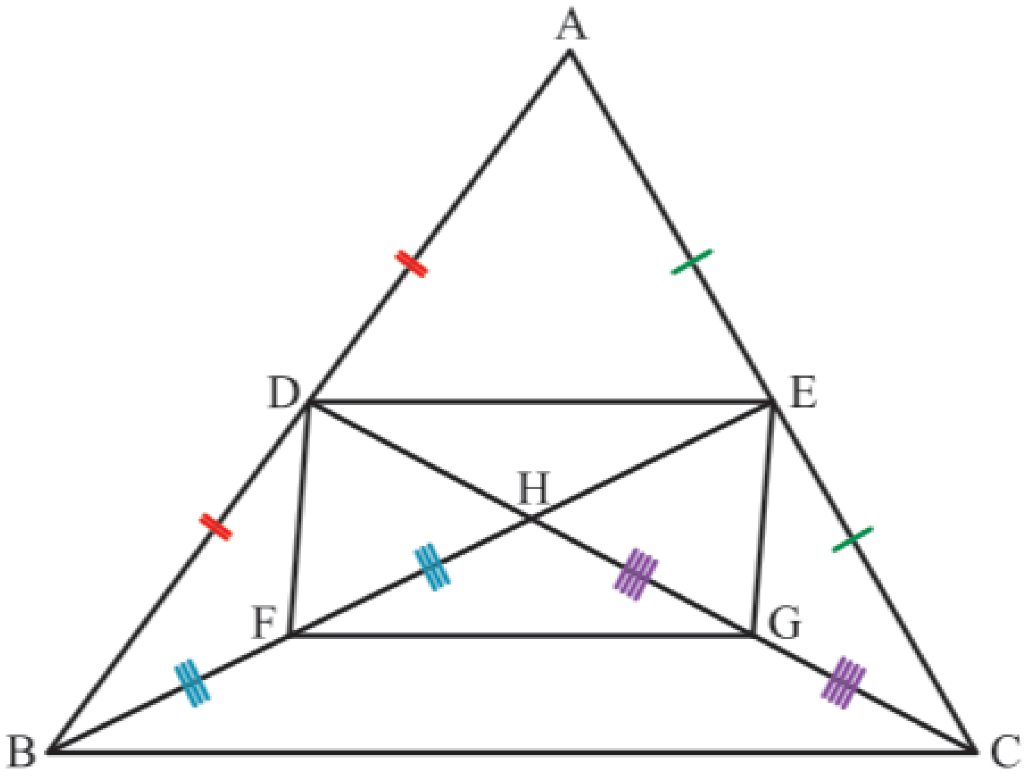
Statement	Reason
In ΔDAE and ΔFCE	
$DE = FE$	by construction
$\hat{E}_1 = \hat{E}_2$	vert. opp. $\angle$'s
$AE = CE$	given
$\therefore \Delta DAE \equiv \Delta FCE$	SAS
$\therefore \hat{D}_1 = \hat{F}$	congruent Δ's , $\Delta DAE \equiv \Delta FCE$
$\therefore AD // FC \rightarrow$ BD is also parallel to FC	alt $\angle$'s are equal
$AD = CF$	congruent Δ's , $\Delta DAE \equiv \Delta FCE$
$BD = AD$	given
$\therefore BD = CF$	
$\therefore$ CFDB is a parm	one pair of opp. sides are = and //
$\therefore DF = BC$ and $DF // BC$	opp. sides of a parm. are = //
$\therefore BC = 2DE$	

Example 1: **In the diagram below, F is the midpoint of AB, G is the midpoint of AE and H is the midpoint of DE. Prove that AD = CE.**



Statement	Reason
AF = FB and AG = GE	given
∴ FG // BE → this means GH // EC	midpt. thm.
∴ CE = 2GH	conv. midpt. thm.
DH = HE and AG = GE	given
∴ AD = 2GH	midpt. thm.
∴ AD = CE	

Example 2: **In the diagram below, D is the midpoint of AB, E is the midpoint of AC, F is the midpoint of BH and G is the midpoint of HC.**
Prove that DFGE is a parallelogram.



Statement	Reason
AD = DB and AE = EC	given
∴ BC = 2DE and DE // BC	midpt. thm.
BF = FH and CG = GH	given
∴ BC = 2FG and BC // FG	midpt. thm.
∴ DE = FG and DE // FG	
∴ DFGE is a parm.	one pair of opp, sides = and //

Complete Exercise 6 on pages 328 and 329.

B. ANALYTICAL GEOMETRY

Analytical Geometry is a mixture of algebra and geometry and deals with the Cartesian coordinate system.
In Grade 10 you will deal with the following concepts:

1. **The Distance Formula**
2. **The Midpoint Formula**
3. **Average Gradient**

The green block below summarises the analytical formula which you are required to know.

FORMULAS

Distance Formula:
The distance between any two points $A(x_1;y_1)$ and $B(x_2;y_2)$ is

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \rightarrow \text{this is also written as } AB^2=(x_2-x_1)^2+(y_2-y_1)^2$$

Midpoint Formula

The coordinates of the midpoint of any line segment AB are: $\left(\frac{x_A+x_B}{2};\frac{y_A+y_B}{2}\right)$

Average Gradient (m):
The average gradient of a line is the ratio between the change in y over the change in x between any two points on the line.

$$\text{Average Gradient}=m=\frac{y_2-y_1}{x_2-x_1}$$

Equation of a Straight Line:
$y=mx+c$ or $(y-y_1)=m(x-x_1)$

Equation of a Horizontal Line:
$y=k \rightarrow$ where k is a constant

Equation of a Vertical Line:
$x=k \rightarrow$ where k is a constant

Gradients of Perpendicular (⊥) and Parallel (//) Lines:

The product of the gradients of perpendicular lines is –1: $m_1\times m_2=-1$ or $m_1=-\frac{1}{m_2}$

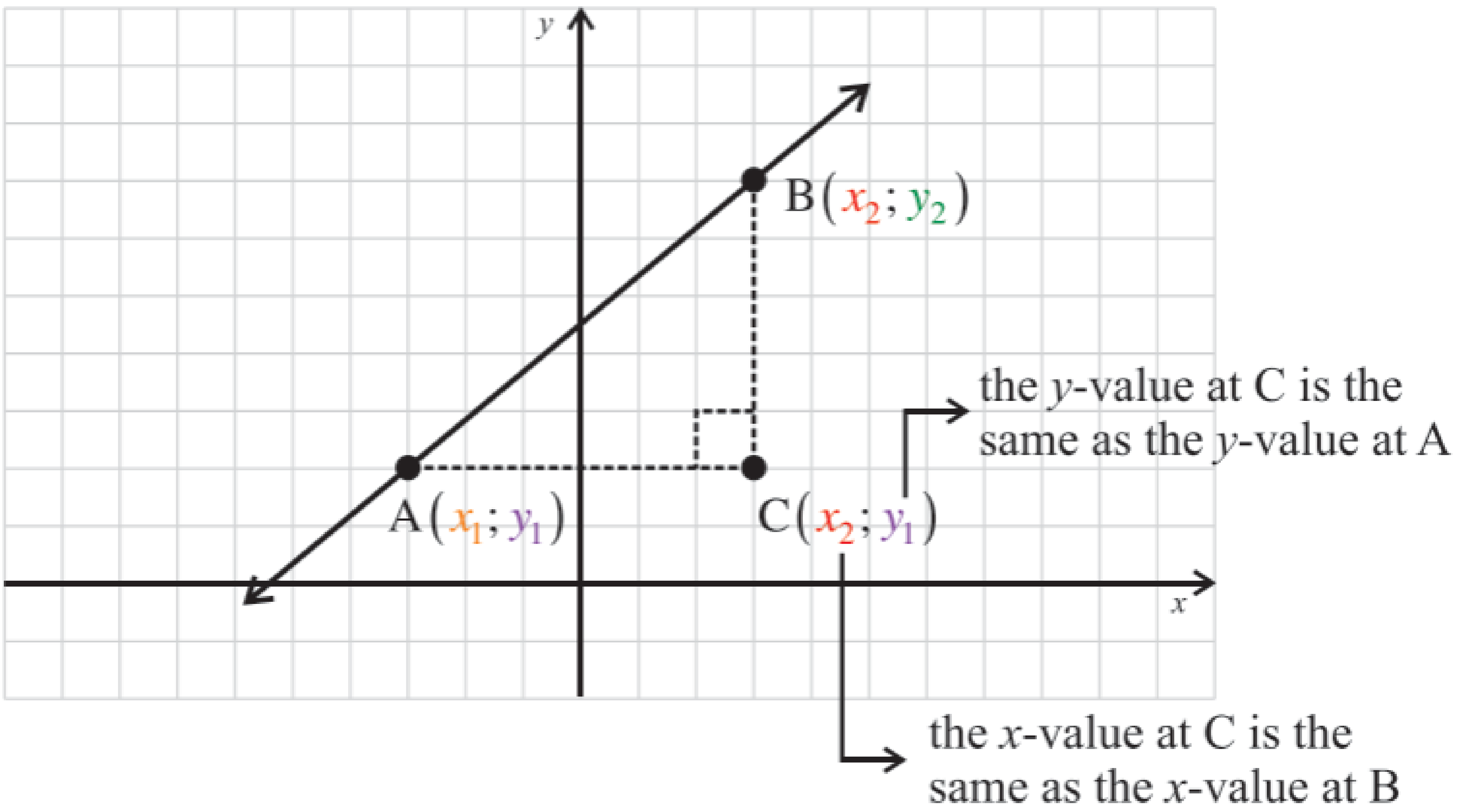
The gradients of parallel lines are equal: $m_1=m_2$

Collinear Points:
If points are collinear it means that they lie on the same straight line.
If points A, B and C are collinear: $m_{AB}=m_{BC}$

THE DISTANCE FORMULA

The distance formula is used to find the length of a line between two points.
The illustrative example below shows the derivation of the distance formula using the theorem of Pythagoras (pg. 142).

Illustrative Example: Determine the length of line segment AB, if A is the point $(x_1;y_1)$ and B is the point $(x_2;y_2)$.



y
x
B$(x_2;y_2)$
A$(x_1;y_1)$
C$(x_2;y_1)$
the y-value at C is the same as the y-value at A
the x-value at C is the same as the x-value at B

The first step in calculating the length of AB is to calculate the length of the lines AC and BC.

$$\begin{aligned} AC &= x_{\text{right}} - x_{\text{left}} \rightarrow \text{AC is a horizontal line} \\ &= x_2 - x_1 \end{aligned}$$

$$\begin{aligned} BC &= y_{\text{top}} - y_{\text{bottom}} \rightarrow \text{BC is a vertical line} \\ &= y_2 - y_1 \end{aligned}$$

You can now use the theorem of Pythagoras to calculate the length of AB.

$$AB^2 = AC^2 + BC^2 \rightarrow \text{pythag}$$

$$\therefore AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This is the general formula for the distance between any two points.

Distance Formula:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

By squaring both sides this formula can also be written as:

$$(\text{Distance})^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Example 1: **Given the points $A(3;-2)$ and $B(-5;4)$, determine the length of line segment AB.**

$$\begin{aligned}
AB &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\
&= \sqrt{(-5-3)^2+[4-(-2)]^2} \\
&= \sqrt{64+36} \\
&= \sqrt{100} \\
&= 10 \text{ units}
\end{aligned}$$

Example 2: **Given the points $A(4;10)$ and $B(-3;-8)$, determine the length of line segment AB. Give your answer correct to one decimal place.**

$$\begin{aligned}
AB &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\
&= \sqrt{(-3-4)^2+(-8-10)^2} \\
&= \sqrt{49+324} \\
&= \sqrt{373} \\
&= 19{,}3 \text{ units}
\end{aligned}$$

Example 3: **Given the points $P(-5;6)$ and $Q(-9;y)$, determine the value(s) of y if $PQ = 5\,\text{cm}$.**

$$\begin{aligned}
PQ &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\
\therefore 5 &= \sqrt{[-9-(-5)]^2+(y-6)^2} \\
\therefore 5 &= \sqrt{(-4)^2+(y-6)^2} \rightarrow \text{square both sides} \\
\therefore 25 &= (-4)^2+(y-6)^2 \\
\therefore 25 &= 16+y^2-12y+36 \rightarrow \text{dont forget the middle term} \\
\therefore 25 &= y^2-12y+52 \\
\therefore 0 &= y^2-12y+27 \\
\therefore 0 &= (y-3)(y-9) \\
\therefore y &= 3 \quad \text{or} \quad y = 9
\end{aligned}$$

The distance formula can also be written as: $AB^2 = (x_2-x_1)^2+(y_2-y_1)^2$

Complete Exercise 7 number 1 on page 329.

THE MIDPOINT FORMULA

The midpoint of a line segment is the point halfway between two given points.
The midpoint is generally denoted as: $M(x_M; y_M)$

Midpoint Formula:

$$M(x_M; y_M) = M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

You can also work with each part of the formula separately to determine the x and y-coordinates of the midpoint.

$$x_M = \frac{x_1 + x_2}{2} \qquad y_M = \frac{y_1 + y_2}{2}$$

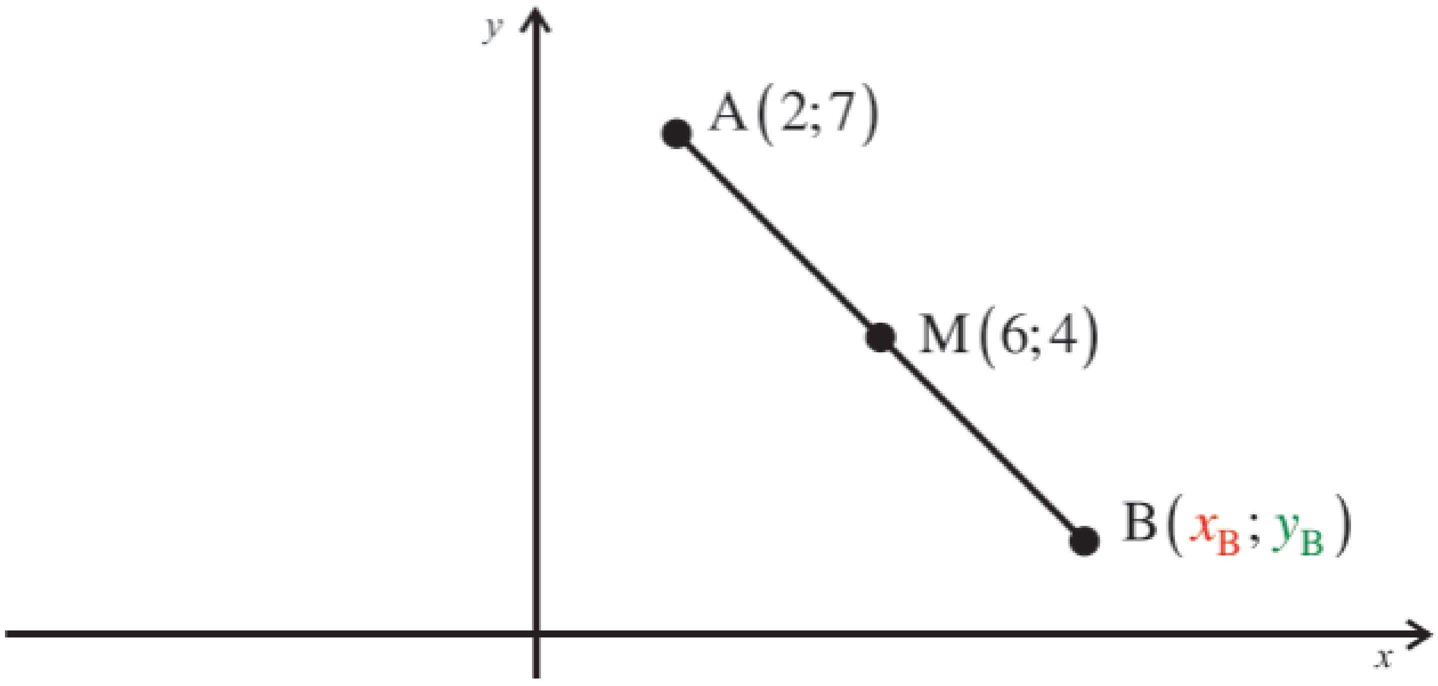
Example 1: **Determine the midpoint of line AB if A is the point (5;6) and B is the point (–3;–2).**

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$= M\left(\frac{5 + (-3)}{2}; \frac{6 + (-2)}{2}\right)$$

$$= M(1;2)$$

Example 2: **Determine the coordinates of B, if A is the point (2; 7) and the coordinates of M, the midpoint of AB are (6;4).**



$$x_M = \frac{x_1 + x_2}{2} \qquad y_M = \frac{y_1 + y_2}{2}$$

$$\therefore 6 = \frac{2 + x_B}{2} \qquad 4 = \frac{7 + y_B}{2}$$

$$\therefore 12 = 2 + x_B \qquad 8 = 7 + y_B$$

$$\therefore x_B = 10 \qquad y_B = 1$$

$\therefore$ B is the point $(10;1)$

Complete Exercise 7 numbers 2 and 3 on page 329.

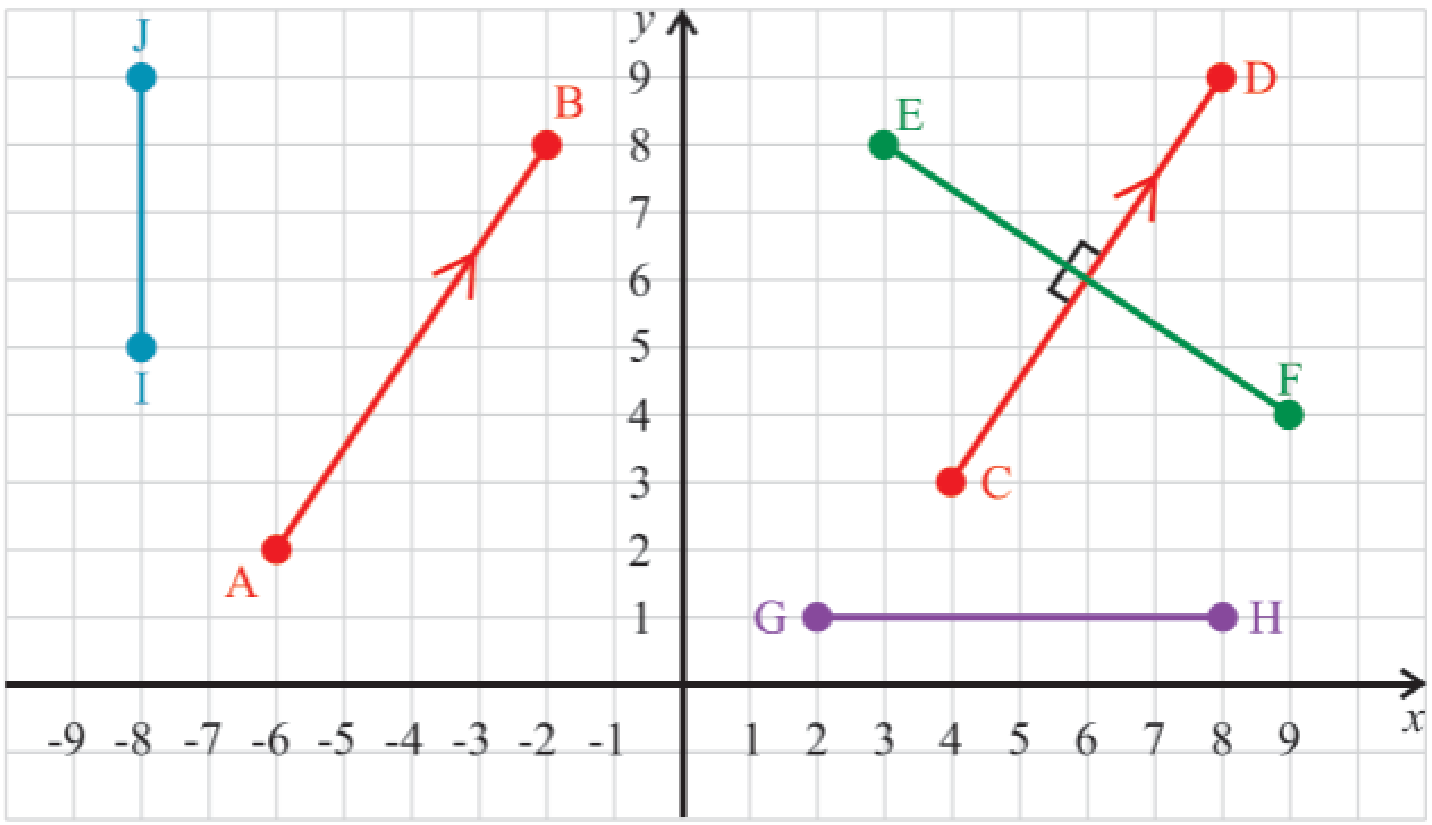
AVERAGE GRADIENT

You have already been introduced to the concept of average gradient in the linear function (pg. 99). Average gradient is the ratio of the change in *y*-values (the dependent variable) to the change in *x*-values (the independent variable).

Average Gradient:

$$\text{gradient} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow \frac{\text{change in } y}{\text{change in } x}$$

Example: Determine the gradient of the lines AB, CD, EF, GH and IJ shown below:



$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{-2-(-6)} = \frac{3}{2}$$

$$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-3}{8-4} = \frac{3}{2}$$

$$m_{EF} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{3-9} = -\frac{2}{3}$$

$$m_{GH} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-1}{8-2} = 0$$

$$m_{IJ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-5}{-8-(-8)} = \frac{4}{0} = \text{undefined} \rightarrow \text{you cannot divide by 0}$$

From the Above examples it can be seen that:

- **Parallel lines have equal gradients.**
- **The gradients of perpendicular lines are negative reciprocals of each other.**

  In other words $m_1 \times m_2 = -1$ **or** $m_1 = -\dfrac{1}{m_2}$.
- **The gradient of a horizontal line is always equal to 0.**
- **The gradient of a vertical line is undefined.**

Complete Exercise 7 numbers 4 and 5 on page 329.

MIXED EXAMPLES

Example 1: Given the points $A(2;3)$, $B(-5;1)$, $C(-12;y)$ and $D(-3;10)$.

Determine the value of y if:

a) B is the midpoint of AC.
b) AC is parallel to BD
c) AB is perpendicular to CD
d) $AC = 14$ units.

a) $$\frac{y_A + y_C}{2} = y_B$$

$$\therefore \frac{3+y}{2} = 1 \rightarrow \text{multiply both sides by 2}$$

$$\therefore 3 + y = 2$$

$$\therefore y = -1$$

b) $$m_{AC} = m_{BD}$$

$$\therefore \frac{y-3}{-12-2} = \frac{10-1}{-3-(-5)}$$

$$\therefore \frac{y-3}{-14} = \frac{9}{2} \rightarrow \text{multiply both sides by } -14$$

$$\therefore y - 3 = -63$$

$$\therefore y = -60$$

c)

Method 1	**Method 2**
$m_{CD} = -\frac{1}{m_{AB}}$ $m_{CD} = \frac{y-10}{-12-(-3)}$ and $m_{AB} = \frac{1-3}{-5-2}$ $= \frac{y-10}{-9}$ $\qquad = \frac{2}{7}$ $\therefore \frac{y-10}{-9} = -\frac{7}{2} \rightarrow -\frac{1}{m_{AB}} = -\left(1 \div \frac{2}{7}\right) = -\frac{7}{2}$ $\therefore y - 10 = \frac{63}{2}$ $\therefore y = \frac{83}{2}$	$m_{CD} \times m_{AB} = -1$ $m_{CD} = \frac{y-10}{-12-(-3)}$ and $m_{AB} = \frac{1-3}{-5-2}$ $= \frac{y-10}{-9}$ $\qquad = \frac{2}{7}$ $\therefore \frac{y-10}{-9} \times \frac{2}{7} = -1$ $\therefore \frac{2y-20}{-63} = -1$ $\therefore 2y - 20 = 63$ $\therefore 2y = 83$ $\therefore y = \frac{83}{2}$

d) $$AC^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

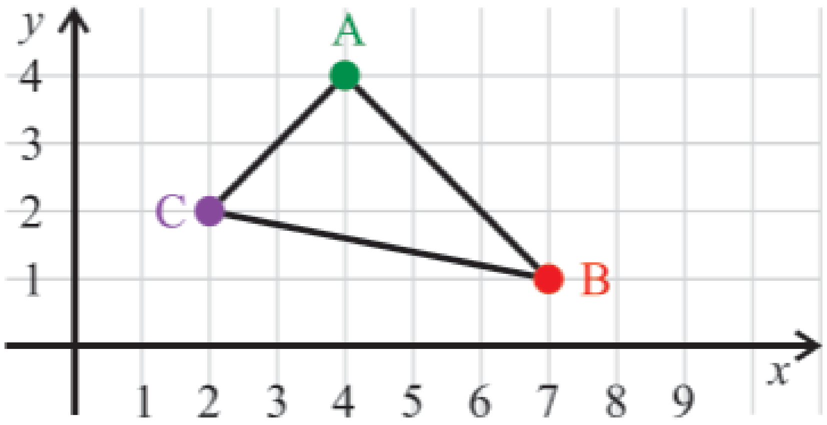
$$\therefore 14^2 = (-12-2)^2 + (y-3)^2$$

$$\therefore 14^2 = 14^2 + (y-3)^2$$

$$\therefore 0 = (y-3)^2$$

$$\therefore y = 3$$

Example 2: **Determine if ΔABC with vertices A(4;4), B(7;1) and C(2;2) is a right angled triangle.**



There are two methods that can be used to solve this problem:

Method 1: Use Pythagoras and the distance formula.

$$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(2-4)^2+(2-4)^2}$$
$$=\sqrt{8}\text{ units}$$

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(7-4)^2+(1-4)^2}$$
$$=\sqrt{18}\text{ units}$$

$$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(2-7)^2+(2-1)^2}$$
$$=\sqrt{26}\text{ units}$$

BC is the longest side. In a right angled triangle the hypotenuse is always the longest side. Therefore, for ΔABC to be a right angled triangle: $BC^2=AB^2+AC^2$

$$\text{LHS}=\left(\sqrt{26}\right)^2=26$$

$$\text{RHS}=\left(\sqrt{8}\right)^2+\left(\sqrt{18}\right)^2=26=\text{LHS}$$

$\therefore \Delta ABC$ is right angled at $\hat{A}$

Method 2: Use the properties of perpendicular lines and their gradients.

$$m_{AC}=\frac{y_2-y_1}{x_2-x_1}=\frac{2-4}{2-4}=1$$

$$m_{AB}=\frac{y_2-y_1}{x_2-x_1}=\frac{1-4}{7-4}=-1$$

$$m_{BC}=\frac{y_2-y_1}{x_2-x_1}=\frac{2-1}{2-7}=\frac{-1}{5}$$

If two lines are perpendicular: $m_1\times m_2=-1$

$m_{AC}\times m_{AB}=-1$

$\therefore$ AC is perpendicular to AB, $\therefore \Delta ABC$ is right angled at $\hat{A}$

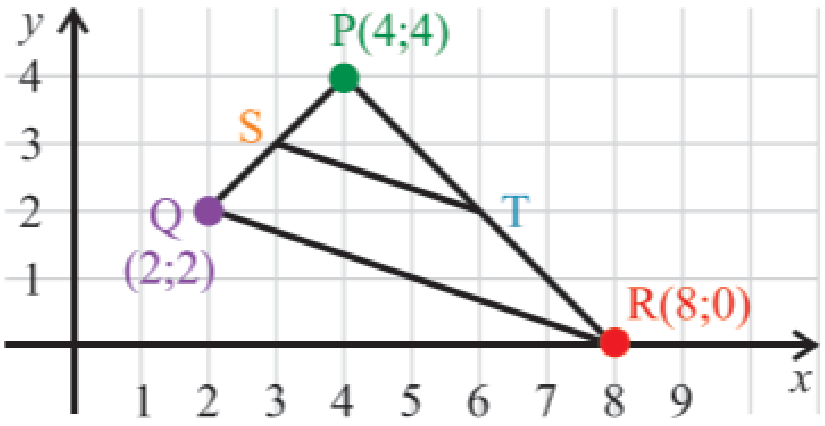
Example 3: **ΔPQR with vertices P(4;4), Q(2;2) and R(8;0) is sketched below.**

a) Determine the coordinates of S and T, if they are the midpoints of PQ and PR respectively.

b) Show that the points P, T and R are collinear.

Collinear points are points that lie on the same straight line.

c) Prove that ST is half the length of QR.



a) Midpoint of PQ:

$$S\left(\frac{x_2+x_1}{2};\frac{y_2+y_1}{2}\right)=S\left(\frac{4+2}{2};\frac{4+2}{2}\right)=S(3;3)$$

Midpoint of PR:

$$T\left(\frac{x_2+x_1}{2};\frac{y_2+y_1}{2}\right)=T\left(\frac{4+8}{2};\frac{4+0}{2}\right)=T(6;2)$$

$\therefore$ S is the point $(3;3)$ and T is the point $(6;2)$

b) For P, T and R to be collinear the gradient of PT must be equal to the gradient of TR.

$$m_{PT}=\frac{y_2-y_1}{x_2-x_1}=\frac{4-2}{4-6}=-\frac{2}{2}=-1 \qquad m_{TR}=\frac{y_2-y_1}{x_2-x_1}=\frac{2-0}{6-8}=-\frac{2}{2}=-1$$

$\therefore m_{PT}=m_{TR}$

$\therefore$ the points P, T and R are collinear

c)

$$\begin{aligned}QR&=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\\&=\sqrt{(8-2)^2+(0-2)^2}\\&=\sqrt{40}\rightarrow\sqrt{40}=\sqrt{4\times10}\\&=2\sqrt{10}\text{ units}\end{aligned}$$

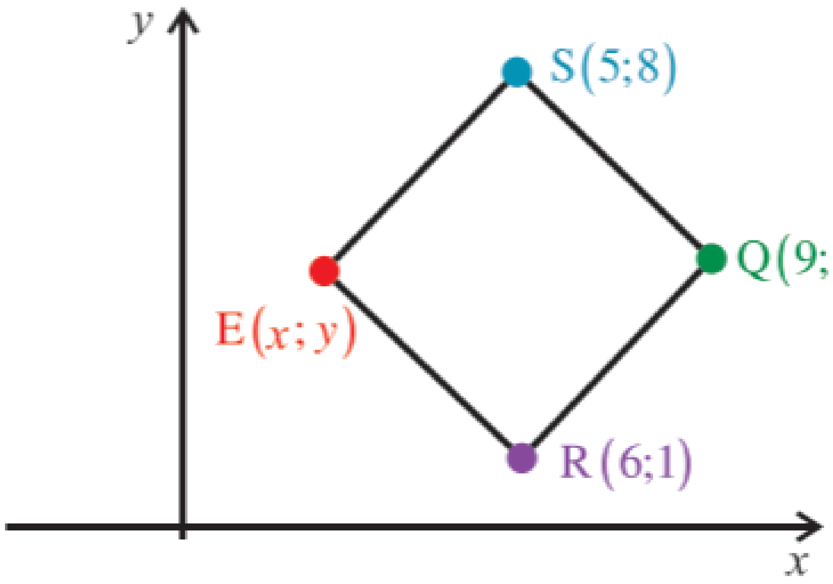
$$\begin{aligned}ST&=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\\&=\sqrt{(6-3)^2+(2-3)^2}\\&=\sqrt{10}\text{ units}\end{aligned}$$

$$\therefore ST=\frac{1}{2}QR$$

Note that if points P, T, and R are collinear, PT + TR = PR where P and R are endpoints of line PTR. This is a longer method of proving that points are collinear.

Complete Exercise 7 numbers 6 and 7 on page 329.

Example 4: **A square has vertices S(5;8), Q(9;5), R(6;1) and E(x;y).**



a) Using the fact that the diagonals of a square bisect each other, determine the coordinates of E.

b) If E is the point (2;4), prove that the diagonals of a square are equal in length.

a) As the diagonals of a square bisect each other they have the same midpoint.

$\therefore$ the midpoint of SR = the midpoint of QE.

$$\therefore \mathrm{M}\left(\frac{x_S + x_R}{2}; \frac{y_S + y_R}{2}\right)$$

$$= \mathrm{M}\left(\frac{5+6}{2}; \frac{8+1}{2}\right)$$

$$= \mathrm{M}\left(\frac{11}{2}; \frac{9}{2}\right)$$

$\therefore$ the x-value of the midpoint of line QE is $\frac{11}{2}$ and the y-value is $\frac{9}{2}$

$\therefore \frac{x_E + x_Q}{2} = \frac{11}{2}$

$\therefore \frac{x_E + 9}{2} = \frac{11}{2}$

$\therefore x_E = 2$

$\therefore \frac{y_E + y_Q}{2} = \frac{9}{2}$

$\therefore \frac{y_E + 5}{2} = \frac{9}{2}$

$\therefore y_E = 4$

$\therefore$ E is the point $(2;4)$

b) $\mathrm{EQ} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(9-2)^2 + (5-4)^2}$

$= \sqrt{50} \rightarrow \sqrt{50} = \sqrt{25 \times 2}$

$= 5\sqrt{2}$ units

$\mathrm{SR} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5-6)^2 + (8-1)^2}$

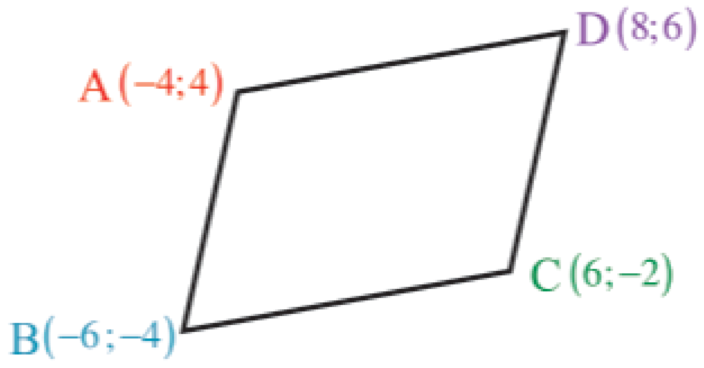
$= \sqrt{50} \rightarrow \sqrt{50} = \sqrt{25 \times 2}$

$= 5\sqrt{2}$ units

$\therefore \mathrm{EQ} = \mathrm{SR}$

$\therefore$ Diagonals of a square are equal in length.

Example 5: **John draws quadrilateral A(–4;4), B(–6;–4), C(6;–2) and D(8;6) and claims that his quadrilateral is a rhombus. Thembi performs some calculations and claims that the quadrilateral which John drew is a parallelogram and cannot be a rhombus. Who is correct?**



The first step is to determine whether or not the quadrilateral is a parallelogram.

The easiest method of proving a shape is a parallelogram is to prove that the diagonals bisect each other (that they have the same midpoint).

$$M_{AC} = \left(\frac{x_2+x_1}{2};\frac{y_2+y_1}{2}\right) \qquad M_{BD} = \left(\frac{x_2+x_1}{2};\frac{y_2+y_1}{2}\right)$$

$$= \left(\frac{6-4}{2};\frac{-2+4}{2}\right) \qquad = \left(\frac{8-6}{2};\frac{6-4}{2}\right)$$

$$= (1;1) \qquad = (1;1)$$

$$\therefore M_{AC} = M_{BD}$$

$\therefore$ The quadrilateral is a parallelogram as the diagonals bisect each other.

However, this does not mean that John is wrong, as a rhombus is also a parallelogram. In order to prove that the shape can't be a rhombus, one of the properties of a rhombus has to be disproved.

In order for a quadrilateral to be a rhombus the diagonals have to bisect each other at 90º.

$$\therefore m_{AC} \times m_{BD} = -1$$

$$m_{AC} = \frac{y_2-y_1}{x_2-x_1} \qquad M_{BD} = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{-2-4}{6-(-4)} \qquad = \frac{-4-6}{-6-8}$$

$$= -\frac{3}{5} \qquad = \frac{5}{7}$$

$$\therefore m_{AC} \times m_{BD} = -\frac{3}{7} \neq -1$$

$\therefore$ the diagonals do not intersect 90^{o}

$\therefore$ the quadrilateral is not a rhombus and Thembi is correct.

Complete Exercise 8 on page 330.

DETERMINING THE EQUATION OF A STRAIGHT LINE

You have already had to determine the equation of a straight line in chapter 6. There are 2 ways of representing the equation of a straight line.
A straight line can be represented as either $y = mx + c$ or $y - y_1 = m(x - x_1)$.

Midpoint Formula:

$$y = mx + c \quad \text{or} \quad y - y_1 = m(x - x_1)$$

Where: m → **Gradient of the line**
c → **y-intercept**
(x_1 ; y_1) → A point on the line

Example 1: **Determine the equation of a straight line which passes through the points A(3;4) and B(5;10).**

You can determine the equation of the line using either of the above equations. In both methods you first have to determine the gradient of the line.

$$m = gradient = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 4}{5 - 3} = 3$$

Now substitute the gradient and a point on the graph into one of the above equations:

Method 1: using $y = mx + c$

$y = 3x + c \rightarrow m = 3$

Sub A$(3;4)$ → B could also be used

$\therefore 4 = 3(3) + c$

$\therefore c = -5$

$\therefore y = 3x - 5$

Method 2: using $y - y_1 = m(x - x_1)$

$y - y_1 = 3(x - x_1) \rightarrow m = 3$

Sub A$(3;4)$ → B could also be used

$\therefore y - 4 = 3(x - 3)$

$\therefore y - 4 = 3x - 9$

$\therefore y = 3x - 5$

Example 2: **Determine the equation of the straight line which is:**

a) Parallel to the line $y = 2x + 5$, passing through the point A(4;5).

b) Perpendicular to the line $y = \frac{1}{3}x + 5$, with a y-intercept of 9.

c) Parallel to the x-axis and passes through the point (1;4).

d) Parallel to the y-axis and passes through the point (1;4).

a) $y = 2x + c \rightarrow$ line is // to $y = 2x + 5$

sub A$(4;5)$

$\therefore 5 = 2(4) + c$

$\therefore c = -3$

$\therefore y = 2x - 3$

b) $y = -3x + c \rightarrow$ line is $\perp$ to $y = \frac{1}{3}x + 5$

sub $c = 9 \rightarrow$ y-intercept is given as 9

$\therefore y = -3x + 9$

c) $y = 4$

A line parallel to the x-axis is a horizontal line. The equation of a horizontal line is always in the form of: $y = k \rightarrow$ where k is a constant.

d) $x = 1$

A line parallel to the y-axis is a vertical line. The equation of a vertical line is always in the form of: $x = k \rightarrow$ where k is a constant

Complete the Mixed Exercise on pages 330, 331, 332 and 333.

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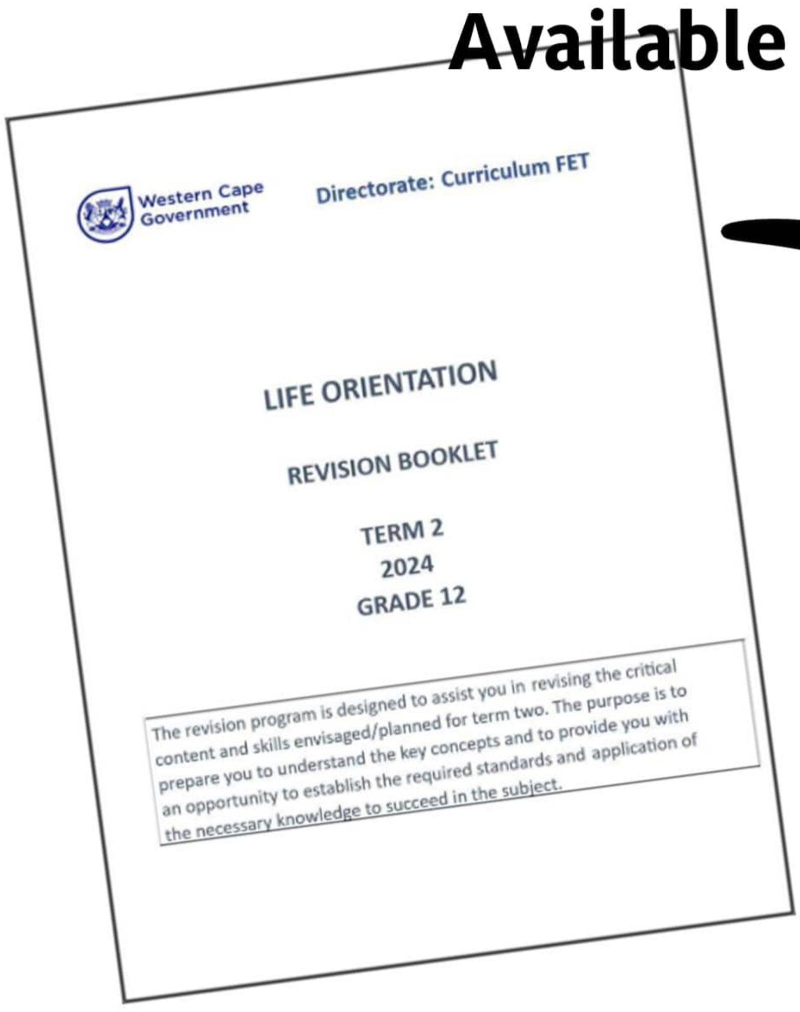

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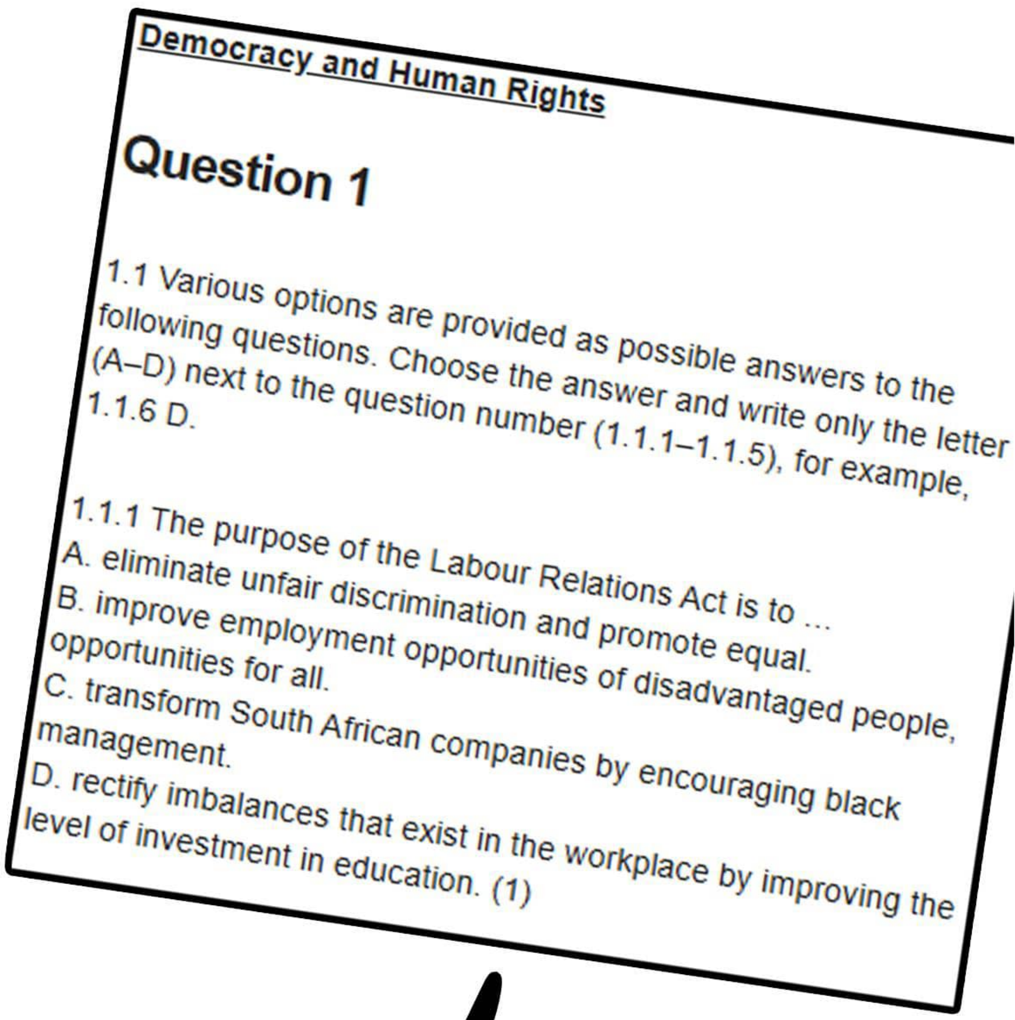
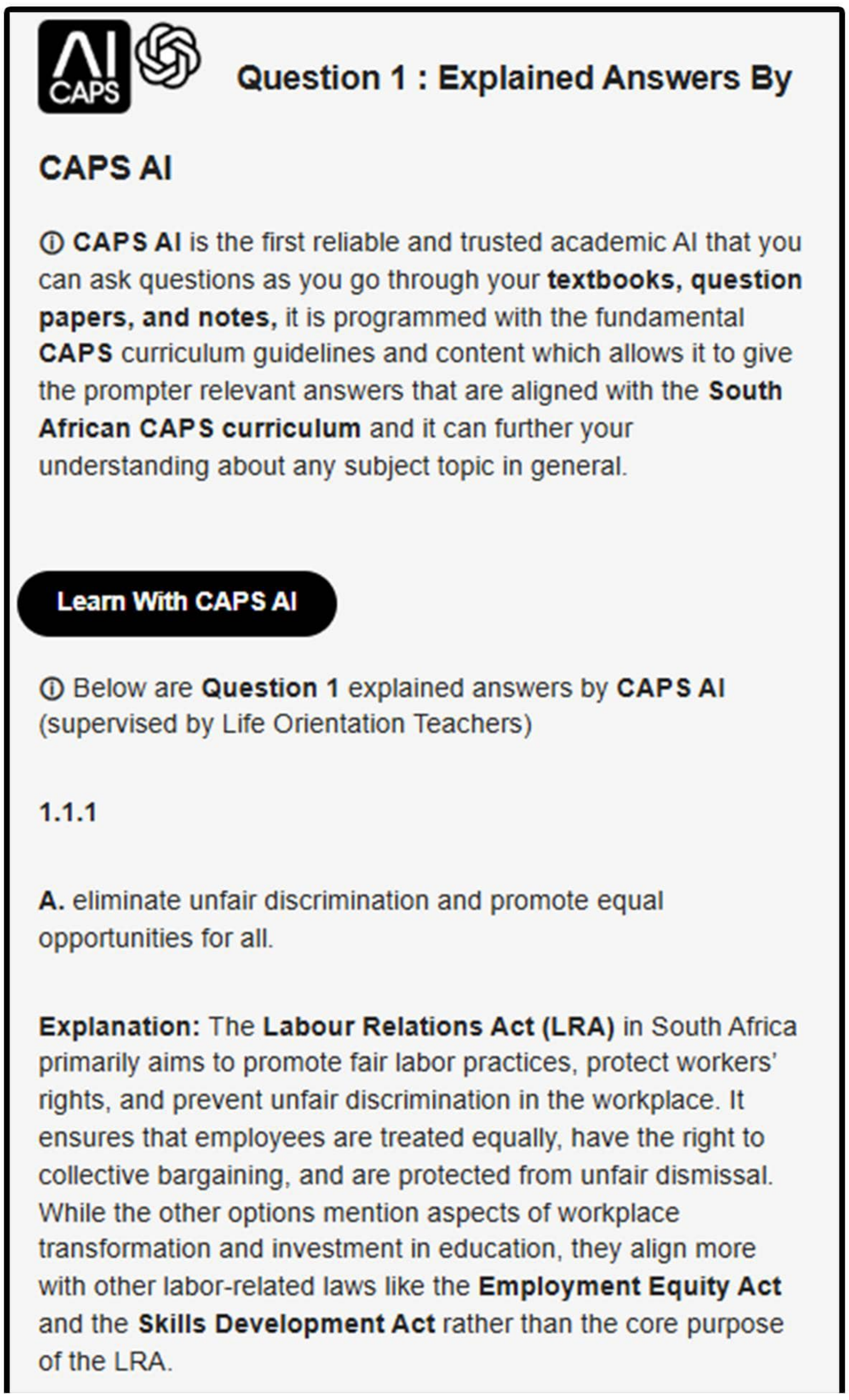


Democracy and Human Rights

Question 1

1.1 Various options are provided as possible answers to the following questions. Choose the answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5), for example, 1.1.6 D.

1.1.1 The purpose of the Labour Relations Act is to ...
A. eliminate unfair discrimination and promote equal opportunities for all.
B. improve employment opportunities of disadvantaged people.
C. transform South African companies by encouraging black management.
D. rectify imbalances that exist in the workplace by improving the level of investment in education. (1)



Question 1 : Explained Answers By CAPS AI

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1.1.1

A. eliminate unfair discrimination and promote equal opportunities for all.

Explanation: The **Labour Relations Act (LRA)** in South Africa primarily aims to promote fair labor practices, protect workers' rights, and prevent unfair discrimination in the workplace. It ensures that employees are treated equally, have the right to collective bargaining, and are protected from unfair dismissal. While the other options mention aspects of workplace transformation and investment in education, they align more with other labor-related laws like the **Employment Equity Act** and the **Skills Development Act** rather than the core purpose of the LRA.

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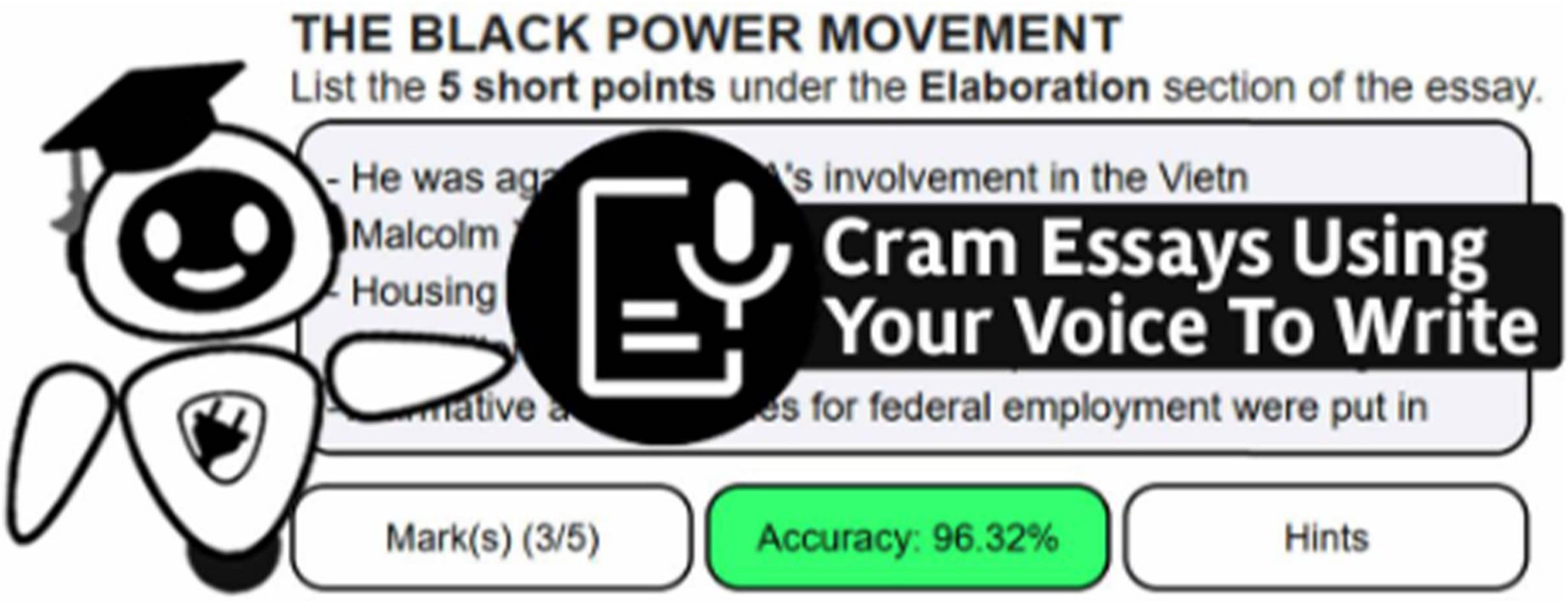

THE BLACK POWER MOVEMENT
List the **5 short points** under the **Elaboration** section of the essay.

- He was ag 's involvement in the Vietn
Malcolm
- Housing
- mative a es for federal employment were put in

Mark(s) (3/5)
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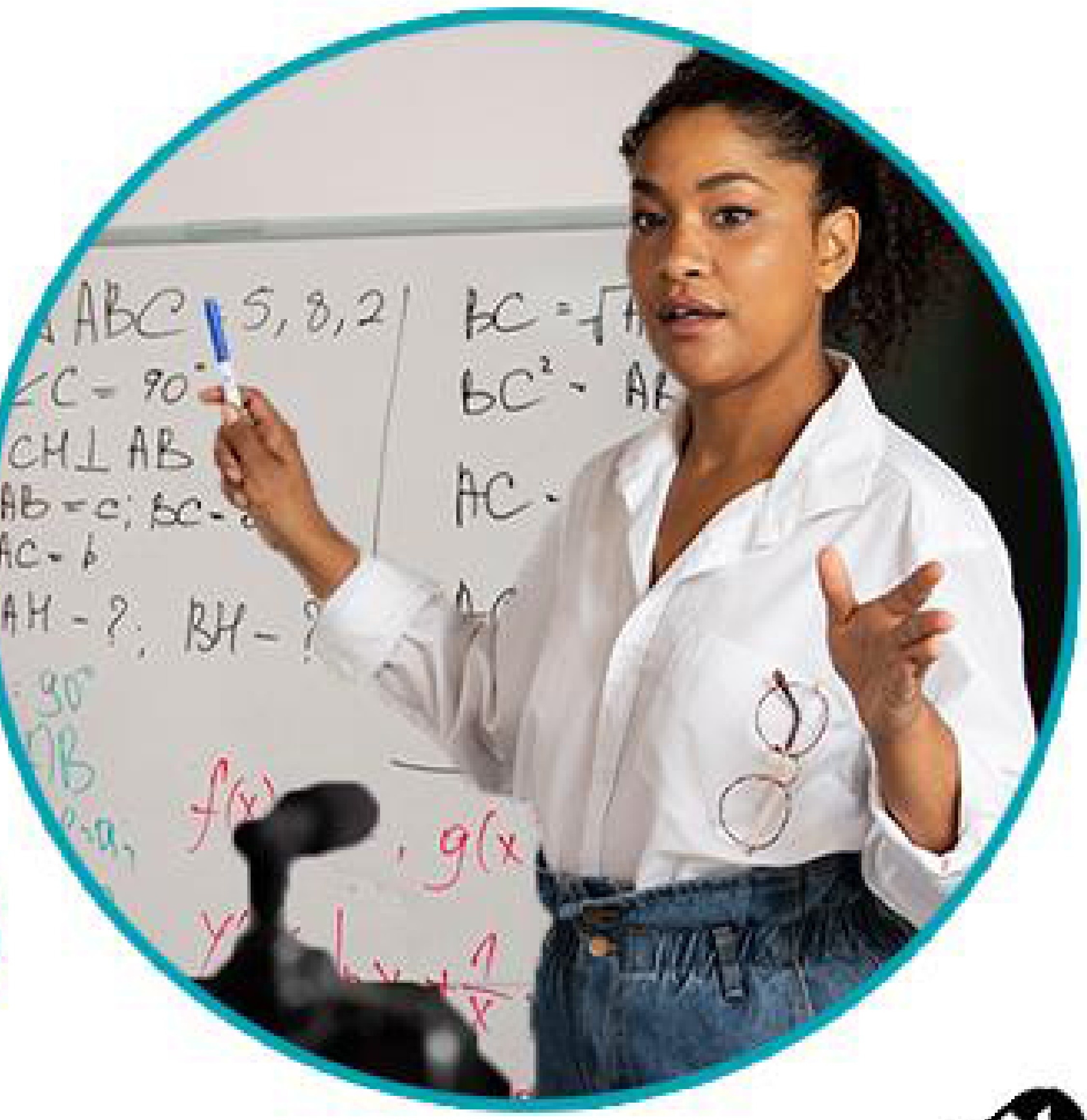

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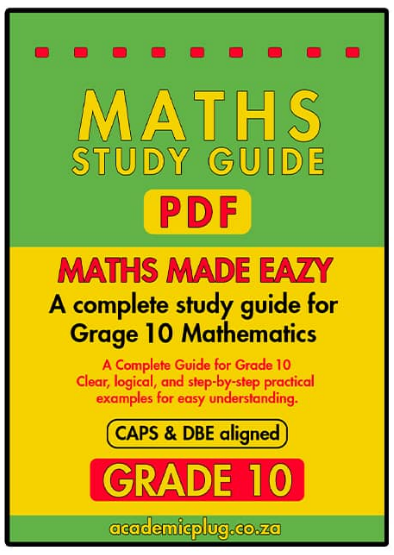

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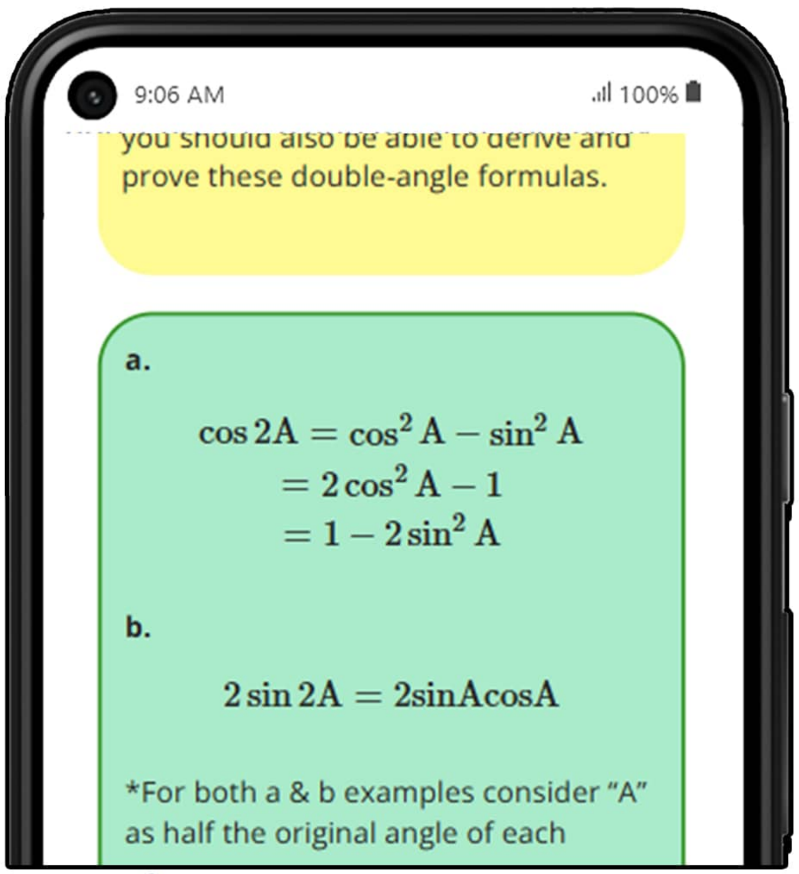

9:06 AM 100%

you should also be able to derive and prove these double-angle formulas.

a.

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

b.

$$2\sin 2A = 2\sin A\cos A$$

*For both a & b examples consider "A" as half the original angle of each

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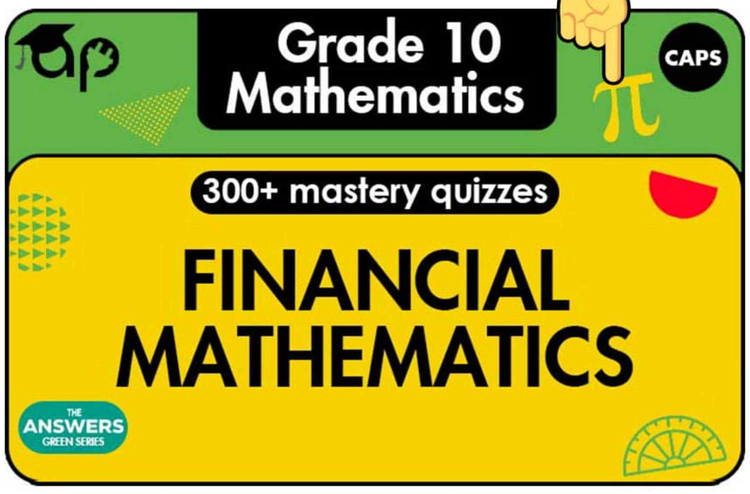

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CHAPTER 8
FINANCIAL MATHEMATICS

WHAT YOU WILL LEARN IN THIS CHAPTER

PAGE	CONTENT	CAPS	ASSESSMENTS YOU WILL BE ABLE TO:
167	**A. Financial Mathematics**	3	• Use the formula for simple and compound growth to solve problems. • Demonstrate an understanding of the implications of fluctuating foreign exchange rates.
167	• Simple Interest/Growth		
169	• Hire Purchase Agreements		
171	• Compound Interest		
173	**B. Time Lines**		
173	• Additional Deposits and Withdrawals		
175	• Changes in Interest Rates		
176	• Mixed Examples		
177	**C. Exchange Rates**		

A. FINANCIAL MATHEMATICS

In Grade 10 there are two types of interest calculations you are required to know:

1. **Simple Interest**
2. **Compound Interest**

1. SIMPLE INTEREST/GROWTH

Simple interest is calculated on the original amount invested or borrowed.
The value of the investment or loan increases by a fixed amount at regular time intervals. The amount that the loan increases by is calculated using the interest rate and the initial investment.

Simple Interest Formula:

$$\mathrm{A}=\mathrm{P}(\mathbf{1}+\boldsymbol{in})$$

Where : $\mathbf{A} \rightarrow$ Final Amount

$\mathbf{P} \rightarrow$ Principal amount / starting value

$\boldsymbol{i} \rightarrow$ Interest rate (written as a decimal $\therefore$ divide the given interest rate by 100)

$\boldsymbol{n} \rightarrow$ Number of time periods (number of times interest is paid)

Example 1: Patricia invests R25 000 into a savings account in order to save money for university. If the interest rate is 8,5% per annum simple interest, calculate:

a) The total amount that Patricia will have after 5 years.
b) The total interest earned.
c) The interest that Patricia received each year.

a) Simple Interest: $\mathrm{A}=\mathrm{P}(1+in)$

$$\therefore \mathrm{A}=25\,000[1+(0{,}085)(5)] \rightarrow \text{interest must be written as a decimal } \left(\frac{8{,}5}{100}\right)$$

$$=25\,000(1{,}425)$$

$$=\mathrm{R}35\,625$$

$\therefore$ Patricia will have R35 625 after 5 years. → always answer the question

b) Interest Earned = Final Amount – Principal Amount

$$\therefore \text{Interest Earned}=35\,625-25\,000$$

$$=\mathrm{R}10\,625$$

$\therefore$ Patricia earned R10 625 interest. → always answer the question

c) As interest is simple the same amount is paid each year:

$$\therefore \text{Yearly Interest}=\frac{\text{Total Interest}}{\text{Number of Years}} \rightarrow \text{this is only true for simple interest}$$

$$=\frac{10\,625}{5}$$

$$=\mathrm{R}2125 \text{ per year}$$

$\therefore$ Patricia received R2125 interest each year. → always answer the question

Example 2: At the end of 2006 Sipho invested R5 000 into a savings account. At the end of 2010 he had a total of R7 400. Calculate the simple interest rate that Sipho received.

Simple Interest: $A = P(1 + in)$

$\therefore 7\,400 = 5\,000\left[1 + (i)(4)\right]$ → the money was invested for a period of 4 years

$\therefore \dfrac{7\,400}{5\,000} = 1 + 4i$ → both sides were divided by 5 000

$\therefore 1{,}48 - 1 = 4i$

$\therefore 0{,}48 = 4i$

$\therefore i = \dfrac{0{,}48}{4}$

$\therefore i = 0{,}12$

$\therefore i = 12\%$

$\therefore$ Sipho received a simple interest rate of 12% per annum. → always answer the question

Example 3: Sarah guessed 4 correct numbers in the Lottery and invested her winnings at a rate of 7,5% per annum simple interest. After two years she had R17 250 in her savings account. How much did Sarah win?

Simple Interest: $A = P(1 + in)$

$\therefore 17\,250 = P\left[1 + (0{,}075)(2)\right]$ → interest must be written as a decimal $\left(\dfrac{7{,}5}{100}\right)$

$\therefore 17\,250 = 1{,}15P$

$\therefore P = \dfrac{17\,250}{1{,}15}$

$\therefore P = 15\,000$

$\therefore$ Sarah won R15 000. → always answer the question

Example 4: Luke wants to save R22 000 for an overseas holiday. If he can afford to invest R15 800 into a savings account that pays 13% simple interest per annum, how long will he have to wait to go on holiday? Give your answer to the nearest year.

Simple Interest: $A = P(1 + in)$

$\therefore 22\,000 = 15\,800\left[1 + (0{,}13)(n)\right]$ → interest must be written as a decimal $\left(\dfrac{13}{100}\right)$

$\therefore 22\,000 = 15\,800 + 2054n$

$\therefore 22\,000 - 15\,800 = 2054n$

$\therefore n = \dfrac{6\,200}{2054}$

$\therefore n = 3$ → the question said you must give your answer to the nearest year

$\therefore$ Luke will have to wait 3 years to go on holiday. → always answer the question

Complete Exercise 1 on page 346.

HIRE PURCHASE AGREEMENTS

Hire Purchase (HP) agreements are always calculated using simple interest.
An HP agreement is essentially a short term loan that many stores offer on expensive items. The simple interest is calculated on the full value of the loan for the agreed upon repayment period. It is common for stores to insist on **cash deposits** and to add on additional **monthly insurance costs**.
No interest is charged on cash deposits or insurance costs.

Example 1: **Sandy purchases a dishwasher for R6 500 on a hire purchase agreement. The agreement is that the loan is to be paid off over a period of 3 years with equal monthly payments at an interest rate of 12 % per annum. Calculate:**

a) The amount that Sandy will pay for the dishwasher.
b) The monthly repayments.
c) The total interest paid.

a) Simple Interest: $A = P(1 + in)$

$$\therefore A = 6\,500\left[1 + (0{,}12)(3)\right] \rightarrow \text{interest must be written as a decimal } \left(\frac{12}{100}\right)$$

$$= 6\,500(1{,}36)$$

$$= R8\,840$$

$\therefore$ Sandy will actually pay R8 840 → always answer the question

b)
$$\text{Monthly repayments} = \frac{\text{Total Paid}}{\text{Number of Months}} \rightarrow \text{this is only true for simple interest}$$

$$= \frac{8\,840}{3(12)}$$

$$= R245{,}56 \text{ per month}$$

$\therefore$ Sandy will pay R245,56 per month → always answer the question

c)
$$\text{Total Interest} = \text{Total Paid} - \text{Cash Price}$$

$$= 8\,840 - 6\,500$$

$$= R2\,340$$

$\therefore$ The total interest paid is R2 340 → always answer the question

Example 2: **Simon purchases a home entertainment system for R12 000 on a hire purchase agreement. The agreement is that the loan is to be paid off over a period of 5 years, with equal monthly payments at an interest rate of 9% per annum. If a 10% cash deposit is required and an additional insurance premium of R25 per month is added on to the monthly payments, calculate:**

a) The amount that Simon actually paid.
b) The monthly repayments.

a) The first step in calculating the total amount paid is to calculate the cash deposit:

$$\begin{aligned}\text{Cash deposit} &= 12\,000 \times 0{,}1\\ &= \text{R}1200\end{aligned}$$

$$\begin{aligned}\therefore \text{Amount owing} &= \text{cash price} - \text{deposit}\\ &= 12\,000 - 1200\\ &= \text{R}10\,800 \rightarrow \text{interest is charged on this amount}\end{aligned}$$

Simple Interest: $\text{A} = \text{P}(1 + in)$

$$\begin{aligned}\therefore \text{A} &= 10\,800\left[1 + (0{,}09)(5)\right] \rightarrow \text{interest must be written as a decimal } \left(\frac{9}{100}\right)\\ &= 10\,800(1{,}45)\\ &= \text{R}15\,660 \rightarrow \text{the amount owing without the additional insurance cost}\end{aligned}$$

You now have to calculate the additional insurance cost

$$\begin{aligned}\text{Insurance cost} &= \text{Monthly premium} \times \text{number of months}\\ &= 25(5 \times 12) \rightarrow \text{ convert years to months}\\ &= \text{R}1500\end{aligned}$$

$$\begin{aligned}\text{Total Cost} &= 1200 + 15\,660 + 1500 \rightarrow \text{no interest was paid on the deposit or insurance}\\ &= \text{R}18\,360\end{aligned}$$

$\therefore$ Simon actually paid R18 360

b)

$$\begin{aligned}\text{Monthly repayments} &= \frac{\text{Total Paid (excluding any deposits)}}{\text{Number of Months}} \rightarrow \text{only for simple interest}\\ &= \frac{15\,660 + 1500}{5(12)}\\ &= \text{R}286 \text{ per month}\end{aligned}$$

$\therefore$ Simon will pay R286 per month → always answer the question

An initial cash deposit is paid at the beginning of a hire purchase agreement and is therefore not included in the monthly repayments.

Complete Exercise 2 on page 346.

2. COMPOUND INTEREST

In compound interest, interest is calculated on the original investment as well as the interest earned in each period. Therefore the amount of interest earned increases each period, unlike simple interest calculations (pg. 167) where interest earned is the same in each period.

Compound Interest Formula:

$$\mathbf{A} = \mathbf{P}(1+\boldsymbol{i})^{\boldsymbol{n}}$$

Where : **A** → Final Amount

P → Principal amount / starting value

i → Interest rate (written as a decimal ∴ divide the given interest rate by 100)

n → Number of time periods (number of times interest is compounded)

Example 1: Mrs Baker has R20 000 which she would like to save for an overseas holiday with her family in three years' time. She invests her money at a rate of 8,5% per annum compounded annually. How much will Mrs Baker have available to spend on her family holiday?

Compound Interest: $A = P(1+i)^n$

$P = 20\,000 \qquad i = \frac{8,5}{100} = 0,085 \qquad n = 3$

$$\therefore A = 20\,000(1+0,085)^3$$
$$= R25\,545,78$$

$\therefore$ Mrs Baker will have $R25\,545,78$ for her holiday → always answer the question

Example 2: Lisa invested money into a savings account which promised an interest rate of 9% per annum compounded annually. After ten years she has R14 204,18 in her account. How much did Lisa initially invest?

Compound Interest: $A = P(1+i)^n$

$\therefore 14\,204,18 = P(1+0,09)^{10} \rightarrow$ divide both sides by $(1+0,09)^{10}$

$$\therefore \frac{14\,204,18}{(1+0,09)^{10}} = P$$

$$\therefore P = R6\,000$$

$\therefore$ Lisa initially invested $R6\,000$ → always answer the question

Do not round off during financial mathematics calculations. Rather use your calculator's memory or enter the whole sum into the calculator.

Example 3: **Lesedi invested R6 550 into a savings account. If the interest was compounded annually, calculate the yearly interest rate that Lesedi received if he had R11 543,34 in his account after 5 years.**

Compound Interest: $A = P(1+i)^n$

$$\therefore 11543,34 = 6550(1+i)^5$$

$$\therefore \frac{11543,34}{6550} = (1+i)^5 \to \text{fifth root } \left(\sqrt[5]{\ }\right) \text{ both sides}$$

$$\therefore \sqrt[5]{\frac{11543,34}{6550}} = 1+i$$

$$\therefore \sqrt[5]{\frac{11543,34}{6550}} - 1 = i$$

$$\therefore i = 0,12$$

$$\therefore i = 12\%$$

Example 4: **If Mr Smit's salary is R25 000 per month and has increased at the same rate as inflation since he started working 20 years ago, calculate:**

a) His salary when he started working, if the average rate of inflation over the last twenty years has been 8,5% per annum.

b) His salary in 5 years' time, if inflation continues at the same rate.

a) Inflation: $A = P(1+i)^n \to$ inflation is always compound interest

$A = 25000 \qquad i = 0,085 \qquad n = 20$

$$\therefore 25000 = P(1+0,085)^{20}$$

$$\therefore P = \frac{25000}{(1+0,085)^{20}}$$

$$= R4890,41$$

$\therefore$ Mr Smit's salary when he started working was R4 890,41 per month

b) $P = 25000 \qquad i = 0,085 \qquad n = 5$

$$\therefore A = 25000(1+0,085)^5$$

$$= R37591,42$$

$\therefore$ Mr Smit's salary will be R37 591,42 per month in 5 years time

Always use compound interest for questions which involve inflation.

Complete Exercise 3 on pages 346 and 347.

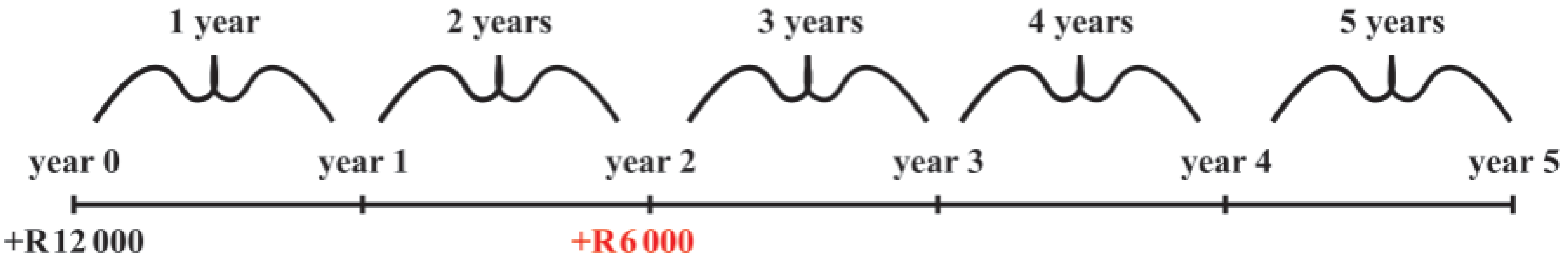
B. TIME LINES

The formulas for simple and compound interest (growth) do not take into account changes in the principal value due to additional deposits or withdrawals.
They also do not take into account changes in interest rates.
Time lines are useful in solving problems where additional deposits or withdrawals are made, as well as problems involving fluctuations in the interest rate.

ADDITIONAL DEPOSITS AND WITHDRAWALS

When dealing with additional deposits or withdrawals, you calculate the effect of each deposit or withdrawal separately.

Example 1: **Mrs Davis deposits R12 000 into a savings account. Two years later she adds an additional R6 000 to her savings. How much will she have after 5 years if she receives an interest rate of 7,5% per annum compounded annually?**



The first step in this example is to calculate the amount that will be in the account after 5 years due to the initial deposit of R12 000.

Compound Interest: $A = P(1+i)^n$

$$A = 12\,000(1+0{,}075)^5 \rightarrow \text{R12 000 was in the account for 5 years}$$
$$= R17\,227{,}55191 \rightarrow \text{do not round off}$$

You can now calculate the amount that would be in the account due to the R6 000 deposit. It is important to note that R6 000 was only in the account for a period of 3 years.

Compound Interest: $A = P(1+i)^n$

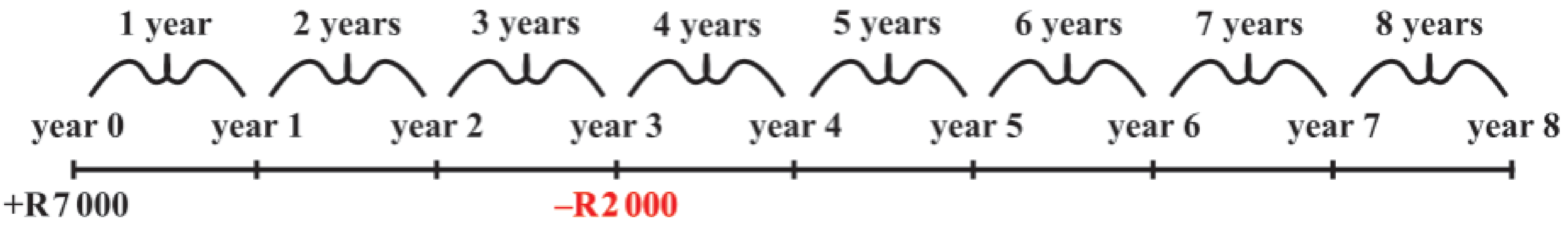
$$A = 6\,000(1+0{,}075)^3 \rightarrow \text{R6 000 was in the account for 3 years}$$
$$= R7\,453{,}78125 \rightarrow \text{do not round off}$$

$$\therefore \text{Total amount in the account after 5 years} = 17\,227{,}55191 + 7\,453{,}78125$$
$$= R24\,681{,}33 \rightarrow \text{only round off at the end}$$

It is important to note that this example can and should be done in one step as shown below:

$$A = 12\,000(1+0{,}075)^5 + 6\,000(1+0{,}075)^3$$
$$= R24\,681{,}33$$

Example 2: **Phindile deposits R7 000 into a savings account which offers an interest rate of 8% p.a. compounded annually. 3 years later she withdraws R2 000. How much will she have in her account at the end of 8 years?**


1 year
2 years
3 years
4 years
5 years
6 years
7 years
8 years
year 0
year 1
year 2
year 3
year 4
year 5
year 6
year 7
year 8
+R7 000
–R2 000

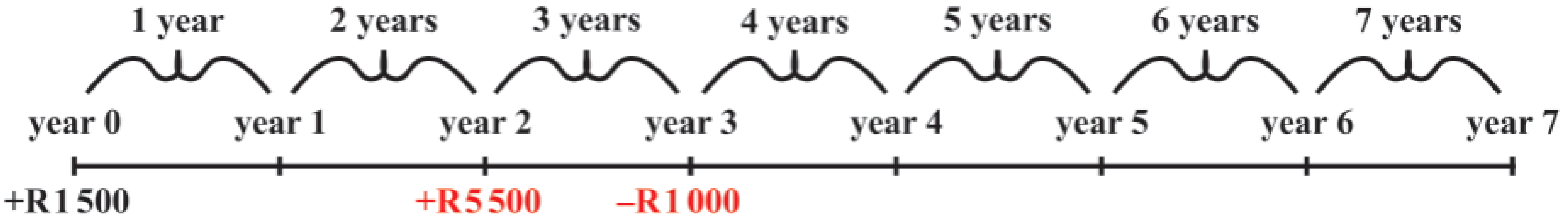
When dealing with a withdrawal you have to remove the withdrawal with its interest for the time that the money was **not** in the account

$$A = 7\,000(1+0,08)^8 - 2\,000(1+0,08)^5 \rightarrow \text{R2 000 was not in the account for 5 years}$$

$$= R10\,017,86$$

$\therefore$ Phindile will have R10 017,86 in her account after 8 years.

Example 3: **Rebecca deposits R1 500 into a savings account which offers an interest rate of 11% p.a. compounded annually. 2 years later she deposits R5 500 into the account and withdraws R1 000 one year later. Determine the amount that Rebecca will have in her account after 7 years.**

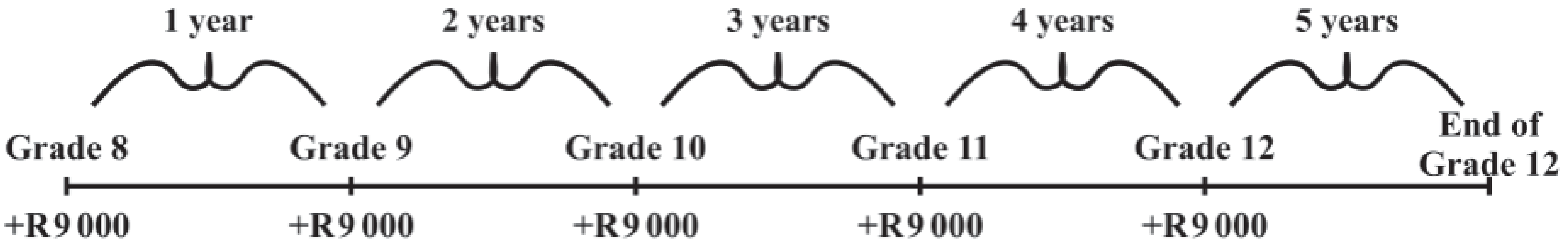

1 year
2 years
3 years
4 years
5 years
6 years
7 years
year 0
year 1
year 2
year 3
year 4
year 5
year 6
year 7
+R1 500
+R5 500
–R1 000

$$A = 1500(1+0,11)^7 + 5\,500(1+0,11)^5 - 1000(1+0,11)^4$$

$$= R10\,863,99$$

$\therefore$ Rebecca will have R10 863,99 in her account after 7 years.

Example 4: **Mrs Reddy deposits R9 000 into a savings account at the beginning of each year in order to save for a car for her son. She makes her first deposit when her son starts grade 8 and her last deposit when he starts Grade 12. If the account offers an interest rate of 10% per annum, how much will Mrs Reddy have saved if she buys a car for her son at the end of Grade 12?**


1 year
2 years
3 years
4 years
5 years
Grade 8
Grade 9
Grade 10
Grade 11
Grade 12
End of Grade 12
+R9 000
+R9 000
+R9 000
+R9 000
+R9 000

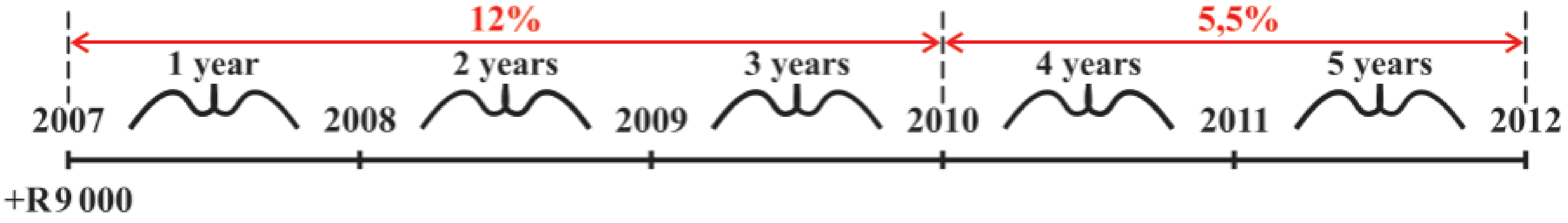
$$A = 9000(1+0,1)^5 + 9000(1+0,1)^4 + 9\,000(1+0,1)^3 + 9\,000(1+0,1)^2 + 9\,000(1+0,1)^1$$

$$= R60\,440,49$$

$\therefore$ Mrs Reddy will have R60 440,49 saved for her son's car.

CHANGES IN INTEREST RATES

Example 1: **At the beginning of 2007, Johannes deposited R9 000 into a Money Market Investment account at an interest rate of 12% compounded annually. At the beginning of 2010 the interest rate dropped to 5,5%. How much will Johannes have in his account at the beginning of 2012?**


12%
5,5%
1 year
2 years
3 years
4 years
5 years
2007
2008
2009
2010
2011
2012
+R9 000

Compound Interest: $A = P(1+i)^n$

2007 - 2010 :

$\therefore A = 9\,000(1+0{,}12)^3 \rightarrow$ interest is compounded yearly, $\therefore$ use the yearly interest rate

$= R12\,644{,}352 \rightarrow$ the amount Johannes will have in his account at the start of 2010

2010 - 2012 : P = the amount in the account at the begining of 2010

$\therefore P = 12\,644{,}352$

$\therefore A = 12\,644{,}352(1+0{,}055)^2 \rightarrow$ there are 2 years remaining at an interest rate of 5,5%

$= R14\,073{,}48$

$\therefore$ Johannes will have R14 073,48 at the beginning of 2012.

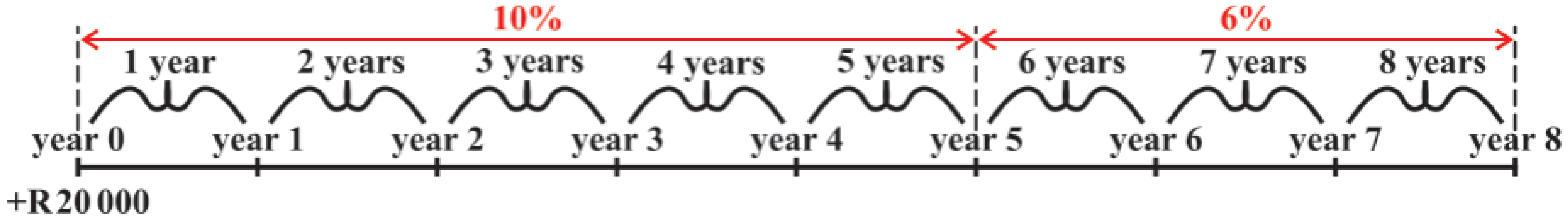
It is important to note that this example can and should be done in one step as shown below:

$$A = 9\,000(1+0{,}12)^3(1+0{,}055)^2$$

$$= R14\,073{,}48$$

$\therefore$ Johannes will have R14 073,48 at the beginning of 2012.

Example 2: **R20 000 is invested into an account which offers an interest rate of 10% p.a. compounded annually. 5 years after the initial investment the interest rate decreases to 6% p.a. compounded annually. How much will be in the account after 8 years.**

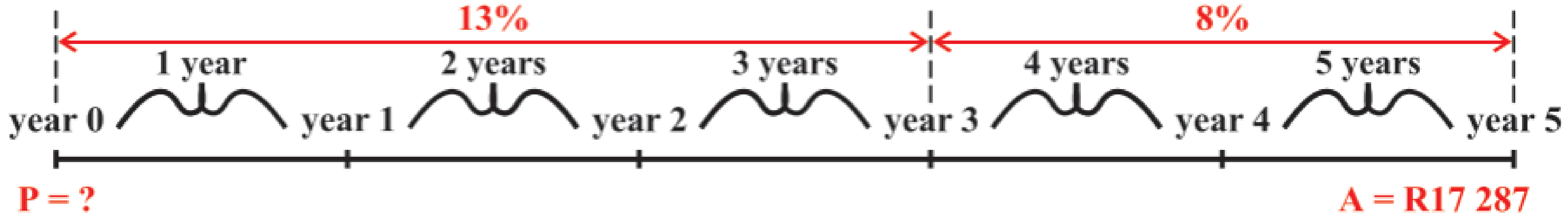

10%
6%
1 year
2 years
3 years
4 years
5 years
6 years
7 years
8 years
year 0
year 1
year 2
year 3
year 4
year 5
year 6
year 7
year 8
+R20 000

$$A = 20\,000(1+0{,}1)^5(1+0{,}06)^3$$

$$= R38\,362{,}86$$

$\therefore$ There will be R38 362,86 after 8 years.

Example 3: **David invests a certain amount of money for 5 years. For the first 3 years the interest rate is 13% per annum compounded annually, after which the interest rate decreases to 8% per annum compounded annually. After 5 years David has R17 287. Calculate how much he initially invested to the nearest rand.**


13%
8%
1 year
2 years
3 years
4 years
5 years
year 0
year 1
year 2
year 3
year 4
year 5
P = ?
A = R17 287

$17\,287 = P(1+0{,}13)^3(1+0{,}08)^2$

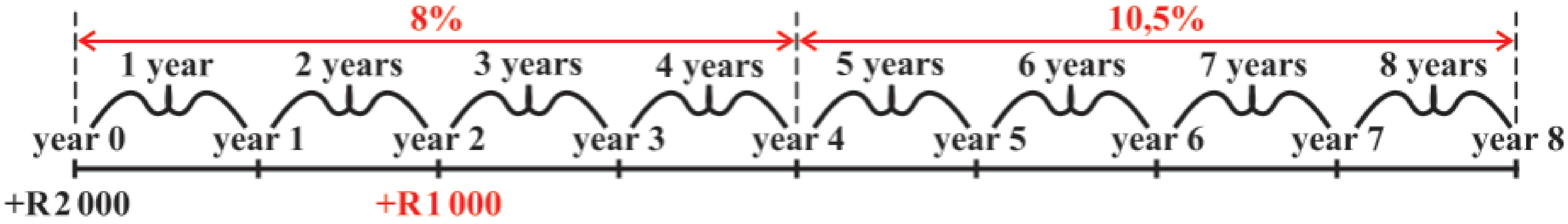
$\therefore P = \dfrac{17\,287}{(1+0{,}13)^3(1+0{,}08)^2}$

$\therefore P = R10\,271{,}56906$

∴ He initially invested R10 272 → the question said to round off to the nearest rand

MIXED EXAMPLES

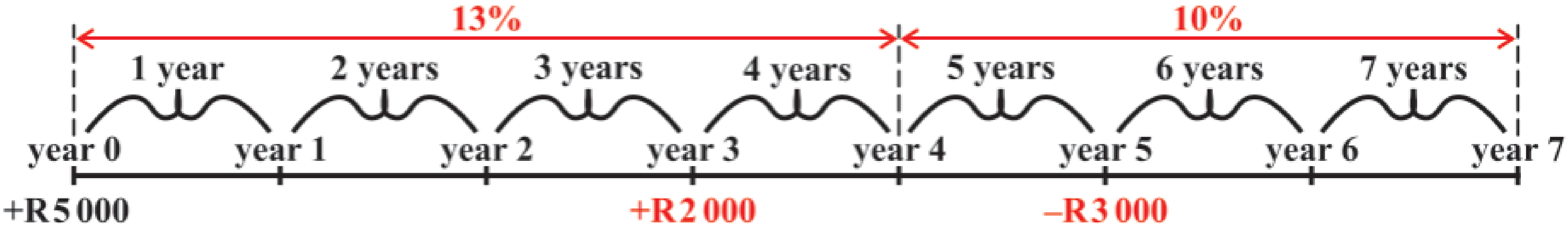
Example 1: **R2 000 is invested into an account which offers an interest rate of 8% p.a. compounded annually. 2 years later an additional R1 000 was deposited into the account. 4 years after the initial investment, the interest rate increased to 10,5% p.a. compounded annually. How much will be in the account after 8 years.**


8%
10,5%
1 year
2 years
3 years
4 years
5 years
6 years
7 years
8 years
year 0
year 1
year 2
year 3
year 4
year 5
year 6
year 7
year 8
+R2 000
+R1 000

$$\begin{aligned} A &= 2\,000(1+0{,}08)^4(1+0{,}105)^4 + 1000(1+0{,}08)^2(1+0{,}105)^4 \\ &= R5\,795{,}70 \end{aligned}$$

∴ There will be R5 795,70 in the account after 8 years.

Example 2: **R5 000 is invested into an account which offers an interest rate of 13% p.a. compounded annually. 3 years later an additional R2 000 is deposited into the account. 2 years after that R3 000 is withdrawn. 4 years after the initial investment the interest rate decreases to 10% p.a. compounded annually. How much will be in the account after 7 years.**


13%
10%
1 year
2 years
3 years
4 years
5 years
6 years
7 years
year 0
year 1
year 2
year 3
year 4
year 5
year 6
year 7
+R5 000
+R2 000
–R3 000

$$\begin{aligned} A &= 5000(1+0{,}13)^4(1+0{,}1)^3 + 2\,000(1+0{,}13)^1(1+0{,}1)^3 - 3\,000(1+0{,}1)^2 \\ &= R10\,228{,}86 \end{aligned}$$

∴ There will be R10 228,86 in the account after 7 years.

Complete Exercise 4 on page 347.

C. EXCHANGE RATES

In Grade 10 you are expected to work with various currencies and understand the implications of fluctuating foreign exchange rates.

Example 1: **Jimmy has grandparents in America and in London. For his birthday his grandparents in America sent him \$150 and his grandparents in London sent him £75. Using the table below, calculate:**

a) How many South African Rands Jimmy received.

b) The number of Euros (€) Jimmy received, if he converted his money into Euros for a holiday in Europe.

	Exchange Rate (Rand to Currency)	Exchange Rate (Currency to Rand)
\$	**0,135278**	**7,392185**
€	**0,089927**	**11,120063**
£	**0,080970**	**12,350123**

a) Using the above table:

$$\begin{aligned}\$150 &= \text{R}\left(7{,}392185 \times 150\right) \rightarrow \text{converting \$ to R } \therefore \text{ use the \$ to R conversion}\\ &= \text{R}1108{,}83 \rightarrow \text{always round off currency to two decimal places}\end{aligned}$$

$$\begin{aligned}£75 &= \text{R}\left(12{,}350123 \times 75\right) \rightarrow \text{converting £ to R } \therefore \text{ use the £ to R conversion}\\ &= \text{R}926{,}26 \rightarrow \text{always round off currency to two decimal places}\end{aligned}$$

$$\begin{aligned}\therefore \text{Total Rands } &= 1108{,}83 + 926{,}26\\ &= \text{R}2\,035{,}09\end{aligned}$$

$\therefore$ Jimmy received R2 035,09

b) Using the above table:

$$\begin{aligned}\text{R}2\,035{,}09 &= €\left(0{,}089927 \times 2\,035.09\right) \rightarrow \text{converting R to €, use the R to € conversion}\\ &= €183{,}01 \rightarrow \text{always round off currency to two decimal places}\end{aligned}$$

$\therefore$ Jimmy will receive €183,01

Example 2: **There are many South Africans who feel that it would be beneficial for the government to take measures to weaken the rand. There are also those who believe that this would be to the detriment of the country. In point form, describe the advantages of a strong Rand as well as the advantages of having a weak Rand.**

ADVANTAGES OF A STRONG RAND

- The **price of petrol decreases** due to the fact that oil is purchased in dollars.
- This results in **decreased transport costs** of essential items such as food, and thus the price of these items decreases.
- **Interest rates decrease**, which means that people such as homeowners will have to pay less on loans and have more disposable income every month.
- **Imported goods** become **cheaper**.
- The **inflation rate decreases** which means that everyday products will become more affordable.

ADVANTAGES OF A WEAK RAND

- As exported goods are paid for in foreign currencies, companies receive **more money for the goods** that they **export**.
- This results in an **increase in the amount of money that comes into South Africa** and helps to improve the country's economy.
- Companies can use the **extra money** to **build and develop** their **businesses**.
- The **expansion of companies** results in a higher demand for workers, which **increases the rate of employment** in the country.

You will generally encounter this type of question when asked to comment on a newspaper article. If asked to speak about the advantages of a weaker or stronger Rand, the above key points should be discussed.

Complete Exercise 5 on page 348 and
The Mixed Exercise on pages 348 and 349.

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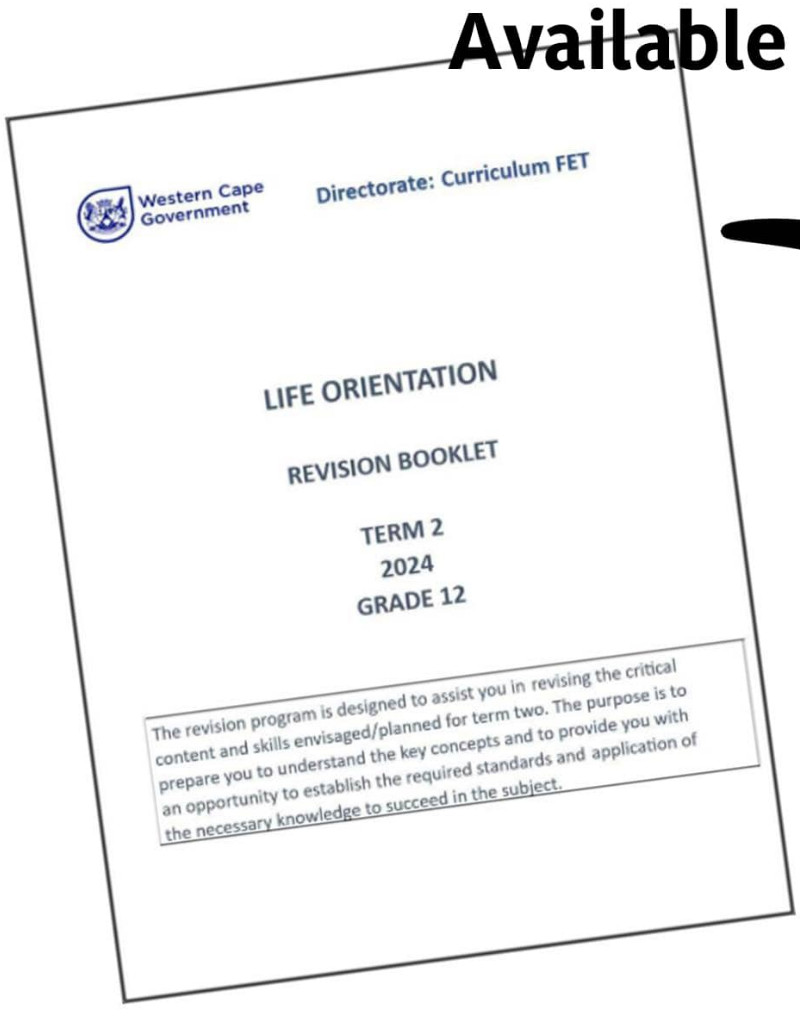

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LIFE ORIENTATION

REVISION BOOKLET

TERM 2
2024
GRADE 12

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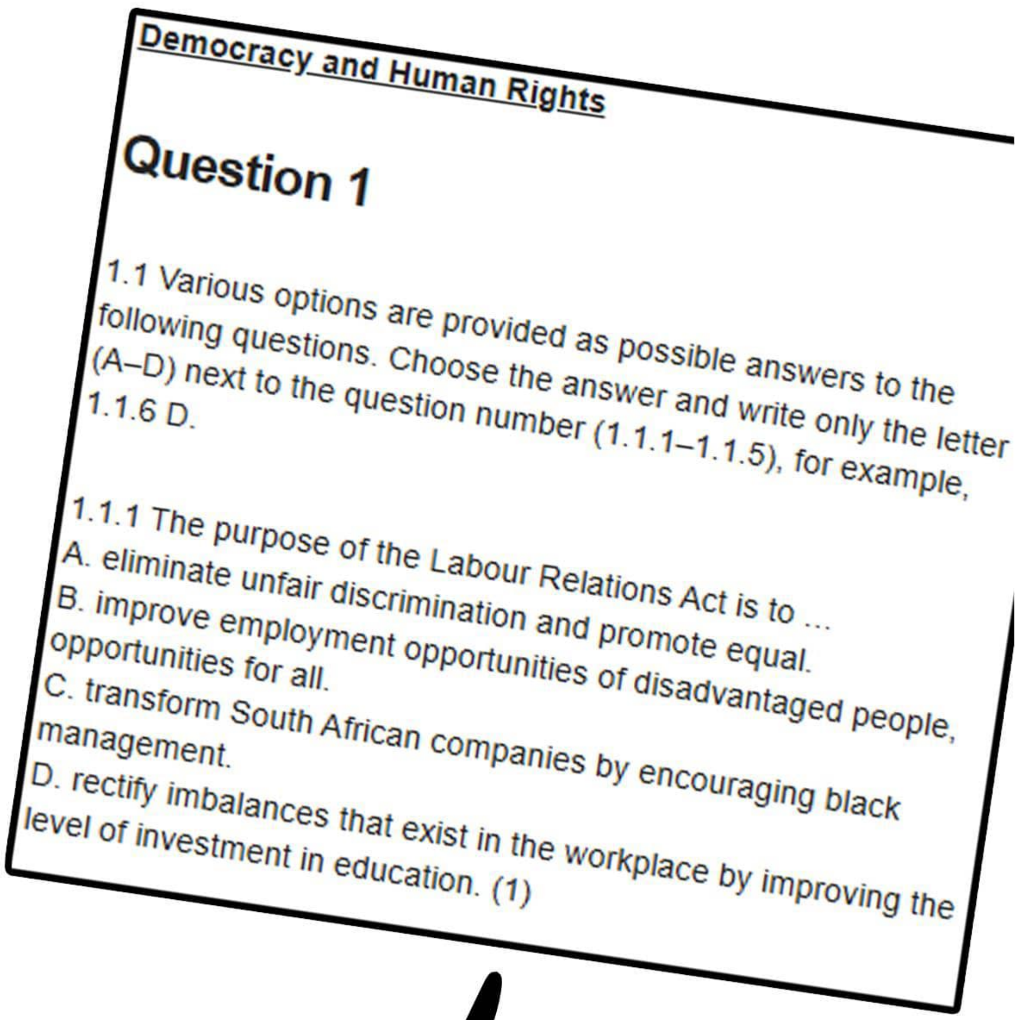
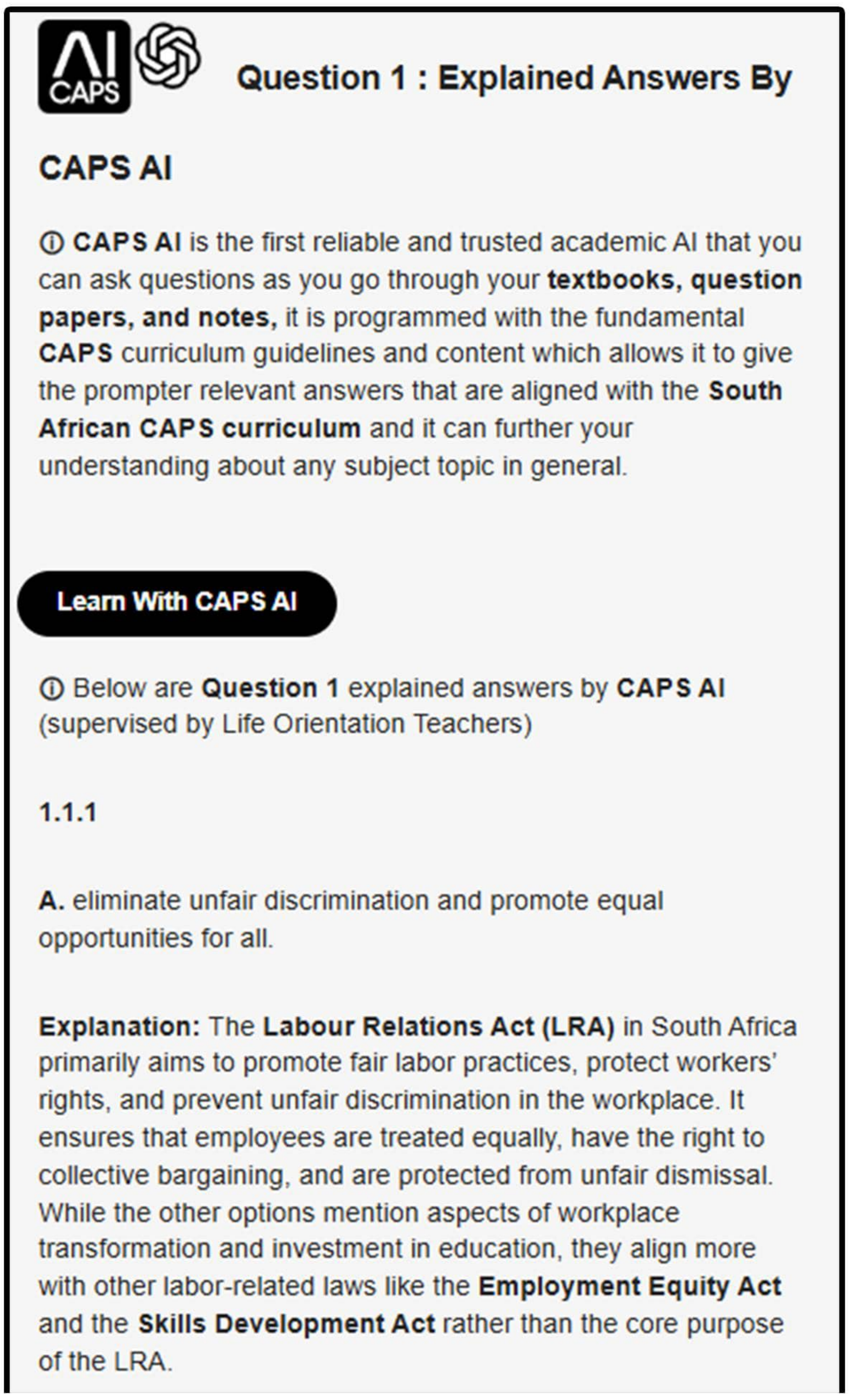


Democracy and Human Rights

Question 1

1.1 Various options are provided as possible answers to the following questions. Choose the answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5), for example, 1.1.6 D.

1.1.1 The purpose of the Labour Relations Act is to ...
A. eliminate unfair discrimination and promote equal opportunities for all.
B. improve employment opportunities of disadvantaged people.
C. transform South African companies by encouraging black management.
D. rectify imbalances that exist in the workplace by improving the level of investment in education. (1)



Question 1 : Explained Answers By CAPS AI

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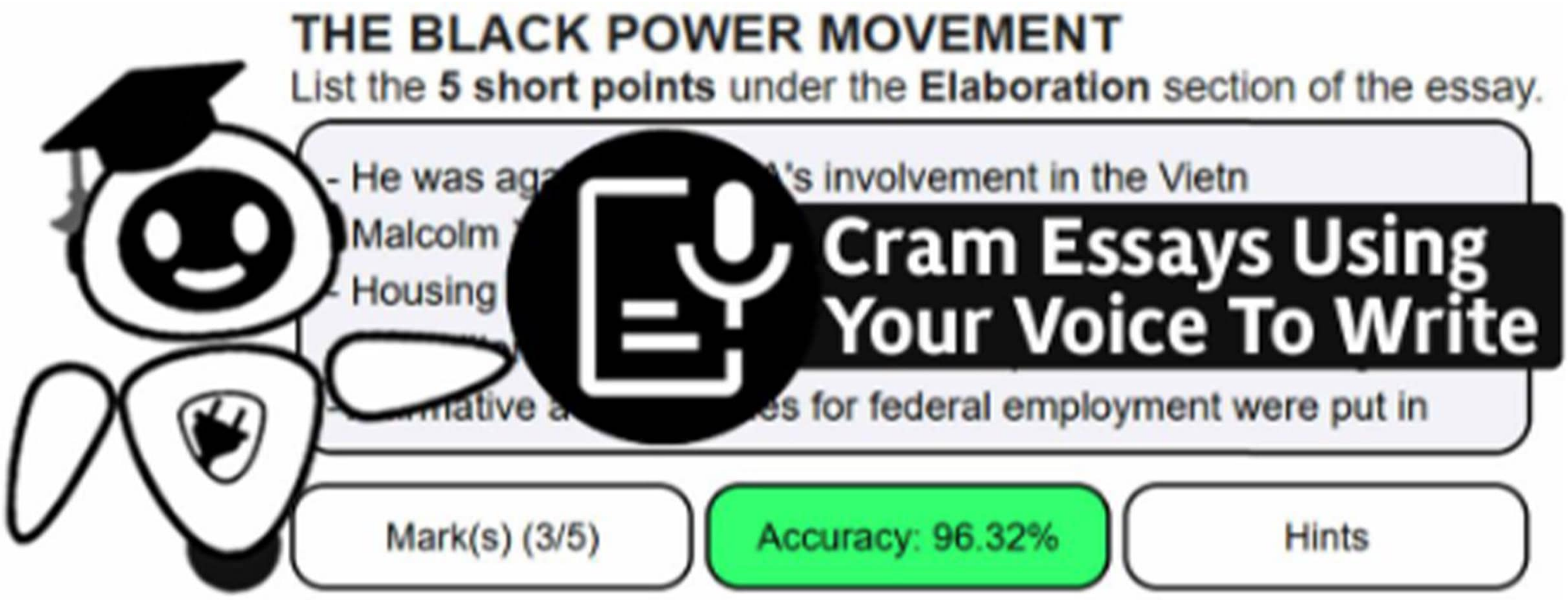
ⓘ Below are **Question 1** explained answers by **CAPS AI** (supervised by Life Orientation Teachers)

1.1.1

A. eliminate unfair discrimination and promote equal opportunities for all.

Explanation: The **Labour Relations Act (LRA)** in South Africa primarily aims to promote fair labor practices, protect workers' rights, and prevent unfair discrimination in the workplace. It ensures that employees are treated equally, have the right to collective bargaining, and are protected from unfair dismissal. While the other options mention aspects of workplace transformation and investment in education, they align more with other labor-related laws like the **Employment Equity Act** and the **Skills Development Act** rather than the core purpose of the LRA.



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THE BLACK POWER MOVEMENT

List the **5 short points** under the **Elaboration** section of the essay.

- He was ag 's involvement in the Vietn
- Malcolm
- Housing
- mative a es for federal employment were put in

Mark(s) (3/5) | Accuracy: 96.32% | Hints

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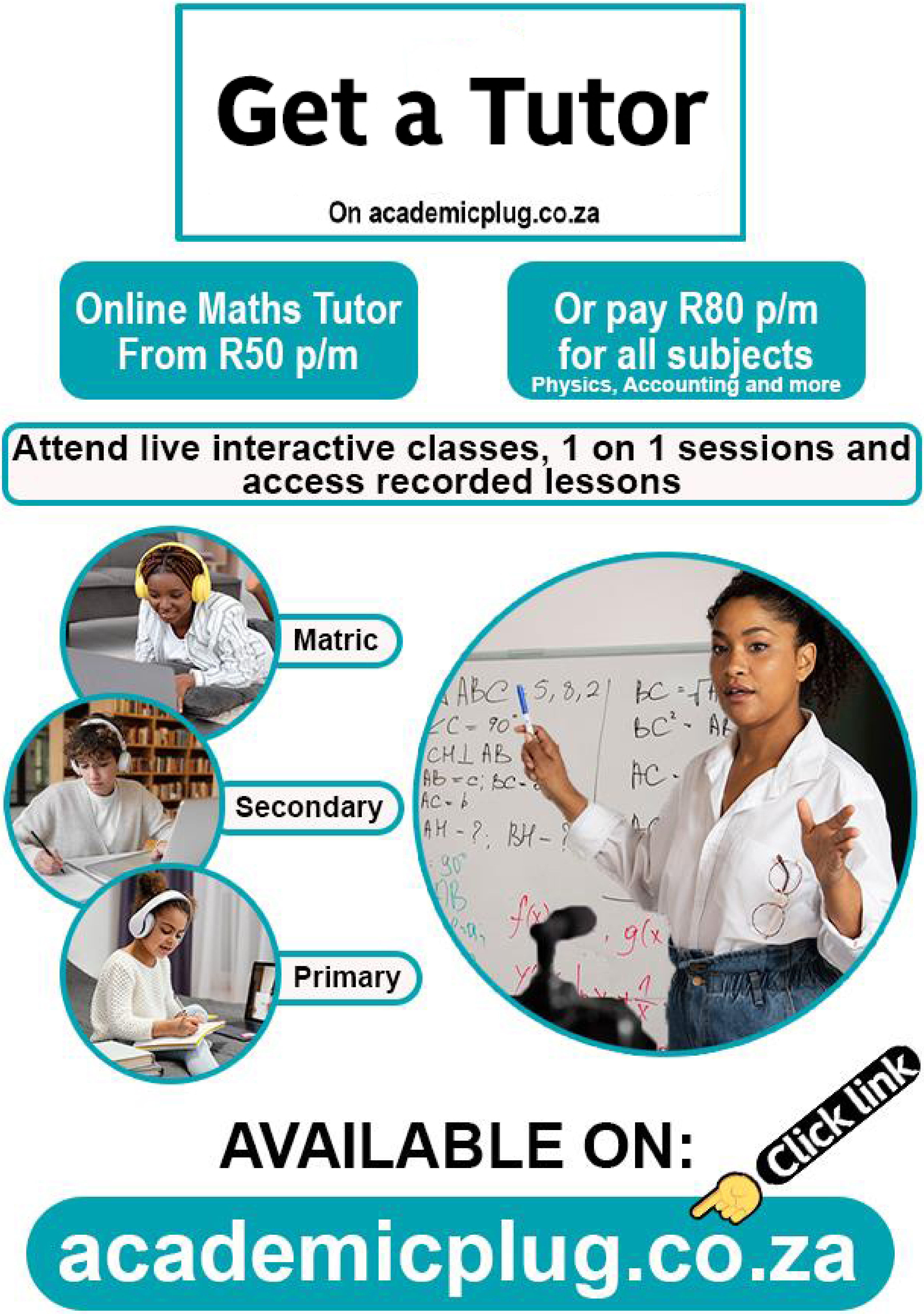

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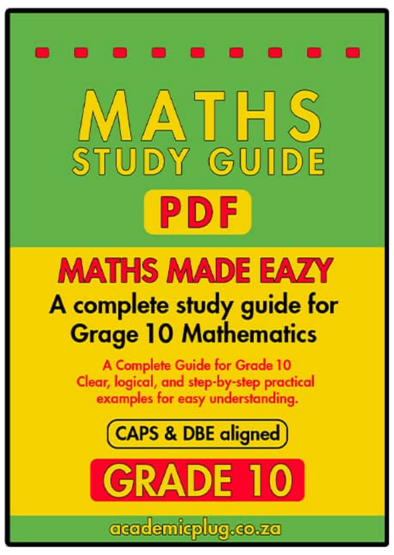

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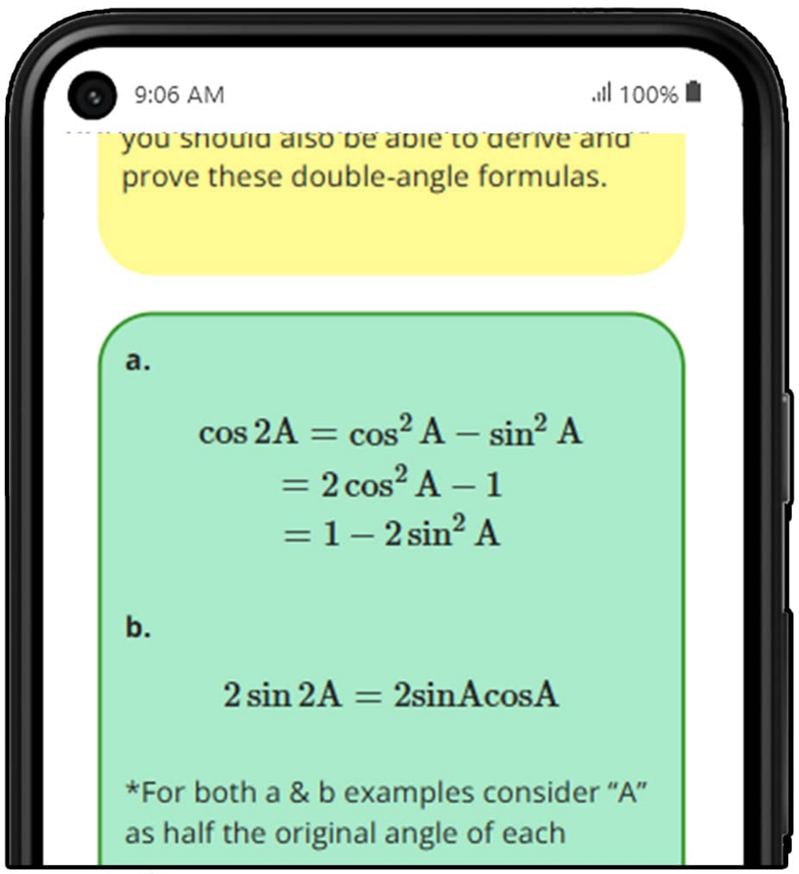

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you should also be able to derive and prove these double-angle formulas.

a.

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

b.

$$2\sin 2A = 2\sin A\cos A$$

*For both a & b examples consider "A" as half the original angle of each

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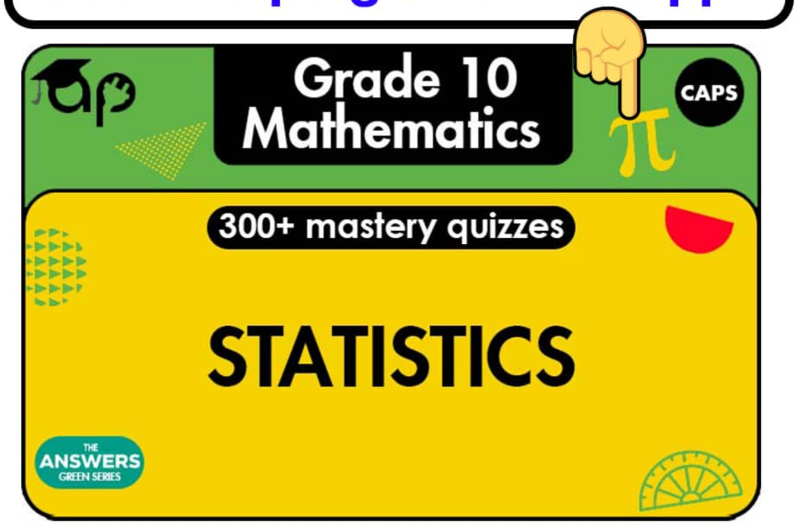

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CHAPTER 9
STATISTICS

WHAT YOU WILL LEARN IN THIS CHAPTER

PAGE	CONTENT	CAPS	ASSESSMENTS YOU WILL BE ABLE TO:
180 180 182	**A. Representation of Data** • Frequency Tables • Stem and Leaf Plots	2	• Collect, organise and interpret univariate numerical data in order to determine measures of central tendency of grouped and ungrouped data and represent these by a five number summary and a box and whisker diagram. Determine which is the most appropriate measure of central tendency under given conditions. • Determine percentiles, quartiles, deciles, inter-quartile and semi-inter-quartile range. Understand the above measures of dispersion.
183 183 184 185 187 188	**B. Measures of Central Tendency** • Mean • Outliers and the Mean • Median • Mode • Measures of Central Tendency Summary		
189 189 190 192 192 193 194	**C. Measures of Dispersion** • Range • Quartiles • Inter-Quartile Range • Semi-Inter-Quartile • Percentiles • Working With Grouped Data		
196 196	**D. The Five Number Summary** • Box and Whisker Diagrams		

A. REPRESENTATION OF DATA

When a survey or research is performed, information called data is collected. Without a meaningful way of representing this information the data would be useless.
This section deals with different ways of collecting, organising, representing and interpreting numerical data.

FREQUENCY TABLES

The number of times a particular data value occurs is known as the frequency.
A frequency table is used to organise data and determine the number of times a particular data value occurs.

Example 1: **A call centre which handles company complaints conducts a survey to evaluate how well the call centre handles customer complaints. They conduct the survey by asking callers to rate the quality of their service from 1 to 5 after their call, where 1 is the lowest rating and 5 is the highest rating. The table below shows the results of the survey for one day. Organise the data using a frequency table.**

1	1	2	2	1	4	3	1	1	2
3	5	4	2	3	1	2	3	2	2
5	4	3	1	3	3	2	4	1	2

Rating	Tally	Frequency
1	𝍸 |||	8
2	𝍸 ||||	9
3	𝍸 ||	7
4	||||	4
5	||	2

The tally column is a simple way of counting the number of times a particular value occurs in a data set. A tally mark (𝍸) is made for every five occurrences of a particular value.

Example 2: The table below shows the marks that a Grade 10 class obtained in a History test. Draw up a frequency table for the data below.

33	23	38	34	90	51	80	87	43	37
58	65	17	45	70	53	91	23	37	96
89	15	17	64	96	1	57	6	13	74
80	98	63	88	81	78	81	88	93	68

It can be seen that the marks vary from 1 to 98. It would clearly be impractical to write down every mark from 1 to 98 in the frequency table. In this case you can split the range of marks into intervals. **As a general rule a frequency table should contain between five and 10 intervals.**

Mark (M)	Tally	Frequency
$0 < M \leq 20$	𝍸 |	6
$20 < M \leq 40$	𝍸 ||	7
$40 < M \leq 60$	𝍸 |	6
$60 < M \leq 80$	𝍸 ||||	9
$80 < M \leq 100$	𝍸 𝍸 ||	12

STEM AND LEAF PLOTS

In a stem and leaf plot the values of the data are split into a stem and leaf (or leaves in back to back stem and leaf plots). The **stem** contains the **primary digits** of the values in the data, whilst the **leaves contain the secondary digit**. For example, the number 136 would be split into the **stem** and **leaf** as follows:

Stem → 13

Leaf → 6

Example 1: **Construct a stem and leaf plot from the following marks that fifteen learners obtained in a Mathematics test out of 100 marks.**

70	**72**	**78**	**80**	**84**	**84**	**86**	**88**	**90**	**90**	**94**	**98**	**100**	**100**	**100**

Stem	Leaf
7	0, 2, 8
8	0, 4, 4, 6, 8
9	0, 0, 4, 8
10	0, 0, 0
Key: 7|0 means a mark of 70	

You have to include a key in order to indicate the meaning of the stem and leaf numbers. Always arrange the leaves in order from smallest to largest from the stem.

Example 2: **Construct a stem and leaf plot from the following two sets of data:**

Data Set A:	**1, 2, 9, 24, 28, 30, 45, 46**
Data Set B:	**1,3, 7, 20, 25, 30, 31, 46**

Leaf Data Set A	Stem	Leaf Data Set B
9, 2, 1	0	1, 3,7
8, 4	2	0, 5
0	3	0, 1
6, 5	4	6
Key: 4|6 means the number 46		

When two sets of data are given, one central stem can be used with two separate leaves for each data set. This is known as a back to back stem and leaf plot.

Complete Exercise 1 on page 355.

B. MEASURES OF CENTRAL TENDENCY

It may be very difficult to compare data using only tables and graphs. When sets of data need to be compared to each other a measure of central tendency is generally used.

In Grade 10 there are three measures of central tendency you are required to know:

1. **Mean (average)**
2. **Median (middle value of a data set)**
3. **Mode (most frequent value in a data set)**

1. MEAN

The mean is the most commonly used measure of central tendency. It is commonly known as the average. The mean is the sum of the data values in the data set, divided by the number of data points in the data set.

Mean ($\bar{x}$):

$$\bar{x} = \frac{\text{sum of the values in the data set}}{\text{number of elements in the data set}}$$

Example 1: The table below shows the heights of 16 plants (in cm) that are being grown in a biology laboratory. Determine the mean height of the plants.

67	78	21	42	80	71	33	99
78	46	98	96	22	98	43	46

$$\text{mean} = \frac{67+78+21+42+80+71+33+99+78+46+98+96+22+98+43+46}{16}$$

$$= 63,63\,\text{cm}$$

$\therefore$ The mean (average) height of the plants is 63,63cm.

Example 2: The table below shows the marks (in percent) of 14 Grade 10 mathematics learners. Determine the mean mark obtained by the learners.

74	71	49	80	49	44	42
58	59	50	79	74	56	55

$$\text{mean} = \frac{74+71+49+80+49+44+42+58+59+50+79+74+56+55}{14}$$

$$= 60\,\%$$

$\therefore$ The mean (average) mark is 60%.

$\bar{x}$ is the symbol for the mean and is called x bar.

Example 3: **The frequency table below shows the number of goals scored by a soccer team in its matches for the season. Determine the mean number of goals scored per game by the team.**

Goals	Frequency
0	1
1	2
2	4
3	2
4	1

In order to calculate the mean from a frequency table you have to add an extra column and row to the frequency table.

Goals	Frequency	Goals × Frequency
0	1	0
1	2	2
2	4	8
3	2	6
4	1	4
Total	10	20

The total of the frequency column is the total number of data points in the data set.
In other words it is the total number of games that were played.
The total of the third column is the total number of goals scored.

$$\therefore \text{mean} = \frac{20}{10} = 2$$

$\therefore$ The team scored an average of 2 goals per game during the season.

OUTLIERS AND THE MEAN

In Grade 10 you can consider an outlier as a point in the data set that is much smaller or larger than the other values in the data set.
Outliers will be formally defined in Grade 12.
It is important to note that in a small set of data outliers can skew the mean.

Example: **In a school there are two classes that take Home Economics. The table below shows the end of year results for each class. Determine the mean mark for each class and comment on each result.**

Class 1 marks (%)	65	72	63	60	58	71	65	62	73
Class 2 marks (%)	73	70	69	75	5	62			

$$\text{mean}_{\text{class 1}} = \frac{65+72+63+60+58+71+65+62+73}{9} = 65{,}44\%$$

$$\text{mean}_{\text{class 2}} = \frac{73+70+69+75+5+62}{6} = 59\%$$

By comparing the mean of each class, it would appear the first class performed better than the second. However, it is clear from looking at the results that the outlier who obtained 5% lowered the mean of the second class. In this case, the mean is not a good indication of the average performance of the second class.

2. MEDIAN

The median is the middle value of an ordered data set. When determining the median of a data set the **data must always be ordered. The median divides a data set in half.**

The formula below is used to determine the position of the median:

$$\textbf{Position of median} = \frac{n+1}{2}$$

Where: *n* is the number of elements in the data set

Example 1: Determine the median of the data set below:

1	**5**	**6**	**2**	**10**	**9**	**16**

Before determining the median the data has to be written in order as shown below:

Position 1	Position 2	Position 3	**Position 4**	Position 5	Position 6	Position 7
1	3	5	**6**	9	10	16

You can now use the formula to determine the position of the median:

$$\begin{aligned}\text{Position of median} &= \frac{n+1}{2}\\ &= \frac{7+1}{2} \rightarrow \text{there are7 elements in the data set}\\ &= 4 \rightarrow \text{the } 4^{\text{th}} \text{ number in the data set is the median}\end{aligned}$$

$$\therefore \text{median} = 6$$

Example 2: Determine the median of the data set below:

3	**8**	**4**	**5**	**6**	**9**	**7**	**1**

Before determining the median the data has to be written in order as shown below:

Position 1	Position 2	Position 3	**Position 4**	**Position 5**	Position 6	Position 7	Position 8
1	3	4	**5**	**6**	7	8	9

You can now use the formula to determine the position of the median:

$$\begin{aligned}\text{Position of median} &= \frac{n+1}{2}\\ &= \frac{8+1}{2} \rightarrow \text{there are 8 elements in the data set}\\ &= 4,5 \rightarrow \text{the median lies halfway between the } 4^{\text{th}} \text{ and } 5^{\text{th}} \text{ term}\end{aligned}$$

In this case there are an even number of terms. This results in two terms being left in the middle. The median will lie half way between the two middle terms. Therefore, the median will be the sum of the middle terms divided by two:

$$\therefore \text{median} = \frac{5+6}{2} = 5,5$$

If there are an odd number of terms, the median will be the middle term in the ordered data set.
If there are an even number of terms, the median will lie halfway between the two middle terms in the ordered data set.

Example 3: **In a school there are two classes that take History. The table below shows the end of year results for each class:**

Class 1 marks (%)	**65**	**62**	**70**	**68**	**75**	**63**	**64**	**72**	**78**
Class 2 marks (%)	**78**	**82**	**20**	**68**	**75**	**60**	**73**	**62**	**76**

a) Determine the median of each data set.
b) Determine the mean percentage for each class.
c) Comment on whether the mean or median is the most reliable measure of central tendency in this situation.

a)

Position of learner	1	2	3	4	**5**	6	7	8	9
Class 1 marks (%)	62	63	64	65	**68**	70	72	75	78

Position of learner	1	2	3	4	**5**	6	7	8	9
Class 2 marks (%)	20	60	62	68	**73**	75	76	78	82

You can now use the formula to determine the position of the median:

Class 1	**Class 2**
$\text{Position of median} = \frac{n+1}{2}$ $= \frac{9+1}{2}$ $= 5$	$\text{Position of median} = \frac{n+1}{2}$ $= \frac{9+1}{2}$ $= 5$
∴ The learner in the 5th position in each class is the median	
∴ The median for class 1 is 68%	**∴ The median for class 2 is 73%**

b) Mean calculation:

$$\text{mean}_{\text{class 1}} = \frac{62+63+64+65+68+70+72+75+78}{9}$$
$$= 68{,}56\%$$

$$\text{mean}_{\text{class 2}} = \frac{20+60+62+68+73+75+76+78+82}{9}$$
$$= 66\%$$

c) It can be seen from the median that **half of class 2 achieved marks above 73%** while **half of class 1 achieved marks above 68%**. The median indicates that, as a whole, class 2 performed better.

However, the mean shows that on average, class 1 performed better than class 2. This is because the outlier of 20% in class 2 skewed the mean. Therefore, the median would be a more reliable measure of central tendency.

The above example shows how the mean can be misleading in small samples with outliers.

3. MODE

The mode of a data set is the value that occurs most often. It is possible for there to be more than one mode or none at all.

Example 1: A new manager of a clothing store has to order jerseys for winter and is unsure of how many of each size to order. The frequency table below shows the number of each size sold the previous year. Which size would you advise the manager to purchase the most stock of?

SIZE	FREQUENCY	SIZE	FREQUENCY
XXS	7	**L**	**100**
XS	12	XL	25
S	30	XXL	18
M	58	XXXL	10

From the table it can be seen that the size large is the mode for this set of data. Therefore, the manager should purchase more large jerseys than any other size.

Example 2: The table below shows the various shoe sizes sold in one day in a men's shoe store. Summarise the data according to shoe size and hence determine the mode.

10	8	9	7	9	7	9	9	10	10
10	12	8	9	9	10	10	8	10	10
9	9	9	8	8	11	11	8	11	11
12	8	10	9	10	11	10	9	9	10

	FREQUENCY
7	2
8	7
9	**12**
10	**12**
11	5
12	2

In this data set there are two modes. These modes are size 9 and size 10.

If there are two modes in the data set, the set is said to be bimodal.

Example 3: **The frequency table below shows the number of points and how many times that particular number of points was scored over the basketball season. Determine the modal class for the given data.**

NUMBER OF POINTS (P)	FREQUENCY
$0 \leq p < 5$	3
$5 \leq p < 10$	6
$10 \leq p < 15$	16
$15 \leq p < 20$	**20**
$20 \leq p < 25$	5

The interval of $15 \leq p < 20$ has a frequency of 20. Therefore, the modal class for this set of data is the interval of $15 \leq p < 20$.

MEASURES OF CENTRAL TENDENCY SUMMARY

DATA REPRESENTATION	USE/EXPLANATION
Mean **($\overline{x}$)**	**The mean determines the average of a data set. The data does not have to be ordered to use the mean. Small samples with outliers can skew the mean and cause it to be misleading.**
Median	**The median is the middle value in an ordered data set. The data has to be ordered to determine the median. Outliers do not affect the median.**
Mode	**The mode is the data in a set which has the highest frequency. It is possible for there to be more than one mode, or for there to be no mode at all in a data set.**

Complete Exercise 2 on pages 355 and 356.

C. MEASURES OF DISPERSION

Measures of dispersion show how the data in a data set is spread around the mean or median. There are five measures of dispersion that you will study in Grade 10:

1. **Range**
2. **Quartiles**
3. **Inter-quartile range**
4. **Semi-inter-quartile range**
5. **Percentiles**

If the value of dispersion is small, the data values in the set are closely spread around the median and each other.
If the value of dispersion is large, the data values in the set are more widely spread around the median and there are most likely outliers in the set.

In most cases, a small value of dispersion indicates that the data points in a set are close to each other and the set does not contain outliers. Therefore, the mean would give a relatively accurate view of the data set.

A large value of dispersion indicates that the data points in a set are spread and it is most likely that the set contains outliers. Therefore, the mean would not provide an accurate representation of the data set.

1. RANGE

The range of a data set is defined as the difference between the largest value and smallest value in the data set. It is important to note that the range cannot be calculated for grouped data.

Grouped data is data that is arranged in groups or classes rather than showing individual values.

The formula below is used to determine range of a data set:

Range = Largest Value – Smallest Value

Example: **Determine the range of data sets A and B below.**

Data Set A	18	62	63	68	72	60	62	65	73	72	76	64
Data Set B	60	63	62	68	75	76	62	71	68	69	74	73

$$\text{Range}_{\text{Set A}} = 76 - 18 = 58$$

$$\text{Range}_{\text{Set B}} = 76 - 60 = 16$$

It can be seen from the above example that a single outlier (18) can cause the measure of spread to be misleading. The range does not take into account the distribution of data between the largest and smallest values in the set.

2. QUARTILES

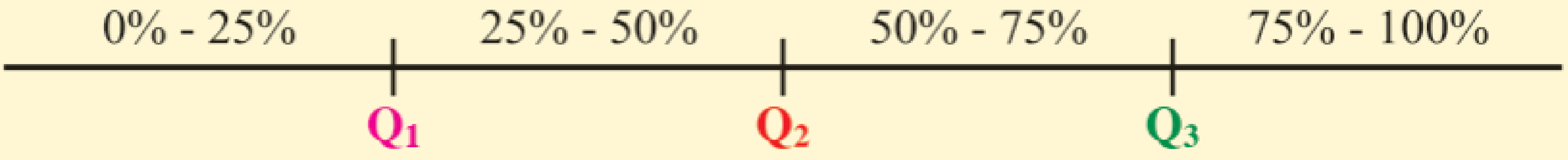
Quartiles subdivide a data set into quarters (four groups). The Quartiles provide an indication of the dispersion around the median and are especially useful in a data set which contains outliers. **The data has to be ordered to determine the quartiles**.

There are three quartiles:

Lower Quartile (Q_1): The median of the lower half of the values in the data set

The Median (Q_2): The middle value of the data set (pg. 185)

Upper Quartile (Q_3): The median of the upper half of the values in the data set



STEPS TO CALCULATING QUARTILES

1. **Write the data in order and determine the median (Q_2).**
2. **Determine the lower quartile (Q_1) by finding the median of the lower half.**
3. **Determine the upper quartile (Q_3) by finding the median of the upper half.**

Example 1: **The table below shows the percentages that 7 students obtained in a Mathematics test. Determine the median as well as the upper and lower quartiles for the data set.**

75	**95**	**60**	**82**	**52**	**78**	**50**

Step 1: Write the data in order and determine the median.

Lower Half				Upper Half		
Position 1	**Position 2**	Position 3	**Position 4**	Position 5	**Position 6**	Position 7
50	**52**	60	**75**	78	**82**	95

$$\text{Position of median} = \frac{n+1}{2}$$

$$= \frac{7+1}{2} \rightarrow \text{there are 7 elements in the data set}$$

$$= 4 \rightarrow \text{the median is the } 4^{\text{th}} \text{ term}$$

$$\therefore Q_2 = \text{median} = 75$$

Step 2 and 3: determine the lower and upper quartiles.
As the median is in position 4 there are 3 terms in the lower half and three terms in the upper half. Note that the lower quartile is simply the median of the lower half and the upper quartile is the median of the upper half.

Lower quartile calculation (Q_1)	**Upper quartile calculation (Q_3)**
$\text{Position of } Q_1 = \frac{n+1}{2}$ $= \frac{3+1}{2} \rightarrow \text{3 values in the lower half}$ $= 2 \rightarrow Q_1 \text{ is the } 2^{\text{nd}} \text{ term}$	$\text{Position of } Q_3 = 4 + \frac{n+1}{2}$ $= 4 + \frac{3+1}{2} \rightarrow \text{4 values before the upper half}$ $= 6 \rightarrow Q_3 \text{ is the } 6^{\text{th}} \text{ term}$

$$\therefore Q_1 = 52 \quad \text{and} \quad Q_2 = 75 \quad \text{and} \quad Q_3 = 82$$

Example 2: **The table below shows the percentages that 12 learners obtained in a History test. Determine the median as well as the upper and lower quartiles for the data set.**

50	60	52	73	75	54	64	70	62	58	71	78

Step 1: Write the data in order and determine the median.

						63						
Position	1	2	3	4	5	**6**	**7**	8	9	10	11	12
Term	50	52	54	58	60	**62**	**64**	70	71	73	75	78

$\text{Position of median} = \dfrac{n+1}{2}$

$= \dfrac{12+1}{2} \rightarrow$ there are 12 terms in the data set

$= 6{,}5 \rightarrow$ the median is half way between the 6^{th} and 7^{th} term

$\therefore Q_2 = \text{median} = \dfrac{62+64}{2} = 63$

Step 2 and 3: determine the lower and upper quartiles.

63

	Lower Half						Upper Half					
Position	1	2	**3**	**4**	5	**6**	**7**	8	9	10	11	12
Term	50	52	**54**	**58**	60	**62**	**64**	70	71	73	75	78

$\text{Position of } Q_1 = \dfrac{n+1}{2}$

$= \dfrac{6+1}{2} \rightarrow$ there are 6 terms in the lower half

$= 3{,}5 \rightarrow Q_1$ is half way between the 3^{rd} and 4^{th} term

$\therefore Q_1 = \dfrac{54+58}{2} = 56$

$\text{Position of } Q_3 = 6 + \dfrac{n+1}{2} \rightarrow$ there are 6 terms before the upper half

$= 6 + \dfrac{6+1}{2} \rightarrow$ there are 6 terms in the upper half

$= 9{,}5 \rightarrow Q_3$ is half way between the 9^{th} and 10^{th} term

$\therefore Q_3 = \dfrac{71+73}{2} = 72$

$\therefore Q_1 = 56\%$ and $Q_2 = 63\%$ and $Q_3 = 72\%$

The quartiles show that:

- **A quarter of the results are below 56%**
- **Half of the marks are below 63%**
- **Three quarters of the marks are below 72%.**

3. INTER-QUARTILE RANGE

The Inter-Quartile Range (IQR) is the difference between the lower and upper quartiles. It is a good indication of how data is spread about the median and is not affected by outliers as it is based on the middle half of the data. For this reason it is a more accurate measure of dispersion than the range.

Inter-Quartile Range:

$$\text{IQR} = \text{Upper Quartile } (Q_3) - \text{Lower Quartile } (Q_1)$$

Example: The table below shows the percentages that 7 learners obtained in a test. Determine the inter-quartile range for the data.

60	**65**	**68**	**72**	**70**	**75**	**63**

In order to determine the IQR you first have to write the data in order and determine the upper and lower quartiles:

Step 1: Write the data in order and determine the median.

Position 1	**Position 2**	Position 3	**Position 4**	Position 5	**Position 6**	Position 7
60	**63**	65	**68**	70	**72**	75

$$\text{Position of median} = \frac{n+1}{2} = \frac{7+1}{2} = 4$$

$$\therefore Q_2 = 68$$

Step 2 and 3: determine the lower and upper quartiles.

Lower quartile calculation (Q_1)	**Upper quartile calculation (Q_3)**
Position of $Q_1 = \frac{n+1}{2}$ $= \frac{3+1}{2} \rightarrow$ 3 values in the lower half $= 2 \rightarrow Q_1$ is the 2nd term	Position of $Q_3 = 4 + \frac{n+1}{2}$ $= 4 + \frac{3+1}{2} \rightarrow$ 4 values before the upper half $= 6 \rightarrow Q_3$ is the 6th term

$$\therefore Q_1 = 63 \quad \text{and} \quad Q_3 = 72$$

$$\therefore \text{IQR} = Q_3 - Q_1 = 72 - 63$$
$$= 9$$

4. SEMI-INTER-QUARTILE RANGE

The Semi-Inter-Quartile Range is half the Inter-Quartile Range.

Semi-Inter-Quartile Range:

$$\textbf{Semi-Inter-Quartile Range} = \frac{(Q_3) - (Q_1)}{2} = \frac{\text{IQR}}{2}$$

Example: Given that the IQR in the above example is 9, determine the semi-inter-quartile-range.

$$\text{Semi-Inter-Quartile Range} = \frac{\text{IQR}}{2} = \frac{9}{2} = 4{,}5$$

5. PERCENTILES

Percentiles divide a data set into intervals, such as tenths or hundredths.
The percentiles tell you what percentage of data lies below and above a specific data point in a data set. In other words, if a data point is in the 80th percentile, it means that 80% of the data is equal to or below that value and that 20% of the data lies above it.

Percentiles:

$$\textbf{Position of Percentile} = \frac{\textbf{Percentile}}{\textbf{100}} \times \textbf{number of values in the data set}$$

Steps to calculating percentiles

1. **Write the data in order.**
2. **Determine the position of the percentile asked for using the above formula.**
3. **Determine the actual value of the percentile by counting up the ordered data set.**

Example: **Determine the 30th and 65th percentiles of the data set below.**

18	**12**	**5**	**32**	**22**	**20**	**24**	**26**	**25**	**29**

Step 1: Write the data in order.

5	12	18	20	22	24	25	26	29	32

Step 2: Determine the position of the 30th and 60th percentiles.

$$\text{Position} = \frac{\text{Percentile}}{100} \times \text{Number of values in data set}$$

30th percentile: Position $= \frac{30}{100} \times 10$

$= 3$

65th percentile: Position $= \frac{65}{100} \times 10$

$= 6{,}5$

$\approx 7 \rightarrow$ round off to the nearest integer

Step 3: Determine the values of the percentiles.

Position 1	Position 2	Position 3	Position 4	Position 5	Position 6	Position 7	Position 8	Position 9	Position 10
5	12	**18**	20	22	24	**25**	26	29	32

This means that **30%** **of the data values are less than or equal to** **18** **and that** **65%** **of the data values are less than or equal to** 25.

The lower quartile, median and upper quartile are the 25th percentile, 50th percentile and 75th percentile respectively.

If the position of the percentile is decimal, round off to the nearest integer.

Complete Exercise 3 on pages 356 and 357.

WORKING WITH GROUPED DATA

In grade 10 you are required to be able to identify the intervals in which the lower quartile, median, upper quartile and any percentile lies in grouped data.
In addition, you are required to calculate the approximate mean for grouped data.
Grouped data is data that is arranged in groups or classes rather than showing individual figures.

Example: **The following table shows the percentage that 199 Grade 10 learners obtained in an exam.**

Percentage Obtained (x)	Number of Learners
$0 < x \le 20$	4
$20 < x \le 40$	47
$40 < x \le 60$	79
$60 < x \le 80$	50
$80 < x \le 100$	19

a) Determine the interval in which the median lies.
b) Determine the interval in which the lower quartile lies.
c) Determine the interval in which the upper quartile lies.
d) Determine the interval in which the 40th percentile lies.
e) Determine the estimated mean percentage of the examination.

When determining which interval quartiles and percentiles lie within grouped data, you have to add in a cumulative frequency column.

Percentage Obtained (x)	Number of Learners	Cumulative Frequency
$0 < x \le 20$	4	4
$20 < x \le 40$	47	51 → (4+47)
$40 < x \le 60$	79	130 → (51+79)
$60 < x \le 80$	50	180 → (130+50)
$80 < x \le 100$	19	199 → (180+19)

a) The position of the **median** learner is at $\frac{n+1}{2}$:

$$\text{median position} = \frac{199+1}{2} = 100$$

$\therefore$ the 100th learner is in the median position.

From the table and cumulative frequency it can be seen that there are 51 learners up to the interval of $20 < x \le 40$ and 130 learners in the interval up to $40 < x \le 60$.

$\therefore$ The median (100th) learner must lie in the interval $40 < x \le 60$.

b) The position of the **lower quartile** learner is the median of the lower half of the data. As the median is in position 100 there are 99 learners in the lower half.

$$\therefore \text{position of lower quartile} = \frac{99+1}{2} = 50$$

$\therefore$ the 50th learner is in the lower quartile position.

From the table and cumulative frequency it can be seen that there are 4 learners up to the interval of $0 < x \le 20$ and 51 learners in the interval up to $20 < x \le 40$.

$\therefore$ The lower quartile (50th learner) must lie in the interval of $20 < x \le 40$.

c) The position of the **upper quartile** learner is the median of the upper half of the data. As the median is in position 100 there are 99 learners in the upper half.

$$\therefore \text{position of upper quartile} = 100 + \frac{99+1}{2} = 150$$

$\therefore$ the 150^{th} learner is in the upper quartile position.

From the table and cumulative frequency it can be seen that there are 130 learners up to the interval of $40 < x \leq 60$ and 180 learners in the interval up to $60 < x \leq 80$.

$\therefore$ The upper quartile (150^{th} learner) must lie in the interval of $60 < x \leq 80$.

d) Calculate the position of the 40^{th} percentile:

$$\text{Position} = \frac{40}{100} \times 199 \approx 80 \rightarrow 40\% = \frac{40}{100}\text{ , there are 199 learners in the data set}$$

$\therefore$ the 40^{th} percentile is the 80^{th} learner

$\therefore$ the 40^{th} percentile lies in the interval of $40 < x \leq 60$.

e) The mean percentage of the data is the total percentage obtained by all of the learners divided by the number of learners. As you don't know the exact percentage obtained by the learners you can assume that each learner obtained the middle value of each interval. In order to calculate the mean you draw up a new table as shown below:

Percentage Obtained (x)	Number of Learners	Midpoint of Interval	Learners × Midpoint
$0 < x \leq 20$	4	10	**40**
$20 < x \leq 40$	47	30	**1 410**
$40 < x \leq 60$	79	50	**3 950**
$60 < x \leq 80$	50	70	**3 500**
$80 < x \leq 100$	19	90	**1 710**
Sum	**199**	**Total**	**10 610**

$$\therefore \text{ Estimated mean} = \frac{10\,610}{199} = 53{,}32\%$$

When working with grouped data, if you are asked for the value of the median, lower, upper quartile or percentile, the approximate value will be the midpoint of the interval. So for the above example the values of the:

- **lower quartile would be 30**
- **median would be 50**
- **upper quartile would be 70**
- **40^{th} percentiles would be 50**

Complete Exercise 4 on page 357.

D. THE FIVE NUMBER SUMMARY

The different measures of dispersion that you have learnt so far can be represented and analysed graphically on a box and whisker diagram using the Five Number Summary. The five number summary is:

1) **Lowest value in the data set.**
   The first of the five number summary, is the lowest (minimum) value in the ordered data set.

2) **Lower Quartile / First Quartile / 25th percentile (Q_1).**
   The lower quartile is the median of the lower half of the ordered data set.

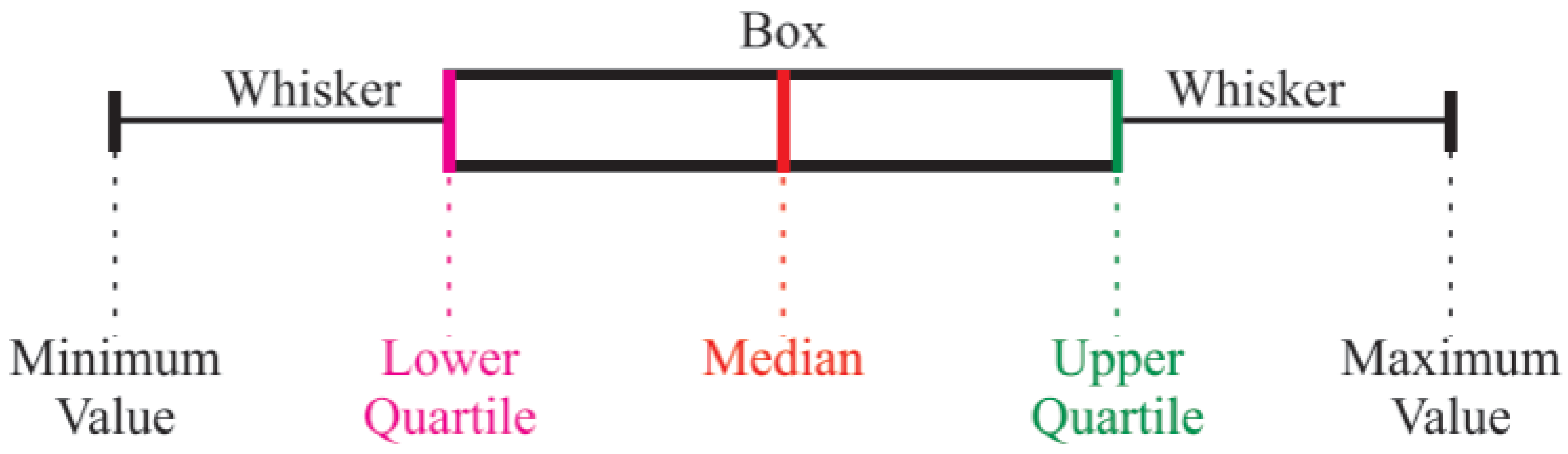
3) **Median / Second Quartile / 50th percentile (Q_2).**
   The median is the middle value in an ordered set of data. If there is an even number of terms, the median will be the sum of the two terms in the middle divided by two.

4) **Upper Quartile / Third Quartile / 75th percentile (Q_3).**
   The upper quartile is the median of the upper half of the ordered data set.

5) **Highest value in the data set.**
   The last of the five number summary, is the highest (maximum) value in the ordered data set.

BOX AND WHISKER DIAGRAMS

A box and whisker plot is a graphical representation of the Five Number summary.
A box and whisker plot can assist in analysing the spread of data about the median.



It is important to note the following about the box and whisker diagrams:
- **half of the values lie between the minimum value and the median**
- **half of the values lie between the median and the maximum value**
- **half of the values lie between the lower quartile and upper quartile**
- **a quarter of the values lie between the minimum value and lower quartile**
- **a quarter of the values lie between the lower quartile and median**
- **a quarter of the values lie between the median and upper quartile**
- **a quarter of the values lie between the upper quartile and maximum value**

Steps to drawing box and whisker diagrams:

1. **Write the data set in order and determine the median.**
2. **Determine the upper and lower quartiles.**
3. **Draw an accurately scaled number line with a box on top of it from the lower to upper quartile and mark the median in the box.**
4. **Draw whiskers from the lowest value in the set to the box and from the highest value in the set to the box.**
5. **If asked, determine if the data is evenly distributed, skewed to the left or the right.**

Example: **The following data represents the percentages obtained in a Maths test for 7 students (75, 90, 50, 25, 60, 30, 10). Draw a box and whisker diagram for the given data.**

Step 1: Write the data in order and determine the median.

Position 1	**Position 2**	Position 3	**Position 4**	Position 5	**Position 6**	Position 7
10	**25**	30	**50**	60	**75**	90

$$\text{Position of median} = \frac{n+1}{2} = \frac{7+1}{2} = 4$$

$$\therefore Q_2 = 50$$

Step 2: Determine the upper and lower quartiles.

$$\text{Position of } Q_1 = \frac{n+1}{2}$$

$$= \frac{3+1}{2} \rightarrow 3 \text{ values in the lower half}$$

$$= 2 \rightarrow Q_1 \text{ is the } 2^{\text{nd}} \text{ term}$$

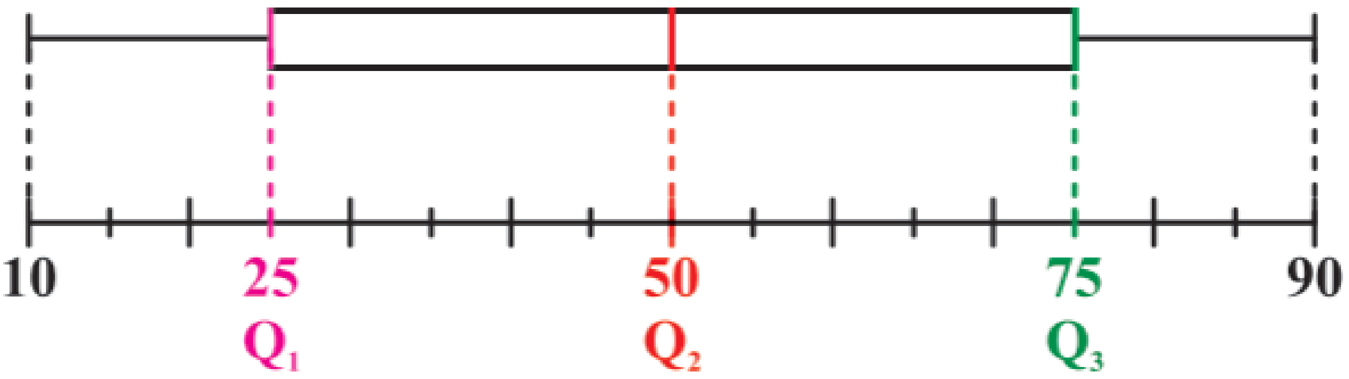
$$\text{Position of } Q_3 = 4 + \frac{n+1}{2}$$

$$= 4 + \frac{3+1}{2} \rightarrow 4 \text{ values before the upper half}$$

$$= 6 \rightarrow Q_3 \text{ is the } 6^{\text{th}} \text{ term}$$

$$\therefore Q_1 = 25 \quad \text{and} \quad Q_3 = 75$$

Step 3, 4 and 5:



A box and whisker diagram has to be drawn accurately and to scale.

Complete Exercise 5 on pages 357 and 358 and
The Mixed Exercise on pages 358 and 359.

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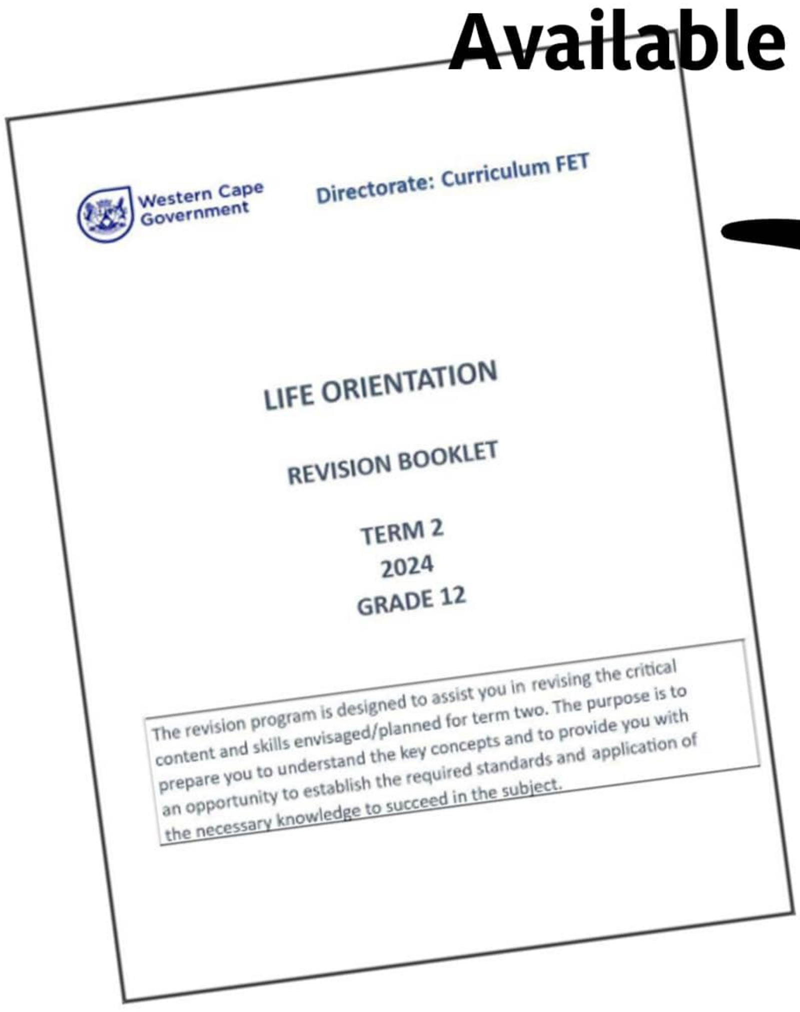

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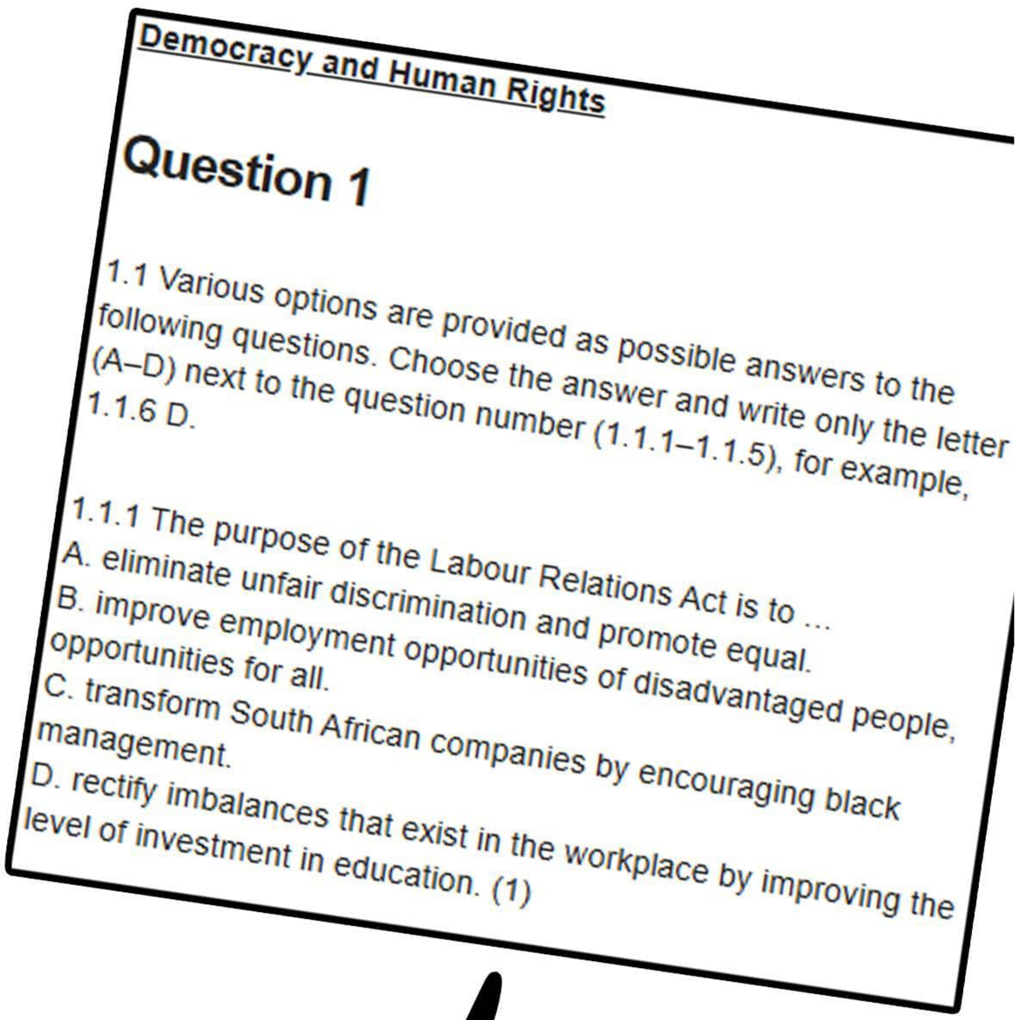
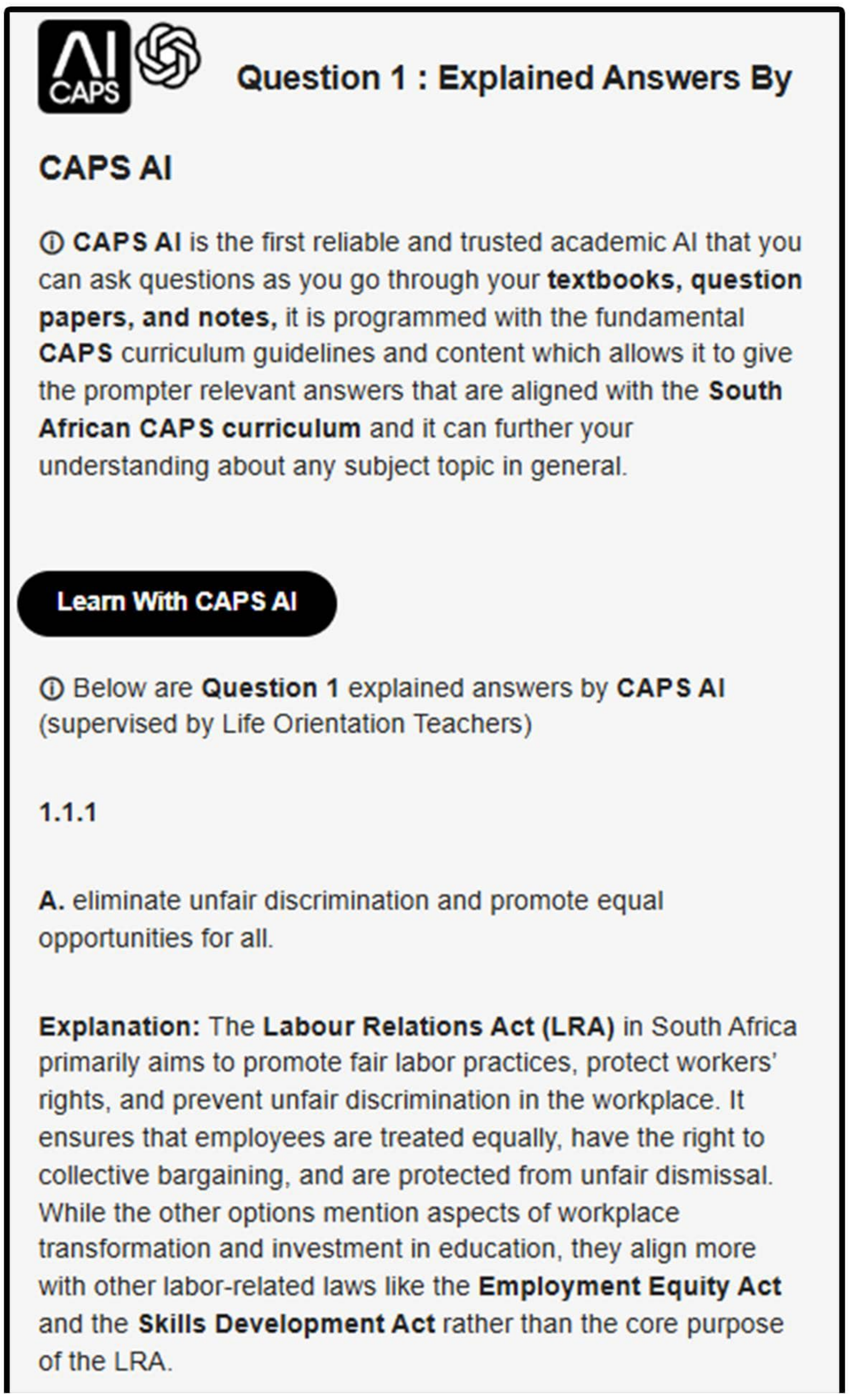


Democracy and Human Rights

Question 1

1.1 Various options are provided as possible answers to the following questions. Choose the answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5), for example, 1.1.6 D.

1.1.1 The purpose of the Labour Relations Act is to ...
A. eliminate unfair discrimination and promote equal opportunities for all.
B. improve employment opportunities of disadvantaged people.
C. transform South African companies by encouraging black management.
D. rectify imbalances that exist in the workplace by improving the level of investment in education. (1)



Question 1 : Explained Answers By CAPS AI

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ⓘ Below are **Question 1** explained answers by **CAPS AI** (supervised by Life Orientation Teachers)

1.1.1

A. eliminate unfair discrimination and promote equal opportunities for all.

Explanation: The **Labour Relations Act (LRA)** in South Africa primarily aims to promote fair labor practices, protect workers' rights, and prevent unfair discrimination in the workplace. It ensures that employees are treated equally, have the right to collective bargaining, and are protected from unfair dismissal. While the other options mention aspects of workplace transformation and investment in education, they align more with other labor-related laws like the **Employment Equity Act** and the **Skills Development Act** rather than the core purpose of the LRA.

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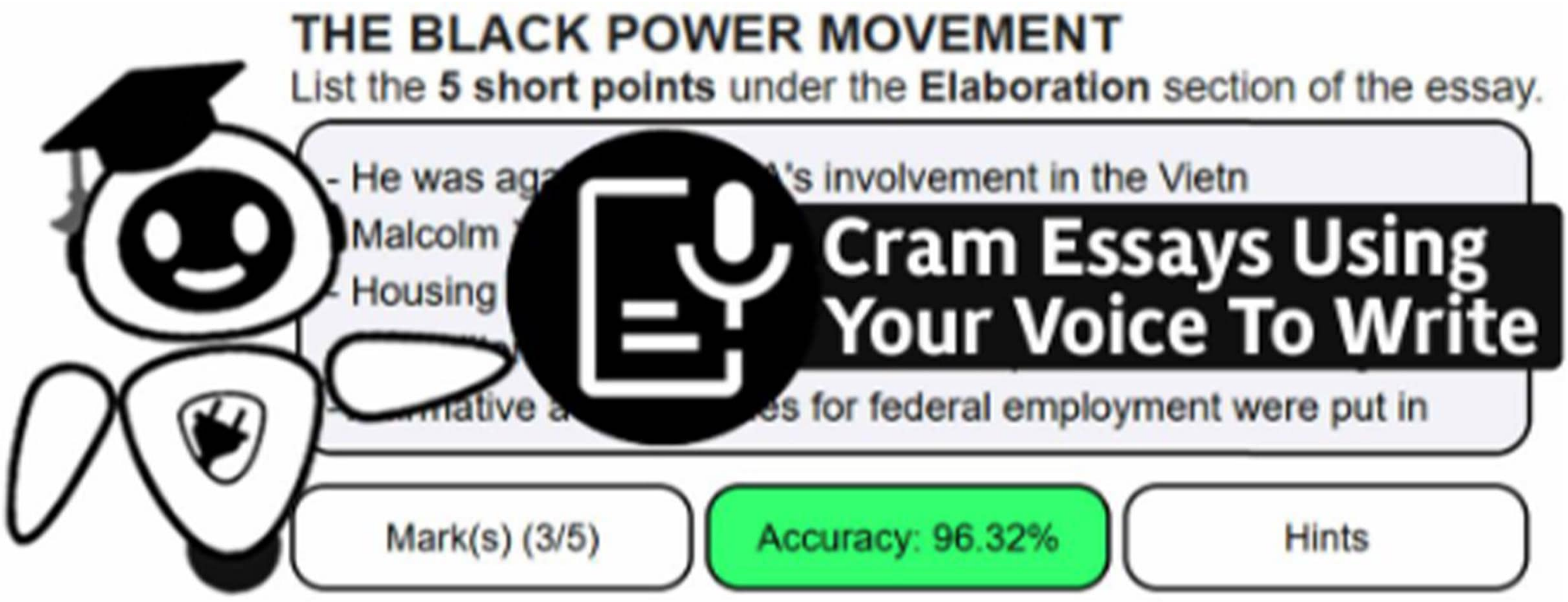

THE BLACK POWER MOVEMENT

List the **5 short points** under the **Elaboration** section of the essay.

- He was ag 's involvement in the Vietn
- Malcolm
- Housing
- mative a es for federal employment were put in

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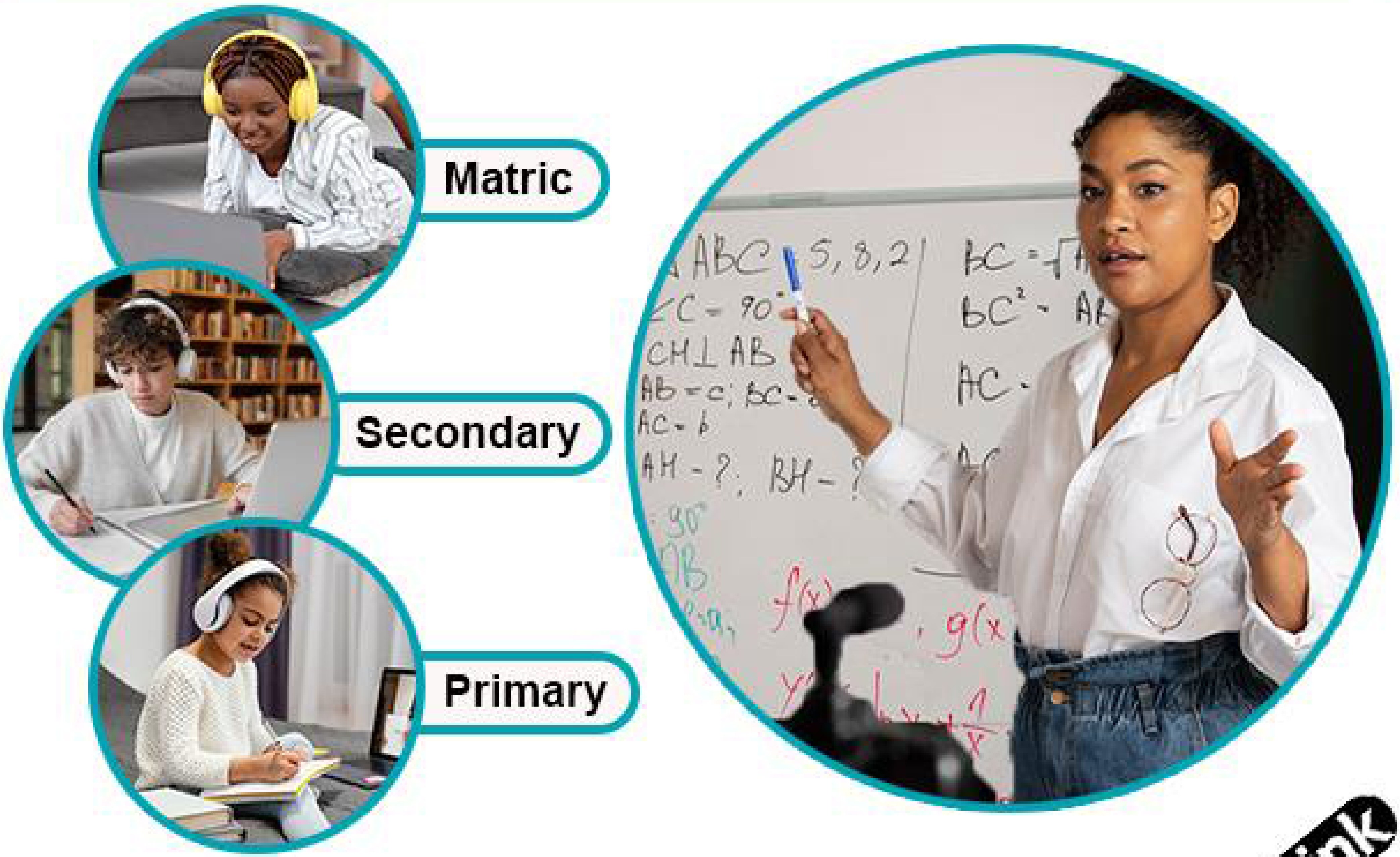

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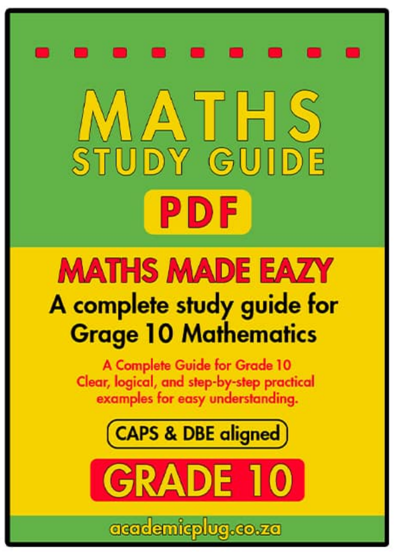
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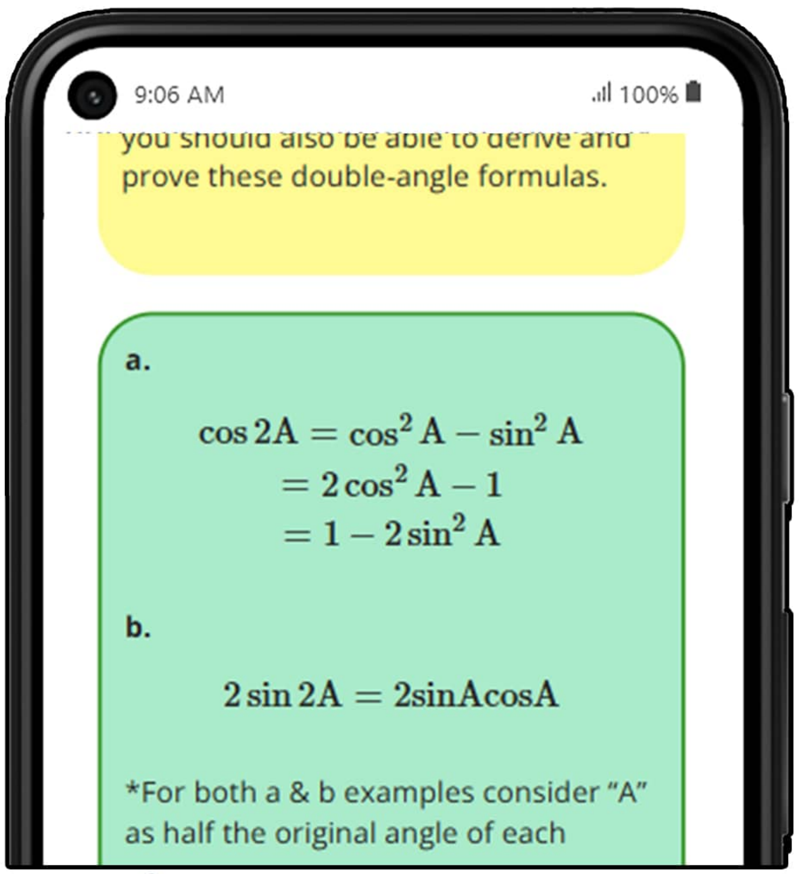

you should also be able to derive and prove these double-angle formulas.

a.

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A\end{aligned}$$

b.

$$2\sin 2A = 2\sin A\cos A$$

*For both a & b examples consider "A" as half the original angle of each

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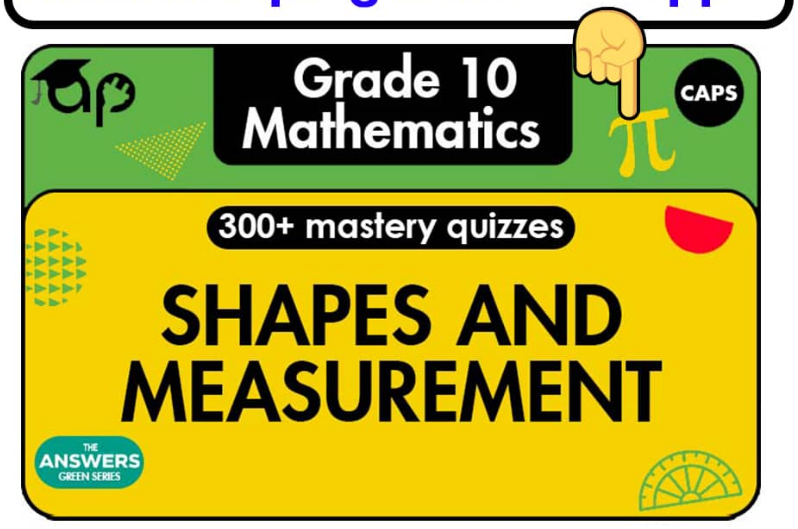
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CHAPTER 10
SHAPES AND MEASUREMENT

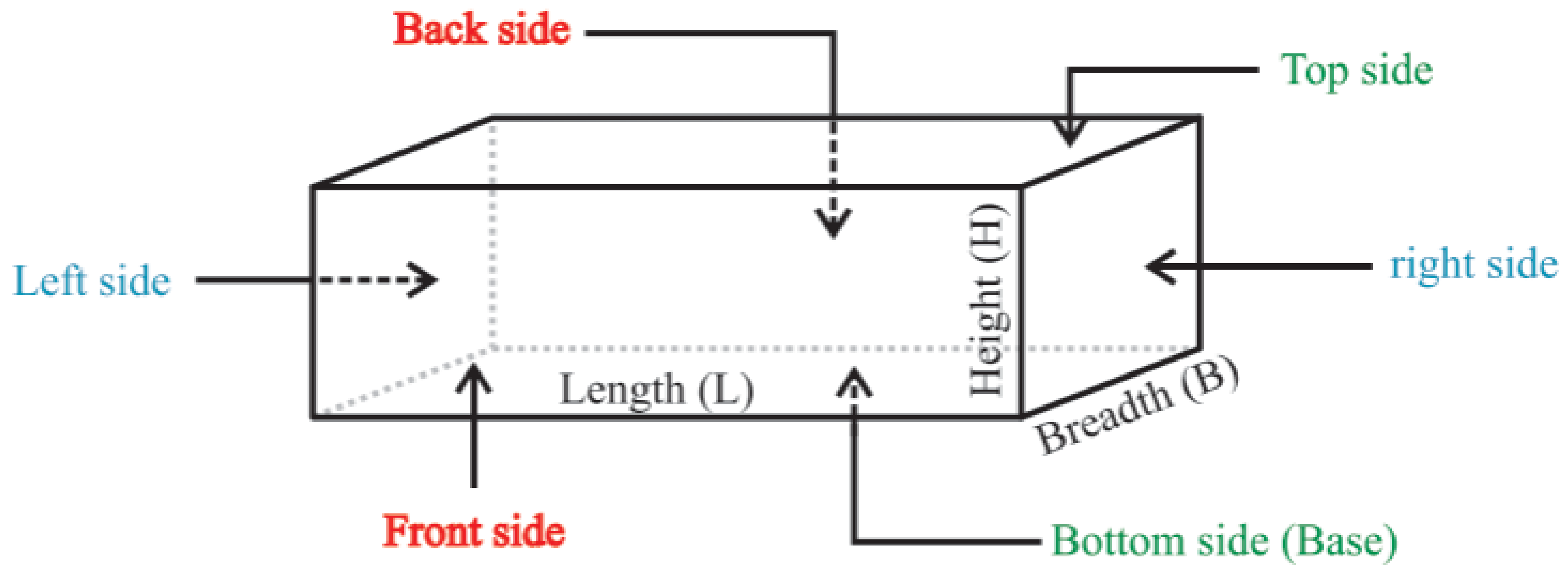
WHAT YOU WILL LEARN IN THIS CHAPTER

PAGE	CONTENT	CAPS	ASSESSMENTS YOU WILL BE ABLE TO:
199	**A. Surface Area and Volume**	2	• Solve problems involving volume and surface area of solids studied in earlier grades and combinations of those objects to form more complex shaped solids. • Understand and determine the effect that multiplying any dimension of a right prism or cylinder has on the volume and surface area of the shapes.
199	• Surface Area of Right Prisms		
200	• Volume of a Right Prism		
200	• Scaling Factors of Right Prisms		
202	• Surface Area of a Cylinder		
202	• Volume of a Cylinder		
205	• Surface Area and Volume of a Sphere		
205	• Surface Area and Volume of a Hemisphere		
206	• Surface Area and Volume of Pyramids		
207	• Surface Area and Volume of Cones		
208	• Mixed Examples		

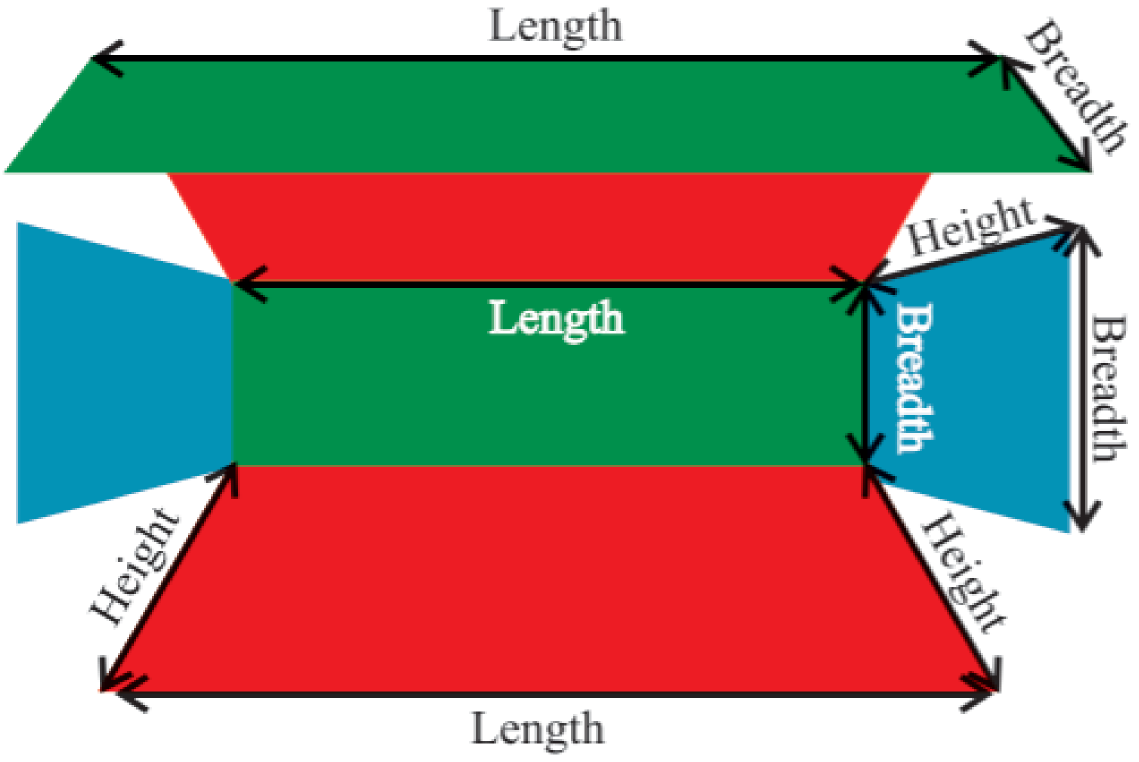
A. SURFACE AREA AND VOLUME

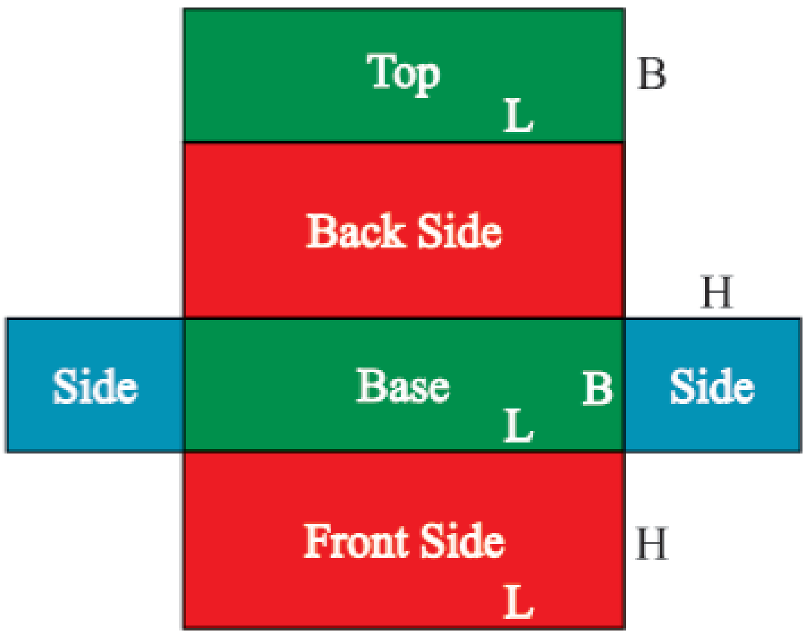
SURFACE AREA OF RIGHT PRISMS

The surface area of a right prism is the sum of the areas of each individual face that makes up the surface of the prism.


Back side
Top side
Left side
right side
Height (H)
Length (L)
Breadth (B)
Front side
Bottom side (Base)

By unfolding the box each individual side that makes up the prism can be seen.


Length
Breadth
Height
Length
Breadth
Breadth
Height
Height
Length


Top
L
B
Back Side
H
Side
Base
B
L
Side
Front Side
H
L

$$\begin{aligned} SA &= \left(A_{\text{Left Side}} + A_{\text{Right Side}}\right) + \left(A_{\text{Front Side}} + A_{\text{Back Side}}\right) + \left(A_{\text{Bottom Side}} + A_{\text{Top Side}}\right) \\ &= (BH + BH) + (LH + LH) + (LB + LB) \\ &= 2BH + 2LH + 2LB \end{aligned}$$

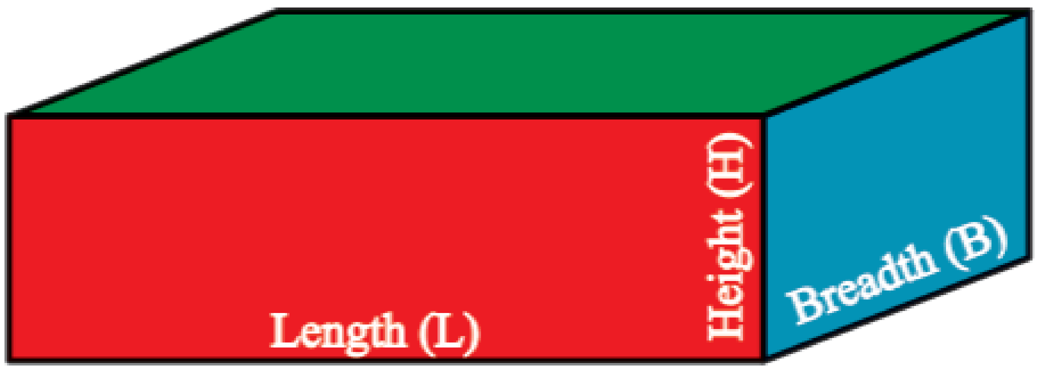
Surface Area of a Right Prism:

$$SA = 2BH + 2LH + 2LB$$

A prism is a three dimensional shape which has identical ends that are parallel to each other.

VOLUME OF A RIGHT PRISM

The volume of any 3 dimensional shape is the area of the base shape multiplied by the height that it is extended by.



Volume of a Right Prism:

$$V = L \times B \times H$$

SCALING FACTORS OF RIGHT PRISMS

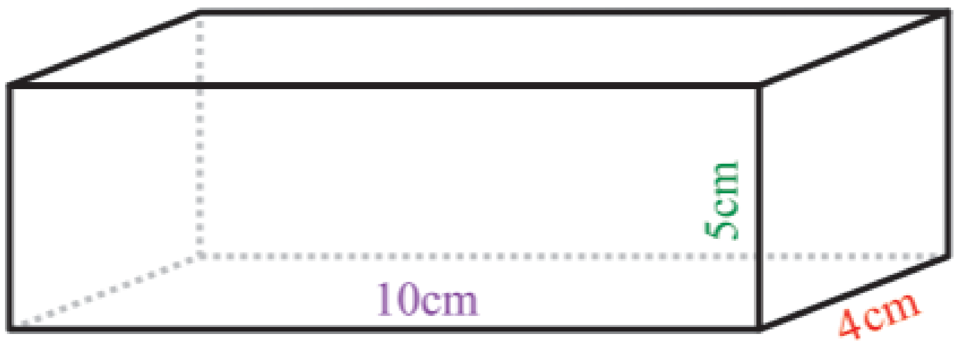
You are expected to be able to determine the effect that multiplying any of the dimensions of the prism by a factor of *k* will have on the area and volume of the prism.

Change	Effect on Surface Area
Length, Breadth and Height multiplied by a factor of k	Surface area increases by a factor of k^2

Change	Effect on Volume
Length **or** Breadth **or** Height multiplied by a factor of k	Volume is increased by a factor of **k**
Length and Breadth or **Length and Height** or **Breadth and Height** multiplied by a factor of k	Volume is increased by a factor of k^2
Length and Breadth and Height multiplied by factor of k	Volume is increased by a factor of k^3

There is no general rule for the change in surface area if one or two sides are multiplied by a factor of *k*. In this case the area has to be calculated from first principles using all of the formulas that you know.

Example: **For the rectangular prism shown below, determine:**
a) The surface area of the prism.
b) The volume of the prism.
c) The effect on the volume and surface area if all of the dimensions are increased by a factor of *k*.



a) $\text{SA} = 2\text{BH} + 2\text{LH} + 2\text{LB}$

$= 2(4)(5) + 2(10)(5) + 2(10)(4)$

$= 40 + 100 + 80$

$= 220\,\text{cm}^2 \rightarrow$ area is always in units squared

b) $\text{V} = \text{L} \times \text{B} \times \text{H}$

$= 10 \times 4 \times 5$

$= 200\,\text{cm}^3 \rightarrow$ volume is always in units cubed

c) If all of the dimensions are multiplied by a factor of ***k*** the dimensions are:

$\text{Length} = 10k$

$\text{Breadth} = 4k$

$\text{Height} = 5k$

$$\begin{aligned}\therefore \text{SA}_{\text{new}} &= 2(4k)(5k) + 2(10k)(5k) + 2(10k)(4k)\\ &= 40k^2 + 100k^2 + 80k^2\\ &= 220k^2\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\therefore \text{V}_{\text{new}} &= (10k)(4k)(5k)\\ &= 200k^3\ \text{cm}^3\end{aligned}$$

If all of the dimensions are multiplied by a factor of *k* the surface area is increased by a factor of k^2 and the volume is increased by a factor of k^3.

Area and Surface area are always expressed in units^2.
Volume is always expressed in units^3.

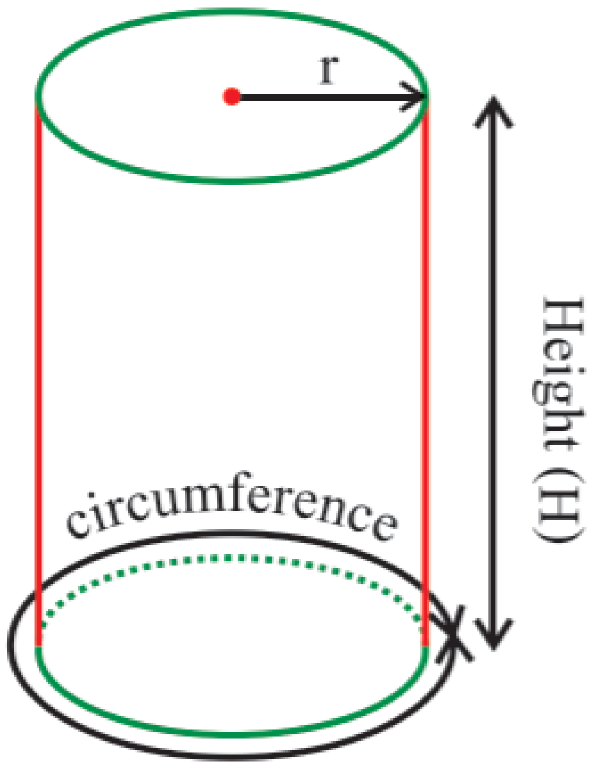
Complete Exercise 1 on page 366.

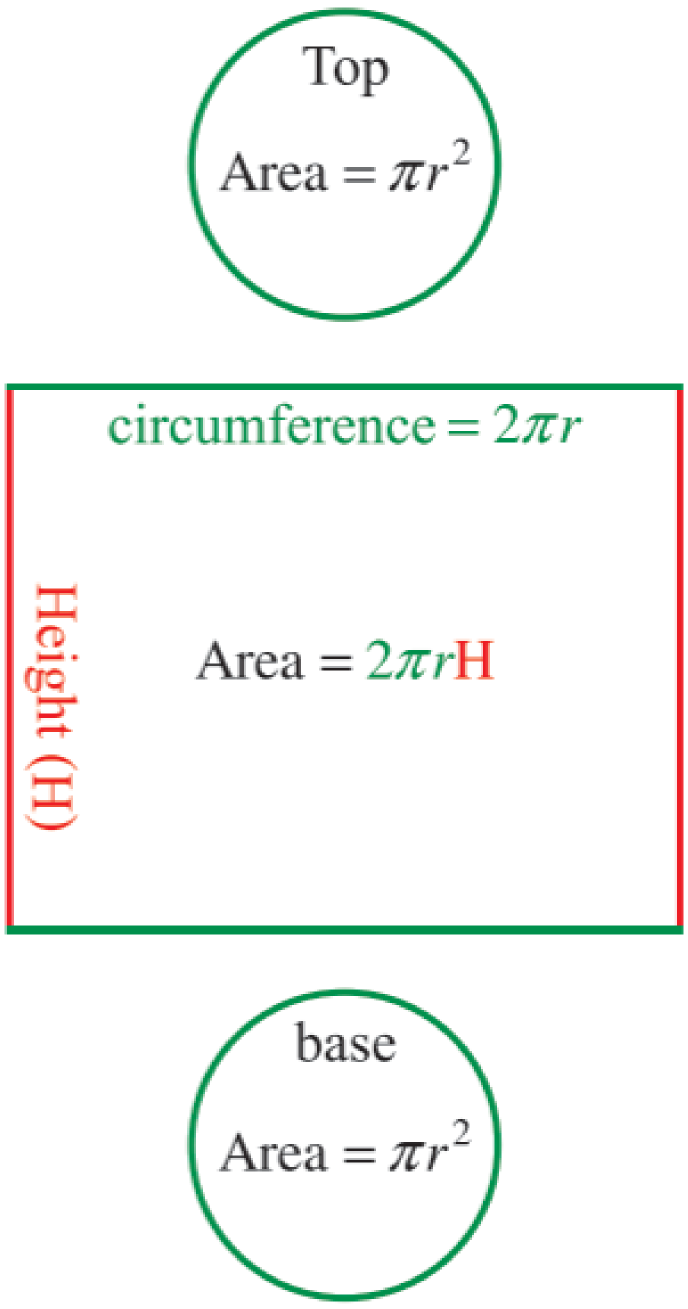
SURFACE AREA OF A CYLINDER

A cylinder is made up of two circles and a rectangle. The area of the rectangle is the circumference of the circle multiplied by the height of the cylinder.

Be careful to take note whether the cylinder is open or closed. If the cylinder is open the formula for the surface area has to be adjusted.

The glossary at the back of this book defines the radius, diameter and circumference of a circle.


r
Height (H)
circumference


Top
Area = πr^2
circumference = $2\pi r$
Height (H)
Area = $2\pi r$H
base
Area = πr^2

$$\begin{aligned} SA &= A_{rectangle} + A_{base} + A_{top} \\ &= 2\pi rH + \pi r^2 + \pi r^2 \\ &= 2\pi rH + 2\pi r^2 \end{aligned}$$

Surface Area of a Cylinder:

$SA = 2\pi rH + 2\pi r^2$ → **closed cylinder**

$SA = 2\pi rH + \pi r^2$ → **cylinder open on one end**

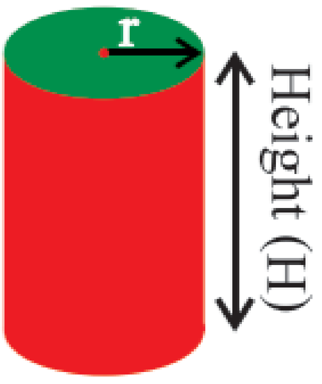
$SA = 2\pi rH$ → **cylinder open on both ends**

VOLUME OF A CYLINDER

The volume of most three dimensional shapes is the area of the base shape multiplied by the height that it is extended by.

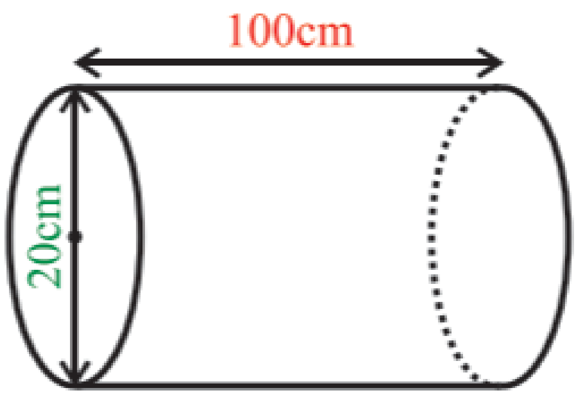
Volume of a Cylinder:

$$V = \pi r^2 \times H$$



Example 1: **For the cylinder shown below, determine:**

a) The surface area of the cylinder if it is closed.
b) The surface area of the cylinder if it is open on one end.
c) The volume of the cylinder.
d) The effect on the volume and surface area of the closed cylinder if the radius and height are increased by a factor of *k*.



Recap: (pg 202)

Surface Area of a cylinder

$\mathbf{SA = 2\pi rH + 2\pi r^2}$

$\mathbf{SA = 2\pi rH + \pi r^2}$

$\mathbf{SA = 2\pi rH}$

a) It is important to note that the diameter of the cylinder is 20cm.
$\therefore$ the radius is 10 cm.

$$\begin{aligned} SA &= 2\pi(10)(100) + 2\pi(10)^2 \\ &= 2200\pi \\ &= 6\,911{,}50\ \text{cm}^2 \rightarrow \text{area is always in units squared} \end{aligned}$$

b) If the cylinder is open on one end, then the surface area of one circle is used.

$$\begin{aligned} SA &= 2\pi(10)(100) + \pi(10)^2 \rightarrow \text{only add on the surface area for one end} \\ &= 2100\pi \\ &= 6597{,}34\ \text{cm}^2 \rightarrow \text{area is always in units squared} \end{aligned}$$

c)
$$\begin{aligned} V &= \pi r^2 \times H \\ &= \pi(10)^2(100) \\ &= 10\,000\pi \\ &= 31415{,}93\ \text{cm}^3 \rightarrow \text{volume is always in units cubed} \end{aligned}$$

d) If the radius and height are multiplied by a factor of k the dimensions are:
Radius $= 10k$
Height $= 100k$

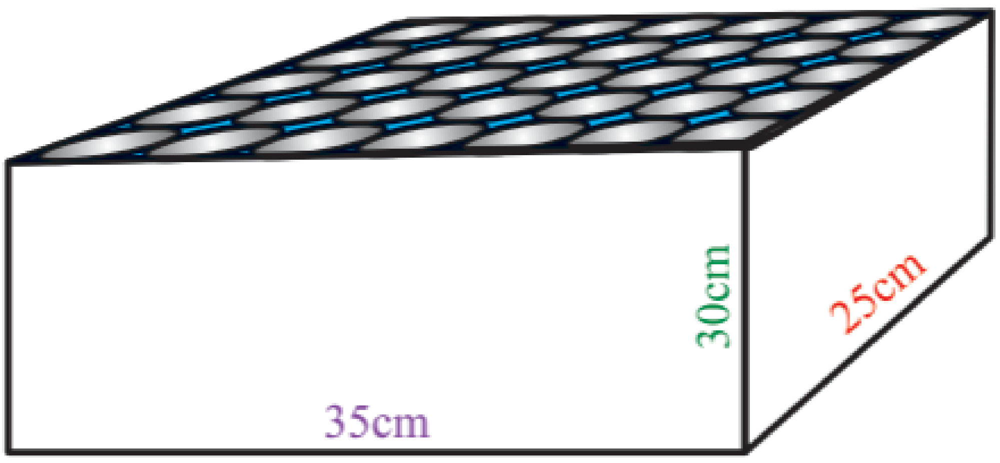
$$\begin{aligned} SA_{\text{new}} &= 2\pi(10k)(100k) + 2\pi(10k)^2 \\ &= 2200\pi k^2 \\ &= 6911{,}50k^2\ \text{cm}^2 \rightarrow \text{area is always in units squared} \end{aligned}$$

$$\begin{aligned} \therefore V_{\text{new}} &= \pi(10k)^2(100k) \\ &= 10\,000\pi k^3 \\ &= 31415{,}93k^3\ \text{cm}^3 \rightarrow \text{volume is always in units cubed} \end{aligned}$$

If all of the dimensions are multiplied by a factor of *k* the surface area is increased by a factor of k^2 and the volume will be increased by a factor of k^3.

Example 2: **A newspaper company's packaging machine rolls newspapers into rolls and packs a total of 35 newspapers into a rectangular box with dimensions as shown in the diagram below. The machine packs 7 newspapers in 5 rows. Calculate:**

a) The radius of each roll of newspaper if they fit exactly into the box.
b) The volume that the newspapers occupy in the box.
c) The volume of the gaps between the newspapers in the box.



a) The box is 35cm in length and holds 7 newspapers in each row.

$\therefore$ the diameter of each newspaper is:

$35 \div 7 = 5\,\text{cm}$

$\therefore$ the radius of each newspaper is half the diameter:

$\therefore r = 2{,}5\,\text{cm}$

b) The box is 30cm high. Therefore, each newspaper is 30cm in height.

$$\begin{aligned} V &= \pi r^2 \times H \\ &= \pi(2{,}5)^2(30) \\ &= 589{,}05\,\text{cm}^3 \rightarrow \text{volume is measured in cubic units} \end{aligned}$$

$\therefore$ the volume of a single newspaper is $589{,}05\,\text{cm}^3$

$\therefore V_{\text{total}} = (35)(589{,}05) = 20\,616{,}75\,\text{cm}^3 \rightarrow$ there are 35 newspapers in the box

c) $V_{\text{gaps}} = V_{\text{box}} - V_{\text{newspapers}}$

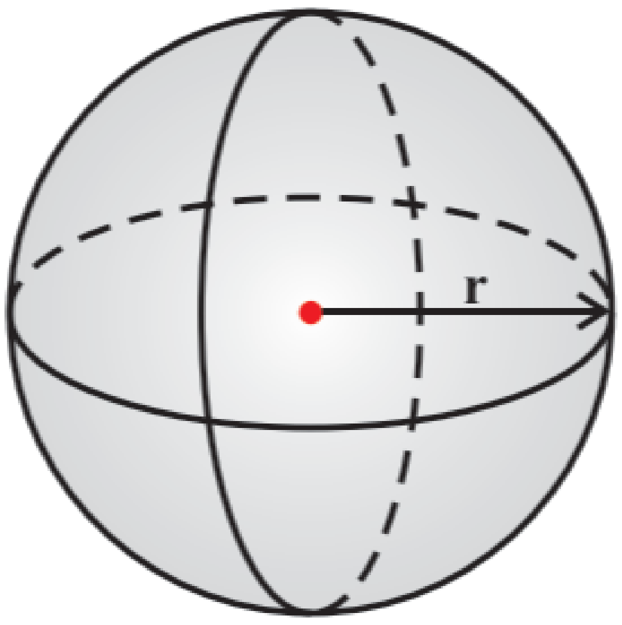
$$\begin{aligned} V_{\text{box}} &= L \times B \times H \\ &= (35)(25)(30) \\ &= 26\,250\,\text{cm}^3 \end{aligned}$$

$$\begin{aligned} \therefore V_{\text{gaps}} &= 26\,250 - 20\,616{,}75 \\ &= 5\,633{,}25\,\text{cm}^3 \end{aligned}$$

Complete Exercise 2 on page 366.

SURFACE AREA AND VOLUME OF A SPHERE

A sphere is a perfectly symmetrical ball with all points on the surface of the sphere being equidistant from the centre. The formulas for surface area and volume of a sphere are shown below.

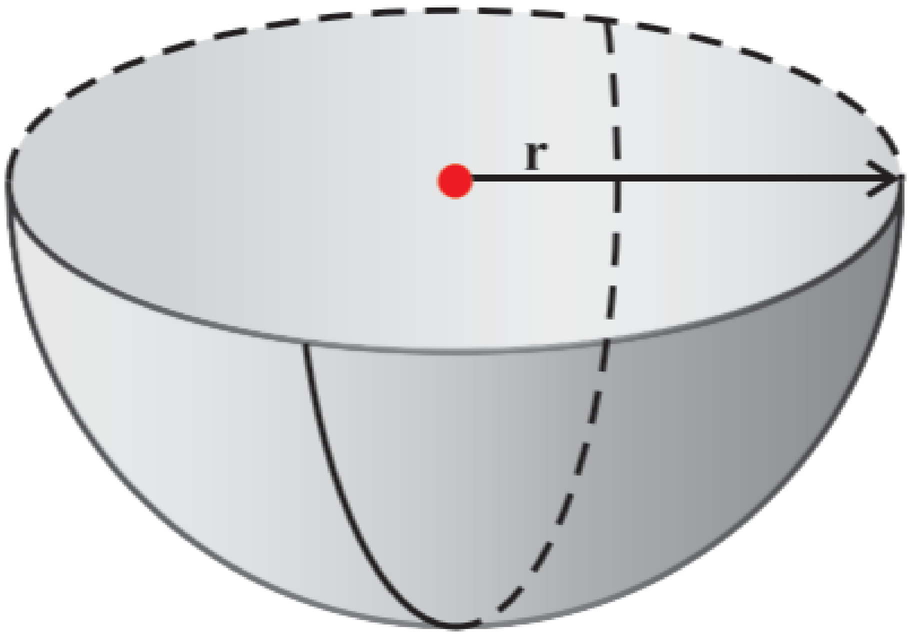


Surface Area and Volume of a Sphere:

Surface Area	Volume
$SA = 4\pi r^2$	$\text{Volume} = \frac{4}{3}\pi r^3$

SURFACE AREA AND VOLUME OF A HEMISPHERE

A hemisphere is half a sphere. Therefore, the volume of a hemisphere is simply half that of a sphere. However, if the hemisphere is closed you have to take into account the area of the base of the hemisphere **when calculating the surface area** of a hemisphere. **If the hemisphere is closed or solid the area of the base (πr^2) has to be added.**



Surface Area and Volume of a Closed/Solid Hemisphere:

Surface Area	Volume
$SA = \frac{1}{2}\left(4\pi r^2\right) + \pi r^2 = 3\pi r^2$	$\text{Volume} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$

Surface Area and Volume of an Open Hemisphere:

Surface Area	Volume
$SA = \frac{1}{2}\left(4\pi r^2\right) = 2\pi r^2$	$\text{Volume} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$

SURFACE AREA AND VOLUME OF PYRAMIDS

In grade 10 you will be expected to calculate the volume and surface area of two types of pyramids:

1) **Triangular Pyramids**
2) **Rectangular Pyramids**

The methods and formulas for calculating the surface area and volume of pyramids do not change for the different types of pyramids.

Volume and Surface Area of a Pyramid:

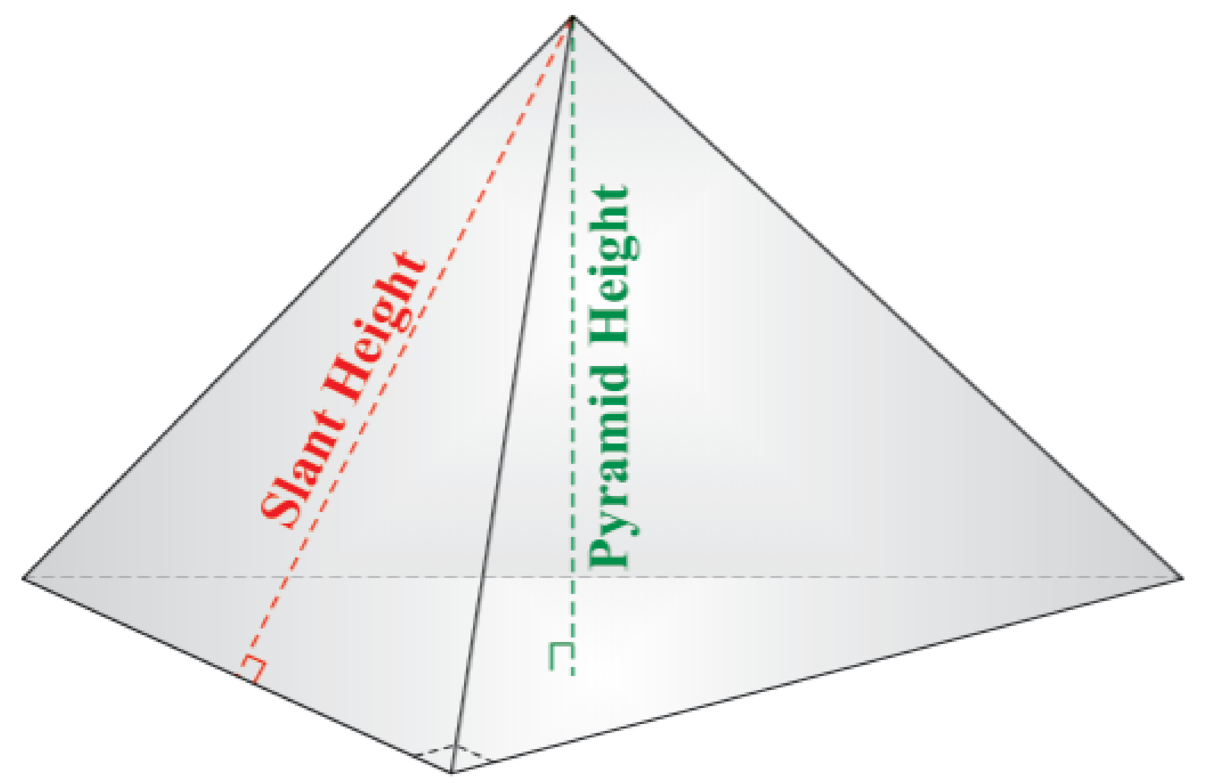
$$\text{Volume} = \frac{\textit{area of base} \times \textit{height}}{3}$$

$$\text{Surface area} = \text{area base} + \text{area of triangular faces}$$

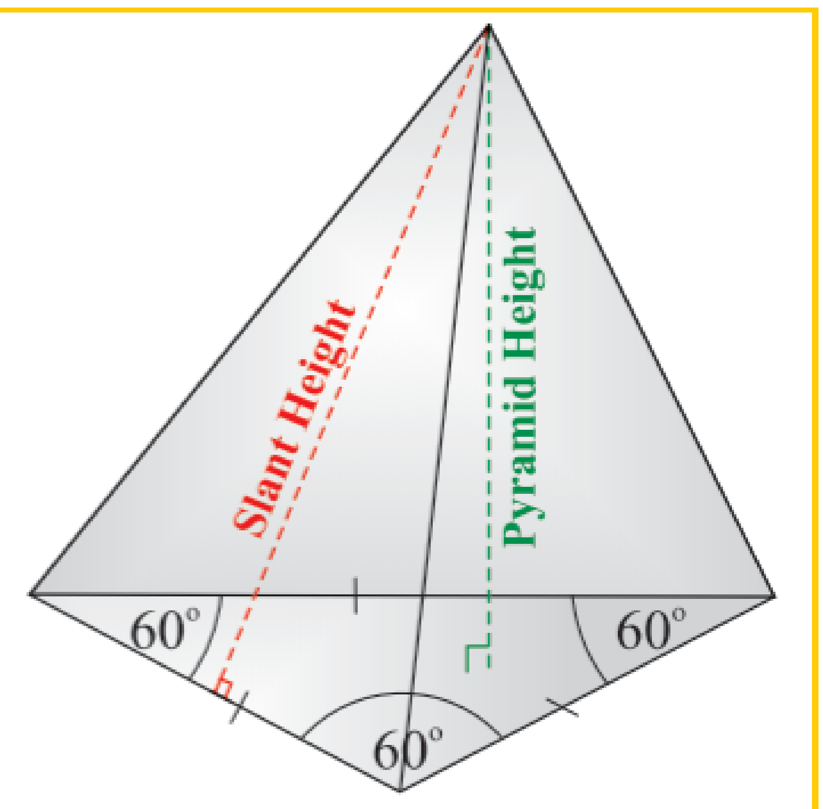
$$\text{Area of triangular face} = \frac{1}{2}\textit{base} \times \perp h_s \ (\textit{slant}\ \text{height})$$

1. Triangular Pyramids

A triangular pyramid is a pyramid that has a triangle as a base. In Grade 10 you will deal with right triangle base pyramids and equilateral base pyramids.



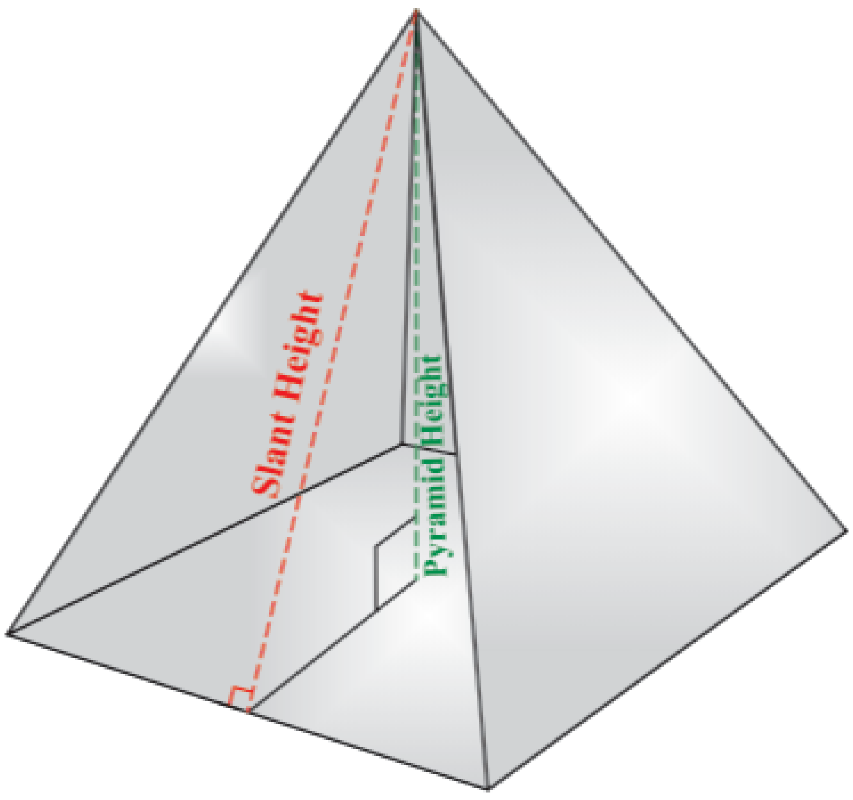
Right triangle based pyramid



Equilateral based Pyramid

2. Rectangular Pyramids

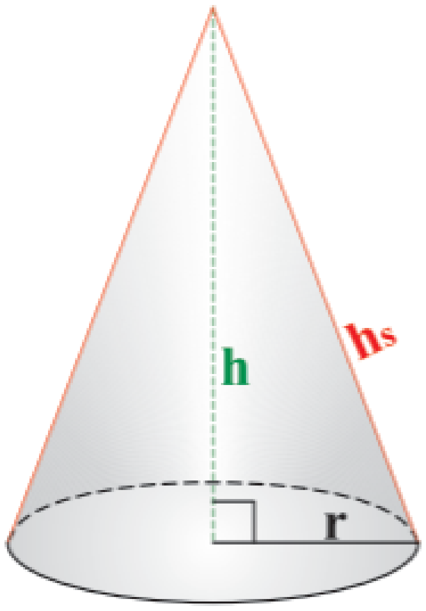
A rectangular pyramid is a pyramid that has a rectangle as a base. It is important to note that if the base is a square, the area of all the faces will be the same.
Pythagoras (pg. 142) will often have to be used to calculate the pyramid height or slant height.

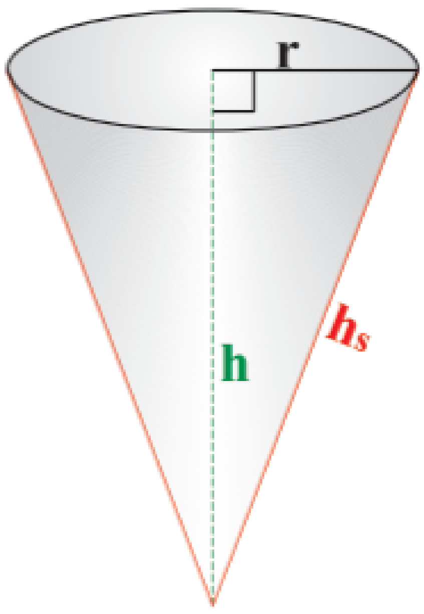


SURFACE AREA AND VOLUME OF CONES

In Grade 10 you are also expected to calculate the surface area and volume of cones. As with hemispheres it is important to take note whether the cone is closed (solid) or open (hollow). If the cone is closed, the area of the base has to be taken into account when calculating the surface area of cones.

Pythagoras (pg. 142) will often have to be used to calculate the cone height or slant height.





Volume and Surface Area of a Closed/Solid Cone:

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Surface Area} = \frac{1}{2}\text{circumference of the base} \times \text{slant height} + \text{area of the base}$$

$$= \frac{1}{2}(2\pi r)(h_s) + \pi r^2$$

$$= \pi r h_s + \pi r^2$$

Volume and Surface Area of an Open/Hollow Cone:

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Surface Area} = \frac{1}{2}\text{circumference of the base} \times \text{slant height}$$

$$= \frac{1}{2}(2\pi r)(h_s)$$

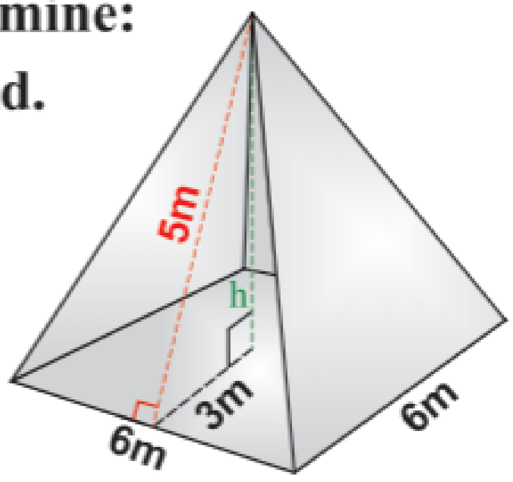
$$= \pi r h_s$$

MIXED EXAMPLES

Example 1: A pyramid stands on a square base which is 6m in length. The slant height of the pyramid is 5m. Determine:

a) The surface area of the pyramid.

b) The volume of the pyramid.



a) $\text{Surface Area} = A_{base} + 4A_{face} \rightarrow$ as the base is square the area of all faces are the same

$$A_{base} = l \times b = 6 \times 6 = 36\,\text{m}^2$$

$$A_{face} = 0,5(base)(h_s) = 0,5(6)(5) = 15\,\text{m}^2$$

$$\therefore \text{Surface Area} = 36 + 4(15) = 96\,\text{m}^2$$

b) In order to calculate the volume of the pyramid, the **pyramid height** is needed. This can be calculated using Pythagoras:

$h^2 = 5^2 - 3^2 \rightarrow \text{Pythag}$

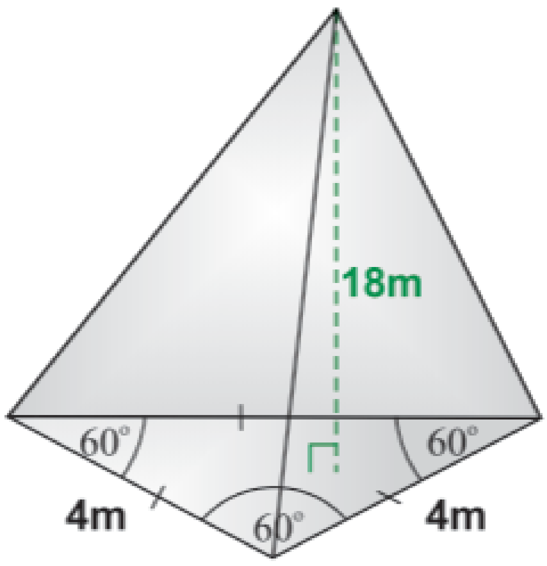
$\therefore h = \sqrt{16}$

$\therefore h = 4\,\text{m} \rightarrow$ note that h is the pyramid height not the hypotenuse

$$\therefore \text{Volume} = \frac{area\ of\ base \times h}{3} = \frac{36 \times 4}{3} = 48\,\text{m}^3$$

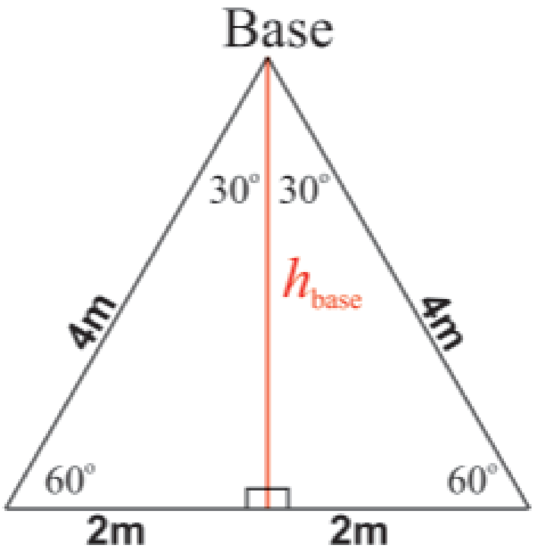
> **In this context *h* is the pyramid height not the hypotenuse.**

Example 2: Calculate the volume of the equilateral based pyramid shown below, if the pyramid has a height of 18m and the length of the base is 4m.



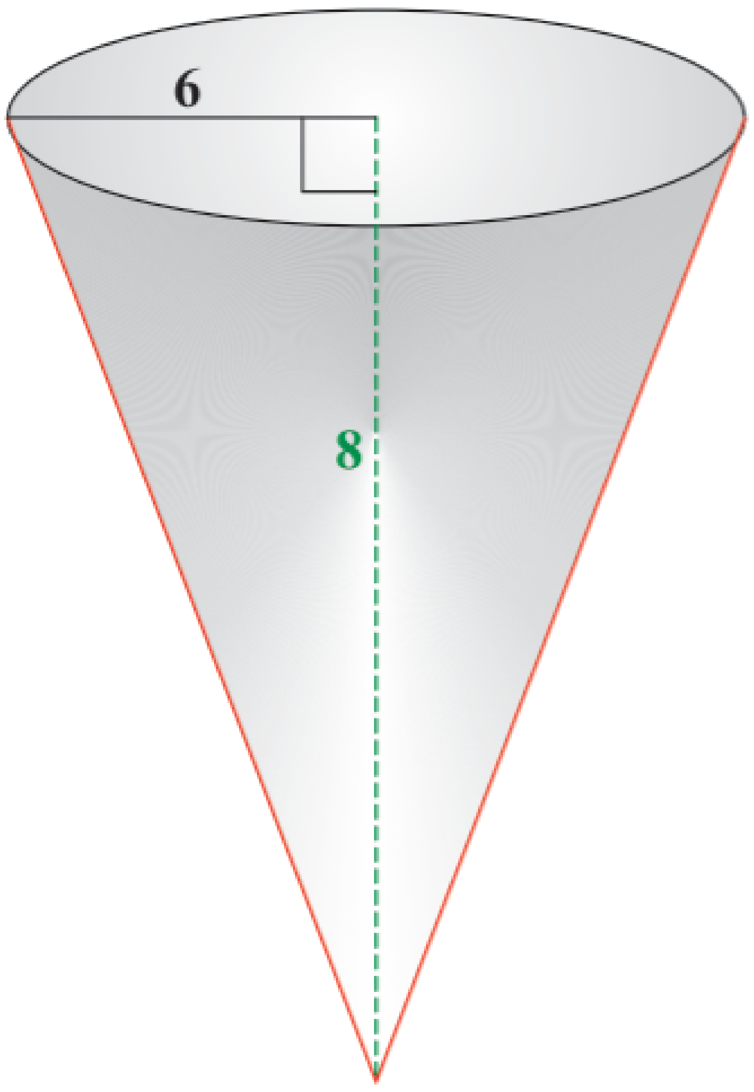
In order to calculate the volume of the pyramid, the area of the base is needed. As the base is an equilateral triangle, it can be divided in half and Pythagoras can be used to solve for the height of the base and hence the area.

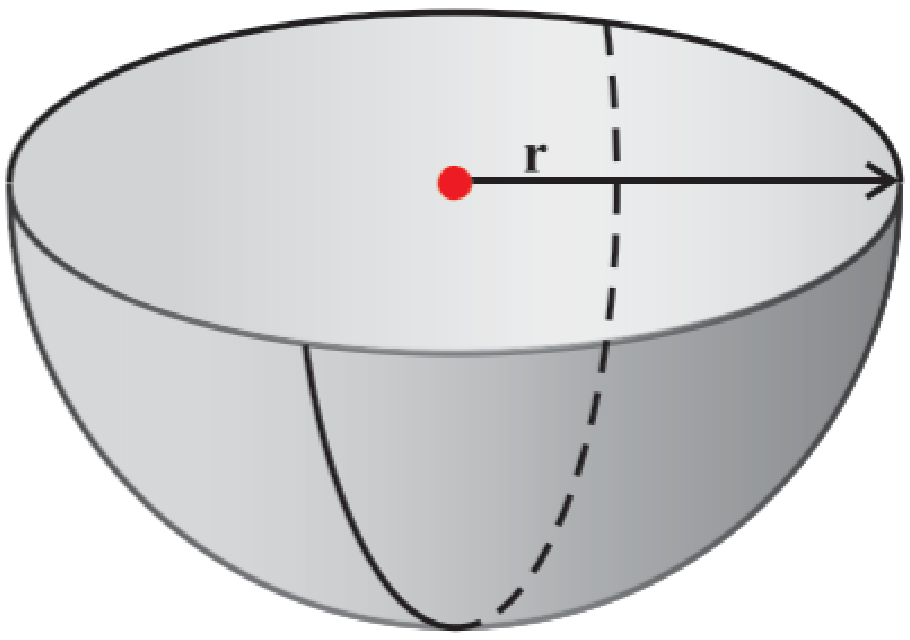
$$h^2_{base} = 4^2 - 2^2 \rightarrow \text{Pythag}$$
$$= 12$$
$$h_{base} = \sqrt{12}\,\text{m}$$
$$A_{base} = 0,5 \times \text{base} \times \perp \text{height}$$
$$= 0,5 \times 4 \times \sqrt{12}$$
$$= 2\sqrt{12}\,\text{m}^2$$



$$\text{Volume} = \frac{1}{3} \times A_{base} \times \textbf{height}$$
$$= \frac{1}{3}(2\sqrt{12})(18)$$
$$= 24\sqrt{3}$$
$$= 41,57\,\text{m}^3$$

Example 3: **An ice-cream cone display is constructed from a solid cone and a hollow hemisphere, as shown below. Calculate the surface area of the display. (Hint: the ice-cream has to fit exactly on the cone)**





The first step is to calculate the slant height of the cone:

$$h_s{}^2 = 6^2 + h^2 \rightarrow \text{Pythagoras}$$

$$\therefore h_s{}^2 = 6^2 + 8^2$$

$$\therefore h_s = 10$$

$$\text{Surface Area} = \text{SA}_{cone} + \text{SA}_{Hemisphere}$$

$$\begin{aligned}\text{SA}_{cone} &= \frac{1}{2} \textit{circumference of the base} \times \textit{slant height} + \textit{area of the base} \\ &= \pi r h_s + \pi r^2 \\ &= \pi(6)(10) + \pi(6)^2 \\ &= 301{,}59\end{aligned}$$

$$\begin{aligned}\text{SA}_{Hemisphere} &= 2\pi r^2 \rightarrow \text{the hemisphere is hollow} \\ &= 2\pi(6)^2 \\ &= 226{,}19\end{aligned}$$

$$\begin{aligned}\therefore \text{Surface Area} &= 301{,}59 + 226{,}19 \\ &= 527{,}78 \text{ units}^2 \rightarrow \text{ always put in the units, whether they are given or not}\end{aligned}$$

In this context *h* is not the hypotenuse, it is the height of the cone.

Complete Exercise 3 on page 367 and
The Mixed Exercise on pages 368 and 369.

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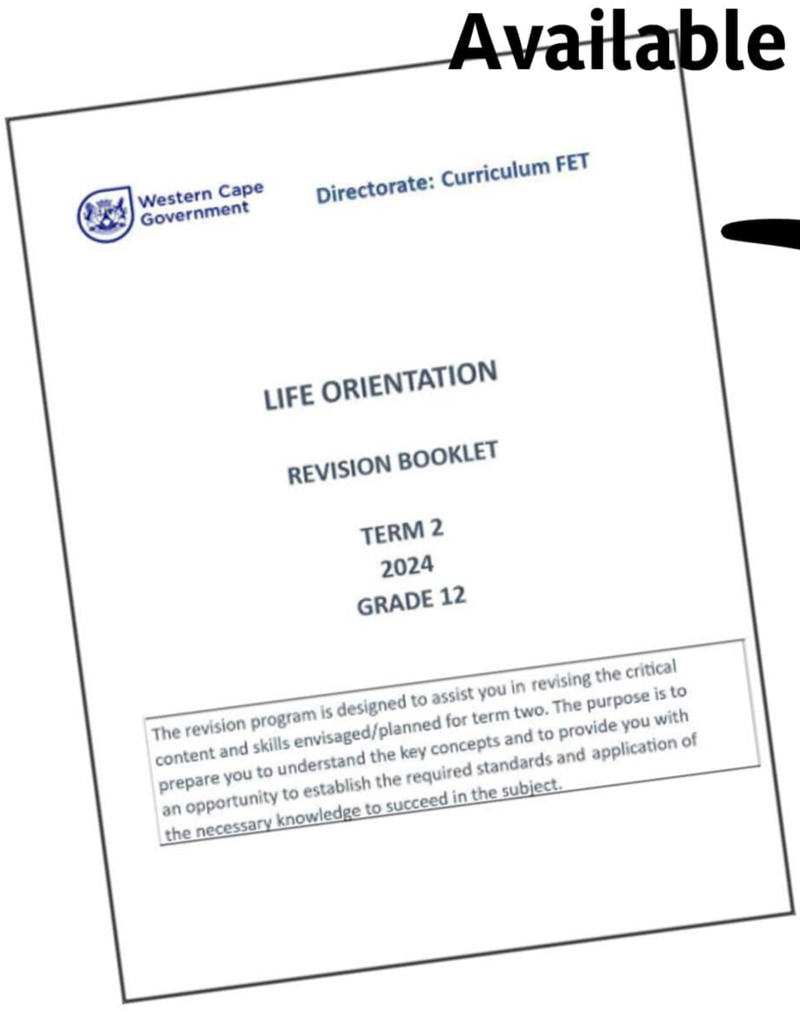

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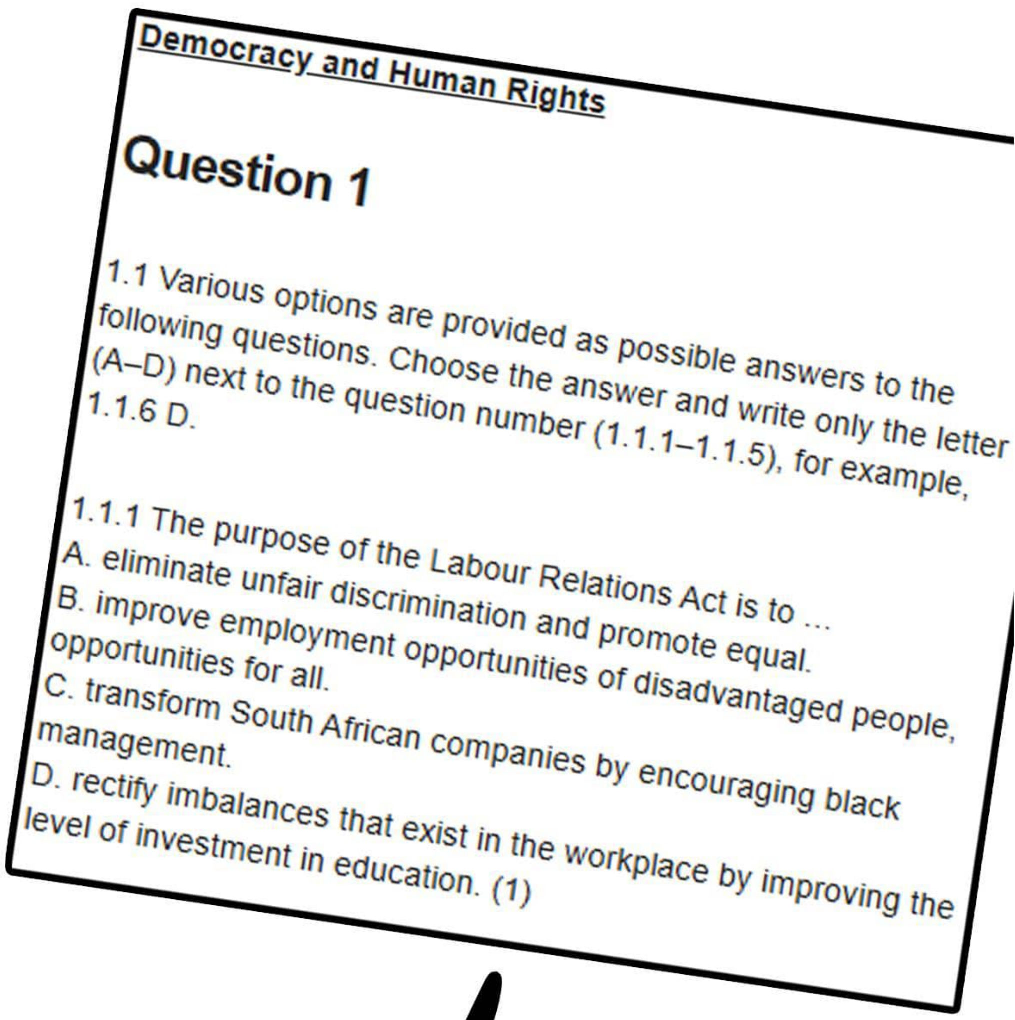
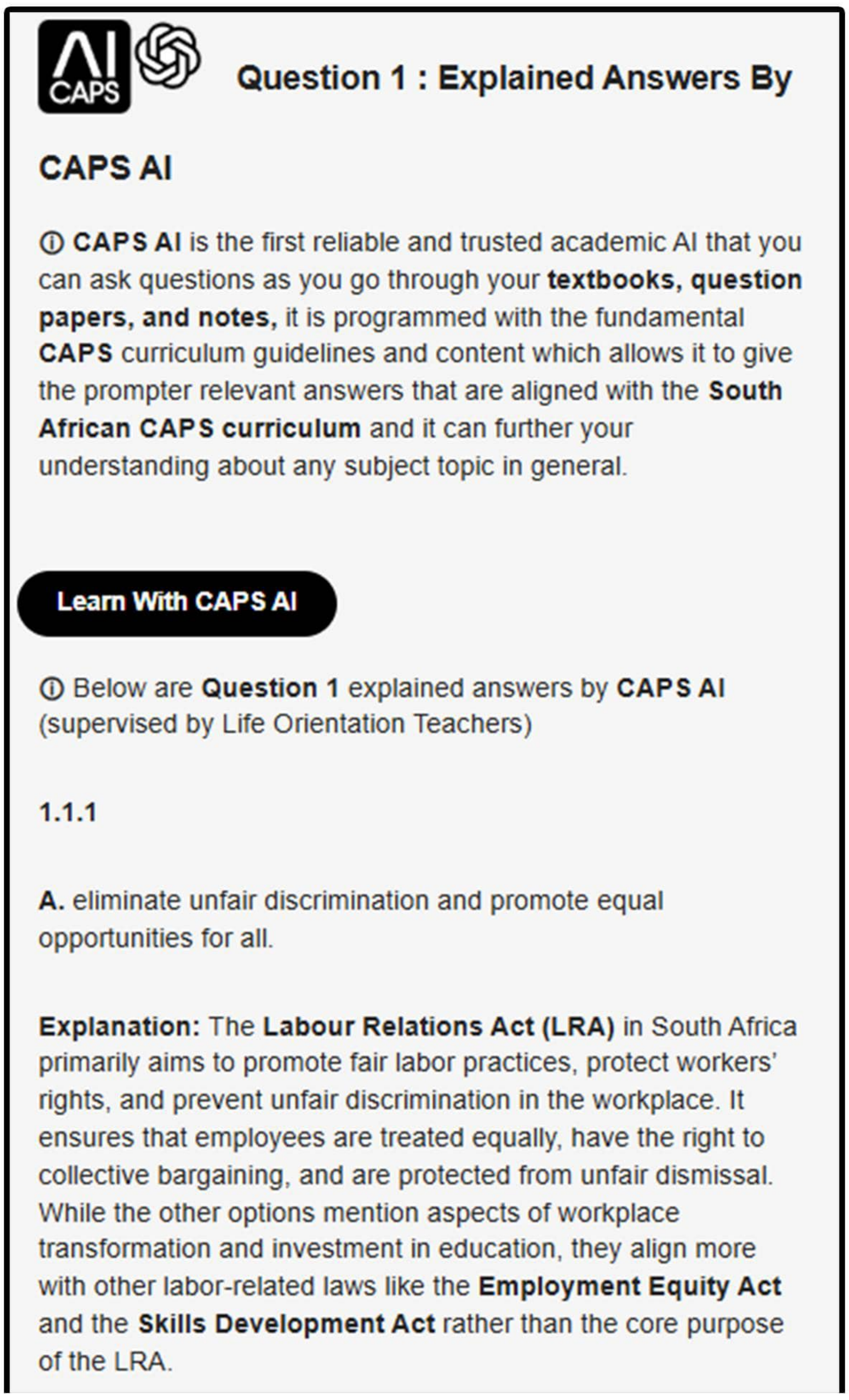


Democracy and Human Rights

Question 1

1.1 Various options are provided as possible answers to the following questions. Choose the answer and write only the letter (A–D) next to the question number (1.1.1–1.1.5), for example, 1.1.6 D.

1.1.1 The purpose of the Labour Relations Act is to ...
A. eliminate unfair discrimination and promote equal opportunities for all.
B. improve employment opportunities of disadvantaged people.
C. transform South African companies by encouraging black management.
D. rectify imbalances that exist in the workplace by improving the level of investment in education. (1)



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1.1.1

A. eliminate unfair discrimination and promote equal opportunities for all.

Explanation: The **Labour Relations Act (LRA)** in South Africa primarily aims to promote fair labor practices, protect workers' rights, and prevent unfair discrimination in the workplace. It ensures that employees are treated equally, have the right to collective bargaining, and are protected from unfair dismissal. While the other options mention aspects of workplace transformation and investment in education, they align more with other labor-related laws like the **Employment Equity Act** and the **Skills Development Act** rather than the core purpose of the LRA.

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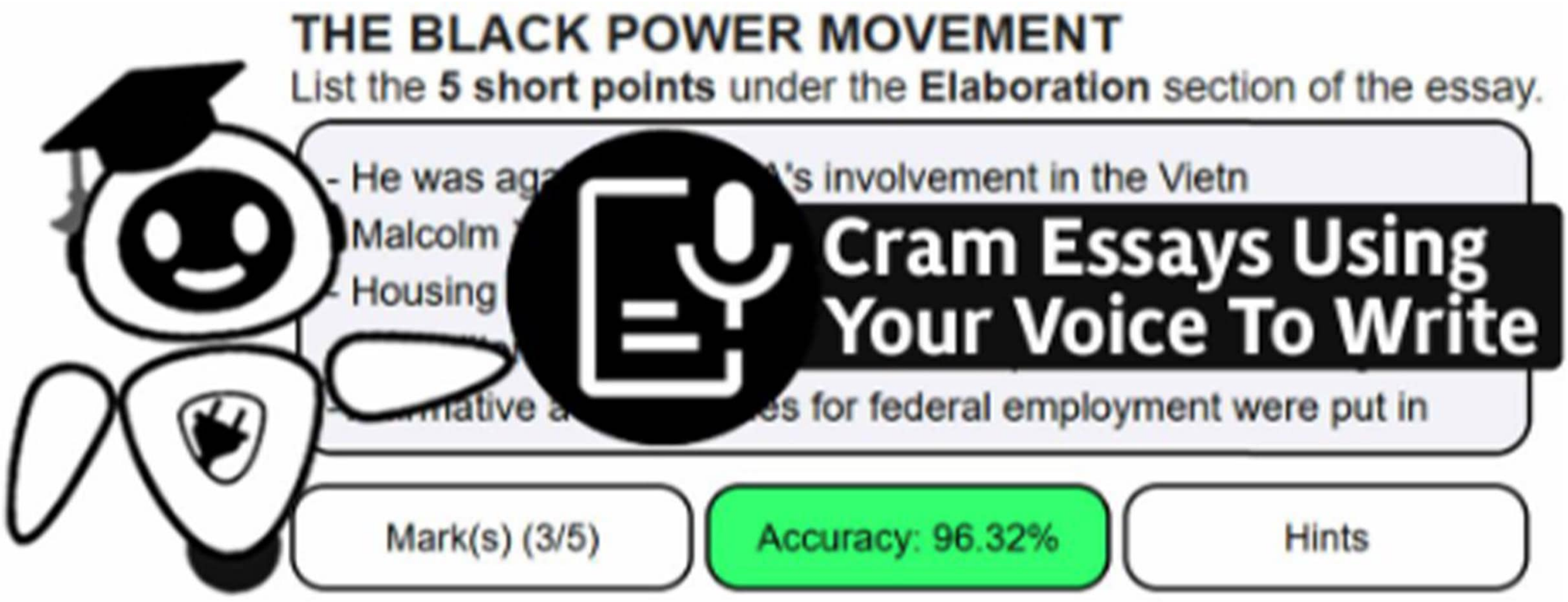

THE BLACK POWER MOVEMENT

List the **5 short points** under the **Elaboration** section of the essay.

- He was ag 's involvement in the Vietn
Malcolm
- Housing
- mative a es for federal employment were put in

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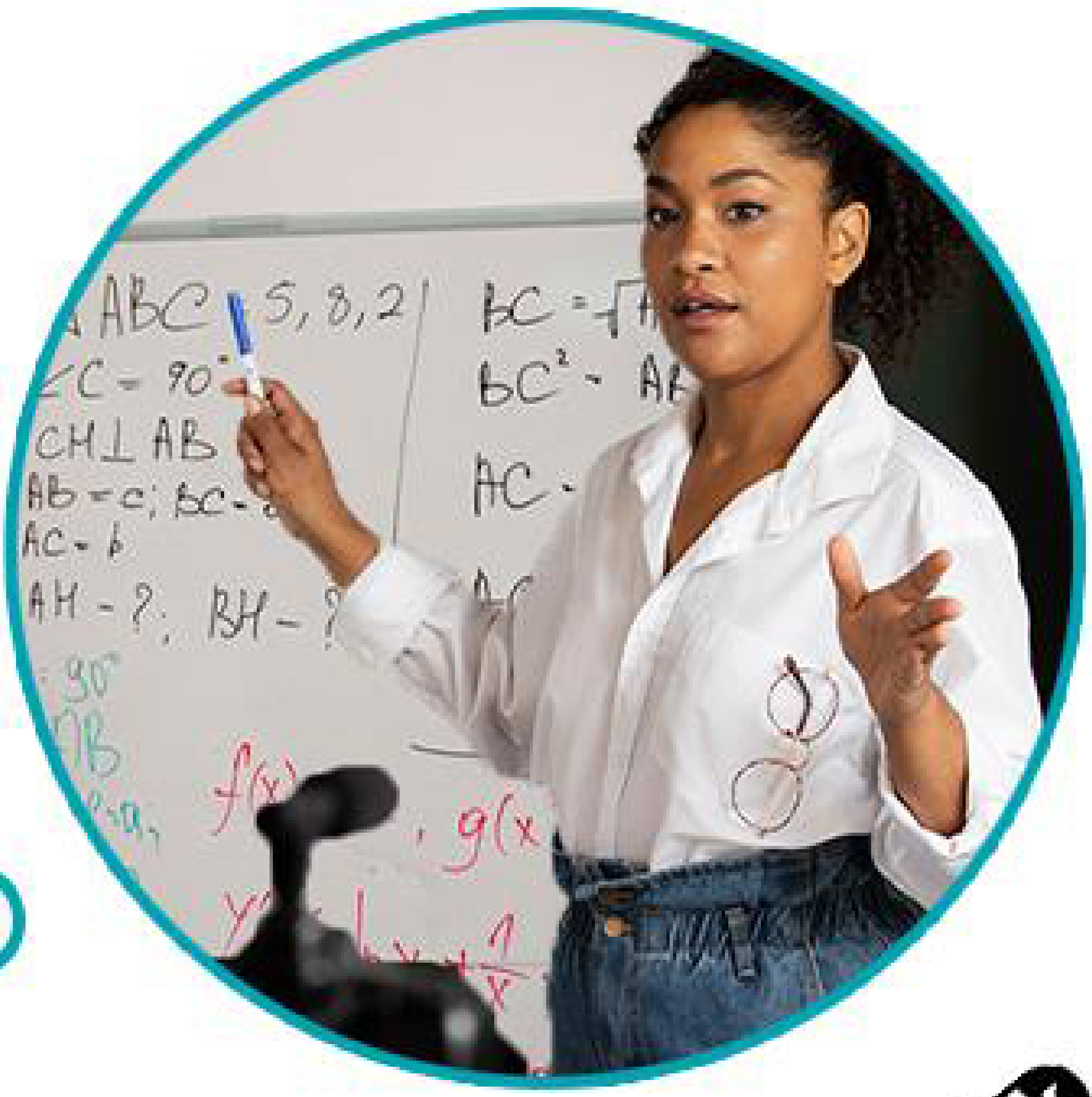

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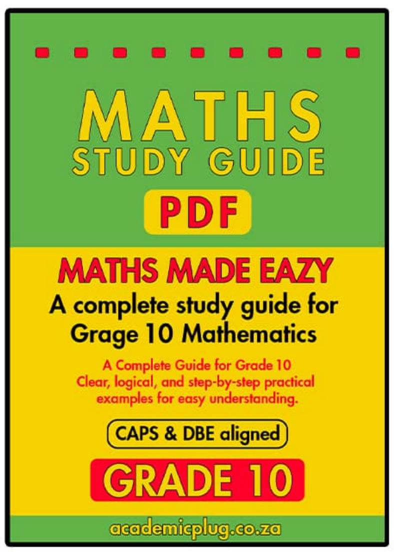

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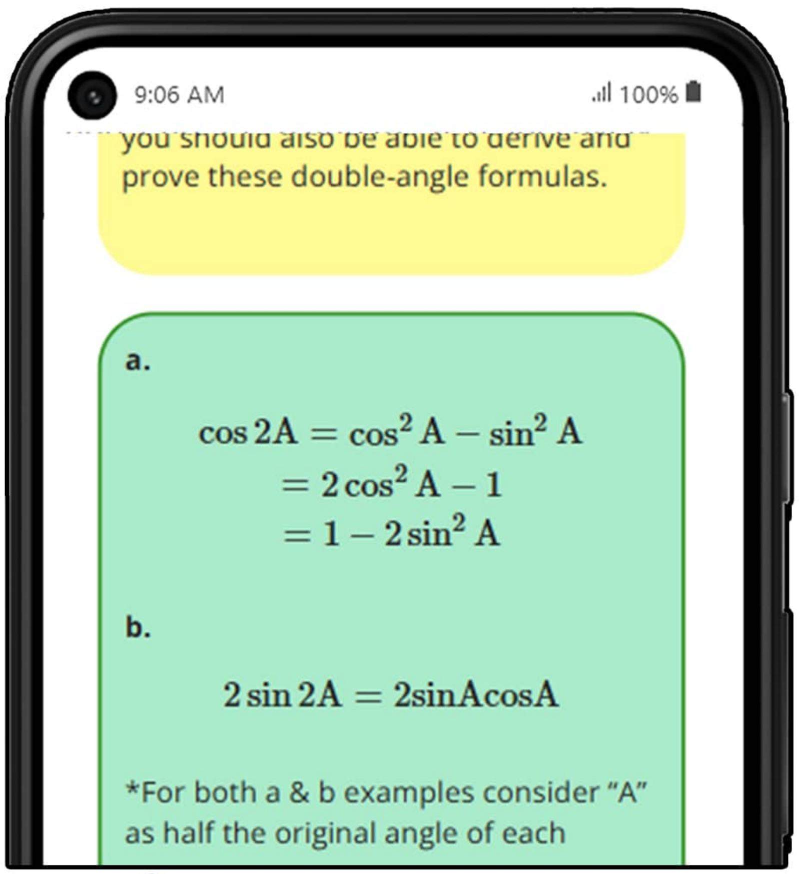

9:06 AM 100%

you should also be able to derive and prove these double-angle formulas.

a.

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

b.

$$2\sin 2A = 2\sin A\cos A$$

*For both a & b examples consider "A" as half the original angle of each

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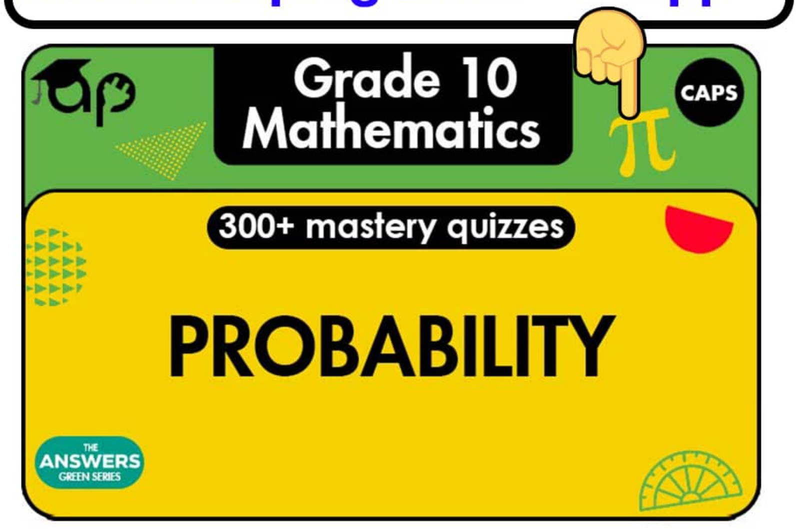

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CHAPTER 11
PROBABILITY

WHAT YOU WILL LEARN IN THIS CHAPTER

PAGE	CONTENT	CAPS	ASSESSMENTS YOU WILL BE ABLE TO:
211 212	**A. Terminology** • Types of Events	6	• Compare the relative frequency of an experimental outcome with the theoretical probability of the outcome.
213 214 214 214	**B. Calculating Probability** • Methods of Determining Probability • Theoretical Probability • Experimental Probability		• Use Venn diagrams as an aid to solving probability problems. • Understand and define mutually exclusive and complementary events.
215 218 218	**C. Venn Diagrams** • Probability Summary • Mixed Examples		• Use the identity for any two events A and B: **P(A or B) = P(A) + P(B) – P(A and B)**

A. TERMINOLOGY

Probability is a branch of Mathematics that studies the likelihood of something happening. It is important that you understand and are familiar with the terminology used throughout this section.

Probability

Probability is the likelihood of something happening.

Experiment

An experiment is a method of testing probability. An example of an experiment is tossing a coin or throwing a dice.

Trial

In an experiment of tossing a coin or throwing a dice, each toss of the coin or throw of a dice would be a trial. A single experiment normally involves many trials.

Outcome

The result of each trial is called an outcome. If a coin were tossed the outcome would be a head or tail.

Sample Space

The sample space is a list (set) of all the possible outcomes. If two coins are tossed there are 4 possible outcomes:

1) **Head** and **Head**
2) **Head** and **Tail**
3) **Tail** and **Head**
4) **Tail** and **Tail**

The sample space is written as: S = {HH, HT, TH, TT}

The number of outcomes in the sample space is written as: $\boldsymbol{n}\,\mathbf{(S) = 4}$

Event

An event is a set of specific outcomes (a subset of the sample space). The event that you would like to occur in an experiment is known as a favourable outcome.

In an experiment where a dice is thrown the sample space is: $S = \{1, 2, 3, 4, 5, 6\}$

If the event you are looking for is obtaining an odd number, the set A, of favourable outcomes would be written as: $A = \{1, 3, 5\}$

Example: **In an experiment a coin is tossed and a dice is thrown. A favourable outcome is obtaining a tail and an odd number.**

a) Using a table, write the sample space for the above experiment.

b) Write the set A of favourable outcomes.

a)

Dice	1	2	3	4	5	6
Coin	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

$\therefore S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

b) $A = \{T1, T3, T5\}$

TYPES OF EVENTS

There are special names given to specific events. In grade 10 you are required to understand the names of the events listed below.

Impossible Events

Impossible events have a probability of 0. An example of an impossible event would be obtaining a 10 when rolling a single die. Impossible events are represented by the null or empty set and are written as:

$$A = \{\text{obtaining a 10 when rolling a die}\}$$

$$\therefore A = \phi \quad \text{or} \quad A = \{\ \} \rightarrow \text{null or empty set}$$

Even Chance Events

Even events are events which have a 50% chance (probability) of occurring. Tossing a fair coin is an even chance event as there is 50% chance of obtaining a head or tail.

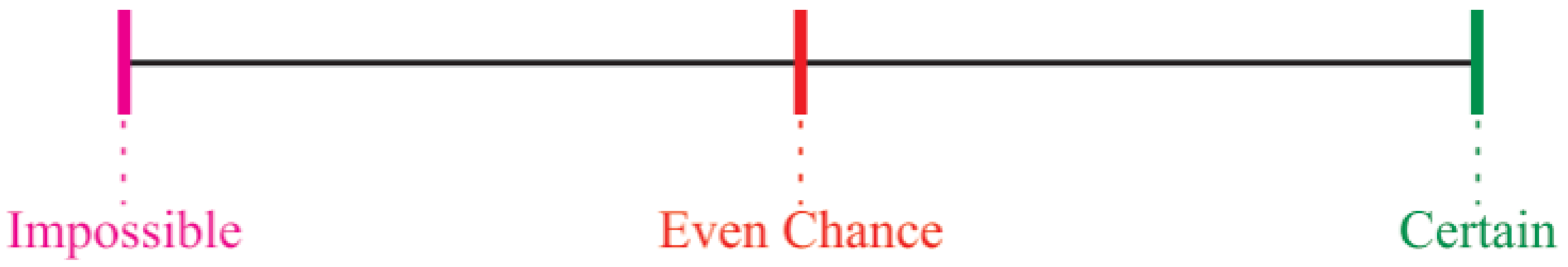
Certain Events

Certain events have a probability of 1 (100%). An example of a certain event is that the sun will rise every morning in South Africa.

Impossible, even and certain events can be represented on a probability line as shown below. Probability can be expressed as a fraction, decimal or percentage.

Probability written as a:

Fraction	0	$\frac{1}{2}$	1
Decimal	0	0,5	1
Percentage	0%	50%	100%



Mutually Exclusive (disjoint) Events

If two events are mutually exclusive it means that they cannot happen in the same trial. In other words, they have no elements in common. Mutually Exclusive events will be discussed in more detail further in the chapter.

Complementary Events

The complementary event of the set A is the same as not obtaining the set A.
If the sample set $S = \{1, 2, 3, 4, 5, 6\}$ and A is the set $A = \{1, 5\}$ then the complement of set A is $A' = \{2, 3, 4, 6\}$.
For two events to be complementary they have to be mutually exclusive.

The smaller the fraction, decimal or percentage the less likely it is for an event to occur. The larger the fraction, decimal or percentage the more likely it is for an event to occur.

B. CALCULATING PROBABILITY

If all the outcomes of an experiment are equally likely to occur you can calculate the probability of an event occurring using the formula below.

$$\textbf{Formula: } \mathbf{P(E)=\frac{\mathit{n}(E)}{\mathit{n}(S)}}$$

$$\begin{aligned} \text{Where: } \mathrm{P(E)} &\rightarrow \text{Probability of the event E occurring} \\ n(\mathrm{E}) &\rightarrow \text{Total number of favourable outcomes} \\ n(\mathrm{S}) &\rightarrow \text{Total number of possible outcomes} \end{aligned}$$

Example 1: **In an experiment a fair dice is thrown.**

a) Write down the sample space for the experiment.
b) Determine the probability of getting a 5.
c) Determine the probability of getting an odd number.
d) If event E is getting an even number determine the probability of event E occurring.
e) Determine the probability of getting a number greater than 6.
f) Determine the probability of getting a number less than 3.

a) $\mathrm{S}=\{1,2,3,4,5,6\} \rightarrow n(\mathrm{S})=6$

b) In this case there is only one favourable outcome: $\mathrm{E}=\{5\} \rightarrow n(\mathrm{E})=1$

$$\therefore \mathrm{P}(5)=\mathrm{P}(\mathrm{E})=\frac{n(\mathrm{E})}{n(\mathrm{S})}=\frac{1}{6}$$

c) Favourable outcomes: $\mathrm{E}=\{1,3,5\} \rightarrow n(\mathrm{E})=3$

$$\therefore \mathrm{P}(\text{an odd number})=\mathrm{P}(\mathrm{E})=\frac{n(\mathrm{E})}{n(\mathrm{S})}=\frac{3}{6}=\frac{1}{2}$$

d) Favourable outcomes: $\mathrm{E}=\{2,4,6\} \rightarrow n(\mathrm{E})=3$

$$\therefore \mathrm{P}(\text{an even number})=\mathrm{P}(\mathrm{E})=\frac{n(\mathrm{E})}{n(\mathrm{S})}=\frac{3}{6}=\frac{1}{2}$$

e) Favourable outcomes: $\mathrm{E}=\phi \rightarrow n(\mathrm{E})=0$

$$\therefore \mathrm{P}(\text{greater than } 6)=\mathrm{P}(\mathrm{E})=\frac{n(\mathrm{E})}{n(\mathrm{S})}=\frac{0}{6}=0 \rightarrow \text{this is an impossible event}$$

f) Favourable outcomes: $\mathrm{E}=\{1,2\} \rightarrow n(\mathrm{E})=2$

$$\therefore \mathrm{P}(\text{less than } 3)=\mathrm{P}(\mathrm{E})=\frac{n(\mathrm{E})}{n(\mathrm{S})}=\frac{2}{6}=\frac{1}{3}$$

In probability, a fair dice is one with six sides that has equal probability of landing on any number. Unless otherwise stated, you can always assume that a dice is fair.

Example 2: **In an experiment, 8 balls numbered from 1 to 8 are placed in a bag.**

a) Write down the sample space for the experiment.

b) If A is the event drawing a prime number, determine $P(A)$.

c) If B is the event drawing a multiple of 2, determine $P(B)$.

d) If C is the event drawing an odd number, determine $P(C)$.

e) If D is the event drawing a number greater than or equal to 6, determine $P(D)$.

a) $S=\{1,2,3,4,5,6,7,8\} \rightarrow n(S)=8$

b) Favourable outcomes: $A=\{\text{drawing a prime}\}=\{2,3,5,7\} \rightarrow n(A)=4$

$$\therefore P(A)=\frac{n(A)}{n(S)}=\frac{4}{8}=\frac{1}{2}$$

c) Favourable outcomes: $B=\{\text{Drawing a factor of 2}\}=\{2,4,6,8\} \rightarrow n(B)=4$

$$\therefore P(B)=\frac{n(B)}{n(S)}=\frac{4}{8}=\frac{1}{2}$$

d) Favourable outcomes: $C=\{\text{Drawing an odd number}\}=\{1,3,5,7\} \rightarrow n(C)=4$

$$\therefore P(C)=\frac{n(C)}{n(S)}=\frac{4}{8}=\frac{1}{2}$$

e) Favourable outcomes: $D=\{\text{drawing a number } \geq \text{ to } 6\}=\{6,7,8\} \rightarrow n(D)=3$

$$\therefore P(D)=\frac{n(D)}{n(S)}=\frac{3}{8}$$

METHODS OF DETERMINING PROBABILITY

There two methods that are used to determine probability:

1) **Theoretical Probability**
2) **Experimental Probability**

THEORETICAL PROBABILITY

This method uses logical thought to determine probability. It is logical that if a fair coin is tossed the probability of getting a head is 50% and the probability of getting a tail is 50%.

EXPERIMENTAL PROBABILITY

This method uses trials to determine probability. In order for this method to be accurate you need many trials. If you toss a coin 10 times you will not necessarily get 5 heads and 5 tails. However, if you toss a coin 1000 times you will get closer to the 50% probability of a head or tail.

In most cases it takes a large number of trials before experimental and theoretical probability approach the same answers.

Complete Exercise 1 on page 374.

C. VENN DIAGRAMS

Venn Diagrams are a graphical way of representing a sample space and its events.

Illustrative Example 1: Suppose 9 cards are placed in a bag with the numbers 1 through to 9 written on them and a single card is selected.

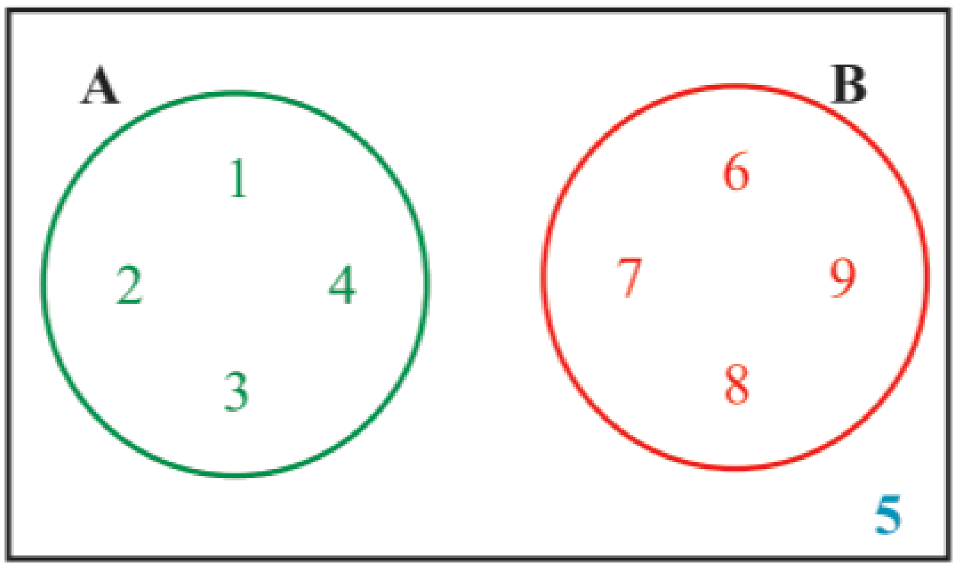
a) Write down the sample space for this experiment.
b) Write down the set of event A, selecting a number less than 5.
c) Write down the set of event B, selecting a number greater than 5.
d) Illustrate the above information on a Venn diagram.
e) Calculate the P(A).
f) Calculate the P(B).
g) Calculate the probability of getting A or B.

a) $S=\{1,2,3,4,5,6,7,8,9\}$

b) $A=\{1,2,3,4\}$

c) $B=\{6,7,8,9\}$

d)



e) $P(A)=\frac{4}{9}$

f) $P(B)=\frac{4}{9}$

g) $P(A \text{ or } B)=P(A\cup B)=\frac{4}{9}+\frac{4}{9}=\frac{8}{9}$

Explanation:
From the Venn diagram it can be seen that **set A** and **set B** have **no elements in common** and are therefore **mutually exclusive** events. In probability the symbol '$\cup$' means **or**. This is known as the **union** of two sets. The **union** of **A** and **B** is the event consisting of all the outcomes in **set A** and **set B**. In the above example the union of sets A and B is written as:
$A \text{ or } B=A\cup B=\{1,2,3,4,6,7,8,9\}$

As **5** is not in set A or set B it is only part of the sample space. Elements in the sample space but not in any subsets such as A and B are written in the rectangle as shown above.

For Mutually Exclusive events: $P(A \text{ or } B)=P(A\cup B)=P(A)+P(B)$

Illustrative Example 2: Suppose 18 discs are placed in a bag with the numbers 1 through to 18 written on them and a single disc is selected.

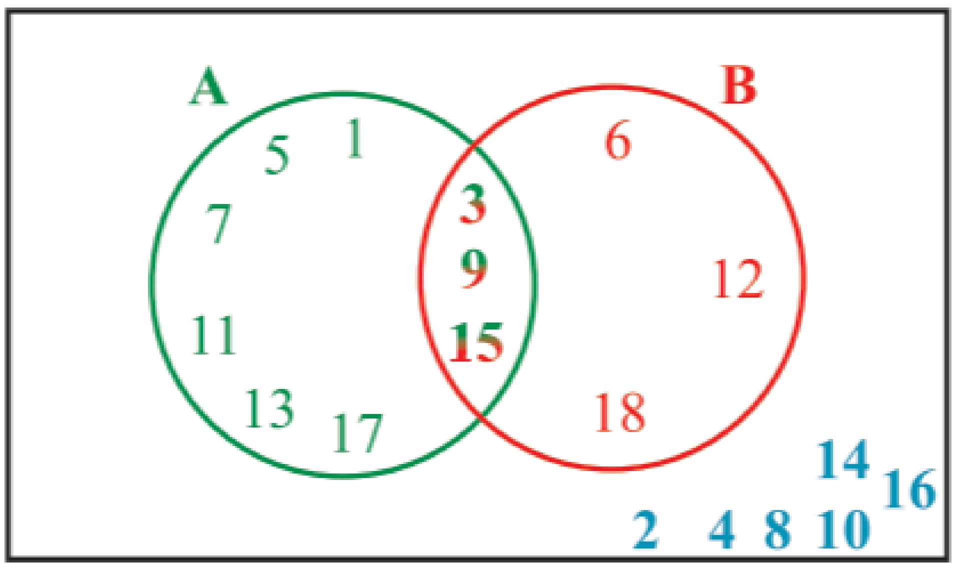
- **a) Write down the sample space for this experiment.**
- **b) Write down the set of event A, selecting an odd number.**
- **c) Write down the set of event B, selecting a multiple of 3.**
- **d) Illustrate the above information on a Venn diagram.**
- **e) Calculate the P(A).**
- **f) Calculate the P(B).**
- **g) Calculate the probability of getting A or B.**

a) $S=\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18\}$

b) $A=\{1,3,5,7,9,11,13,15,17\}$

c) $B=\{3,6,9,12,15,18\}$

d)



e) $P(A)=\frac{9}{18}=\frac{1}{2}$

f) $P(B)=\frac{6}{18}=\frac{1}{3}$

g) $P(A \text{ or } B)=P(A\cup B)=\frac{6}{18}+\frac{3}{18}+\frac{3}{18}$ → the elements common to both sets

$$=\frac{12}{18}=\frac{2}{3}$$

Explanation:
Set A and set B are **not** mutually exclusive as they have events that are common to both sets. In probability the symbol '$\cap$' means **and**. This is known as the **intersection** of two sets. The **intersection** of **A** and **B** is the event consisting of all the outcomes common to **set A** and **set B.** In the above example the intersection of sets A and B is written as: $A \text{ and } B = A\cap B=\{3,9,15\}$

When events are **not** mutually exclusive $P(A\overset{or}{\cup}B)\neq P(A)+P(B)$ as you would be including the events that are common to both sets twice.

For events which are not Mutually Exclusive:

$$P(A \text{ or } B)=P(A\cup B)=P(A)+P(B)-P(A\overset{and}{\cap}B)$$

In the above example: $P(A\cup B)=\frac{9}{18}+\frac{6}{18}-\frac{3}{18}=\frac{12}{18}$ → the intersection of A and B

Illustrative Example 3: **A bag contains 4 green balls, 3 red balls and 2 purple balls. If a single ball is drawn from the bag at random:**

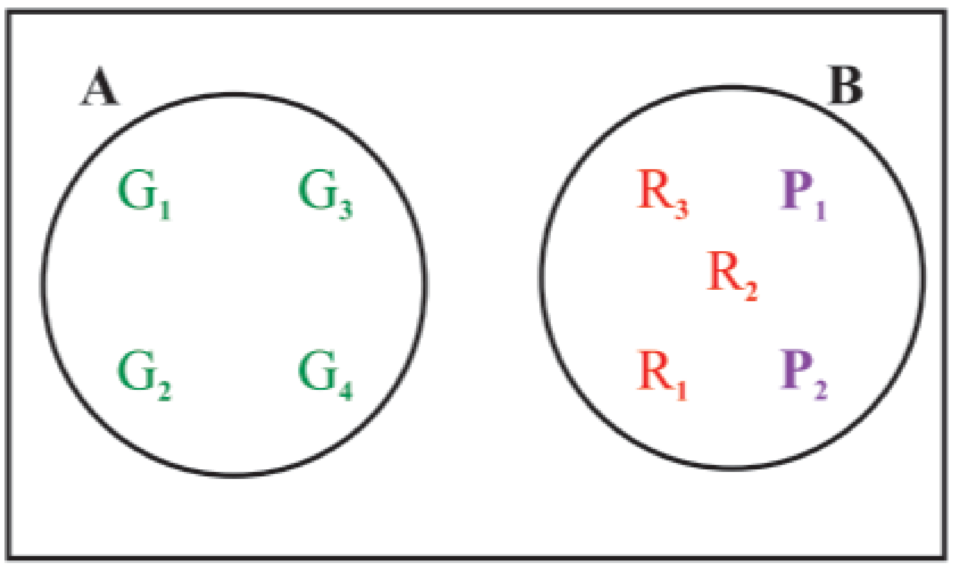
a) Write down the sample space for this experiment.
b) Write down the set of event A, selecting a green ball.
c) Write down the set of event B, not selecting a green ball.
d) Illustrate the above information on a Venn diagram.
e) Calculate the P(A).
f) Calculate the P(B).
g) Calculate the $P(A \cup B)$.

a) $S = \{G_1, G_2, G_3, G_4, R_1, R_2, R_3, P_1, P_2\}$

b) $A = \{G_1, G_2, G_3, G_4\}$

c) $B = \{R_1, R_2, R_3, P_1, P_2\}$

d)



e) $P(A) = \frac{4}{9}$

f) $P(B) = \frac{5}{9}$

g) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{4}{9} + \frac{5}{9} - 0 = 1 \rightarrow 0 \text{ as A and B are mutually exclusive events}$$

Explanation:
Set A and set B are mutually exclusive as they have no elements common to both sets. A and B are **complementary** because they are **mutually exclusive** and $\mathbf{P(A) + P(B) = 1}$.

Set B can be written as $\mathbf{B = A'}$ and P(B) can be written as $\mathbf{P(B) = P(A')}$.

For Complementary Events:

$P(A') = 1 - P(A)$

$P(A')$ reads as: the probability of not getting A

In the above example: $P(A') = 1 - P(A) = 1 - \frac{4}{9} = \frac{5}{9}$

PROBABILITY SUMMARY

Mutually Exclusive Events
$\mathbf{P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)}$ $\mathbf{P(A \text{ and } B) = P(A \cap B) = 0}$
Non-Mutually Exclusive Events
$\mathbf{P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)}$ $\mathbf{P(A \cup B) \neq P(A) + P(B)}$
Complementary Events
$\mathbf{P(\text{not } A) = P(A') = 1 - P(A)}$

MIXED EXAMPLES

Example 1: A card is drawn from a fair pack of playing cards.

a) Determine the probability of drawing a king.

b) Determine the probability of drawing a red card.

c) Determine the probability of drawing a red king.

d) Explain whether or not drawing a king and drawing a red card are mutually exclusive events.

e) Hence, determine the percentage probability of drawing a king or a red card, correct to 2 decimal places.

a) $\text{P}(\text{drawing a king}) = \frac{4}{52} = \frac{1}{13} \rightarrow \text{there are 52 cards in a fair pack}$

b) $\text{P}(\text{red card}) = \frac{26}{52} = \frac{1}{2} \rightarrow \text{there are 26 red cards in a fair pack}$

c) $\text{P}(\text{king} \cap \text{red}) = \frac{2}{52} = \frac{1}{26} \rightarrow \text{there are 2 red kings in a fair pack}$

d) It is possible to draw a king and a red card in the same trial. Therefore, the events are not mutually exclusive.

e)

$$
\begin{aligned}
\text{P}(\text{king} \cup \text{red}) &= \text{P}(\text{king}) + \text{P}(\text{red}) - \text{P}(\text{king} \cap \text{red}) \\
&= \frac{4}{52} + \frac{26}{52} - \frac{2}{52} \\
&= \frac{28}{52} \\
&= 0{,}5385 \\
&= 53{,}85\% \rightarrow \text{mulitply by 100 to convert a decimal to a percentage}
\end{aligned}
$$

Complete Exercise 2 numbers 1 and 2 on pages 374 and 375.

Example 2: **A Grade 10 learner asks 200 people whether they like milk or cream in their coffee. He found that 72 people liked cream, 110 liked milk and 40 liked milk and cream.**

a) **How many people liked only cream?**

b) **How many people liked only milk?**

c) **How many people did not like cream or milk in their coffee?**

d) **Record the above information in a Venn diagram.**

e) **What is the probability that a person chosen at random will like milk or cream?**

f) **What is the probability that a person chosen at random will not like either cream or milk?**

g) **What is the probability that a person chosen at random will not like cream?**

a) The 72 people that liked cream include the 40 that liked milk and cream:
$\therefore$ No. of people that liked only cream $= 72 - 40 = 32$

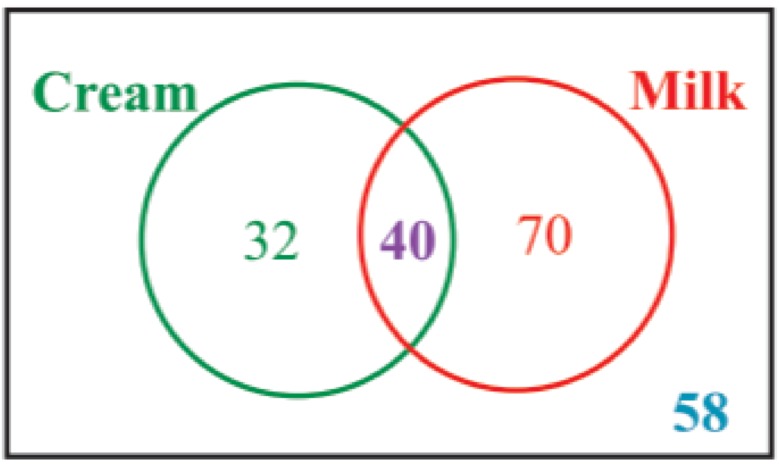
b) The 110 people that liked cream include the 40 that liked milk and cream:
$\therefore$ No. of people that liked only milk $= 110 - 40 = 70$

c) You first need to calculate the total number of people that liked cream or milk:
cream: 32 milk: 70 cream & milk: 40

Total: $32 + 70 + 40 = 142$

$\therefore$ No. of people that did not like milk or cream $= 200 - 142 = 58$

d)



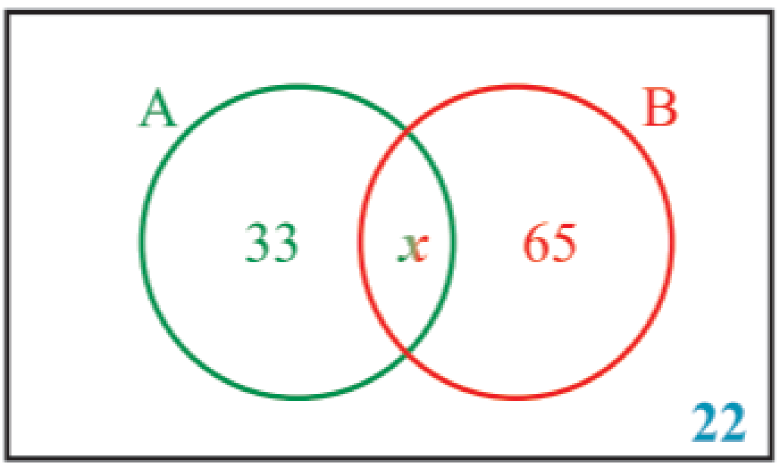
e) $\mathrm{P}(\text{milk or cream}) = \frac{32}{200} + \frac{40}{200} + \frac{70}{200} = \frac{142}{200} = 0{,}71 = 71\%$

f) $\mathrm{P}(\text{not cream or milk}) = \frac{58}{200} = 0{,}29 = 29\%$

g) $\mathrm{P}(\text{not cream}) = \frac{70}{200} + \frac{58}{200} = \frac{128}{200} = 0{,}64 = 64\%$ → people who don't like cream or milk

The formulas which you have learnt could also have been used to calculate the above probability. However, it easier to read off of the Venn diagram.

Example 3: **Samantha polls 160 people to determine if they like soft drinks A or B. Her results are shown in the Venn diagram below.**



a) How many people like both drinks A and B?

b) Determine the probability of randomly picking a person that likes brands A or B?

c) Determine the probability of randomly picking a person that likes brand A and doesn't like Brand B.

d) Determine the probability of randomly picking a person that doesn't like brand A.

a) $x = 160 - (33 + 65 + 22) \rightarrow$ don't forget the 22 in the sample space

$\therefore x = 40$

b) $$\mathrm{P}(\mathrm{A \text{ or } B}) = \frac{33}{160} + \frac{65}{160} + \frac{40}{160}$$

$$= \frac{69}{80}$$

$$= 0{,}8625$$

$$= 86{,}25\%$$

c) $$\mathrm{P}(\mathrm{A \text{ and } B'}) = \frac{33}{160}$$

$$= 0{,}2063$$

$$= 20{,}63\%$$

d) $$\mathrm{P}(\mathrm{A'}) = \frac{65}{160} + \frac{22}{160}$$

$$= \frac{87}{160}$$

$$= 0{,}5438$$

$$= 54{,}38\%$$

Example 4: **In a marketing survey 160 people were asked if they owned two different brands of denim jeans. The choice was between Guest jeans and Tringle jeans. The results of the survey are given below:**

- **85 people owned Guest**
- **62 people owned Tringle**
- **136 people owned at least one of the brands**
- ***x* people owned both brands**

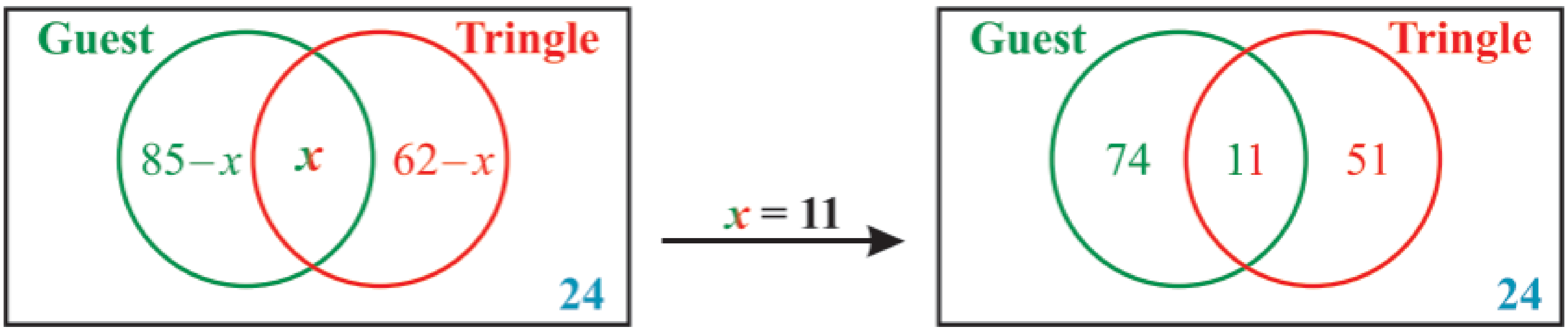
a) Record the above information in a Venn diagram.

b) How many people own Guest but not Tringle?

c) What is the probability that a person selected at random owns only one of the brands?

d) What is the probability that a person selected at random owns one of the brands?

a)



Method:

Step 1: Determine the number of people that wear none of the brands. There are 160 people in the sample space and 136 people who wear at least one of the brands.
$\therefore$ the number of people who wear non of the brands $= 160 - 136 = 24$

Step 2: It is given that **85** people own Guest. However this includes the ***x*** number of people that own both Guest and Tringle.
$\therefore$ Guest only $= 85 - x$

Step 3: It is given that **62** people own Tringle. However this includes the ***x*** number of people that own both Guest and Tringle.
$\therefore$ Tringle only $= 62 - x$

Step 4: You can now determine the value of x. There were 160 people in the survey.
$\therefore 85 - x + 62 - x + x + 24 = 160$
$\therefore x = 11$

b) 74

c)
$$\begin{aligned} P(\text{only Guest or Only Tringle}) &= \frac{74}{160} + \frac{51}{160} \\ &= \frac{25}{32} \\ &= 0{,}7813 \\ &= 78{,}13\% \end{aligned}$$

d)
$$\begin{aligned} P(\text{Guest or Tringle}) &= \frac{74}{160} + \frac{51}{160} + \frac{11}{160} \\ &= \frac{17}{20} \\ &= 0{,}85 \\ &= 85\% \end{aligned}$$

Complete Exercise 2 number 3 on page 375 and the Mixed Exercise on pages 375 and 376.

CHAPTER 12

MAKING THE MOST OF MATHS

WHAT YOU WILL LEARN IN THIS CHAPTER

PAGE	CONTENT	CAPS	ASSESSMENTS YOU WILL BE ABLE TO:
223 223 223 224	A. Maths in the Classroom B. Exam Study Techniques C. The Morning of the Exam D. Exam Writing Techniques	2	• Get the most out of mathematics in your classroom • Study with confidence and use your time effectively. • Relax and walk into your exam with confidence. • Write your exam neatly and use you time efficiently.

A. MATHS IN THE CLASSROOM

- Up to **80% of learning** should be achieved in the **classroom**. Try to sit as close as possible to the front of the classroom where you can see and hear properly.

- **Concentrate** and avoid the temptation to talk to your friends. Missing an explanation could result in a misunderstanding of an entire section.

- **Do not be shy!** If you don't understand an explanation that your teacher has given, put your hand up and say so. **Ask** your teacher to explain it again until you do understand. If you did not understand a concept, it is likely that the majority of your class is in the same position.

B. EXAM STUDY TECHNIQUES

- **Practising past exam papers** is the most helpful and practical form of revision. Past papers expose you to the required level of work, exam format and the necessary time limitations imposed on you.

- **Make sure you start studying early.** You should start with past papers at least two weeks before the exam. Make sure that as you do each question you check your answer. If you check your answers after you have completed the whole paper, you cannot remember what each question was about. This means that you do not learn from your mistakes.

- **In order to achieve the correct pace**, while timing yourself, do an entire past paper and only check your answers at the end of the paper. This should be done two to three days before your exam.

- Ensure the room that you are studying in is **brightly lit** and **properly ventilated**.

- **Cellular phones** can be your worst enemy whilst studying. **Turn your phone off** and check your messages only when you have finished studying.

C. THE MORNING OF THE EXAM

- Make sure that **everything you need for your exam is packed the night before** and ready to take in the morning. Ensure your **calculator is in the correct mode** with charged batteries. It is a good idea to take a spare calculator into your exam.

- **Sleep is very important**. Avoid going to bed too late or waking up very early to study. A maths test or exam requires a fresh brain.

- **Eat Breakfast!** Hunger affects concentration.

- **Avoid discussing work** before the exam. This is likely to cause confusion.

- You've worked hard and are **prepared** - now relax and enjoy the exam.

D. EXAM WRITING TECHNIQUES

- **Read through the entire question paper before you start**. You can answer the questions in any order you like. Answer the questions that you are most confident in first. However, you must answer an entire question at once. Don't answer parts of a question in different places in your answer book. If you are unsure of a certain part of the question, leave it out and come back to it.

- **If you leave out part of a question, make sure you leave enough space to answer it**. Always leave at least half a page open if the question is worth more than two marks. If you are not sure how much space will be needed, be safe and leave a full page.

- **Mark questions** that you have **left out** on the question paper, so that you remember to come back to them.

- **Neatness is crucial**. Keeping your solutions **neat** and **legible** during an exam may result in up to a 5% increase in your results compared to an untidy and illegible answer book. Neat workings also assist you, as it is easier to check answers and find mistakes. In addition, neatness helps to avoid silly mistakes.

- **Do not** attempt to **save space** in your exams! Cramped work is messy and difficult to mark. Leave at least one line between answers. Similarly, do not write complicated fractions on a single line.

- **Never cross out an answer, unless** you have already written another answer to replace it. Once you have what you feel is the correct answer, only then cross out the incorrect workings.

- **Do not use** any type of **correction fluid** (eg. tippex) in a test or exam.
  Rather **cross out your answer neatly** and carry on.

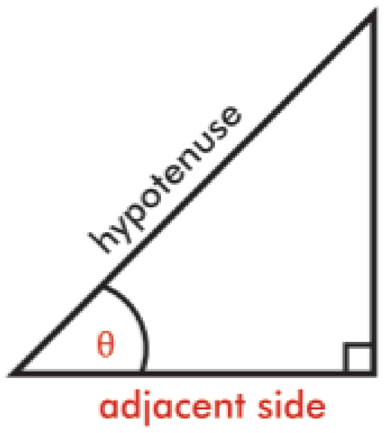
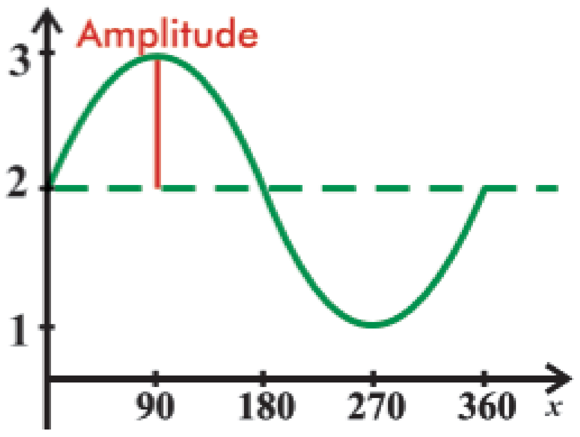
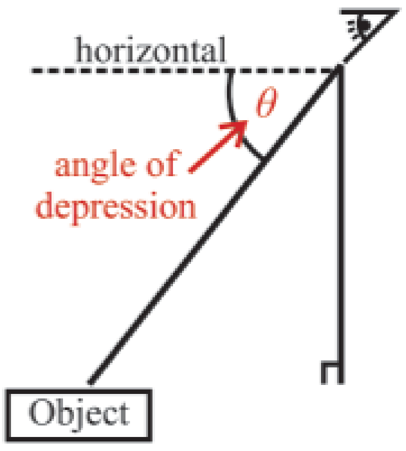
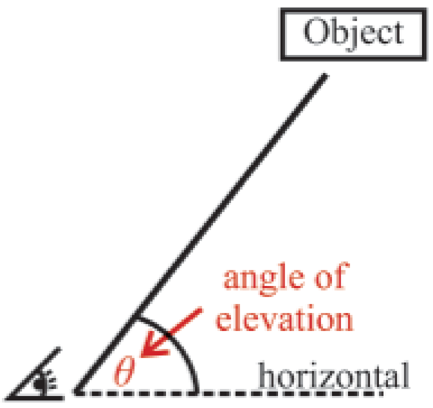
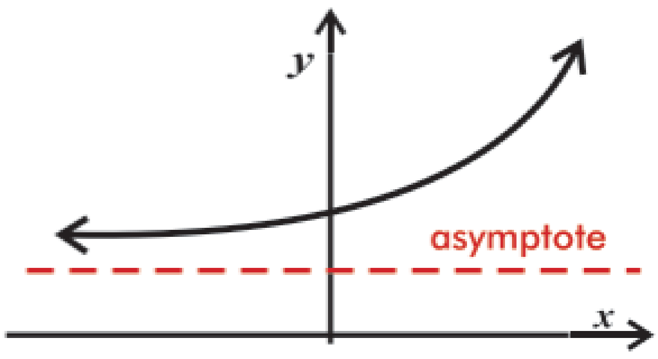
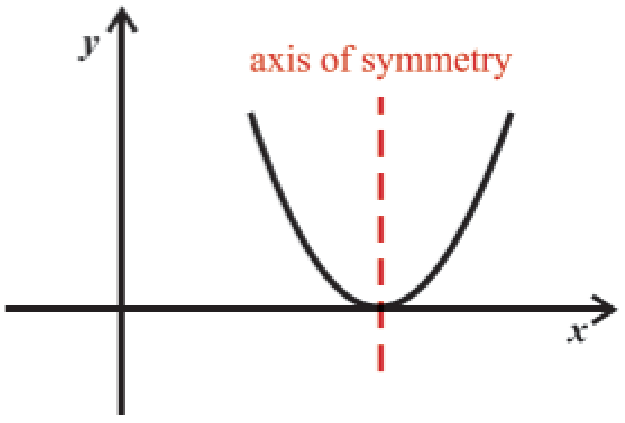
- **Follow the layout** shown in this handbook. Keep to the top to bottom layout and the exam will be a pleasure to write and mark.

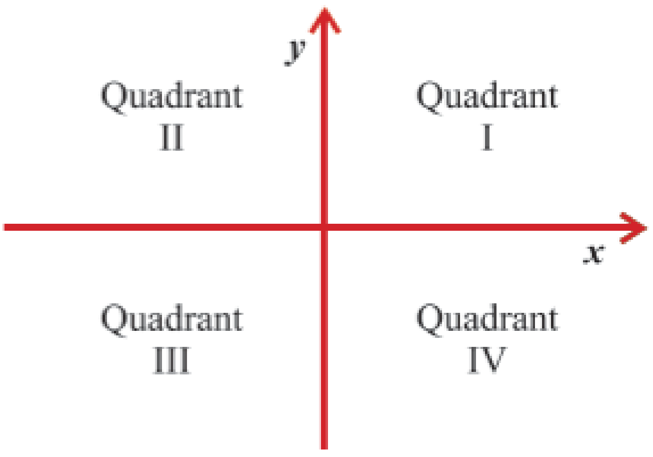
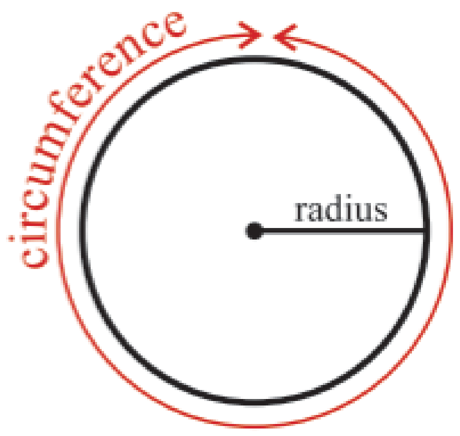
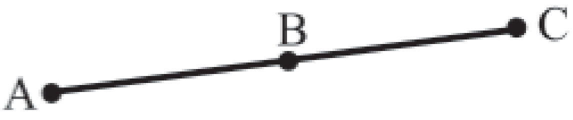
- An answer book that has the final answer of each question **underlined with an arrow** is very pleasant for a teacher to mark. However, this can be very costly in terms of time. **If you are a slow or untidy worker, use a ruler** and neatly underline your answers with arrows, **only once you are finished** your test or exam and have **checked** through **your solutions**. Make sure that you don't cross out your answer when doing this.

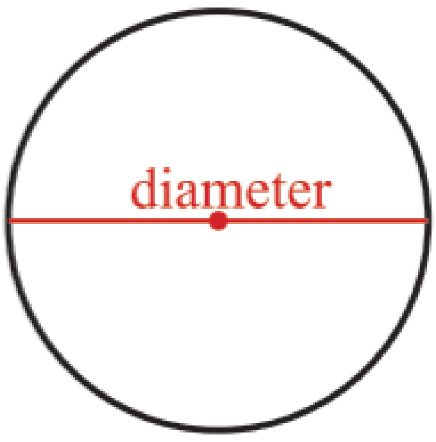
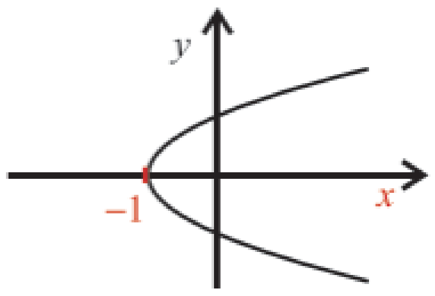
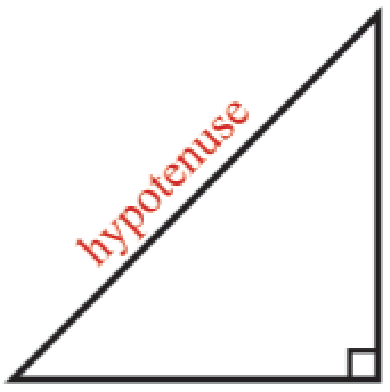
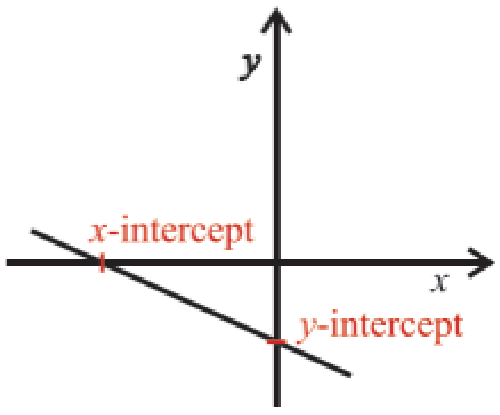
- Finally, **relax** and be **positive**. It is important that you **don't panic** at any stage during the exam.

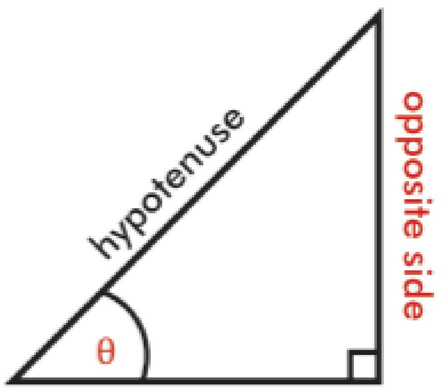
Good luck – you have earned it!

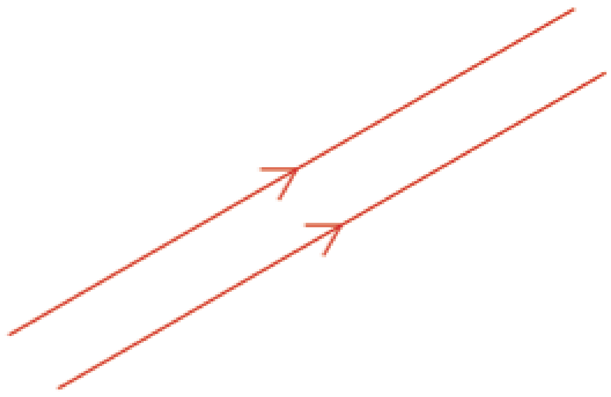
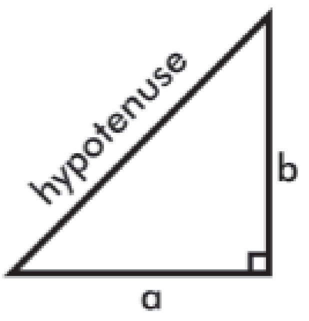
GLOSSARY OF IMPORTANT TERMS AND SYMBOLS

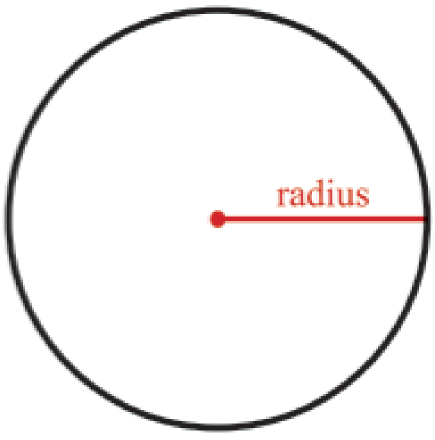
Term	Explanation	Example (where possible)
adjacent side	In a right angled triangle the adjacent side is the side next to the angle. The Adjacent side cannot be the hypotenuse.	 hypotenuse θ adjacent side
amplitude	The amplitude is the length of maximum deviation from the zero line. $\text{Amplitude} = \frac{y_{\max} - y_{\min}}{2}$	
angle of depression	The angle between the horizontal and the line of sight when looking down at an object.	 horizontal θ angle of depression Object
angle of elevation	The angle between the horizontal and the line of sight when looking up at an object.	 Object angle of elevation θ horizontal
asymptote	An imaginary line that a graph approaches but can never touch or cross.	
axis of symmetry	A line drawn so that the graphs on either side of the line are mirror images of each other.	

Term	Explanation	Example (where possible)
bimodal data set	A data set which has two modes.	In the data set 1, 3, 3, 7, 8, 8 The modes are 3 and 8.
binomial	An expression which only has two terms.	The expression $x+1$ Is a binomial.
Cartesian Plane	Four quadrants which are separated by an x and y-axis.	 y Quadrant II Quadrant I x Quadrant III Quadrant IV
circumference	The physical distance around a circle. Formula: $C = 2\pi r \text{ or } C = \pi d$	
coefficient	The number in front of a variable.	In the expression $2x^2+4$ the coefficient of x^2 is 2.
collinear	**Collinear points are points which lie on the same straight line**	 Points A, B and C lie on the same straight line
constant term	The term in an expression that does not contain any variables.	In the expression $2x^2+bx+4$ the constant term is 4.
denominator	The number at the bottom of a fraction.	In the fraction $\frac{3}{x}$, x is the denominator.

Term	Explanation	Example (where possible)
diameter	A line through the centre of the circle that touches the circumference of the circle at two points.	
domain	The domain of a function is the possible x-values that a function can have.	 In the above graph the **domain** is $x \geq -1$
exponent (and base)	The power that a number or variable is raised to.	**In the term x^a, a is the exponent and x is the base.**
exponential equation	An Equation in which the variable is in the exponent.	**$2^x = 4$ is an example of an exponential equation.**
gradient (m)	The rate of change of y with respect to x. $m = \frac{y_2 - y_1}{x_2 - x_1}$	**The average gradient between the points (5;2) and (10;5) is:** $m = \frac{5-2}{10-5} = \frac{3}{5}$
hypotenuse	The side opposite the 90° angle in a right angled triangle. It is the longest side of a right angled triangle.	
intercept (x or y)	The point where a line cuts the x-axis (x-intercept) and the y-axis (y-intercept).	

Term	Explanation	Example (where possible)
linear number pattern	A number pattern where the difference between sequential terms is constant.	In the sequence: 2, 4, 6... The constant difference d is $T_n - T_{n-1} = 2$.
mean ($\bar{x}$)	The average of a data set.	The mean of the set 2, 3, 4, 7 is: $\frac{2+3+4+7}{4} = 4$
median	The middle value in an ordered data set.	The median of the set 2, 4, 6, 8, 10 is 6.
mode	The value with the highest frequency in a data set.	The mode of the data set 2, 3, 9, 3, 3 is 3.
monomial	An expression which only has one term.	The expression x is a monomial.
numerator	The number or expression at the top of a fraction.	In the fraction: $\frac{x+5}{2}$ $x+5$ is the numerator and 2 is the denominator.
opposite side	In a right angled triangle the opposite side is the side directly opposite the angle.	

Term	Explanation	Example (where possible)
outlier (of a data set)	A value in a data set the is much bigger or smaller than any of the other values in the data set.	**2, 100, 125, 126, 130** **In the above set of data points, 2 is an outlier.**
Parallel lines (//)	Lines which have the same gradient. Lines that are parallel (//) never touch.	
percentile	Percentiles divide a data set into hundredths.	**If x is in the n^{th} percentile it means the n% of the values in the data set are** less than x.
period	The number of degrees it takes for a trigonometric function to complete one cycle.	**Graphs of sine and cosine have a period of 360° whilst tan has a period of 180°.**
polygon	A closed chape with 3 or more sides.	**Refer to page 163 for examples of polygons.**
polynomial	Any expression with one or more terms. The largest exponent (with a variable base) will be the degree of the polynomial.	$x^3 + x^2 + x + 2$ **is a 3rd degree polynomial.**
Pythagoras (Theorem of Pythagoras)	In a 90° triangle the (hypotenuse)^2 = the sum of the squares of the other 2 sides.	 $h^2 = a^2 + b^2$

Term	Explanation	Example (where possible)
quadratic equation	An equation where there is a variable that is squared and there are no variables which have exponents greater than 2.	**The general form of a quadratic equation is:** $ax^2 + bx + c = 0$
quadrilateral	A closed four sided figure whose interior angles add up to 360°	Refer to page 159 for examples of quadrilaterals.
radius	A line drawn from the centre of the circle to the circumference of the circle. The radius is half the length of the diameter.	

GLOSSARY OF IMPORTANT SYMBOLS

Symbol	Meaning	Symbol	Meaning
$\sqrt{}$	square root	$\in$	element (of a set)
$\sqrt[n]{}$	nth root	$\neq$	not equal to
$\mathbb{N}$	natural numbers	$\approx$	approximately equal to
$\mathbb{N}_0$	whole numbers	$>$	greater than
$\mathbb{Z}$	integers	$<$	less than
$\mathbb{Q}$	rational numbers	$\geq$	greater than or equal to
$\mathbb{Q}'$	irrational numbers	$\leq$	less than or equal to
$\mathbb{R}$	real numbers	$\therefore$	therefore
$\equiv$	congruent	///	similar

MATHS HANDBOOK and STUDY GUIDE™

EXERCISES & SOLUTIONS

The exercises contained in this section are designed for reinforcement of the material covered in each chapter of the book. At the end of the exercises for each relevant chapter, learners will find a mixed exercise. The mixed exercises are designed to test all of the work in the relevant chapters and are of the same standard that learners would expect to find in a test or exam. Learners are strongly encouraged to make sure that they fully understand all questions within the mixed exercises.

The exercises in this section contain worked through solutions. These are merely suggested solutions and are not necessarily the only methods that can be used to arrive at a correct answer. If there is any doubt in the method which learners have used, they are encouraged to check their methodology with their teachers.

Worked through solutions are provided in order for learners to check their answers and understand errors which they have made. Very little will be gained from viewing solutions before an attempt has been made by the learner to solve the problems themselves. Learners should only look at solutions after a true attempt of the problem has been made.

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Chapter 1 – Number Relationships and Algebraic Expressions

The Number System

Exercise 1: (Theory on pages 3 and 4)

1) State whether the following numbers are rational or irrational:

a) 5

b) 2

c) $\sqrt{16}$

d) $2\frac{3}{4}$

e) $-0,32$

f) π

g) $\sqrt{9}$

h) $\sqrt{5}$

i) $\sqrt{0}$

j) $0,\dot{8}\dot{8}$

k) $0,2587964\ldots$

l) $0,35$

Solutions to Exercise 1 number 1 are on page **235**

2) Classify the following number by placing a tick in the correct column:

Number	$\mathbb{N}$	$\mathbb{N}_0$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{Q}'$
$-5\frac{1}{4}$					
$0,142$					
$\sqrt{16}$					
$\sqrt{7}$					
-2					
$\frac{22}{7}$					
π					
$0,5$					
$8,315\ldots$					
$0,\dot{9}88\dot{7}$					

Solutions to Exercise 1 number 2 are on page **235**

Determining Between Which Two Integers an Irrational Number Lies

Exercise 2: Without using a calculator, determine between which two integers the following irrational numbers lie: (Theory on page 5)

a) $\sqrt{28}$

b) $\sqrt{48}$

c) $\sqrt[3]{49}$

d) $\sqrt[3]{15}$

e) $\sqrt{90}$

f) $\sqrt{115}$

Solutions to Exercise 2 are on page **235**

Rounding Off

Exercise 3: Round off the following numbers to the specified number of decimal places: (Theory on page 6)

a) $5,235865845$ (2 decimal places)

b) $0,256862554$ (4 decimal places)

c) $28,56988712$ (3 decimal places)

d) $\sqrt{5}$ (2 decimal places)

e) $(3,35)^2 \div 2,6$ (4 decimal places)

f) $\pi + \sqrt{14}$ (3 decimal places)

Solutions to Exercise 3 are on page **235**

Rewriting Rational Numbers

Exercise 4: **(Theory on pages 6-7)**

1) Without using a calculator write the following decimals as a fraction in their simplest form:

a) 0,78 **b)** 0,23 **c)** 0,34

Solutions to Exercise 4 number 1 are on page **236**

2) Write the following recurring decimals as fractions:

a) $0,\dot{3}$ **c)** $0,\dot{2}37\dot{5}$ **e)** $0,5\dot{8}$

b) $0,\dot{4}\dot{5}$ **d)** $0,2\dot{5}$ **f)** $1+0,\dot{8}$

Solutions to Exercise 4 number 2 are on page **236**

Mixed Exercise: **(Chapter 1 is on pages 2-8)**

1) Calculate the following without the use of a calculator (grade 8 and 9 revision):

a) $2\frac{1}{2}-5\frac{1}{8}+3\frac{2}{3}$ **d)** $\frac{2}{3}$ of $15\div\left(\frac{7}{8}+2\frac{2}{3}\right)\times1\frac{5}{12}$

b) $2\frac{3}{4}\div\frac{5}{12}\times3\frac{2}{11}$ **e)** $\sqrt{64\times36}$

c) $\sqrt{64+36}$ **f)** $\sqrt[3]{64}-\sqrt{64}$

Solutions to the Mixed Exercise numbers 1a to 1d are on page **236** and number 1 e to 1f are on page **237**

2) A watermelon is cut in half. Sam's half is then cut into 8 slices and Sipho's half is cut into 12 slices. If Sam ate 5 of her slices and Sipho ate 8 of his slices, who ate more watermelon?

Solutions to the Mixed Exercise number 2 are on page **237**

3) Indicate whether each of the following numbers are rational or irrational by placing a tick in the correct block:

Number	Q	Q'
0,9		
$\frac{22}{7}$		
$-2\sqrt{3}$		
$7\sqrt{16}$		
$\sqrt[3]{16}$		

Solutions to the Mixed Exercise number 3 are on page **237**

4) Without using a calculator, determine between which two integers the following irrational numbers lie.

a) $\sqrt{4}\sqrt{21}$ **b)** $\sqrt{62}$ **c)** $\sqrt[3]{60}$

Solutions to the Mixed Exercise number 4 are on page **237**

5) Convert the following to common fractions:

a) $\frac{3}{11}+0,\dot{6}\dot{3}$ **b)** $\frac{3}{18}+0,\dot{7}$

Solutions to the Mixed Exercise numbers 5 are on page **237**

Chapter 1 – Number Relationships and Algebraic Expressions (Solutions)

Exercise 1 Solutions:

1)

a)	Rational	**e)**	Rational	**i)**	Rational
b)	Rational	**f)**	Irrational	**j)**	Rational
c)	Rational	**g)**	Rational	**k)**	Irrational
d)	Rational	**h)**	Irrational	**l)**	Rational

Exercise 1 number 1 is on page **233**

2)

Number	$\mathbb{N}$	$\mathbb{N}_0$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{Q}'$
$-5\frac{1}{4}$				✓	
$0,142$				✓	
$\sqrt{16}$	✓	✓	✓	✓	
$\sqrt{7}$					✓
-2			✓	✓	
$\frac{22}{7}$				✓	
π					✓
$0,5$				✓	
$8,315\ldots$					✓
$0,\dot{9}88\dot{7}$				✓	

Exercise 1 number 2 is on page **233**

Exercise 2 Solutions:

a) $\sqrt{25} < \sqrt{28} < \sqrt{36}$

$\therefore \sqrt{28}$ lies between 5 and 6

b) $\sqrt{36} < \sqrt{48} < \sqrt{49}$

$\therefore \sqrt{48}$ lies between 6 and 7

c) $\sqrt[3]{27} < \sqrt[3]{49} < \sqrt[3]{64}$

$\therefore \sqrt[3]{49}$ lies between 3 and 4

d) $\sqrt[3]{8} < \sqrt[3]{15} < \sqrt[3]{27}$

$\therefore \sqrt[3]{15}$ lies between 2 and 3

e) $\sqrt{81} < \sqrt{90} < \sqrt{100}$

$\therefore \sqrt{90}$ lies between 9 and 10

f) $\sqrt{100} < \sqrt{115} < \sqrt{121}$

$\therefore \sqrt{115}$ lies between 10 and 11

Exercise 2 is on page **233**

Exercise 3 Solutions:

a) 5,24
b) 0,2569
c) 28,570
d) 2,24
e) 4,3163
f) 6,883

Exercise 3 is on page **233**

Exercise 4 Solutions:

1)

a) $0,78=\frac{78}{100}=\frac{39}{50}$

b) $0,23=\frac{23}{100}$

c) $0,34=\frac{34}{100}=\frac{17}{50}$

Exercise 4 number 1 is on page **234**

2)

a) $\text{let } x=0,\dot{3}\rightarrow\boxed{1}$

$\therefore 10x=3,\dot{3}\rightarrow\boxed{2}$

Equation $\boxed{2}$ – Equation $\boxed{1}$

$\therefore 9x=3$

$\therefore x=\frac{3}{9}=\frac{1}{3}$

b) $\text{let } x=0,\dot{4}\dot{5}\rightarrow\boxed{1}$

$\therefore 100x=45,\dot{4}\dot{5}\rightarrow\boxed{2}$

Equation $\boxed{2}$ – Equation $\boxed{1}$

$\therefore 99x=45$

$\therefore x=\frac{45}{99}$

$=\frac{15}{33}$

$=\frac{5}{11}$

c) $\text{let } x=0,\dot{2}37\dot{5}\rightarrow\boxed{1}$

$\therefore 10000x=2375,\dot{2}37\dot{5}\rightarrow\boxed{2}$

Equation $\boxed{2}$ – Equation $\boxed{1}$

$\therefore 9999x=2375$

$\therefore x=\frac{2375}{9999}$

d) $\text{let } x=0,2\dot{5}\rightarrow\boxed{1}$

$\therefore 10x=2,\dot{5}\rightarrow\boxed{2}$

$\therefore 100x=25,\dot{5}\rightarrow\boxed{3}$

Equation $\boxed{3}$ – Equation $\boxed{2}$

$\therefore 90x=23$

$\therefore x=\frac{23}{90}$

e) $\text{let } x=0,5\dot{8}\rightarrow\boxed{1}$

$\therefore 10x=5,\dot{8}\rightarrow\boxed{2}$

$\therefore 100x=58,\dot{8}\rightarrow\boxed{3}$

Equation $\boxed{3}$ – Equation $\boxed{2}$

$\therefore 90x=53$

$\therefore x=\frac{53}{90}$

f) $1+0,\dot{8}\rightarrow\boxed{1}$

$\text{let } x=0,\dot{8}\rightarrow\boxed{1}$

$\therefore 10x=8,\dot{8}\rightarrow\boxed{2}$

Equation $\boxed{2}$ – Equation $\boxed{1}$

$\therefore 9x=8$

$\therefore x=\frac{8}{9}$

$\therefore 1+0,\dot{8}=1+\frac{8}{9}$

$=\frac{9+8}{9}$

$=\frac{17}{9}$

Exercise 4 number 2 is on page **234**

Mixed Exercise Solutions:

1)

a) $2\frac{1}{2}-5\frac{1}{8}+3\frac{2}{3}$

$=\frac{5}{2}-\frac{41}{8}+\frac{11}{3}\rightarrow \text{LCD}=24$

$=\frac{5(12)-41(3)+11(8)}{24}$

$=\frac{25}{24}$

b) $\left(2\frac{3}{4}\div\frac{5}{12}\right)\times 3\frac{2}{11}$

$=\left(\frac{11}{4}\times\frac{12}{5}\right)\times\frac{35}{11}$

$=\frac{84}{4}$

$=21$

c) $\sqrt{64+36}$

$=\sqrt{100}$

$=10$

d) $\frac{2}{3}\text{ of }15\div\left(\frac{7}{8}+2\frac{2}{3}\right)\times 1\frac{5}{12}$

$=\left(\frac{2}{3}\times\frac{15}{1}\right)\div\left(\frac{7}{8}+\frac{8}{3}\right)\times\frac{17}{12}$

$=\left(\frac{10}{1}\div\frac{7(3)+8(8)}{24}\right)\times\frac{17}{12}\rightarrow\text{LCD}$

$=\frac{10}{1}\div\frac{85}{24}\times\frac{17}{12}$

$=\frac{10}{1}\times\frac{24}{85}\times\frac{17}{12}$

$=\frac{2}{1}\times\frac{2}{17}\times\frac{17}{1}$

$=4$

The Mixed Exercise number 1 is on page **234** (solutions are continued on the next page)

e) $\sqrt{64\times 36}$

$=\sqrt{64}\times\sqrt{36}$

$=8\times 6$

$=48$

f) $\sqrt[3]{64}-\sqrt{64}$

$=4-8$

$=-4$

The Mixed Exercise number 1 is on page **234**

2) Sam ate :$\frac{5}{8}$ Sipho ate:$\frac{8}{12}$

In order to compare the fractions they have to have the same denominator.

Sam:$\frac{5}{8}=\frac{15}{24}$ Sipho:$\frac{8}{12}=\frac{16}{24}$

$\therefore$ Sipho ate more watermelon.

The Mixed Exercise number 2 is on page **234**

3)

Number	Q	Q'
0,9	✓	
$\frac{22}{7}$	✓	
$-2\sqrt{3}$		✓
$7\sqrt{16}$	✓	
$\sqrt[3]{16}$		✓

The Mixed Exercise number 3 is on page **234**

4)

a) $\sqrt{4}\times\sqrt{21}=\sqrt{4\times 21}=\sqrt{84}$

$\sqrt{81}<\sqrt{84}<\sqrt{100}$

$\therefore\sqrt{4}\sqrt{21}$ lies between 9 and 10

b) $\sqrt{49}<\sqrt{62}<\sqrt{64}$

$\therefore\sqrt{62}$ lies between 7 and 8

c) $\sqrt[3]{27}<\sqrt[3]{60}<\sqrt[3]{64}$

$\therefore\sqrt[3]{60}$ lies between 3 and 4

The Mixed Exercise number 4 is on page **234**

5)

a) $\frac{3}{11}+0,\dot{6}\dot{3}$

let $x=0,\dot{6}\dot{3}\rightarrow\boxed{1}$

$\therefore 100x=63,\dot{6}\dot{3}\rightarrow\boxed{2}$

Equation $\boxed{2}$ – Equation $\boxed{1}$

$\therefore 99x=63$

$\therefore x=\frac{63}{99}=\frac{7}{11}$

$\therefore\frac{3}{11}+0,\dot{6}\dot{3}$

$=\frac{3}{11}+\frac{7}{11}$

$=\frac{10}{11}$

b) $\frac{3}{18}+0,\dot{7}$

let $x=0,\dot{7}\rightarrow\boxed{1}$

$\therefore 10x=7,\dot{7}\rightarrow\boxed{2}$

Equation $\boxed{2}$ – Equation $\boxed{1}$

$\therefore 9x=7$

$\therefore x=\frac{7}{9}$

$\therefore\frac{3}{18}+0,\dot{7}$

$=\frac{3}{18}+\frac{7}{9}$

$=\frac{3}{18}+\frac{14}{18}$

$=\frac{17}{18}$

The Mixed Exercise number 5 is on page **234**

Chapter 2 – Manipulation of Algebraic Expressions

Grade 9 Revision

Exercise 1:

1) Expand and simplify the following: (Theory on page 10)

a) $3\left(x^2+3x^3-2\right)$

b) $-2\left(p^2-3pq+8a\right)-2(qp-2a)$

c) $5x\left(x^2+y^2-3xy\right)-3y\left(xy+5x^2\right)$

d) $4xy\left(xy+2y^2-3y\right)-2y^2\left(2x^2+4xy+2x\right)$

Solutions to Exercise 1 number 1 are on page **243**

2) Expand and simplify the following: (Theory on page 11)

a) $(x+3)(x-2)$

b) $(x+3)(x-5)$

c) $(y-3)(y-7)$

d) $(2x+3)(3x-2)$

e) $(2x+y)(x-3y)$

f) $(2p+q)(p-q)$

g) $(a+b)(a+b)$

h) $(3a+2b)(5a-3b)$

i) $(2b+3c)(3b-7c)$

Solutions to Exercise 1 number 2 are on page **243**

3) Expand and simplify the following: (Theory on page 12)

a) $(x-y)^2$

b) $(3x+2y)^2$

c) $(7x+2y)^2$

d) $\left(x^3+y^2\right)^2$

e) $\left(a-\frac{b}{2}\right)^2$

f) $\left(\frac{x}{3}-3\right)^2$

g) $\left(x^a+b^y\right)^2$

h) $\left(\frac{x^2}{3}-\frac{3y^3}{2}\right)^2$

Solutions to Exercise 1 number 3 are on page **243**

4) Expand and simplify the following as far as possible: (Theory on page 13)

a) $(a-b)(a+b)$

b) $(xy-3)(xy+3)$

c) $(2a-3b)(2a+3b)$

d) $\left(x^a-y^b\right)\left(x^a+y^b\right)$

e) $\left(a^4-2b^4\right)\left(a^4+2b^4\right)$

f) $(1+2x)(1-2x)$

g) $(2x-y+3)(2x-y-3)$

h) $(3x-y-3p)(3x-y+3p)$

i) $(pqr-3s+b)(pqr-3s-b)$

Solutions to Exercise 1 number 4 are on page **244**

5) Factorise the following: (Theory on page 14)

a) $2x^3+8x^2+16x^2y$

b) $3x^2y-12xy^2+6xy$

c) $2x(x-1)+4x^2(x-1)-12x^3(x-1)$

d) $8(2x-3)-y(3-2x)$

e) $3(2a-3b)(2a+3b)-2x\left(4a^2-9b^2\right)$

f) $3(x-5)+x(10-2x)$

Solutions to Exercise 1 number 5 are on page **244**

6) Factorise the following: (Theory on page 15)

a) x^2-4

b) x^4-16

c) $2a^2-8b^2$

d) $36p^2r-16q^2r$

e) $\left(\frac{2}{x^2}-\frac{8}{y^4}\right)$

f) $\left(\frac{x^2}{9}-\frac{y^2}{16}\right)$

g) $(x-3)^2-36$

h) $2(x+2)^2-8$

i) $16x^2+4y^2$

Solutions to Exercise 1 number 6 are on page **244**

Multiplying a Binomial by a Trinomial

Exercise 2: Expand and simplify the following: (Theory on page 16)

a) $(a+2)\left(2a^2-3a+5\right)$

b) $(3x-2y)\left(2x^2-3xy+5y^2\right)$

c) $\left(x^2-2y\right)(2x+2y+2)$

d) $(a-b)\left(a^2+ab+b^2\right)$

e) $(2x+3y)\left(4x^2-6xy+9y^2\right)$

f) $(p-2q)\left(p^2+2pq+4q^2\right)$

g) $(3x+2y)^3$

h) $\left(4x^2+14x+49\right)(2x-7)$

i) $\left(\frac{x}{2}-\frac{y}{3}\right)\left(\frac{x^2}{4}+\frac{xy}{6}+\frac{y^2}{9}\right)$

j) $\left(\frac{x}{2}+3\right)^3$

k) $\left(p-\frac{2}{q}\right)\left(p^2+\frac{2p}{q}+\frac{4}{q^2}\right)$

Solutions to Exercise 2 are on page **245**

Factorisation of Trinomials

Exercise 3: Factorise the following as far as possible:

1) (Theory on pages 17-18)

a) x^2-5x+6

b) x^2+6x+8

c) $x^2+7x+12$

d) $x^2+8x+15$

e) x^2+5x+4

f) x^2-3x+2

g) $x^2-12x+35$

h) $x^2-9x+18$

i) $2x^2-6x+4$

j) $3x^2-18x+24$

k) $5x^2+15x+10$

l) $3x^3+18x^2+24x$

m) $2x^4-12x^3+10x^2$

n) x^3-10x^2+21x

o) $5x^3y+25x^2y+30xy$

Solutions to Exercise 3 number 1 are on page **245**

2) (Theory on pages 19-20)

a) x^2-x-6

b) x^2+4x-5

c) $x^2-3x-28$

d) $x^2+5x-24$

e) x^2-x-12

f) x^2-x-72

g) $x^2+4x-21$

h) $x^2+9x-36$

i) $2x^3+4x^2-6x$

j) $5x^3-5x^2-60x$

k) $2x^3y+2x^2y-24xy$

l) $x^4+x^3-2x^2$

m) $3x^2-6x-240$

n) $2x^3+4x^2-30x$

o) $x^4y-11x^3y-12x^2y$

Solutions to Exercise 3 number 2 are on page **246**

3) (Theory on page 21)

a) $2x^2+x-3$

b) $2x^2-x-3$

c) $3x^2+7x+2$

d) $10x^2+3x-1$

e) $3x^2+5x-2$

f) $8x^2+2x-3$

g) $3x^2-19x+20$

h) $5x^2-8x-36$

i) $24x^2-2xy-y^2$

j) $5a^2+13ab-28b^2$

k) $5p^2+28pq+15q^2$

l) $8x^2-2xy-y^2$

m) $22x^2+13xy+y^2$

n) $21x^2+81xy+54y^2$

o) $40x^2+6xy-18y^2$

Solutions to Exercise 3 number 3 are on page **246**

4) (Theory on page 22)

a) x^2-y^2+x+y

b) x^2-y^2-x-y

c) $10x^2+5xy+6xy+3y^2$

d) $5x^2y-20y+2x^3-8x$

e) $27a^2x-12b^2x-18a^2y+8b^2y$

f) $18x^2+3x-8y^2+2y$

g) $16x^2-9y^2+(4x-3y)(x+2y)$

h) $3x^2-2y+9x^4-4y^2$

i) $5p^2-2q-5q^2+2p$

j) $x^3-9xy^2+xy-3y^2$

Solutions to Exercise 3 numbers 4a to 4d are on page **246** and numbers 4e to 4j are on page **247**

5) (Theory on page 24)

a) $x^3 - y^3$

b) $x^3 + y^3$

c) $8a^3 - 64b^3$

d) $54x^5y - 250x^2y$

e) $2ax^3 + 3bx^3 - 2ay^3 - 3by^3$

f) $\dfrac{p^6}{27} - \dfrac{q^3}{8}$

Solutions to Exercise 3 number 5 are on page **247**

Multiplication and Division of Fractions (Theory on pages 25-26)

Exercise 4: Simplify the following (you may assume that denominators are non-zero):

a) $\dfrac{5}{3a} \times \dfrac{9a^2}{15}$

b) $\dfrac{5x}{3} \div \dfrac{x}{6}$

c) $\dfrac{5x^2}{2y} \div \dfrac{4x^2}{y}$

d) $\dfrac{4x^2}{6y^2} \div \left(\dfrac{16x^3}{6y^3} \times \dfrac{2y}{32x} \right)$

e) $\dfrac{x+5}{2y} \times \dfrac{4y^2}{1} \div \dfrac{2y}{x-5}$

f) $\dfrac{9}{3y^2} \times \dfrac{x-y}{x} \div \dfrac{9(x-y)}{y^2}$

g) $\dfrac{3(a^2+a-2)}{a^2+5a+6} \div \dfrac{a^2-1}{a^2+4a+3}$

h) $\dfrac{2(p+q)}{p^2+pq+q^2} \div \dfrac{p^2-q^2}{p^3-q^3}$

Solutions to Exercise 4 *a* to *c* are on page **247** and *d* to *h* are on page **248**

Addition and Subtraction of Fractions (Theory on pages 27-29)

Exercise 5: Simplify the following (you may assume that denominators are non-zero):

a) $2 - \dfrac{1}{2x^2} + \dfrac{3}{x}$

b) $\dfrac{5}{6} - \dfrac{4q}{p} + \dfrac{4p}{q}$

c) $\dfrac{8a^2+4a}{4a}$

d) $\dfrac{5x+2}{5} - \dfrac{3x+10}{3}$

e) $\dfrac{2}{3} + \dfrac{3x+2}{x} - \dfrac{x+4}{2x}$

f) $\dfrac{2x+3}{x} + \dfrac{7}{4x^2} - \dfrac{4x+1}{3}$

Solutions to Exercise 5 are on page **248**

Addition, Subtraction, Multiplication and Division of Fractions (Theory on pages 30-33)

Exercise 6: Simplify the following (you can assume that denominators are non-zero):

a) $\dfrac{5}{4x+12} \div \dfrac{3}{2x+6}$

b) $\dfrac{4x^3}{5} \times \dfrac{1}{12x^4 - 4x^3}$

c) $\dfrac{p^2+pq}{q^2} \div \dfrac{p}{q}$

d) $\dfrac{x^2-2xy}{x^2y} \div \dfrac{x^2y-4y^3}{3xy^2}$

e) $\dfrac{6a^2}{3a^2-24a+48} \times \dfrac{a^2-4a}{a^3}$

f) $\dfrac{x^2-36}{x^2-7x} \div \dfrac{x^2+4x-12}{x^2-9x+14}$

g) $\dfrac{m^2-49}{m^2-5m-14} \times \dfrac{2m^2-8}{m^2-2m}$

h) $\dfrac{2x^2-3x-20}{x^2-8x+16} \times \dfrac{7x-x^2-12}{2x^2-x-15}$

i) $\dfrac{2}{3(x-5)} - \dfrac{1}{x(5-x)}$

j) $\dfrac{a+2}{2(a-6)} + \dfrac{a+6}{3(6-a)}$

k) $\dfrac{p-5}{5(p-2)} - \dfrac{p+1}{p(2-p)}$

l) $\dfrac{1}{x-y} - \dfrac{1}{x}$

m) $\dfrac{x+y}{xy} - \dfrac{2}{x+y}$

n) $\dfrac{p}{p-1} - \dfrac{1}{4p-4}$

o) $\dfrac{7}{3x^2-6x} + \dfrac{2}{x}$

p) $\dfrac{y^2+1}{2y+8} - \dfrac{2(y-1)}{3y+12}$

Solutions to Exercise 6 *a* to *d* are on page **248** and *e* to *p* are on page **249**

Mixed Exercise: (Chapter 2 is on pages 9-33)

1) Simplify the following:

a) $(2x-3)^2$

b) $(2x-y)^2-(x-2y)(x+2y)$

c) $(3x-5)\left(2x^2+x-4\right)$

d) $(8m-3n)(4m+n)-(n-3m)(n+3m)$

e) $(4xy-3)^2+24xy$

f) $2xy\left(3x^2-7xy+10y^2\right)$

g) $3(x-1)^2-2(x+3)(2x-1)$

h) $(-x+2)\left(x^2-4x+1\right)$

i) $(x+5)\left(x^2-3x-9\right)$

j) $\left(\frac{4}{x}-\frac{3}{y}\right)\left(\frac{4}{x}+\frac{3}{y}\right)$

k) $-x^2\left(\frac{1}{x}+2x\right)\left(\frac{2}{x}-3x\right)$

l) $\left(\frac{x}{4}-3\right)\left(\frac{2x}{3}+5\right)$

m) $2(2x+3)(2x-3)+(2x-1)\left(x^2-3x+4\right)$

n) $\frac{3p^2}{q}(4q-2pq)-\frac{p}{5}\left(5p^2+10p\right)\left(5p^2-10p\right)$

o) $(2x+3)(x-7)(2x-3)(x+7)$

p) $(2a-3b)\left(8a^3+27b^3\right)\left(4a^2+6ab+9b^2\right)$

Solutions to the Mixed Exercise Number 1 are on page **250**

2) Factorise the following completely:

a) xy^2-x

b) $72-8a^2$

c) $6a^2b^3-2ab^2$

d) x^3+16x

e) $\frac{a^2b}{4}-\frac{9b^3}{25}$

f) $2xy+x-10y-5$

g) $x^3-3x^2-4x+12$

h) $3x^2-x-10$

i) $3x^2-9x-xy+3y$

j) $x^2-3x-10$

k) p^3-3p^2-p+3

l) $8x^2+18xy-5y^2$

m) $8p^2+14p-15$

n) $pr-(r+p)x+x^2$

o) $4x^2-2(5x+3)$

p) $xy-xz-y+z$

q) $8p^3-27r^3$

r) $-4x^4y^4-7x^2y^2+2$

s) $36(a-2)^2-9$

t) $3x^3-5x^2+12xy^2-20y^2$

u) $18p^2-24pq+8q^2-32$

v) $x^2\left(y^2-1\right)+5x\left(1-y^2\right)-6y^2+6$

Solutions to the Mixed Exercise Number 2a and 2b are on page **250** and 2c to 2v are on page **251**

3)

a) $1+\frac{2}{3x}-\frac{3}{2x}$

b) $\frac{5x^2+3x}{x}$

c) $\left(1-\frac{a}{b}\right)\div\frac{a-b}{b}$

d) $\frac{a-3}{6-2a}$

e) $\frac{4p^2-8pq}{8pq}$

f) $\frac{a-3}{3}-\frac{2(a+1)}{3}-\frac{3a-5}{12}$

g) $\frac{4a^2-1}{3}\times\frac{1}{2a-1}$

h) $\frac{a+1}{5}-\frac{a+3}{2}$

i) $\frac{8p^3}{3q^2}\times\frac{9q^3}{p^4}\div\frac{16p^2}{q}$

j) $\frac{2p^2-8}{27}\times\frac{9p-27}{p^2-p-6}$

k) $\frac{2+x}{6-3x}\div\frac{x^2+x-2}{2x-4}$

l) $\frac{x^2-3x-4}{6x^2-6}\div\frac{2x^2-9x+4}{6-3x}\times\frac{x-1}{2}$

m) $\frac{x^2+5xy+6y^2}{x^2-36}\div\frac{3x+9y}{x^2-3x-18}\times\frac{x+6}{x+2y}$

n) $\frac{3a^2}{a^2-a-2}+\frac{2}{2-a}-1\frac{2}{3}$

Solutions to the Mixed Exercise Number 3a to 3*l* are on page **252** and 3m and 3n are on page **253**

4) What must be added to $(x+y)^2$ so that it will be equal to x^2+y^2?

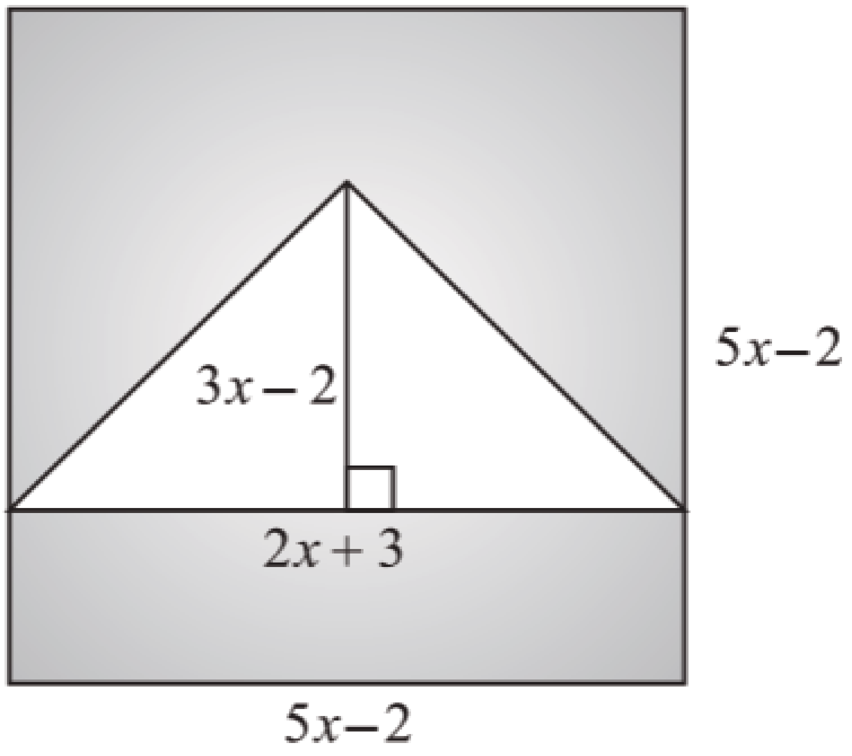
Solutions to the Mixed Exercise Number 4 are on page **253**

5) What must be added to $(x+4)(x-4)$ so that it will be equal to $(x-1)(x+16)$?

Solutions to the Mixed Exercise Number 5 are on page **253**

6) Given the following formulas, determine the area of the shaded region below:

Area of a square $=(length)^2$ Area of a triangle $=\frac{1}{2}\times base\times perpendicular\ height$



Solutions to the Mixed Exercise Number 6 are on page **253**

7) A question in a test read: factorise the following expression $(2x-1)^2-(2x-1)(x-1)$

David answered the question as shown below. However, he was awarded no marks. Explain in words why he received no marks and write down the correct solution to the question.

David's answer:

$$(2x-1)^2-(2x-1)(x-1)$$
$$=4x^2-4x+1-\left(2x^2-3x+1\right)$$
$$=4x^2-4x+1-2x^2+3x-1$$
$$=2x^2-x$$

Solutions to the Mixed Exercise Number 7 are on pages **253**

Chapter 2 – Manipulation of Algebraic Expressions (Solutions)

Exercise 1 Solutions:

1)

a) $3\left(x^2+3x^3-2\right)$

$=3x^2+9x^3-6$

b) $-2\left(p^2-3pq+8a\right)-2\left(qp-2a\right)$

$=-2p^2+6pq-16a-2qp+4a$

$=-2p^2+4pq-12a$

c) $5x\left(x^2+y^2-3xy\right)-3y\left(xy+5x^2\right)$

$=5x^3+5xy^2-15x^2y-3xy^2-15x^2y$

$=5x^3+2xy^2-30x^2y$

d) $4xy\left(xy+2y^2-3y\right)-2y^2\left(2x^2+4xy+2x\right)$

$=4x^2y^2+8xy^3-12xy^2-4x^2y^2-8xy^3-4xy^2$

$=-16xy^2$

Exercise 1 number 1 is on page 238

2)

a) $(x+3)(x-2)$

$=x^2-2x+3x-6$

$=x^2+x-6$

b) $(x+3)(x-5)$

$=x^2-5x+3x-15$

$=x^2-2x-15$

c) $(y-3)(y-7)$

$=y^2-7y-3y+21$

$=y^2-10y+21$

d) $(2x+3)(3x-2)$

$=6x^2-4x+9x-6$

$=6x^2+5x-6$

e) $(2x+y)(x-3y)$

$=2x^2-6xy+xy-3y^2$

$=2x^2-5xy-3y^2$

f) $(2p+q)(p-q)$

$=2p^2-2pq+pq-q^2$

$=2p^2-pq-q^2$

g) $(a+b)(a+b)$

$=a^2+ab+ab+b^2$

$=a^2+2ab+b^2$

h) $(3a+2b)(5a-3b)$

$=15a^2-9ab+10ab-6b^2$

$=15a^2+ab-6b^2$

i) $(2b+3c)(3b-7c)$

$=6b^2-14bc+9bc-21c^2$

$=6b^2-5bc-21c^2$

Exercise 1 number 2 is on page 238

3)

a) $(x-y)^2$

$=x^2-2xy+y^2$

b) $(3x+2y)^2$

$=9x^2+12xy+4y^2$

c) $(7x+2y)^2$

$=49x^2+28xy+4y^2$

d) $\left(x^3+y^2\right)^2$

$=x^6+2x^3y^2+y^4$

e) $\left(a-\frac{b}{2}\right)^2$

$=a^2-ab+\frac{b^2}{4}$

f) $\left(\frac{x}{3}-3\right)^2$

$=\frac{x^2}{9}-2x+9$

g) $\left(x^a+b^y\right)^2$

$=x^{2a}+2x^ab^y+b^{2y}$

h) $\left(\frac{x^2}{3}-\frac{3y^3}{2}\right)^2$

$=\frac{x^4}{9}-x^2y^3+\frac{9y^6}{4}$

Exercise 1 number 3 is on page 238

4)

a) $(a-b)(a+b)$

$= a^2 - b^2$

b) $(xy-3)(xy+3)$

$= x^2y^2 - 9$

c) $(2a-3b)(2a+3b)$

$= 4a^2 - 9b^2$

d) $\left(x^a - y^b\right)\left(x^a + y^b\right)$

$= x^{2a} - y^{2b}$

e) $\left(a^4 - 2b^4\right)\left(a^4 + 2b^4\right)$

$= a^8 - 4b^8$

f) $(1+2x)(1-2x)$

$= 1 - 4x^2$

g) $(2x-y+3)(2x-y-3)$

$= \left[(2x-y)^2 - 9\right]$

$= 4x^2 - 4xy + y^2 - 9$

h) $(3x-y-3p)(3x-y+3p)$

$= \left[(3x-y)^2 - 9p^2\right]$

$= 9x^2 - 6xy + y^2 - 9p^2$

i) $(pqr-3s+b)(pqr-3s-b)$

$= \left[(pqr-3s)^2 - b^2\right]$

$= p^2q^2r^2 - 6pqrs + 9s^2 - b^2$

Exercise 1 number 4 is on page **238**

5)

a) $2x^3 + 8x^2 + 16x^2y$

$= 2x^2(x + 4 + 8y)$

b) $3x^2y - 12xy^2 + 6xy$

$= 3xy(x - 4y + 2)$

c) $2x(x-1) + 4x^2(x-1) - 12x^3(x-1)$

$= 2x(x-1)\left(1 + 2x - 6x^2\right)$

d) $8(2x-3) - y(3-2x)$

$= 8(2x-3) + y(2x-3)$

$= (2x-3)(8+y)$

e) $3(2a-3b)(2a+3b) - 2x\left(4a^2 - 9b^2\right)$

$= 3\left(4a^2 - 9b^2\right) - 2x\left(4a^2 - 9b^2\right)$

$= \left(4a^2 - 9b^2\right)(3-2x)$

$= (2a-3b)(2a+3b)(3-2x)$

f) $3(x-5) + x(10-2x)$

$= 3(x-5) - 2x(x-5)$

$= (x-5)(3-2x)$

Exercise 1 number 5 is on page **238**

6)

a) $x^2 - 4$

$= (x-2)(x+2)$

b) $x^4 - 16$

$= \left(x^2 - 4\right)\left(x^2 + 4\right)$

$= (x-2)(x+2)\left(x^2 + 4\right)$

c) $2a^2 - 8b^2$

$= 2\left(a^2 - 4b^2\right)$

$= 2(a-2b)(a+2b)$

d) $36p^2r - 16q^2r$

$= 4r\left(9p^2 - 4q^2\right)$

$= 4r(3p-2q)(3p+2q)$

e) $\left(\dfrac{2}{x^2} - \dfrac{8}{y^4}\right)$

$= 2\left(\dfrac{1}{x^2} - \dfrac{4}{y^4}\right)$

$= 2\left(\dfrac{1}{x} - \dfrac{2}{y^2}\right)\left(\dfrac{1}{x} + \dfrac{2}{y^2}\right)$

f) $\left(\dfrac{x^2}{9} - \dfrac{y^2}{16}\right)$

$= \left(\dfrac{x}{3} - \dfrac{y}{4}\right)\left(\dfrac{x}{3} + \dfrac{y}{4}\right)$

g) $(x-3)^2 - 36$

$= [(x-3)-6][(x-3)+6]$

$= (x-9)(x+3)$

h) $2(x+2)^2 - 8$

$= 2\left[(x+2)^2 - 4\right]$

$= 2[(x+2)-2][(x+2)+2]$

$= 2x(x+4)$

i) $16x^2 + 4y^2$

$= 4\left(4x^2 + y^2\right)$

Exercise 1 number 6 is on page **238**

Exercise 2 Solutions:

a) $(a+2)\left(2a^2-3a+5\right)$
$=2a^3-3a^2+5a+4a^2-6a+10$
$=2a^3+a^2-a+10$

b) $(3x-2y)\left(2x^2-3xy+5y^2\right)$
$=6x^3-9x^2y+15xy^2-4x^2y+6xy^2-10y^3$
$=6x^3-13x^2y+21xy^2-10y^3$

c) $\left(x^2-2y\right)(2x+2y+2)$
$=2x^3+2x^2y+2x^2-4xy-4y^2-4y$

d) $(a-b)\left(a^2+ab+b^2\right)$
$=a^3+a^2b+ab^2-a^2b-ab^2-b^3$
$=a^3-b^3$

e) $(2x+3y)\left(4x^2-6xy+9y^2\right)$
$=8x^3-12x^2y+18xy^2+12x^2y-18xy^2+27y^3$
$=8x^3+27y^3$

f) $(p-2q)\left(p^2+2pq+4q^2\right)$
$=p^3+2p^2q+4pq^2-2p^2q-4pq^2-8q^3$
$=p^3-8q^3$

g) $(3x+2y)^3$
$=(3x+2y)\left(9x^2+12xy+4y^2\right)$
$=27x^3+36x^2y+12xy^2+18x^2y+24xy^2+8y^3$
$=27x^3+54x^2y+36xy^2+8y^3$

h) $\left(4x^2+14x+49\right)(2x-7)$
$=8x^3+28x^2+98x-28x^2-98x-343$
$=8x^3-343$

i) $\left(\frac{x}{2}-\frac{y}{3}\right)\left(\frac{x^2}{4}+\frac{xy}{6}+\frac{y^2}{9}\right)$
$=\frac{x^3}{8}+\frac{x^2y}{12}+\frac{xy^2}{18}-\frac{x^2y}{12}-\frac{xy^2}{18}-\frac{y^3}{27}$
$=\frac{x^3}{8}-\frac{y^3}{27}$

j) $\left(\frac{x}{2}+3\right)^3$
$=\left(\frac{x}{2}+3\right)\left(\frac{x^2}{4}+3x+9\right)$
$=\frac{x^3}{8}+\frac{3x^2}{2}+\frac{9x}{2}+\frac{3x^2}{4}+9x+27$
$=\frac{x^3}{8}+\frac{9x^2}{4}+\frac{27x}{2}+27$

k) $\left(p-\frac{2}{q}\right)\left(p^2+\frac{2p}{q}+\frac{4}{q^2}\right)$
$=p^3+\frac{2p^2}{q}+\frac{4p}{q^2}-\frac{2p^2}{q}-\frac{4p}{q^2}-\frac{8}{q^3}$
$=p^3-\frac{8}{q^3}$

Exercise 2 is on page 239

Exercise 3 Solutions:

1)

a) x^2-5x+6
$=(x-2)(x-3)$

b) x^2+6x+8
$=(x+2)(x+4)$

c) $x^2+7x+12$
$=(x+3)(x+4)$

d) $x^2+8x+15$
$=(x+3)(x+5)$

e) x^2+5x+4
$=(x+4)(x+1)$

f) x^2-3x+2
$=(x-2)(x-1)$

g) $x^2-12x+35$
$=(x-5)(x-7)$

h) $x^2-9x+18$
$=(x-3)(x-6)$

i) $2x^2-6x+4$
$=2\left(x^2-3x+2\right)$
$=2(x-2)(x-1)$

j) $3x^2-18x+24$
$=3\left(x^2-6x+8\right)$
$=3(x-4)(x-2)$

k) $5x^2+15x+10$
$=5\left(x^2+3x+2\right)$
$=5(x+1)(x+2)$

l) $3x^3+18x^2+24x$
$=3x\left(x^2+6x+8\right)$
$=3x(x+2)(x+4)$

m) $2x^4-12x^3+10x^2$
$=2x^2\left(x^2-6x+5\right)$
$=2x^2(x-1)(x-5)$

n) x^3-10x^2+21x
$=x\left(x^2-10x+21\right)$
$=x(x-7)(x-3)$

o) $5x^3y+25x^2y+30xy$
$=5xy\left(x^2+5x+6\right)$
$=5xy(x+3)(x+2)$

Exercise 3 number 1 is on page 239

2)

a) $x^2 - x - 6$
$= (x+2)(x-3)$

b) $x^2 + 4x - 5$
$= (x-1)(x+5)$

c) $x^2 - 3x - 28$
$= (x-7)(x+4)$

d) $x^2 + 5x - 24$
$= (x-3)(x+8)$

e) $x^2 - x - 12$
$= (x+3)(x-4)$

f) $x^2 - x - 72$
$= (x+8)(x-9)$

g) $x^2 + 4x - 21$
$= (x+7)(x-3)$

h) $x^2 + 9x - 36$
$= (x-3)(x+12)$

i) $2x^3 + 4x^2 - 6x$
$= 2x\left(x^2 + 2x - 3\right)$
$= 2x(x+3)(x-1)$

j) $5x^3 - 5x^2 - 60x$
$= 5x\left(x^2 - x - 12\right)$
$= 5x(x-4)(x+3)$

k) $2x^3y + 2x^2y - 24xy$
$= 2xy\left(x^2 + x - 12\right)$
$= 2xy(x+4)(x-3)$

l) $x^4 + x^3 - 2x^2$
$= x^2\left(x^2 + x - 2\right)$
$= x^2(x+2)(x-1)$

m) $3x^2 - 6x - 240$
$= 3\left(x^2 - 2x - 80\right)$
$= 3(x+8)(x-10)$

n) $2x^3 + 4x^2 - 30x$
$= 2x\left(x^2 + 2x - 15\right)$
$= 2x(x-3)(x+5)$

o) $x^4y - 11x^3y - 12x^2y$
$= x^2y\left(x^2 - 11x - 12\right)$
$= x^2y(x+1)(x-12)$

Exercise 3 number 2 is on page **239**

3)

a) $2x^2 + x - 3$
$= (2x+3)(x-1)$

b) $2x^2 - x - 3$
$= (2x-3)(x+1)$

c) $3x^2 + 7x + 2$
$= (3x+1)(x+2)$

d) $10x^2 + 3x - 1$
$= (5x-1)(2x+1)$

e) $3x^2 + 5x - 2$
$= (3x-1)(x+2)$

f) $8x^2 + 2x - 3$
$= (4x+3)(2x-1)$

g) $3x^2 - 19x + 20$
$= (3x-4)(x-5)$

h) $5x^2 - 8x - 36$
$= (5x-18)(x+2)$

i) $24x^2 - 2xy - y^2$
$= (4x-y)(6x+y)$

j) $5a^2 + 13ab - 28b^2$
$= (5a-7b)(a+4b)$

k) $5p^2 + 28pq + 15q^2$
$= (5p+3q)(p+5q)$

l) $8x^2 - 2xy - y^2$
$= (2x-y)(4x+y)$

m) $22x^2 + 13xy + y^2$
$= (11x+y)(2x+y)$

n) $21x^2 + 81xy + 54y^2$
$= 3\left(7x^2 + 27xy + 18y^2\right)$
$= 3(7x+6y)(x+3y)$

o) $40x^2 + 6xy - 18y^2$
$= 2\left(20x^2 + 3xy - 9y^2\right)$
$= 2(5x-3y)(4x+3y)$

Exercise 3 number 3 is on page **239**

4)

a) $x^2 - y^2 + x + y$
$= (x-y)(x+y) + (x+y)$
$= (x+y)(x-y+1)$

b) $x^2 - y^2 - x - y$
$= (x-y)(x+y) - (x+y)$
$= (x+y)(x-y-1)$

c) $10x^2 + 5xy + 6xy + 3y^2$
$= 5x(2x+y) + 3y(2x+y)$
$= (2x+y)(5x+3y)$

d) $5x^2y - 20y + 2x^3 - 8x$
$= 5y\left(x^2 - 4\right) + 2x\left(x^2 - 4\right)$
$= \left(x^2 - 4\right)(5y+2x)$
$= (x-2)(x+2)(5y+2x)$

Exercise 3 number 4 is on page **239** (solutions are continued on the next page)

e) $27a^2x-12b^2x-18a^2y+8b^2y$
$=3x\left(9a^2-4b^2\right)-2y\left(9a^2-4b^2\right)$
$=\left(9a^2-4b^2\right)(3x-2y)$
$=(3a-2b)(3a+2b)(3x-2y)$

f) $18x^2+3x-8y^2+2y$
$=2\left(9x^2-4y^2\right)+3x+2y$
$=2(3x-2y)(3x+2y)+(3x+2y)$
$=(3x+2y)\left[2(3x-2y)+1\right]$
$=(3x+2y)(6x-4y+1)$

g) $16x^2-9y^2+(4x-3y)(x+2y)$
$=(4x-3y)(4x+3y)+(4x-3y)(x+2y)$
$=(4x-3y)(4x+3y+x+2y)$
$=(4x-3y)(5x+5y)$
$=5(4x-3y)(x+y)$

h) $3x^2-2y+9x^4-4y^2$
$=\left(3x^2-2y\right)+\left(3x^2-2y\right)\left(3x^2+2y\right)$
$=\left(3x^2-2y\right)\left(1+3x^2+2y\right)$

i) $5p^2-2q-5q^2+2p$
$=5\left(p^2-q^2\right)+2(p-q)$
$=5(p-q)(p+q)+2(p-q)$
$=(p-q)\left[5(p+q)+2\right]$
$=(p-q)(5p+5q+2)$

j) $x^3-9xy^2+xy-3y^2$
$=x\left(x^2-9y^2\right)+y(x-3y)$
$=x(x-3y)(x+3y)+y(x-3y)$
$=(x-3y)\left[x(x+3y)+y\right]$
$=(x-3y)\left(x^2+3xy+y\right)$

Exercise 3 number 4 is on page **239**

5)

a) x^3-y^3
$(x-y)\left(x^2+xy+y^2\right)$

b) x^3+y^3
$=(x+y)\left(x^2-xy+y^2\right)$

c) $8a^3-64b^3$
$=8\left(a^3-8b^3\right)$
$=8(a-2b)\left(a^2+2ab+4b^2\right)$

d) $54x^5y-250x^2y$
$=2x^2y\left(27x^3-125\right)$
$=2x^2y(3x-5)\left(9x^2+15x+25\right)$

e) $2ax^3+3bx^3-2ay^3-3by^3$
$=x^3(2a+3b)-y^3(2a+3b)$
$=(2a+3b)\left(x^3-y^3\right)$
$=(2a+3b)(x-y)\left(x^2+xy+y^2\right)$

f) $\dfrac{p^6}{27}-\dfrac{q^3}{8}$
$=\left(\dfrac{p^2}{3}-\dfrac{q}{2}\right)\left(\dfrac{p^4}{9}+\dfrac{p^2q}{6}+\dfrac{q^2}{4}\right)$

Exercise 3 number 5 is on page **240**

Exercise 4 Solutions:

a) $\dfrac{5}{3a}\times\dfrac{9a^2}{15}$
$=\dfrac{1}{1}\times\dfrac{3a}{3}$
$=a$

b) $\dfrac{5x}{3}\div\dfrac{x}{6}$
$=\dfrac{5x}{3}\times\dfrac{6}{x}$
$=10$

c) $\dfrac{5x^2}{2y}\div\dfrac{4x^2}{y}$
$=\dfrac{5x^2}{2y}\times\dfrac{y}{4x^2}$
$=\dfrac{5}{8}$

Exercise 4 is on page **240** (solutions are continued on the next page)

d)
$$\frac{4x^2}{6y^2}\div\left(\frac{16x^3}{6y^3}\times\frac{2y}{32x}\right)$$
$$=\frac{4x^2}{6y^2}\div\left(\frac{x^2}{6y^2}\right)$$
$$=\frac{4x^2}{6y^2}\times\frac{6y^2}{x^2}=4$$

e)
$$\frac{x+5}{2y}\times\frac{4y^2}{1}\div\frac{2y}{x-5}$$
$$=\frac{x+5}{2y}\times\frac{4y^2}{1}\times\frac{x-5}{2y}$$
$$=(x+5)(x-5)$$
$$=x^2-25$$

f)
$$\frac{9}{3y^2}\times\frac{x-y}{x}\div\frac{9(x-y)}{y^2}$$
$$=\frac{9}{3y^2}\times\frac{x-y}{x}\times\frac{y^2}{9(x-y)}$$
$$=\frac{1}{3x}$$

g)
$$\frac{3\left(a^2+a-2\right)}{a^2+5a+6}\div\frac{a^2-1}{a^2+4a+3}$$
$$=\frac{3(a-1)(a+2)}{(a+3)(a+2)}\times\frac{(a+1)(a+3)}{(a-1)(a+1)}$$
$$=3$$

h)
$$\frac{2(p+q)}{p^2+pq+q^2}\div\frac{p^2-q^2}{p^3-q^3}$$
$$=\frac{2(p+q)}{p^2+pq+q^2}\times\frac{(p-q)\left(p^2+pq+q^2\right)}{(p-q)(p+q)}$$
$$=2$$

Exercise 4 is on page **240**

Exercise 5 Solutions:

a)
$$2-\frac{1}{2x^2}+\frac{3}{x}$$
$$=\frac{2\left(2x^2\right)-1+3(2x)}{2x^2}$$
$$=\frac{4x^2-1+6x}{2x^2}$$

b)
$$\frac{5}{6}-\frac{4q}{p}+\frac{4p}{q}$$
$$=\frac{5(pq)-4q(6q)+4p(6p)}{6pq}$$
$$=\frac{5pq-24q^2+24p^2}{6pq}$$

c)
$$\frac{8a^2+4a}{4a}$$
$$=\frac{4a(2a+1)}{4a}$$
$$=2a+1$$

d)
$$\frac{5x+2}{5}-\frac{3x+10}{3}$$
$$=\frac{3(5x+2)-5(3x+10)}{15}$$
$$=\frac{15x+6-15x-50}{15}$$
$$=-\frac{44}{15}$$

e)
$$\frac{2}{3}+\frac{3x+2}{x}-\frac{x+4}{2x}$$
$$=\frac{2(2x)+6(3x+2)-3(x+4)}{6x}$$
$$=\frac{4x+18x+12-3x-12}{6x}$$
$$=\frac{19x}{6x}=\frac{19}{6}$$

f)
$$\frac{2x+3}{x}+\frac{7}{4x^2}-\frac{4x+1}{3}$$
$$=\frac{12x(2x+3)+3(7)-4x^2(4x+1)}{12x^2}$$
$$=\frac{24x^2+36x+21-16x^3-4x^2}{12x^2}$$
$$=\frac{-16x^3+20x^2+36x+21}{12x^2}$$

Exercise 5 is on page **240**

Exercise 6 Solutions:

a)
$$\frac{5}{4x+12}\div\frac{3}{2x+6}$$
$$=\frac{5}{4(x+3)}\times\frac{2(x+3)}{3}$$
$$=\frac{5}{6}$$

b)
$$\frac{4x^3}{5}\times\frac{1}{12x^4-4x^3}$$
$$=\frac{4x^3}{5}\times\frac{1}{4x^3(3x-1)}$$
$$=\frac{1}{5(3x-1)}$$
$$=\frac{1}{15x-5}$$

c)
$$\frac{p^2+pq}{q^2}\div\frac{p}{q}$$
$$=\frac{p(p+q)}{q^2}\times\frac{q}{p}$$
$$=\frac{p+q}{q}$$

d)
$$\frac{x^2-2xy}{x^2y}\div\frac{x^2y-4y^3}{3xy^2}$$
$$=\frac{x(x-2y)}{x^2y}\times\frac{3xy^2}{y\left(x^2-4y^2\right)}$$
$$=\frac{x-2y}{1}\times\frac{3}{(x-2y)(x+2y)}$$
$$=\frac{3}{x+2y}$$

Exercise 6 is on page **240** (solutions are continued on the next page)

e) $$\frac{6a^2}{3a^2-24a+48}\times\frac{a^2-4a}{a^3}$$
$$=\frac{6a^2}{3\left(a^2-8a+16\right)}\times\frac{a(a-4)}{a^3}$$
$$=\frac{2}{(a-4)(a-4)}\times\frac{a-4}{1}$$
$$=\frac{2}{a-4}$$

f) $$\frac{x^2-36}{x^2-7x}\div\frac{x^2+4x-12}{x^2-9x+14}$$
$$=\frac{(x-6)(x+6)}{x(x-7)}\times\frac{(x-7)(x-2)}{(x+6)(x-2)}$$
$$=\frac{x-6}{x}$$

g) $$\frac{m^2-49}{m^2-5m-14}\times\frac{2m^2-8}{m^2-2m}$$
$$=\frac{(m-7)(m+7)}{(m-7)(m+2)}\times\frac{2(m-2)(m+2)}{m(m-2)}$$
$$=\frac{2(m+7)}{m}$$
$$=\frac{2m+14}{m}$$

h) $$\frac{2x^2-3x-20}{x^2-8x+16}\times\frac{7x-x^2-12}{2x^2-x-15}$$
$$=\frac{(2x+5)(x-4)}{(x-4)(x-4)}\times\frac{-\left(x^2-7x+12\right)}{(2x+5)(x-3)}$$
$$=\frac{(2x+5)(x-4)}{(x-4)(x-4)}\times\frac{-(x-4)(x-3)}{(2x+5)(x-3)}$$
$$=-1$$

i) $$\frac{2}{3(x-5)}-\frac{1}{x(5-x)}$$
$$=\frac{2}{3(x-5)}+\frac{1}{x(x-5)}$$
$$=\frac{2x+3}{3x(x-5)}$$

j) $$\frac{a+2}{2(a-6)}+\frac{a+6}{3(6-a)}$$
$$=\frac{a+2}{2(a-6)}-\frac{a+6}{3(a-6)}$$
$$=\frac{3(a+2)-2(a+6)}{6(a-6)}$$
$$=\frac{3a+6-2a-12}{6(a-6)}$$
$$=\frac{a-6}{6(a-6)}=\frac{1}{6}$$

Exercise 6 is on page 240

k) $$\frac{p-5}{5(p-2)}-\frac{p+1}{p(2-p)}$$
$$=\frac{p-5}{5(p-2)}+\frac{p+1}{p(p-2)}$$
$$=\frac{p(p-5)+5(p+1)}{5p(p-2)}$$
$$=\frac{p^2-5p+5p+5}{5p(p-2)}=\frac{p^2+5}{5p(p-2)}$$

l) $$\frac{1}{x-y}-\frac{1}{x}$$
$$=\frac{x-(x-y)}{x(x-y)}$$
$$=\frac{y}{x(x-y)}$$

m) $$\frac{x+y}{xy}-\frac{2}{x+y}$$
$$=\frac{(x+y)^2-2xy}{xy(x+y)}$$
$$=\frac{x^2+2xy+y^2-2xy}{xy(x+y)}=\frac{x^2+y^2}{xy(x+y)}$$

n) $$\frac{p}{p-1}-\frac{1}{4p-4}$$
$$=\frac{p}{p-1}-\frac{1}{4(p-1)}$$
$$=\frac{4p-1}{4p-4}$$

o) $$\frac{7}{3x^2-6x}+\frac{2}{x}$$
$$=\frac{7}{3x(x-2)}+\frac{2}{x}$$
$$=\frac{7+2(3)(x-2)}{3x(x-2)}$$
$$=\frac{7+6x-12}{3x(x-2)}=\frac{6x-5}{3x^2-6x}$$

p) $$\frac{y^2+1}{2y+8}-\frac{2(y-1)}{3y+12}$$
$$=\frac{y^2+1}{2(y+4)}-\frac{2(y-1)}{3(y+4)}$$
$$=\frac{3\left(y^2+1\right)-4(y-1)}{6(y+4)}$$
$$=\frac{3y^2+3-4y+4}{6(y+4)}=\frac{3y^2-4y+7}{6y+24}$$

Mixed Exercise Solutions:

1)

a) $(2x-3)^2$

$= 4x^2 - 12x + 9$

b) $(2x-y)^2 - (x-2y)(x+2y)$

$= 4x^2 - 4xy + y^2 - (x^2 - 4y^2)$

$= 4x^2 - 4xy + y^2 - x^2 + 4y^2$

$= 3x^2 - 4xy + 5y^2$

c) $(3x-5)(2x^2 + x - 4)$

$= 6x^3 + 3x^2 - 12x - 10x^2 - 5x + 20$

$= 6x^3 - 7x^2 - 17x + 20$

d) $(8m-3n)(4m+n) - (n-3m)(n+3m)$

$= 32m^2 - 4mn - 3n^2 - (n^2 - 9m^2)$

$= 32m^2 - 4mn - 3n^2 - n^2 + 9m^2$

$= 41m^2 - 4mn - 4n^2$

e) $(4xy-3)^2 + 24xy$

$= 16x^2y^2 - 24xy + 9 + 24xy$

$= 16x^2y^2 + 9$

f) $2xy(3x^2 - 7xy + 10y^2)$

$= 6x^3y - 14x^2y^2 + 20xy^3$

g) $3(x-1)^2 - 2(x+3)(2x-1)$

$= 3(x^2 - 2x + 1) - 2(2x^2 + 5x - 3)$

$= 3x^2 - 6x + 3 - 4x^2 - 10x + 6$

$= -x^2 - 16x + 9$

h) $(-x+2)(x^2 - 4x + 1)$

$= -x^3 + 4x^2 - x + 2x^2 - 8x + 2$

$= -x^3 + 6x^2 - 9x + 2$

i) $(x+5)(x^2 - 3x - 9)$

$= x^3 - 3x^2 - 9x + 5x^2 - 15x - 45$

$= x^3 + 2x^2 - 24x - 45$

j) $\left(\frac{4}{x} - \frac{3}{y}\right)\left(\frac{4}{x} + \frac{3}{y}\right)$

$= \frac{16}{x^2} - \frac{9}{y^2}$

k) $-x^2\left(\frac{1}{x} + 2x\right)\left(\frac{2}{x} - 3x\right)$

$= -x^2\left(\frac{2}{x^2} - 3 + 4 - 6x^2\right)$

$= 6x^4 - x^2 - 2$

l) $\left(\frac{x}{4} - 3\right)\left(\frac{2x}{3} + 5\right)$

$= \frac{2x^2}{12} + \frac{5x}{4} - 2x - 15$

$= \frac{2x^2 + 3(5x) - 12(2x) - 12(15)}{12}$

$= \frac{2x^2 + 15x - 24x - 180}{12}$

$= \frac{2x^2 - 9x - 180}{12}$

m) $2(2x+3)(2x-3) + (2x-1)(x^2 - 3x + 4)$

$= 2(4x^2 - 9) + 2x^3 - 6x^2 + 8x - x^2 + 3x - 4$

$= 8x^2 - 18 + 2x^3 - 6x^2 + 8x - x^2 + 3x - 4$

$= 2x^3 + x^2 + 11x - 22$

n) $\frac{3p^2}{q}(4q - 2pq) - \frac{p}{5}(5p^2 + 10p)(5p^2 - 10p)$

$= 12p^2 - 6p^3 - \frac{p}{5}(25p^4 - 100p^2)$

$= 12p^2 - 6p^3 - 5p^5 + 20p^3$

$= -5p^5 + 14p^3 + 12p^2$

o) $(2x+3)(x-7)(2x-3)(x+7)$

$= (4x^2 - 9)(x^2 - 49)$

$= 4x^4 - 205x^2 + 441$

p) $(2a-3b)(8a^3 + 27b^3)(4a^2 + 6ab + 9b^2)$

$= (8a^3 + 27b^3)(8a^3 - 27b^3)$

$= 64a^6 - 729b^6$

The mixed Exercise number 1 is on page **241**

2)

a) $xy^2 - x$

$= x(y^2 - 1)$

$= x(y-1)(y+1)$

b) $72 - 8a^2$

$= 8(9 - a^2)$

$= 8(3-a)(3+a)$

The mixed Exercise number 2 is on page **241**

c)
$$\begin{aligned}&6a^2b^3-2ab^2\\&=2ab^2(3ab-1)\end{aligned}$$

d)
$$\begin{aligned}&x^3+16x\\&=x\left(x^2+16\right)\to\text{cannot factorise further}\end{aligned}$$

e)
$$\begin{aligned}&\frac{a^2b}{4}-\frac{9b^3}{25}\\&=b\left(\frac{a^2}{4}-\frac{9b^2}{25}\right)\\&=b\left(\frac{a}{2}-\frac{3b}{5}\right)\left(\frac{a}{2}+\frac{3b}{5}\right)\end{aligned}$$

f)
$$\begin{aligned}&2xy+x-10y-5\\&=x(2y+1)-5(2y+1)\\&=(2y+1)(x-5)\end{aligned}$$

g)
$$\begin{aligned}&x^3-3x^2-4x+12\\&=x^2(x-3)-4(x-3)\\&=(x-3)\left(x^2-4\right)\\&=(x-3)(x-2)(x+2)\end{aligned}$$

h)
$$\begin{aligned}&3x^2-x-10\\&=(x-2)(3x+5)\end{aligned}$$

i)
$$\begin{aligned}&3x^2-9x-xy+3y\\&=3x(x-3)-y(x-3)\\&=(x-3)(3x-y)\end{aligned}$$

j)
$$\begin{aligned}&x^2-3x-10\\&=(x-5)(x+2)\end{aligned}$$

k)
$$\begin{aligned}&p^3-3p^2-p+3\\&=p^2(p-3)-(p-3)\\&=(p-3)\left(p^2-1\right)\\&=(p-3)(p-1)(p+1)\end{aligned}$$

l)
$$\begin{aligned}&8x^2+18xy-5y^2\\&=(4x-y)(2x+5y)\end{aligned}$$

m)
$$\begin{aligned}&8p^2+14p-15\\&=(4p-3)(2p+5)\end{aligned}$$

n)
$$\begin{aligned}&pr-(r+p)x+x^2\\&=pr-rx-px+x^2\\&=r(p-x)-x(p-x)\\&=(p-x)(r-x)\end{aligned}$$

o)
$$\begin{aligned}&4x^2-2(5x+3)\\&=4x^2-10x-6\\&=2\left(2x^2-5x-3\right)\\&=2(2x+1)(x-3)\end{aligned}$$

p)
$$\begin{aligned}&xy-xz-y+z\\&=x(y-z)-(y-z)\\&=(y-z)(x-1)\end{aligned}$$

q)
$$\begin{aligned}&8p^3-27r^3\\&=(2p-3r)\left(4p^2+6pr+9r^2\right)\end{aligned}$$

r)
$$\begin{aligned}&-4x^4y^4-7x^2y^2+2\\&=-\left(4x^4y^4+7x^2y^2-2\right)\\&=-\left(4x^2y^2-1\right)\left(x^2y^2+2\right)\\&=-(2xy-1)(2xy+1)(2x+5)\end{aligned}$$

s)
$$\begin{aligned}&36(a-2)^2-9\\&=[6(a-2)-3][6(a-2)+3]\\&=(6a-12-3)(6a-12+3)\\&=(6a-15)(6a-9)\\&=9(2a-5)(2a-3)\end{aligned}$$

t)
$$\begin{aligned}&3x^3-5x^2+12xy^2-20y^2\\&=x^2(3x-5)+4y^2(3x-5)\\&=(3x-5)\left(x^2+4y^2\right)\end{aligned}$$

u)
$$\begin{aligned}&18p^2-24pq+8q^2-32\\&=2\left(9p^2-12pq+4q^2-16\right)\\&=2\left[(3p-2q)^2-16\right]\\&=2[(3p-2q-4)(3p-2q+4)]\end{aligned}$$

v)
$$\begin{aligned}&x^2\left(y^2-1\right)+5x\left(1-y^2\right)-6y^2+6\\&=x^2\left(y^2-1\right)-5x\left(y^2-1\right)-6\left(y^2-1\right)\\&=\left(y^2-1\right)\left(x^2-5x-6\right)\\&=\left(y^2-1\right)(x-6)(x+1)\\&=(y-1)(y+1)(x-6)(x+1)\end{aligned}$$

The mixed Exercise number 2 is on page **241**

3)

a)
$$1+\frac{2}{3x}-\frac{3}{2x}$$
$$=\frac{6x+2(2)-3(3)}{6x}$$
$$=\frac{6x-5}{6x}$$

b)
$$\frac{5x^2+3x}{x}$$
$$=\frac{x(5x+3)}{x}$$
$$=5x+3$$

c)
$$\left(1-\frac{a}{b}\right)\div\frac{a-b}{b}$$
$$=\frac{b-a}{b}\times\frac{b}{a-b}$$
$$=\frac{-(a-b)}{b}\times\frac{b}{a-b}$$
$$=-1$$

d)
$$\frac{a-3}{6-2a}$$
$$=\frac{a-3}{-2(a-3)}$$
$$=-\frac{1}{2}$$

e)
$$\frac{4p^2-8pq}{8pq}$$
$$=\frac{4p(p-2q)}{8pq}$$
$$=\frac{p-2q}{2q}$$

f)
$$\frac{a-3}{3}-\frac{2(a+1)}{3}-\frac{3a-5}{12}$$
$$=\frac{4(a-3)-4(2)(a+1)-(3a-5)}{12}$$
$$=\frac{4a-12-8a-8-3a+5}{12}$$
$$=\frac{-7a-15}{12}$$

g)
$$\frac{4a^2-1}{3}\times\frac{1}{2a-1}$$
$$=\frac{(2a-1)(2a+1)}{3}\times\frac{1}{2a-1}$$
$$=\frac{2a+1}{3}$$

h)
$$\frac{a+1}{5}-\frac{a+3}{2}$$
$$=\frac{2(a+1)-5(a+3)}{10}$$
$$=\frac{2a+2-5a-15}{10}$$
$$=\frac{-3a-13}{10}$$

i)
$$\frac{8p^3}{3q^2}\times\frac{9q^3}{p^4}\div\frac{16p^2}{q}$$
$$=\frac{8p^3}{3q^2}\times\frac{9q^3}{p^4}\times\frac{q}{16p^2}$$
$$=\frac{3q^2}{2p^3}$$

j)
$$\frac{2p^2-8}{27}\times\frac{9p-27}{p^2-p-6}$$
$$=\frac{2\left(p^2-4\right)}{27}\times\frac{9(p-3)}{(p-3)(p+2)}$$
$$=\frac{2(p-2)(p+2)}{3}\times\frac{1}{p+2}$$
$$=\frac{2p-4}{3}$$

k)
$$\frac{2+x}{6-3x}\div\frac{x^2+x-2}{2x-4}$$
$$=\frac{2+x}{3(2-x)}\times\frac{2(x-2)}{(x+2)(x-1)}$$
$$=\frac{x+2}{-3(x-2)}\times\frac{2(x-2)}{(x+2)(x-1)}$$
$$=\frac{2}{-3(x-1)}$$

l)
$$\frac{x^2-3x-4}{6x^2-6}\div\frac{2x^2-9x+4}{6-3x}\times\frac{x-1}{2}$$
$$=\frac{(x-4)(x+1)}{6\left(x^2-1\right)}\times\frac{3(2-x)}{(2x-1)(x-4)}\times\frac{x-1}{2}$$
$$=\frac{(x-4)(x+1)}{6(x-1)(x+1)}\times\frac{3(2-x)}{(2x-1)(x-4)}\times\frac{x-1}{2}$$
$$=\frac{2-x}{8x-4}$$

The mixed Exercise number 3 is on page **242** (solutions are continued on the next page)

m) $\frac{x^2+5xy+6y^2}{x^2-36} \div \frac{3x+9y}{x^2-3x-18} \times \frac{x+6}{x+2y}$

$$= \frac{(x+3y)(x+2y)}{(x-6)(x+6)} \times \frac{(x-6)(x+3)}{3(x+3y)} \times \frac{x+6}{x+2y}$$

$$= \frac{x+3}{3}$$

n) $\frac{3a^2}{a^2-a-2} + \frac{2}{2-a} - 1\frac{2}{3}$

$$= \frac{3a^2}{(a-2)(a+1)} + \frac{2}{2-a} - \frac{5}{3}$$

$$= \frac{3a^2}{(a-2)(a+1)} - \frac{2}{a-2} - \frac{5}{3}$$

$$= \frac{3(3a^2)-2(3)(a+1)-5(a-2)(a+1)}{3(a-2)(a+1)}$$

$$= \frac{9a^2-6a-6-5(a^2-a-2)}{3(a-2)(a+1)}$$

$$= \frac{9a^2-6a-6-5a^2+5a+10}{3(a-2)(a+1)}$$

$$= \frac{4a^2-a+4}{3a^2-3a-6}$$

The mixed Exercise number 3 is on page **242**

4) $(x+y)^2 = x^2+2xy+y^2$

$\therefore -2xy$ must be added to x^2+y^2 so that x^2+y^2 will be equal to $(x+y)^2$

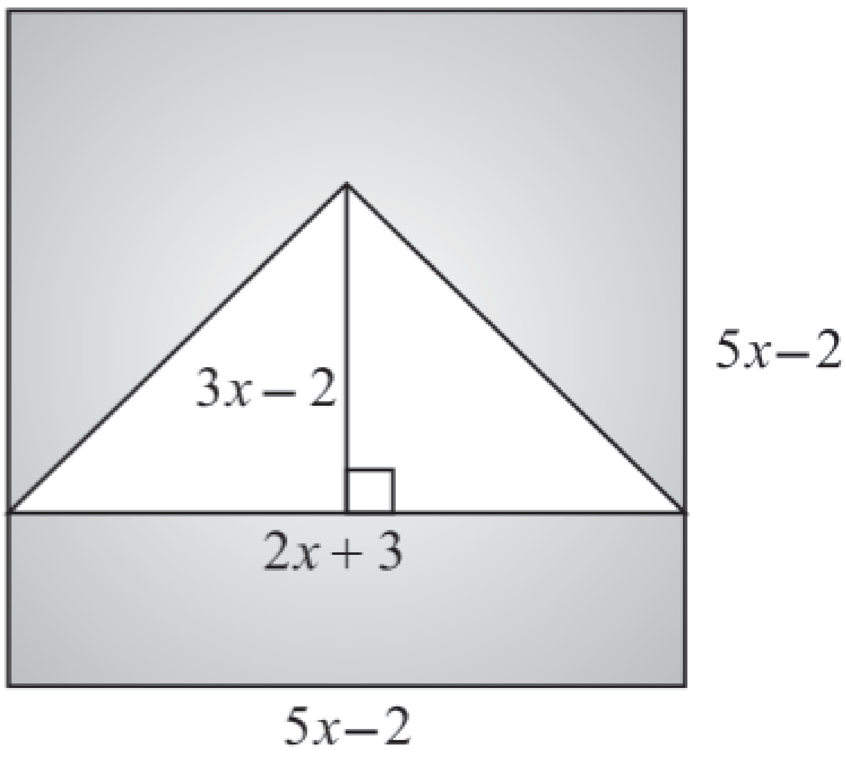
The mixed Exercise number 4 is on page **242**

5) $(x+4)(x-4) = x^2-16$ and $(x-1)(x+16) = x^2+15x-16$

$\therefore 15x$ must be added to $(x+4)(x-4)$ so that $(x+4)(x-4)$ will be equal to $(x-1)(x+16)$

The mixed Exercise number 5 is on page **242**

6)



$$\text{A}_{square} = (5x-2)^2$$

$$= 25x^2-20x+4 \text{ units}^2$$

$$\text{A}_{triangle} = \frac{1}{2}(2x+3)(3x-2)$$

$$= \frac{1}{2}(6x^2+5x-6)$$

$$= 3x^2+\frac{5}{2}x-3 \text{ units}^2$$

$$A_{shaded\ region} = A_{square} - A_{triangle}$$

$$= 25x^2-20x+4-\left(3x^2+\frac{5}{2}x-3\right)$$

$$= 25x^2-20x+4-3x^2-\frac{5}{2}x+3$$

$$= 22x^2-20x-\frac{5}{2}x+7$$

$$= \frac{44x^2-45x+14}{2} \text{ units}^2$$

The mixed Exercise number 6 is on page **242**

7) He simplified the expression rather than factorising it. The solution should have been:

$$(2x-1)^2-(2x-1)(x-1)$$

$$= (2x-1)[2x-1-(x-1)]$$

$$= (2x-1)(x)$$

The mixed Exercise number 7 is on page **242**

Chapter 3 – Exponents

Definitions and Laws of Exponents (Theory on pages 35-36)

Exercise 1: **Simplify the following as far as possible and write your answers with positive exponents:**

a) 5^0

b) $5x^0$

c) $(5x)^0$

d) 2^{-2}

e) 5^{-2}

f) 3^{-2}

g) 2.3^{-2}

h) $(2.3)^{-2}$

i) $\frac{1}{3^{-2}}$

j) $\frac{1}{2.3^{-2}}$

k) $\frac{1}{2x^{-2}y^3}$

l) $\left(\frac{x^2}{y}\right)^{-2}$

m) $x.x.x$

n) $2x\times 3x$

o) $\left(x^3y^2\right)\left(x^2y^3\right)$

p) $\left(x^3y^2\right)^{-1}\left(x^2y^3\right)$

q) $\left[\left(x^3y^2\right)\left(x^2y^3\right)\right]^{-1}$

r) $\frac{2}{\left(\frac{1}{8}\right)^{-1}}$

s) $\left(\frac{-1}{8}\right)^{-2}$

t) $\left(\frac{x+y}{3}\right)^{-1}$

u) $\frac{-4x^{-2}}{2x^{-5}}$

Solutions to Exercise 1 are on page **257**

Multiplication and Division of Exponents (Theory on page 37)

Exercise 2: **Simplify the following as far as possible and write your answers with positive exponents:**

a) $\frac{2^x.4^{x-1}}{2.8^x}$

b) $\frac{2^{2n+1}.2^{2n-2}.2}{2^{4n-2}}$

c) $\frac{5^{2x-1}.25^{x+1}.5}{25^{2x}}$

d) $\frac{9^{x-1}.12^{x+1}}{27^x.4^{x-1}}$

e) $\frac{2^{2x+1}.3^{2x+3}}{6^{2x}}$

f) $\frac{3^y.15^{y-1}.5^{2y}}{125^y.9^{y-1}}$

g) $\frac{4^{n-1}.24^{-2n}.4}{6^{-n}.3^{-n}}$

h) $\frac{x^{2n}\left(xy\right)^n y^n}{y^{2n-1}\left(x^3\right)^n}$

i) $\frac{\left(xy^2\right)^n\left(x^3\right)^{n-1}}{\left(y^3\right)^{n+1}\left(x^2\right)^{2n-1}}$

j) $\frac{6^n.9^{n-1}.2.3^{1-n}}{18^{n+1}}$

k) $\left(\frac{2}{3}\right)^{2x+2y}\left(\frac{9}{4}\right)^{-x-y}$

l) $\frac{(27)^{x-2}(15)^x}{405^x}$

Solutions to Exercise 2 *a* to *c* are on page **257** and *d* to *l* are on page **258**

Addition and Subtraction of Exponents (Theory on pages 38-39)

Exercise 3: **Simplify the following as far as possible and write your answers with positive exponents:**

a) $\frac{9^n.3^{n+2}-27^n}{3^2.3^{3n}}$

b) $\frac{5^x+5^{x+2}}{5^{x-1}}$

c) $\frac{2^{x+1}-2^{x+2}}{2^x+2^{x+2}}$

d) $\frac{5^{2n}-5^{2n+2}}{25^{n+1}-5^{2n}}$

e) $\frac{-2\times 9^n+3^{2n+1}}{3^{n+3}-3^{2+n}}$

f) $\frac{5.2^{4x}-10.2^{3x}}{5.2^{3x}}$

Solutions to Exercise 3 are on page **259**

Rational Exponents (Theory on page 40)

Exercise 4: **Round off the following numbers to the specified number of decimal places:**

a) $2^{\frac{1}{2}}\times 8^{\frac{1}{2}}$

b) $4^{\frac{3}{2}}$

c) $4^{\frac{2}{3}}$

Solutions to Exercise 4 are on page **259**

Exponential Equations

Exercise 5:

1) Solve for *x* in the following equations: (Theory on pages 41-43)

a) $2^x = 8$

b) $3^x = 81$

c) $4^x = 64$

d) $2^x = \frac{1}{4}$

e) $5^{2x-1} = 125$

f) $8^{x+5} = 4^{2x+3}$

g) $8^{x-2} = 16^{2x+1}$

h) $2^{\frac{3x+1}{2}} = \frac{1}{16}$

i) $19^{x-1} = 1$

j) $2^{x+1} = 0,25^{x-3}$

k) $5.4^{x-2} = 80$

l) $4^{x-2}.8^x = 2$

m) $27^{-x} = 3.9^{x-1}$

n) $2\left(\frac{1}{4}\right)^{5x} = 64$

o) $4^{x-2} = 0,5^{2x-4}$

p) $16^{2x+1} = 8^{2x-2}$

q) $9.27^x = 81$

r) $2 \times 8^{2x-4} = 4^{-2x}$

Solutions to Exercise 5 number 1a to 1h are on page **259** and 1i to 1r are on page **260**

2) Solve for *x* in the following equations correct to two decimal place: (Theory on page 44)

a) $2^x = 18$

b) $5^x = 100$

c) $3^{2x} = 78$

Solutions to Exercise 5 number 2 are on page **260**

Mixed Exercise: (Chapter 3 is on pages 34-44)

1) Simplify the following as far as possible and write your answers with positive exponents:

a) $\frac{2x^{-2}}{y^{-2}}$

b) $\left(-3x^3y^{-2}\right)^2$

c) $-\left(3x^3y^{-2}\right)^2$

d) $\frac{\left(a^2\right)^{x-1} \times a^2 \times \left(a^6 b\right)^{-x}}{b^{-x} \times (ab)^{-3x}}$

e) $\sqrt{18^4 x^6 y^8}$

f) $\left(\frac{x+y}{x}\right)^{-1}$

g) $\frac{1}{x^0 + y^0}$

h) $\frac{25^{2n}.5^{n+1}.25^2}{5^3}$

i) $\frac{9^x - 3^{2x+1}}{3^{2x}}$

Solutions to the Mixed Exercise number 1a to 1d are on page **260** and 1e to 1i are on page **261**

2) Determine the value of:

a) Half of 2^5

b) One fifth of 5^4

c) Two Thirds of 81

d) Three quarters of 2^6

e) $2^{(212)^x}$ if $x = 0$

f) $1^9 + 1^{99} + 1^{999} + \ldots + 1^{9999999}$

Solutions to the Mixed Exercise number 2 are on page **261**

3) Determine the value of:

a) $\frac{3^{2008} + 3^{2009}}{3^{2010}}$

b) $\frac{5^{1010} + 3 \times 5^{1008}}{2 \times 5^{1009}}$

Solutions to the Mixed Exercise number 3 are on page **261**

4) Given that $12^x = 10$ determine the value of $\frac{48^x}{3^{-x}}$ without solving for *x*.

Solution to the Mixed Exercise number 4 is on page **262**

5) Solve for the unknown variable in each of the following equations:

a) $7^x = 49$

b) $9^{3r-1} = 27$

c) $9^{2x} = \frac{1}{81}$

d) $4^{2m} = 16^{m-1}$

Solutions to the Mixed Exercise number 5 are on page **262**

6) Given that $3^x = p$ express 3^{x+1} in terms of p.

Solution to the Mixed Exercise number 6 is on page **262**

7) The table below shows solutions given by a Grade 10 learner. In each case errors have been made. Fill in the table correcting the errors. The first row has been completed for you.

Question and Solution	Error	Correct solution
Simplify $a^2 \times a^3$ $a^2 \times a^3 = a^6$	When multiplying like bases exponents should be added not multiplied.	$a^2 \times a^3 = a^{2+5} = a^5$
a) Simplify $\left(2x^2y^3\right)^2$ $\left(2x^2y^3\right)^2 = 2x^4y^6$		
b) Solve for x if $2 \times 2^x = 8$ $2 \times 2^x = 8$ $\therefore 4^x = 8$ $\therefore 2^{2x} = 2^3$ $\therefore 2x = 3$ $\therefore x = \frac{3}{2}$		
c) Simplify $\left(9x^4 + 16x^4\right)^{\frac{1}{2}}$ $\left(9x^4 + 16x^4\right)^{\frac{1}{2}} = 3x^2 + 4x^2$ $= 7x^2$		
d) Solve for x if $2^{x+2} = \frac{1}{4}$ $2^{x+2} = \frac{1}{2^2}$ $\therefore x + 2 = 2$ $\therefore x = 0$		
e) Solve for x if $2^{x+1} + 2^{x+1} = 16$ $2^{x+1} + 2^{x+1} = 16$ $\therefore 4^{x+1} = 4^2$ $\therefore x + 1 = 2$ $\therefore x = 1$		

Solutions to the Mixed Exercise number 6 is on page **263**

Chapter 3 – Exponents (Solutions)

Exercise 1 Solutions:

a) $5^0 = 1$

b) $5x^0 = 5(1) = 5$

c) $(5x)^0 = 1$

d) $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

e) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

f) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

g) $2.3^{-2} = \frac{2}{3^2} = \frac{2}{9}$

h) $(2.3)^{-2}$

$= 2^{-2}.3^{-2}$

$= \frac{1}{2^2.3^2}$

$= \frac{1}{36}$

i) $\frac{1}{3^{-2}}$

$= 3^2$

$= 9$

j) $\frac{1}{2.3^{-2}}$

$= \frac{3^2}{2}$

$= \frac{9}{2}$

k) $\frac{1}{2x^{-2}y^3} = \frac{x^2}{2y^3}$

l) $\left(\frac{x^2}{y}\right)^{-2}$

$= \frac{x^{-4}}{y^{-2}}$

$= \frac{y^2}{x^4}$

m) $x.x.x = x^3$

n) $2x \times 3x = 6x^2$

o) $\left(x^3y^2\right)\left(x^2y^3\right)$

$= x^{3+2}y^{2+3}$

$= x^5y^5$

p) $\left(x^3y^2\right)^{-1}\left(x^2y^3\right)$

$= \left(x^{-3}y^{-2}\right)\left(x^2y^3\right)$

$= x^{-3+2}.y^{-2+3}$

$= x^{-1}.y^1 = \frac{y}{x}$

q) $\left[\left(x^3y^2\right)\left(x^2y^3\right)\right]^{-1}$

$= \left(x^3y^2\right)^{-1}\left(x^2y^3\right)^{-1}$

$= \left(x^{-3}y^{-2}\right)\left(x^{-2}y^{-3}\right)$

$= x^{-3-2}y^{-2-3}$

$= x^{-5}y^{-5}$

$= \frac{1}{x^5y^5}$

r) $\frac{2}{\left(\frac{1}{8}\right)^{-1}}$

$= \frac{2}{\left(\frac{1}{8^{-1}}\right)}$

$= \frac{2}{8}$

$= \frac{1}{4}$

s) $\left(\frac{-1}{8}\right)^{-2}$

$= \frac{(-1)^{-2}}{8^{-2}}$

$= \frac{8^2}{(-1)^2}$

$= \frac{64}{1}$

$= 64$

t) $\left(\frac{x+y}{3}\right)^{-1}$

$= \frac{(x+y)^{-1}}{3^{-1}}$

$= \frac{3}{x+y}$

u) $\frac{-4x^{-2}}{2x^{-5}}$

$= \frac{-2x^5}{x^2}$

$= -2x^{5-2}$

$= -2x^3$

Exercise 1 is on page 254

Exercise 2 Solutions:

a) $\frac{2^x.4^{x-1}}{2.8^x}$

$= \frac{2^x\left(2^2\right)^{x-1}}{2^1\left(2^3\right)^x}$

$= \frac{2^x.2^{2x-2}}{2.2^{3x}}$

$= 2^{x+2x-2-(1+3x)}$

$= 2^{x+2x-2-1-3x}$

$= 2^{-3} = \frac{1}{8}$

b) $\frac{2^{2n+1}.2^{2n-2}.2}{2^{4n-2}}$

$= 2^{2n+1+2n-2+1-(4n-2)}$

$= 2^{2n+1+2n-2+1-4n+2}$

$= 2^2$

$= 4$

c) $\frac{5^{2x-1}.25^{x+1}.5}{25^{2x}}$

$= \frac{5^{2x-1}\left(5^2\right)^{x+1}.5^1}{\left(5^2\right)^{2x}}$

$= \frac{5^{2x-1}.5^{2x+2}.5}{5^{4x}}$

$= 5^{2x-1+2x+2+1-4x}$

$= 5^2$

$= 25$

Exercise 2 is on page 254 (solutions are continued on the next page)

d) $$\frac{9^{x-1}.12^{x+1}}{27^{x}.4^{x-1}}$$

$$=\frac{\left(3^2\right)^{x-1}\left(2^2.3\right)^{x+1}}{\left(3^3\right)^{x}\left(2^2\right)^{x-1}}$$

$$=\frac{3^{2x-2}.2^{2x+2}.3^{x+1}}{3^{3x}.2^{2x-2}}$$

$$=3^{2x-2+x+1-3x}.2^{2x+2-(2x-2)}$$

$$=3^{-1}.2^{2x+2-2x+2}$$

$$=\frac{1}{3}.2^{4}=\frac{16}{3}$$

e) $$\frac{2^{2x+1}.3^{2x+3}}{6^{2x}}$$

$$=\frac{2^{2x+1}.3^{2x+3}}{(3.2)^{2x}}$$

$$=\frac{2^{2x+1}.3^{2x+3}}{3^{2x}.2^{2x}}$$

$$=2^{2x+1-2x}.3^{2x+3-2x}$$

$$=2.3^{+3}=54$$

f) $$\frac{3^{y}.15^{y-1}.5^{2y}}{125^{y}.9^{y-1}}$$

$$=\frac{3^{y}\left(5.3\right)^{y-1}.5^{2y}}{\left(5^3\right)^{y}\left(3^2\right)^{y-1}}$$

$$=\frac{3^{y}.5^{y-1}.3^{y-1}.5^{2y}}{5^{3y}.3^{2y-2}}$$

$$=3^{y+y-1-(2y-2)}.5^{y-1+2y-3y}$$

$$=3^{y+y-1-2y+2}.5^{-1}$$

$$=3^{1}\left(\frac{1}{5}\right)$$

$$=\frac{3}{5}$$

g) $$\frac{4^{n-1}.24^{-2n}.4}{6^{-n}.3^{-n}}$$

$$=\frac{\left(2^2\right)^{n-1}\left(2^3.3^1\right)^{-2n}.2^2}{\left(2.3\right)^{-n}.3^{-n}}$$

$$=\frac{2^{2n-2}.2^{-6n}.3^{-2n}.2^2}{2^{-n}.3^{-n}.3^{-n}}$$

$$=2^{2n-2-6n+2-(-n)}.3^{-2n-(-n-n)}$$

$$=2^{2n-2-6n+2+n}.3^{-2n+2n}$$

$$=2^{-3n}.3^{0}$$

$$=\frac{1}{2^{3n}}$$

h) $$\frac{x^{2n}\left(xy\right)^{n}y^{n}}{y^{2n-1}\left(x^3\right)^{n}}$$

$$=\frac{x^{2n}.x^{n}.y^{n}.y^{n}}{y^{2n-1}.x^{3n}}$$

$$=x^{2n+n-3n}.y^{n+n-(2n-1)}$$

$$=x^{0}.y^{n+n-2n+1}=y$$

i) $$\frac{\left(xy^2\right)^{n}\left(x^3\right)^{n-1}}{\left(y^3\right)^{n+1}\left(x^2\right)^{2n-1}}$$

$$=\frac{x^{n}.y^{2n}.x^{3n-3}}{y^{3n+3}.x^{4n-2}}$$

$$=x^{n+3n-3-(4n-2)}.y^{2n-(3n+3)}$$

$$=x^{n+3n-3-4n+2}.y^{2n-3n-3}$$

$$=x^{-1}.y^{-n-3}=\frac{1}{x.y^{n+3}}$$

j) $$\frac{6^{n}.9^{n-1}.2.3^{1-n}}{18^{n+1}}$$

$$=\frac{(3.2)^{n}\left(3^2\right)^{n-1}.2.3^{1-n}}{\left(3^2.2\right)^{n+1}}$$

$$=\frac{3^{n}.2^{n}.3^{2n-2}.2.3^{1-n}}{3^{2n+2}.2^{n+1}}$$

$$=3^{n+2n-2+1-n-(2n+2)}.2^{n+1-(n+1)}$$

$$=3^{n+2n-2+1-n-2n-2}.2^{n+1-n-1}$$

$$=3^{-3}.2^{0}=\frac{1}{27}$$

k) $$\left(\frac{2}{3}\right)^{2x+2y}\left(\frac{9}{4}\right)^{-x-y}$$

$$=\frac{2^{2x+2y}}{3^{2x+2y}}\times\frac{3^{-2x-2y}}{2^{-2x-2y}}$$

$$=2^{2x+2y-(-2x-2y)}.3^{-2x-2y-(2x+2y)}$$

$$=2^{4x+4y}.3^{-4x-4y}=\frac{2^{4x+4y}}{3^{4x+4y}}$$

l) $$\frac{(27)^{x-2}(15)^{x}}{405^{x}}$$

$$=\frac{\left(3^3\right)^{x-2}(5.3)^{x}}{\left(3^4.5\right)^{x}}$$

$$=\frac{3^{3x-6}.5^{x}.3^{x}}{3^{4x}.5^{x}}$$

$$=3^{3x-6+x-4x}.5^{x-x}$$

$$=3^{-6}.5^{0}=\frac{1}{729}$$

Exercise 2 is on page 254

Exercise 3 Solutions:

a)
$$\begin{aligned}&\frac{9^n.3^{n+2}-27^n}{3^2.3^{3n}}\\&=\frac{3^{2n}.3^n.3^2-3^{3n}}{3^2.3^{3n}}\\&=\frac{3^{3n}\left(3^2-1\right)}{3^2.3^{3n}}\\&=\frac{8}{9}\end{aligned}$$

b)
$$\begin{aligned}&\frac{5^x+5^{x+2}}{5^{x-1}}\\&=\frac{5^x+5^x.5^2}{5^x.5^{-1}}\\&=\frac{5^x\left(1+5^2\right)}{5^x.5^{-1}}\\&=26\times5\\&=130\end{aligned}$$

c)
$$\begin{aligned}&\frac{2^{x+1}-2^{x+2}}{2^x+2^{x+2}}\\&=\frac{2^x.2-2^x.2^2}{2^x+2^x.2^2}\\&=\frac{2^x\left(2-2^2\right)}{2^x\left(1+2^2\right)}\\&=-\frac{2}{5}\end{aligned}$$

d)
$$\begin{aligned}&\frac{5^{2n}-5^{2n+2}}{25^{n+1}-5^{2n}}\\&=\frac{5^{2n}-5^{2n}.5^2}{5^{2n+2}-5^{2n}}\\&=\frac{5^{2n}-5^{2n}.5^2}{5^{2n}.5^2-5^{2n}}\\&=\frac{5^{2n}\left(1-5^2\right)}{5^{2n}\left(5^2-1\right)}\\&=-1\end{aligned}$$

e)
$$\begin{aligned}&\frac{-2\times9^n+3^{2n+1}}{3^{n+3}-3^{2+n}}\\&=\frac{-2\times3^{2n}+3^{2n}\times3^1}{3^n\times3^3-3^2\times3^n}\\&=\frac{3^{2n}\left(-2+3\right)}{3^n\left(3^3-3^2\right)}\\&=3^n\times\frac{1}{18}\\&=\frac{3^n}{18}\end{aligned}$$

f)
$$\begin{aligned}&\frac{5.2^{4x}-10.2^{3x}}{5.2^{3x}}\\&=\frac{5.2^{3x}\left(2^x-2\right)}{5.2^{3x}}\\&=2^x-2\end{aligned}$$

Exercise 3 is on page **254**

Exercise 4 Solutions:

a)
$$\begin{aligned}&2^{\frac{1}{2}}\times8^{\frac{1}{2}}\\&=2^{\frac{1}{2}}\times\left(2^3\right)^{\frac{1}{2}}\\&=2^{\frac{1}{2}}\times2^{\frac{3}{2}}\\&=2^{\frac{1}{2}+\frac{3}{2}}\\&=2^{\frac{4}{2}}=4\end{aligned}$$

b)
$$\begin{aligned}&4^{\frac{3}{2}}\\&=\left(2^2\right)^{\frac{3}{2}}\\&=2^3\\&=8\end{aligned}$$

c)
$$\begin{aligned}&4^{\frac{2}{3}}\\&=\left(2^2\right)^{\frac{2}{3}}\\&=2^{\frac{4}{3}}\\&=\sqrt[3]{2^4}\\&=\sqrt[3]{16}\end{aligned}$$

Exercise 4 is on page **254**

Exercise 5 Solutions:

1)

a)
$$\begin{aligned}&2^x=8\\&\therefore 2^x=2^3\\&\therefore x=3\end{aligned}$$

b)
$$\begin{aligned}&3^x=81\\&\therefore 3^x=3^4\\&\therefore x=4\end{aligned}$$

c)
$$\begin{aligned}&4^x=64\\&\therefore 2^{2x}=2^6\\&\therefore 2x=6\\&\therefore x=3\end{aligned}$$

d)
$$\begin{aligned}&2^x=\frac{1}{4}\\&\therefore 2^x=2^{-2}\\&\therefore x=-2\end{aligned}$$

e)
$$\begin{aligned}&5^{2x-1}=125\\&\therefore 5^{2x-1}=5^3\\&\therefore 2x-1=3\\&\therefore 2x=4\\&\therefore x=2\end{aligned}$$

f)
$$\begin{aligned}&8^{x+5}=4^{2x+3}\\&\left(2^3\right)^{x+5}=\left(2^2\right)^{2x+3}\\&\therefore 2^{3x+15}=2^{4x+6}\\&\therefore 3x+15=4x+6\\&\therefore x=9\end{aligned}$$

g)
$$\begin{aligned}&8^{x-2}=16^{2x+1}\\&\therefore\left(2^3\right)^{x-2}=\left(2^4\right)^{2x+1}\\&\therefore 2^{3x-6}=2^{8x+4}\\&\therefore 3x-6=8x+4\\&\therefore -5x=10\\&\therefore x=-2\end{aligned}$$

h)
$$\begin{aligned}&2^{\frac{3x+1}{2}}=\frac{1}{16}\\&\therefore 2^{\frac{3x+1}{2}}=2^{-4}\\&\therefore\frac{3x+1}{2}=-4\\&\therefore 3x+1=-8\\&\therefore 3x=-9\\&\therefore x=-3\end{aligned}$$

Exercise 5 number 1 is on page **255** (solutions are continued on the next page)

i)
$$\begin{aligned}&19^{x-1}=1\\ &\therefore 19^{x-1}=19^0\\ &\therefore x-1=0\\ &\therefore x=1\end{aligned}$$

j)
$$\begin{aligned}&2^{x+1}=0,25^{x-3}\\ &\therefore 2^{x+1}=\left(2^{-2}\right)^{x-3}\\ &\therefore 2^{x+1}=2^{-2x+6}\\ &\therefore x+1=-2x+6\\ &\therefore 3x=5\\ &\therefore x=\frac{5}{3}\end{aligned}$$

k)
$$\begin{aligned}&5.4^{x-2}=80\\ &\therefore \left(2^2\right)^{x-2}=16\\ &\therefore 2^{2x-4}=2^4\\ &\therefore 2x-4=4\\ &\therefore 2x=8\\ &\therefore x=4\end{aligned}$$

l)
$$\begin{aligned}&4^{x-2}.8^x=2\\ &\left(2^2\right)^{x-2}\left(2^3\right)^x=2^1\\ &\therefore 2^{2x-4+3x}=2^1\\ &\therefore 2x-4+3x=1\\ &\therefore 5x=5\\ &\therefore x=1\end{aligned}$$

m)
$$\begin{aligned}&27^{-x}=3.9^{x-1}\\ &\therefore \left(3^3\right)^{-x}=3\left(3^2\right)^{x-1}\\ &\therefore 3^{-3x}=3^{1+2x-2}\\ &\therefore -3x=1+2x-2\\ &\therefore -5x=-1\\ &\therefore x=\frac{1}{5}\end{aligned}$$

n)
$$\begin{aligned}&2\left(\frac{1}{4}\right)^{5x}=64\\ &\therefore \left(2^{-2}\right)^{5x}=32\\ &\therefore 2^{-10x}=2^5\\ &\therefore -10x=5\\ &\therefore x=-\frac{1}{2}\end{aligned}$$

o)
$$\begin{aligned}&4^{x-2}=0,5^{2x-4}\\ &\therefore \left(2^2\right)^{x-2}=\left(2^{-1}\right)^{2x-4}\\ &\therefore 2^{2x-4}=2^{-2x+4}\\ &\therefore 2x-4=-2x+4\\ &\therefore 4x=8\\ &\therefore x=2\end{aligned}$$

p)
$$\begin{aligned}&16^{2x+1}=8^{2x-2}\\ &\therefore \left(2^4\right)^{2x+1}=\left(2^3\right)^{2x-2}\\ &\therefore 2^{8x+4}=2^{6x-6}\\ &\therefore 8x+4=6x-6\\ &\therefore 2x=-10\\ &\therefore x=-5\end{aligned}$$

q)
$$\begin{aligned}&9.27^x=81\\ &\therefore 3^2.3^{3x}=3^4\\ &\therefore 3^{2+3x}=3^4\\ &\therefore 2+3x=4\\ &\therefore 3x=2\\ &\therefore x=\frac{2}{3}\end{aligned}$$

r)
$$\begin{aligned}&2.8^{2x-4}=4^{-2x}\\ &\therefore 2\left(2^3\right)^{2x-4}=\left(2^2\right)^{-2x}\\ &\therefore 2^1.2^{6x-12}=2^{-4x}\\ &\therefore 2^{6x-12+1}=2^{-4x}\\ &\therefore 2^{6x-11}=2^{-4x}\\ &\therefore 6x-11=-4x\\ &\therefore 10x=11\\ &\therefore x=\frac{11}{10}\end{aligned}$$

Exercise 5 number 1 is on page **255**

2)

a)
$$\begin{aligned}&2^x=18\\ &2^{4,18}=18,13\\ &2^{4,17}=18,00\\ &\therefore x=4,17\end{aligned}$$

b)
$$\begin{aligned}&5^x=100\\ &5^{2,86}=99,78\\ &5^{2,87}=101,40\\ &\therefore x=2,86\end{aligned}$$

c)
$$\begin{aligned}&3^{2x}=78\\ &3^{3,96}=77,52\\ &3^{3,97}=78,37\\ &\therefore 2x=3,97\\ &\therefore x=1,99\end{aligned}$$

Exercise 5 number 2 is on page **255**

Mixed Exercise Solutions:

1)

a)
$$\begin{aligned}&\frac{2x^{-2}}{y^{-2}}\\ &=\frac{2y^2}{x^2}\end{aligned}$$

b)
$$\begin{aligned}&\left(-3x^3y^{-2}\right)^2\\ &=9x^6y^{-4}\\ &=\frac{9x^6}{y^4}\end{aligned}$$

c)
$$\begin{aligned}&-\left(3x^3y^{-2}\right)^2\\ &=-9x^6y^{-4}\\ &=\frac{-9x^6}{y^4}\end{aligned}$$

d)
$$\begin{aligned}&\frac{\left(a^2\right)^{x-1}\times a^2\times\left(a^6b\right)^{-x}}{b^{-x}\times(ab)^{-3x}}\\ &=\frac{a^{2x-2}\times a^2\times a^{-6x}b^{-x}}{b^{-x}\times a^{-3x}\times b^{-3x}}\\ &=a^{2x-2+2-6x-(-3x)}\times b^{-x-(-x-3x)}\\ &=a^{2x-2+2-6x+3x}\times b^{-x+x+3x}\\ &=a^{-x}b^{3x}\\ &=\frac{b^{3x}}{a^x}\end{aligned}$$

The Mixed Exercise number 1 is on page **255** (solutions are continued on the next page)

e)
$$\begin{aligned}&\sqrt{18^4x^6y^8}\\&=18^2x^3y^4\\&=324x^3y^4\end{aligned}$$

f)
$$\begin{aligned}&\left(\frac{x+y}{x}\right)^{-1}\\&=\frac{x}{x+y}\end{aligned}$$

g)
$$\begin{aligned}&\frac{1}{x^0+y^0}\\&=\frac{1}{1+1}\\&=\frac{1}{2}\end{aligned}$$

h)
$$\begin{aligned}&\frac{25^{2n}.5^{n+1}.25^2}{5^3}\\&=\frac{5^{4n}.5^{n+1}.5^4}{5^3}\\&=5^{4n+n+1+4-3}\\&=5^{5n+2}\end{aligned}$$

i)
$$\begin{aligned}&\frac{9^x-3^{2x+1}}{3^{2x}}\\&=\frac{\left(3^2\right)^x-3^{2x}.3}{3^{2x}}\\&=\frac{3^{2x}-3^{2x}.3}{3^{2x}}\\&=\frac{3^{2x}\left(1-3\right)}{3^{2x}}\\&=-2\end{aligned}$$

The Mixed Exercise number 1 is on page **255**

2)

a)
$$\begin{aligned}&\frac{1}{2}\times2^5\\&=2^{-1}\times2^5\\&=2^{-1+5}\\&=2^4\\&=16\end{aligned}$$

b)
$$\begin{aligned}&\frac{1}{5}\times5^4\\&=5^{-1}\times5^4\\&=5^{-1+4}\\&=5^3\\&=125\end{aligned}$$

c)
$$\begin{aligned}&\frac{2}{3}\times81\\&=2\times3^{-1}\times3^4\\&=2\times3^{-1+4}\\&=2\times3^3\\&=54\end{aligned}$$

d)
$$\begin{aligned}&\frac{3}{4}\times2^6\\&=3\times2^{-2}\times2^6\\&=3\times2^{-2+6}\\&=3\times2^4\\&=48\end{aligned}$$

e)
$$\begin{aligned}&2^{(212)^0}\\&=2^1\\&=2\end{aligned}$$

f)
$$\begin{aligned}&1^9+1^{99}+1^{999}+\ldots+1^{9999999}\\&=1+1+1+\ldots\rightarrow 7\text{ times}\\&=7\left(1\right)\\&=7\end{aligned}$$

The Mixed Exercise number 2 is on page **255**

3)

a)
$$\frac{3^{2008}+3^{2009}}{3^{2010}}$$

let $x=2008$

$$\begin{aligned}\therefore\frac{3^{2008}+3^{2009}}{3^{2010}}&=\frac{3^x+3^{x+1}}{3^{x+2}}\\&=\frac{3^x+3^x.3^1}{3^x.3^2}\\&=\frac{3^x\left(1+3\right)}{3^x.3^2}\\&=\frac{4}{9}\end{aligned}$$

b)
$$\frac{5^{1010}+3\times5^{1008}}{2\times5^{1009}}$$

let $x=1008$

$$\begin{aligned}\therefore\frac{5^{1010}+3\times5^{1008}}{2\times5^{1009}}&=\frac{5^{x+2}+\left(3\times5^x\right)}{2\times5^{x+1}}\\&=\frac{5^x.5^2+3.5^x}{2.5^x.5^1}\\&=\frac{5^x\left(5^2+3\right)}{2.5^x.5^1}\\&=\frac{28}{10}\\&=\frac{14}{5}\end{aligned}$$

The Mixed Exercise number 3 is on page **255**

Method 1:

4) $\frac{48^x}{3^{-x}}$

$=\frac{(12\times 4)^x}{3^{-x}}$

$=\frac{12^x\times 4^x}{3^{-x}}$

$=\frac{10\times 4^x}{3^{-x}}$

$=10\times 4^x\times 3^x$

$=10(4\times 3)^x$

$=10\times 12^x$

$=10\times 10$

$=100$

Method 2:

$\frac{48^x}{3^{-x}}$

$=48^x\times 3^x$

$=(48\times 3)^x$

$=144^x$

$=\left(12^2\right)^x$

$=\left(12^x\right)^2$

$=10^2$

$=100$

The Mixed Exercise number 4 is on page **255**

5)

a) $7^x=49$

$\therefore 7^x=7^2$

$\therefore x=2$

b) $9^{3r-1}=27$

$\therefore \left(3^2\right)^{3r-1}=3^3$

$\therefore 3^{6r-2}=3^3$

$\therefore 6r-2=3$

$\therefore 6r=5$

$\therefore r=\frac{5}{6}$

c) $9^{2x}=\frac{1}{81}$

$\therefore \left(3^2\right)^{2x}=3^{-4}$

$\therefore 3^{4x}=3^{-4}$

$\therefore 4x=-4$

$\therefore x=-1$

d) $4^{2m}=16^{m-1}$

$\therefore \left(2^2\right)^{2m}=\left(2^4\right)^{m-1}$

$\therefore 2^{4m}=2^{4m-4}$

$\therefore 4m=4m-4$

$\therefore 0m=-4$

$\therefore$ No solution

The Mixed Exercise number 5 is on page **255**

6) 3^{x+1}

$=3^x(3)$

$=3p$

The Mixed Exercise number 6 is on page **256**

7)

Question and Solution	Error	Correct solution
Simplify $a^2 \times a^3$ $a^2 \times a^3 = a^6$	When multiplying like bases exponents should be added not multiplied.	$a^2 \times a^3 = a^{2+5} = a^5$
a) Simplify $\left(2x^2y^3\right)^2$ $\left(2x^2y^3\right)^2 = 2x^4y^6$	The exponent was only distributed to the variables.	$\left(2x^2y^3\right)^2 = 4x^4y^6$
b) Solve for x if $2 \times 2^x = 8$ $2 \times 2^x = 8$ $\therefore 4^x = 8$ $\therefore 2^{2x} = 2^3$ $\therefore 2x = 3$ $\therefore x = \frac{3}{2}$	You cannot multiply the bases together.	$2 \times 2^x = 8$ $\therefore 2^x = 4$ $\therefore 2^x = 2^2$ $\therefore x = 2$
c) Simplify $\left(9x^4 + 16x^4\right)^{\frac{1}{2}}$ $\left(9x^4 + 16x^4\right)^{\frac{1}{2}} = 3x^2 + 4x^2$ $= 7x^2$	You cannot distribute an exponent over a plus or minus sign.	$\left(9x^4 + 16x^4\right)^{\frac{1}{2}}$ $= \left(25x^2\right)^{\frac{1}{2}}$ $= 5x$
d) Solve for x if $2^{x+2} = \frac{1}{4}$ $2^{x+2} = \frac{1}{2^2}$ $\therefore x + 2 = 2$ $\therefore x = 0$	Bases are not the same on both sides of the equation. Therefore, you cannot equate the exponents.	$2^{x+2} = \frac{1}{4}$ $\therefore 2^{x+2} = 2^{-2}$ $\therefore x + 2 = -2$ $\therefore x = -4$
e) Solve for x if $2^{x+1} + 2^{x+1} = 16$ $2^{x+1} + 2^{x+1} = 16$ $\therefore 4^{x+1} = 4^2$ $\therefore x + 1 = 2$ $\therefore x = 1$	You cannot add the bases of exponents.	$2^{x+1} + 2^{x+1} = 16$ $\therefore 2 \times 2^{x+1} = 16$ $\therefore 2^{x+1} = 8$ $\therefore 2^{x+1} = 2^3$ $\therefore x + 1 = 3$ $\therefore x = 2$

The Mixed Exercise number 6 is on page **256**

Chapter 4 – Number patterns, Equations and Inequalities

Number Patterns (Theory on page 46-49)

Exercise 1:

1)

i) Write down the next three terms of the pattern.
ii) Write down a mathematical formula for the n^th^ term of the pattern.
iii) Hence, determine the 50th term in each pattern.

a) $-4\,;-2\,;0\,;\ldots$

b) $2\,;5\,;8\,;\ldots$

c) $29\,;21\,;13\,;\ldots$

d) $\frac{2}{3}\,;\frac{4}{3}\,;2\,;\ldots$

e) $5\,;\frac{19}{4}\,;\frac{9}{2}\,;\ldots$

f) $\frac{32}{3}\,;\frac{34}{3}\,;12\,;\ldots$

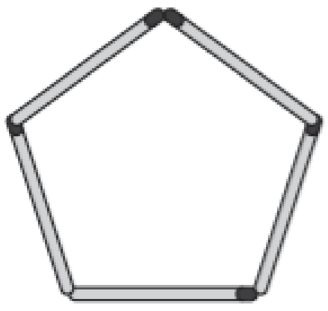
Solutions to Exercise 1 number 1 are on page **271**

2) In the pattern 10 ; 7 ; 4 ; 1 ;…

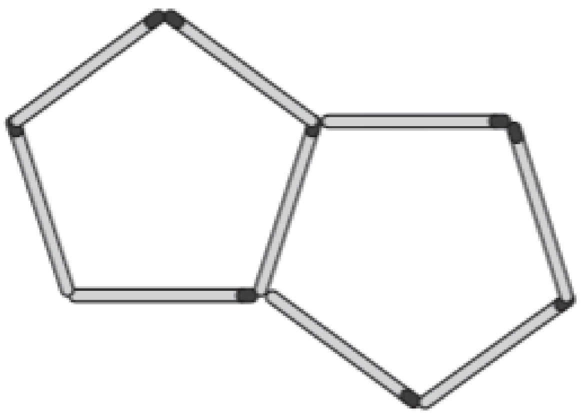
a) Determine the 5th term of the pattern.
b) Determine a formula for the nth term of the pattern.
c) Which term of the pattern will be equal to –11?

Solutions to Exercise 1 number 2 are on page **271**

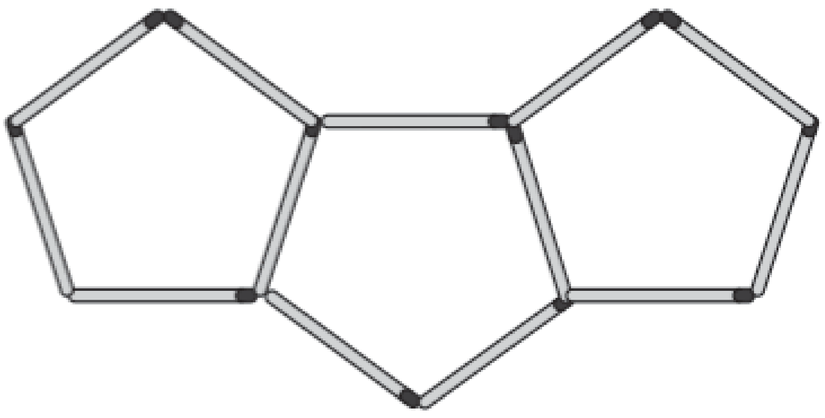
3) Pentagons are constructed using matchsticks as shown below:



1 Pentagon



2 Pentagons



3 Pentagons

a) Fill in the table below:

Number of pentagons (n)	Number of Matches (T_n)
1	
2	
3	
4	
5	

b) Determine a formula for the number of matches used to construct n pentagons.
c) How many matches would be needed to construct 22 pentagons?

Solutions to Exercise 1 number 3 are on page **271**

4) The pattern below shows a series of rectangles made from matchsticks. Determine:

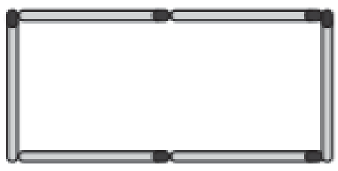


Figure 1

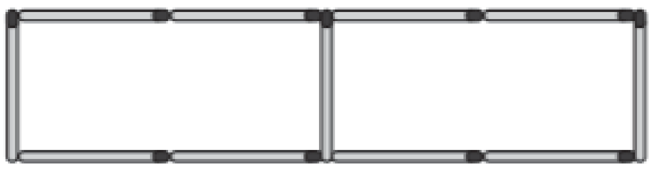


Figure 2

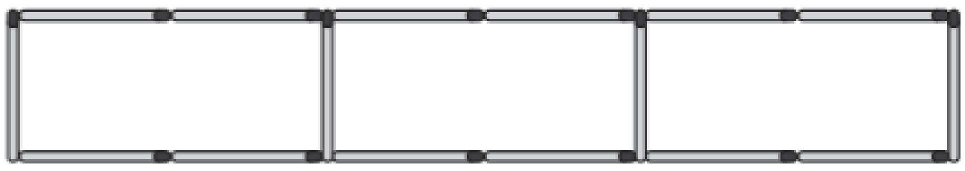


Figure 3

a) The number of matchsticks required to make the fourth figure.
b) A mathematical formula for the number of matchstick required for the nth figure.
c) The number of matchsticks required to make the 10th figure.
d) The figure that will contain 106 matchsticks.

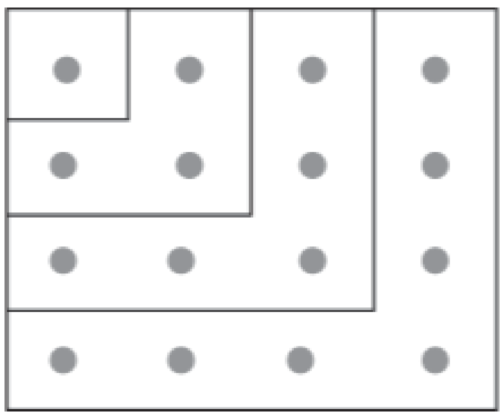
Solutions to Exercise 1 number 4 are on page **272**

(Theory on pages 50-52)

5) If the pattern MATHSMATHSMATHS… is continued what will be the 285th letter?

The solution to Exercise 1 number 5 is on page **272**

6) Thomas is investigating the pattern of dots on the carpet shown below: (Theory on pages 50-52)



He comes up with the following pattern to calculate the number of dots on any carpet of this type:

$\mathbf{1 = 1 = 1^2}$

$\mathbf{1 + 3 = 4 = 2^2}$

$\mathbf{1 + 3 + 5 = 9 = 3^2}$

$\mathbf{1 + 3 + 5 + 7 = 16 = 4^2}$

a) Determine the sum of the first 10 odd numbers.
b) Determine the sum of the odd numbers 1; 3 ;5 ; 7 ; … ; 41.
c) Determine the sum of the odd numbers between 50 and 500.
d) Determine a formula for the sum of the first *n* odd numbers.

The solution to Exercise 1 number 6 is on page **272**

Linear Equations

Exercise 2: (Theory on pages 53 and 54)

1) Solve for the unknown variable in the following equations:

a) $4a + 28 = 10 - 2a$
b) $2x + 30 = 12 - 7x$
c) $7p + 40 = 58 - 2p$
d) $46 - 7y = y + 6$
e) $3(2x+4) = 4(x-1)$
f) $7(n+2) + 4(n+6) = 137$
g) $9x = 18 + 5(x+2)$
h) $-3(2p-4) = 12 - 4(5p-7)$
i) $9(x+6) - 5(x+8) = 30 - (x-4)6$
j) $9r - (5r+3) = 25$
k) $(x+2)^2 = (x+1)(x-3) + 2x + 4$
l) $(b-3)(b+4) + 8b(b-1) = b + (3b-2)^2$

Solutions to Exercise 2 number 1 are on page **272**

2) Solve for the unknown variable in the following equations:

a) $\frac{a}{2} = 14 - \frac{a}{5}$
b) $\frac{5x+3}{12} - \frac{7x-6}{24} = 1$
c) $\frac{5x+10}{4} - \frac{5x-30}{3} = \frac{5}{2}$
d) $\frac{5}{x} + 5 = \frac{11}{x} + 7$
e) $10 - \frac{4(y+4)}{y} = \frac{14}{y}$
f) $\frac{6x-3}{5} - 3x = -15$
g) $\frac{7}{5x} - \frac{4}{x} = \frac{4(x+2)}{5x} + 2$
h) $\frac{6}{x+5} = \frac{3}{x+2}$
i) $\frac{4}{3y} - 2 = \frac{10}{3y} - \frac{2}{y}$
j) $\frac{5}{x+2} + \frac{3}{x^2+3x+2} = \frac{1}{x+1} + \frac{2}{x+2}$

Solutions to Exercise 2 number 2 are on page **273**

3) Solve for the *x* in the following equations and interpret your answers:

a) $2(3x+1) + 4(x-1) = 6x - 2(1-2x)$
b) $(x+2)^2 = x^2 + 4(x+5)$

Solutions to Exercise 2 number 3 are on page **273**

Quadratic Equations (Theory on pages 55- 57)

Exercise 3: Solve for the unknown variable in each of the following equations:

a) $x^2-5x+6=0$

b) $x^2+4x+3=0$

c) $x^2-2x-8=0$

d) $x^2-6x-16=0$

e) $2x^2-2x=4$

f) $x^2=1$

g) $x^2=4$

h) $2x^2=8$

i) $x^2-5x-14=0$

j) $x^2+4x+4=0$

k) $6x\left(x^2-36\right)=0$

l) $(x-3)(x+9)=0$

m) $(x-15)(x+31)=0$

n) $(2x+3)(5x+1)=0$

o) $(x+2)(x-3)=50$

p) $(x-5)(x-7)=3$

q) $x^3-3x^2+2x=0$

r) $3x^3-2x^2-8x=0$

s) $6x^2+7x=-2$

t) $9x^2+24x+15=0$

u) $84x^3+50x^2+6x=0$

v) $2(x+1)^2=(2x+2)(2x+3)$

Solutions to Exercise 3*a* and 3*b* are on page **273** and 3*c* to 3*v* are on page **274**

Literal Equations (Theory on page 58)

Exercise 4:

1) Change the subject in each of the formulas as shown below:

a) $V=l\times b\times h$ $\qquad h=?$

b) $y=mx+c$ $\qquad x=?$

c) $v=u+at$ $\qquad a=?$

d) $R=\dfrac{V}{I}$ $\qquad I=?$

e) $E=mc^2$ $\qquad c=?$

f) $A=\dfrac{1}{2}b.h$ $\qquad h=?$

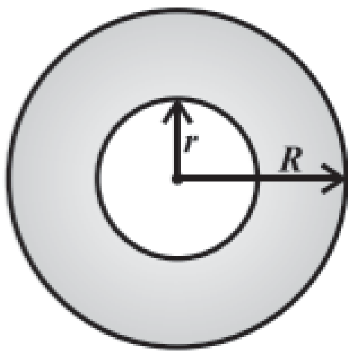
g) $s=ut+\dfrac{1}{2}at^2$ $\qquad a=?$

h) $x=\dfrac{pM}{m+M}$ $\qquad M=?$

i) $s=\dfrac{v+u}{2}\times t$ $\qquad u=?$

Solutions to Exercise 4 number 1 are on page **275**

2) Given the diagram alongside and that the area of a circle is: $A_{\bigcirc}=\pi r^2$



a) Determine the area of the shaded region in terms of *r* and *R*.

b) Make *R* the subject of the formula for the area of the shaded region.

Solutions to Exercise 4 number 2 are on page **275**

Number line Revision (Theory on pages 59 and 60)

Exercise 5: Solve for *x* in each of the following inequalities and illustrate your answer on number line:

1) Represent the following sets on a number line:

a) $\{x : x\geq 3 ; x\in\mathbb{R}\}$

b) $\{x : x<2 ; x\in\mathbb{Z}\}$

c) $\{x : -2<x\leq 3\}$

d) $\{x : -2\leq x<3 ; x\in\mathbb{Z}\}$

e) $\{x : -5\leq x\leq 3 ; x\in\mathbb{R}\}$

f) $\{x : -5\leq x\leq 0 ; x\in\mathbb{R}\}$

g) $\{x : 5\leq x<9 ; x\in\mathbb{N}\}$

h) $\{2x : 1\leq x<5 ; x\in\mathbb{Z}\}$

i) $\{x : x\leq -2 \text{ or } x>1\}$

Solutions to Exercise 5 number 1 are on page **275**

2) Represent the following on a number line:

a) $(-\infty;8)$

b) $(3;8]$

c) $[9;14)$

d) $[-1;4]$

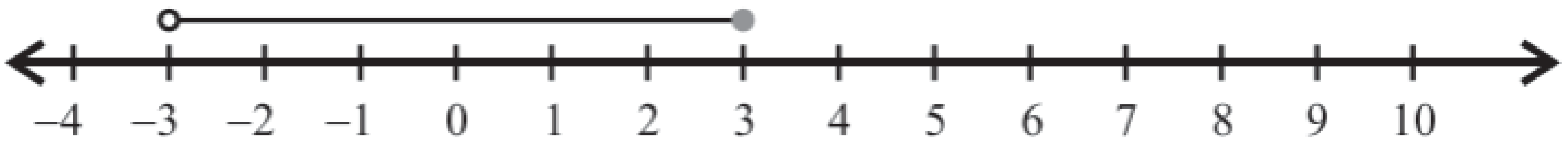
e) $[-5;\infty)$

f) $(-2;3)$

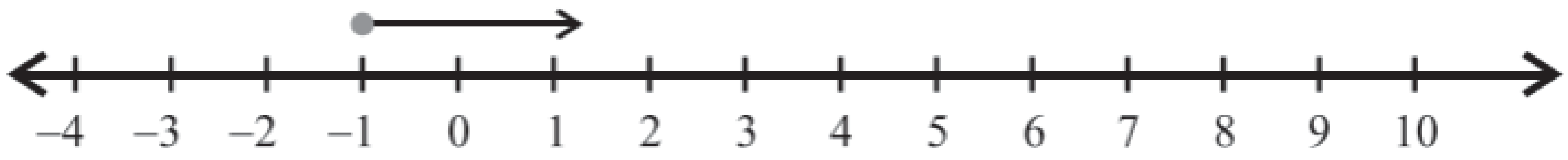
Solutions to Exercise 5 number 2 are on page **275**

3) Represent the following in set builder notation and interval notation (if possible):

a)

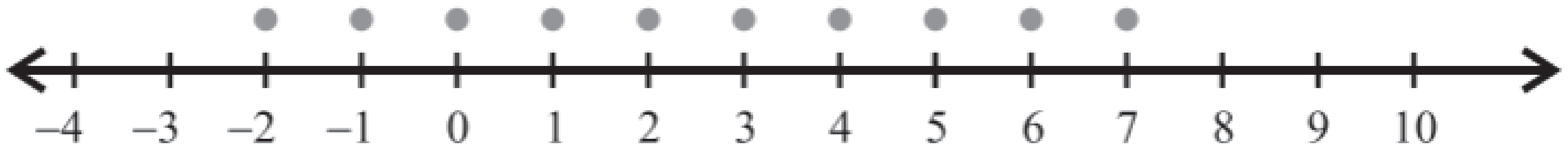


b)

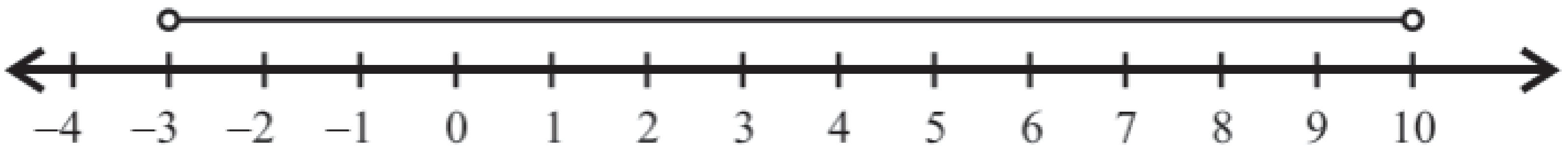


Solutions to Exercise 5 number 3 are on page **276** (Exercises 5 number 3 is continued on the next page)

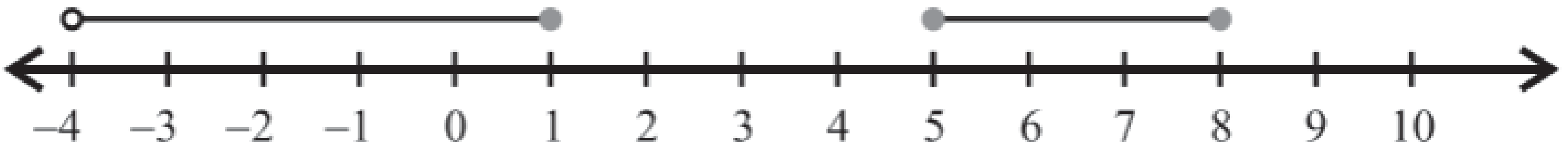
c)



d)



e)



Solutions to Exercise 5 number 3 are on page **276**

Linear Inequalities (Theory on pages 61 and 62)

Exercise 6: Solve for *x* in each of the following inequalities and illustrate your answer on a number line:

a) $7x-5>3x+11$

b) $16x-26\le 14-4x$

c) $-3x\ge 9$

d) $\frac{x-5}{3}-2x+10<\frac{x-5}{2}$

e) $-5x+30<20$

f) $\frac{1}{2}(x-8)<2x-16$

g) $5<x+3\le 9$

h) $-1<3-x<4$

i) $-1<2x-3\le 5$

Solutions to Exercise 6 are on page **276**

Simultaneous Equations

Exercise 7: (Theory on pages 63 and 64)

1) Solve for *x* and *y* in the following equations using the method of elimination:

a) $8x+3y=28$ and $7x-3y=2$

b) $27x+3y=120$ and $5x-3y=8$

c) $3x+2y=22$ and $10x+2y=36$

d) $5x+3y=36$ and $y=x-4$

e) $3x-5y=9$ and $x-y=5$

f) $3x+4y=26$ and $x+2y=12$

Solutions to Exercise 7 numbers 1*a* and 1*b* are on page **276** and 1*c* to 1*f* are on page **277**

(Theory on pages 65 and 66)

2) Solve for *x* and *y* in the following equations using the method of substitution:

a) $2x+4y=20$ and $7x-3y=19$

b) $3x+y=2$ and $5x-3y=-34$

c) $3(x+y)+y=-8$ and $2x+y=-7$

d) $2x+3y=12$ and $2x+8y=62$

e) $\frac{x}{2}+\frac{y}{3}=10$ and $x+\frac{y}{2}=16$

f) $2x+3y=15$ and $5x-4y=-43$

Solutions to Exercise 7 numbers 2*a* and *b* are on page **277** and 2*c* to 2*f* are on page **278**

Mathematical Modelling

Exercise 8: (Theory on page 67)

1) Roger has twice as many sweets as Dan. If Roger gives Dan 5 sweets he will have 5 more sweets than Dan. How many sweets does Dan have?

2) A beef sandwich costs R2 more than a cheese sandwich. If 3 beef and 5 cheese sandwiches cost R78, what is the cost of a beef sandwich?

3) Jenna is 4 times as old as Danni. In 10 years she will be twice as old Danni. How old are Jenna and Danni?

4) Ray is 20 years older than Tim. In 6 years, Ray will be 3 times as old as Tim. How old is Tim?

5) The sum of 2 consecutive numbers is 22 less than 3 times the sum of the numbers. What are the numbers?

6) Twice the sum of two consecutive odd numbers is 22 more than their difference. What are the numbers?

7) Two thirds of a number is 1 more than half the same number. What is the number?

8) If a number multiplied by 8 and then increased by 5 is 20 more than the square of the number, what is the number?

Solutions to Exercise 8 numbers 1 and 2 are on page **278** and numbers 3 to 8 are on page **279**

Exercise 9: (Theory on pages 68 and 69)

1) Tom is raising money for an overseas holiday by selling Tie Dye T-shirts at a flea market. Tom has 60 T-shirts to sell and sells them for R80 per T-shirt. Towards the end of the day he has a sale and sells the remaining T-shirts for R70 each. If Tom made a total of R4 500, how many T-shirts did he sell at R70?

2) Joe buys 30m of plain steel tubing and 24m of galvanised steel tubing at double the price of the plain steel. In total he pays R6 240 for the tubing. Determine the price per meter of each type of steel tubing.

3) Joanne's school has a fun day at which she sells brownies. Each brownie costs her R1,50 to make and she sells them for R5,50 each. If she makes a profit of R120,50 on her sales and she sells all but 5 of her brownies, how many brownies did she make?

4) Daniel buys 1,2kg of sour worms and gobstoppers for a party. Sour worms cost R99 per kilogram and gobstoppers cost R85 per kilogram. How many grams of each sweet did Daniel buy if he paid R109 in total for all his sweets?

Solutions to Exercise 9 numbers 1 and 2 are on page **279** and numbers 3 and 4 are on page **280**

Exercise 10: (Theory on pages 70-72)

1) Tom and Jeff cycle towards each other from their farms which are 40 km apart. Tom cycles at an average speed of 9 km/h and Jeff cycles at an average speed of 11 km/h. How long will it take for them to meet?

2) Ryan took part in a race. The race required him to run a certain distance and then cycle the same distance. If Ryan runs at 8 km/h and cycles at 18 km/h and the race took him 8 hours and 40 min to complete, how long was the race?

3) Jake and his brother have a race. Jake gives his brother a 15 second head start. If Jake can run at 2,5 m/s and his brother can run at 2 m/s, how long will it take Jake to catch his brother?

Solutions to Exercise 10 numbers 1 and 2 are on page **280** and number 3 is on page **281**

Exercise 11: (Theory on pages 73-75)

1) The sum of two numbers is 40 and their difference is 8. What are the numbers?

2) Harry has a rectangular vegetable garden which has a perimeter of 18m. By doubling its length and increasing the width by 3 he doubles the perimeter of the garden. What were the original dimensions of the garden?

3) There are two types of tickets to King of the Dance, a local theatre production. Balcony tickets cost R350 per ticket and floor seats cost R500 per ticket. If the show is sold out and there are 1200 seats available in total, how many of each type of ticket was sold if R480 000 was made on the sale of tickets?

4) If a two digit number is divided by the tens digit the answer is 13. When the digits are reversed the new number exceeds the old number by 36. Determine what the number is by letting the tens digit be x and the units digit be y.

Solutions to Exercise 11 are on page **281**

Mixed Exercise: (Chapter 4 is on pages 45-75)

1) Solve for x if:

a) $x(x+3)=0$

b) $5(x+1)=3(2x-4)$

c) $3(x-2)-2(2x+1)=7-(3x+2)$

d) $(2x-5)(3x+2)=2(3x^2-4x+1)$

e) $2(x-3)-x=-3(x-2)$

f) $(x+1)(x-2)=4$

Solutions to the Mixed Exercise number 1 are on page **282**

2) Solve for x if:

a) $2x(x-1)=4$

b) $4x^2=16x$

c) $3(x+1)-(3-3x)=3x-(10-3x)$

d) $\frac{5x+8}{6}-\frac{x}{4}=\frac{2x-9}{3}$

e) $5+\frac{8}{x}=\frac{2(x+4)}{x}$

f) $(x-3)(x+4)-(3x-2)^2=x-8x(x-1)$

Solutions to the Mixed Exercise number 2a to 2f are on page **282** (questions are continued on the next page)

g) $4x^2-8x=0$

h) $x^2-8x=-12$

i) $3x^2-18x+27=0$

j) $x^4-13x^2+36=0$

k) $\frac{5x+2}{3}+\frac{x}{5}=\frac{3x-5}{15}+x$

l) $\frac{2(x-4)}{3}=x+1-\frac{x}{3}$

m) $(2x+1)(x+8)=27$

Solutions to the Mixed Exercise numbers 2g to 2j are on page **282** and numbers 2k to 2m are on page **283**

3) Solve for *x* in terms of *p* and *q* in the following equations:

a) $\frac{x-p}{2q}=3$ **b)** $p^2+px=q^2+qx$ **c)** $(x-p)^2=(x-q)^2$

Solutions to the Mixed Exercise number 3 are on page **283**

4) Using the following equation it is possible to convert a temperature in degrees Fahrenheit to degrees Celsius:

$$C=\frac{5(F-32)}{9}$$

a) Change the equation to convert degrees Celsius to degrees Fahrenheit.

b) Hence, determine the temperature in Fahrenheit if it is 37°C.

Solutions to the Mixed Exercise number 4 are on page **283**

5) Solve for *x* in the following inequalities:

a) $4(2x-1)>5x+2$

b) $2x-7\geq 4x-10$

c) $\frac{1-2x}{-3}\geq 2x+1$

d) $3<-2x+1<7$

Solutions to the Mixed Exercise number 5 are on page **283**

6) Answer the following questions:

a) Solve for p if: $-4<1-2p\leq 5$

b) Represent your answer on a number line if $p\in\mathbb{R}$.

c) Represent your answer on a number line if $p\in\mathbb{Z}$.

Solutions to the Mixed Exercise number 6 are on page **283**

7) Solve for the unknown variable in each of the following equations:

a) $7^x=49$ **b)** $9^{3r-1}=27$ **c)** $9^{2x}=\frac{1}{81}$ **d)** $4^{2m}=16^{m-1}$

Solutions to the Mixed Exercise number 7 are on page **284**

8) Given that $3^x=p$ express 3^{x+1} in terms of *p*.

Solutions to the Mixed Exercise number 8 are on page **284**

9) Solve for *x* and *y* in the following equations:

a) $y=2x+3$ and $y=-3x-7$

b) $2x-3y=5$ and $3x+6y=11$

Solutions to the Mixed Exercise number 9 are on page **284**

10) Without solving simultaneously, determine the value of $(2p+q)^2$ if $4p^2+q^2=21$ and $pq=2$.

Solutions to the Mixed Exercise number 10 are on page **284**

11) Solve the following problems by making use of appropriate equations:

a) Determine the length and breadth of a rectangle if its length is 5m longer than its breadth and it has a perimeter of 70m.

b) Two motorists undertake a journey by car. One motorist travels at an average speed of 100 km/h and the other travels at an average speed of 120 km/h. If the faster motorist arrives 1,5 hours earlier than the slower motorist, how long did the slower motorist take?

c) An aeroplane can complete a journey of 480 km in 4 hours less than a car which can travel at one fifth of the speed of an aeroplane. How fast can the car travel?

Solutions to the Mixed Exercise number 11a are on page **284** and 11b an 11c are on page **285**

12) A man drives 600km from Johannesburg to Durban. He drives at an average speed of 120km/h. Between Pietermaritzburg and Durban he gets a puncture. Luckily for the man he has new aged run flat tyres and can continue his journey. However, to continue safely he has to decrease his speed to 80km/h. let x be the time he travelled at 120km/h and y be the time he travelled at 80km/h. The entire journey takes him 5 and a half hours.

- **a)** Given that $\text{distance} = \text{speed} \times \text{time}$ determine an equation in terms of x and y for the total distance that the man drove.
- **b)** Determine an equation in terms of x and y for the total time that the man spent driving.
- **c)** Use your equations to determine how far from Durban the man got a puncture.

Solutions to the Mixed Exercise number 12 are on page **285**

13) Consider the following series of triangles below:

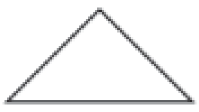


Figure 1

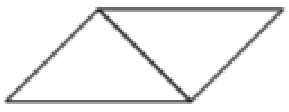


Figure 2

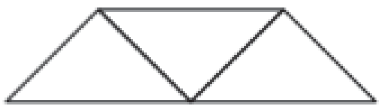
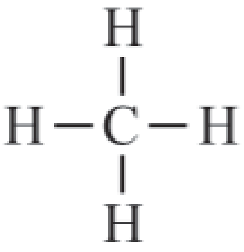


Figure 3

- **a)** How many lines are required to draw the 4th figure?
- **b)** Determine a formula for the number of lines in the n^{th} figure.
- **c)** Use your formula to determine the number of triangles formed if there are 19 lines.

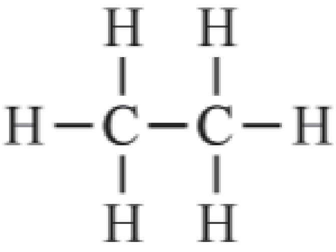
Solutions to the Mixed Exercise number 13 are on page **285**

14) Figures 1 to 3 below show the 3 simplest alkanes possible. In the following alkanes each carbon atom has 4 single bonds with each carbon bonded to a single carbon atom and at least 2 hydrogen atoms as shown below.



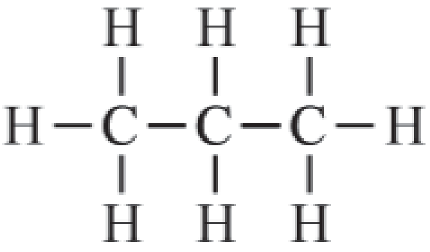
$$\begin{array}{ccccc} & & \mathrm{H} & & \\ & & | & & \\ \mathrm{H} & - & \mathrm{C} & - & \mathrm{H} \\ & & | & & \\ & & \mathrm{H} & & \end{array}$$

Figure 1



$$\begin{array}{ccccccc} & & \mathrm{H} & & \mathrm{H} & & \\ & & | & & | & & \\ \mathrm{H} & - & \mathrm{C} & - & \mathrm{C} & - & \mathrm{H} \\ & & | & & | & & \\ & & \mathrm{H} & & \mathrm{H} & & \end{array}$$

Figure 2



$$\begin{array}{ccccccccc} & & \mathrm{H} & & \mathrm{H} & & \mathrm{H} & & \\ & & | & & | & & | & & \\ \mathrm{H} & - & \mathrm{C} & - & \mathrm{C} & - & \mathrm{C} & - & \mathrm{H} \\ & & | & & | & & | & & \\ & & \mathrm{H} & & \mathrm{H} & & \mathrm{H} & & \end{array}$$

Figure 3

- **a)** Draw an alkane that has 4 carbon atoms.
- **b)** Determine a formula for the number of hydrogen atoms in an alkane (T_n) when there are n carbons.

Solutions to the Mixed Exercise number 14 are on page **286**

15) Shamira is investigating the following number pattern involving consecutive numbers starting at 1:

$$\frac{1+2}{3}=1 \qquad \frac{1+2+3}{4}=\frac{3}{2} \qquad \frac{1+2+3+4}{5}=\frac{4}{2} \qquad \frac{1+2+3+4+5}{6}=\frac{5}{2}$$

a) Write down the answer to the next step in the pattern.

b) Determine the value of : $\dfrac{1+2+3+4+5+6+7+8+9+10}{11}$

c) Determine the value of : $\dfrac{1+2+3+...+40}{41}$

d) Shamira now notices the following:

$$1+2=3 \text{ and } \frac{(2)(3)}{2}=\frac{6}{2}=3 \qquad \text{and} \qquad 1+2+3=6 \text{ and } \frac{(3)(4)}{2}=\frac{12}{2}=6$$

Assist Shamira in determining a formula for the sum of consecutive numbers from 1 to n.

Solutions to the Mixed Exercise number 15 are on page **286**

Chapter 4 – Number patterns, Equations and Inequalities (Solutions)

Exercise 1 Solutions:

1)

a)

i) 2 ; 4 ; 6

ii) $T_n = dn + c$

$\therefore T_n = 2n + c$

Subst $T_1 = -4$

$\therefore -4 = 2(1) + c$

$\therefore c = -6$

$\therefore T_n = 2n - 6$

iii) $T_{50} = 2(50) - 6 = 94$

b)

i) 11 ; 14 ; 17

ii) $T_n = dn + c$

$\therefore T_n = 3n + c$

Subst $T_1 = 2$

$\therefore 2 = 3(1) + c$

$\therefore c = -1$

$\therefore T_n = 3n - 1$

iii) $T_{50} = 3(50) - 1 = 149$

c)

i) 5 ; –3 ; –11

ii) $T_n = dn + c$

$\therefore T_n = -8n + c$

Subst $T_1 = 29$

$\therefore 29 = -8(1) + c$

$\therefore c = 37$

$\therefore T_n = -8n + 37$

iii) $T_{50} = -8(50) + 37 = -363$

d)

i) $\frac{8}{3} ; \frac{10}{3} ; 4$

ii) $T_n = dn + c$

$\therefore T_n = \frac{2}{3}n + c$

Subst $T_1 = \frac{2}{3}$

$\therefore \frac{2}{3} = \frac{2}{3}(1) + c$

$\therefore c = 0$

$\therefore T_n = \frac{2}{3}n$

iii) $T_{50} = \frac{2}{3}(50) = \frac{100}{3}$

e)

i) $\frac{17}{4} ; 4 ; \frac{15}{4}$

ii) $T_n = dn + c$

$\therefore T_n = -\frac{1}{4}n + c$

Subst $T_1 = 5$

$\therefore 5 = -\frac{1}{4}(1) + c$

$\therefore c = \frac{21}{4}$

$\therefore T_n = -\frac{1}{4}n + \frac{21}{4}$

iii) $T_{50} = -\frac{1}{4}(50) + \frac{21}{4} = -\frac{29}{4}$

f)

i) $\frac{38}{3} ; \frac{40}{3} ; 14$

ii) $T_n = dn + c$

$\therefore T_n = \frac{2}{3}n + c$

Subst $T_1 = \frac{32}{3}$

$\therefore \frac{32}{3} = \frac{2}{3}(1) + c$

$\therefore c = 10$

$\therefore T_n = \frac{2}{3}n + 10$

$T_{50} = \frac{2}{3}(50) + 10 = \frac{130}{3}$

Exercise 1 number 1 is on page **264**

2)

a) $T_5 = -2$

b) $T_n = dn + c$

$\therefore T_n = -3n + c$

Subst $T_1 = 10$

$\therefore 10 = -3(1) + c$

$\therefore c = 13$

$\therefore T_n = -3n + 13$

c) $T_n = -11$

$\therefore -11 = -3n + 13$

$\therefore -24 = -3n$

$\therefore n = 8$

$\therefore T_8 = -11$

Exercise 1 number 2 is on page **264**

3)

a)

Number of pentagons (n)	Number of Matches (T_n)
1	5
2	9
3	13
4	17
5	21

a) $T_n = dn + c$

$\therefore T_n = 4n + c$

Subst $T_1 = 5$

$\therefore 5 = 4(1) + c$

$\therefore c = 1$

$\therefore T_n = 4n + 1$

b) $T_{22} = 4(22) + 1$

$= 89$ matchsticks

Exercise 1 number 3 is on page **264**

4)

a) 21

b) $T_n = dn + c$

$\therefore T_n = 5n + c$

Subst $T_1 = 6$

$\therefore 6 = 5(1) + c$

$\therefore c = 1$

$\therefore T_n = 5n + 1$

c) $T_{10} = 5(10) + 1$

$= 51$ matchsticks

d) $T_n = 106$

$\therefore 106 = 5n + 1$

$\therefore 105 = 5n$

$\therefore n = 21$ rectangles

$\therefore$ The 21st figure will contain 106 matchsticks

Exercise 1 number 4 is on page **264**

5) The pattern MATHSMATHS… repeats itself every 5 letters. Therefore, every 5th letter is an S. As 285 is divisible by 5 the 285th letter of the pattern will be an S.

Exercise 1 number 5 is on page **264**

6)

a) Sum of the first 10 odd numbers $= 10^2 = 100$

b) There are 20 odd terms up until 40. Therefore there are 21 odd number between 1 and 41.

$\therefore$ sum of the first 21 odd numbers $= 21^2 = 441$

c) The sum of the odd numbers between 50 and 500 will be the difference between the sum of the odd numbers between 1 and 50 and the sum of the odd numbers between 1 and 500.
There are 25 odd numbers from 1 to 50 and 250 odd numbers from 1 to 500:

$\text{Sum}_{\textit{odd numbers up to 50}} = 25^2 = 625$ $\qquad$ $\text{Sum}_{\textit{odd numbers up to 500}} = 250^2 = 62500$

$\therefore \text{Sum}_{\textit{odd numbers between 50 and 500}} = 62500 - 625 = 61875$

d) $S_n = n^2$

Exercise 1 number 6 is on page **265**

Exercise 2 Solutions:

1)

a) $4a + 28 = 10 - 2a$

$\therefore 6a = -18$

$\therefore a = -3$

b) $2x + 30 = 12 - 7x$

$\therefore 9x = -18$

$\therefore x = -2$

c) $7p + 40 = 58 - 2p$

$\therefore 9p = 18$

$\therefore p = 2$

d) $46 - 7y = y + 6$

$\therefore 40 = 8y$

$\therefore y = 5$

e) $3(2x + 4) = 4(x - 1)$

$\therefore 6x + 12 = 4x - 4$

$\therefore 2x = -16$

$\therefore x = -8$

f) $7(n + 2) + 4(n + 6) = 137$

$\therefore 7n + 14 + 4n + 24 = 137$

$\therefore 11n = 99$

$\therefore n = 9$

g) $9x = 18 + 5(x + 2)$

$\therefore 9x = 18 + 5x + 10$

$\therefore 4x = 28$

$\therefore x = 7$

h) $-3(2p - 4) = 12 - 4(5p - 7)$

$\therefore -6p + 12 = 12 - 20p + 28$

$\therefore 14p = 28$

$\therefore p = 2$

i) $9(x + 6) - 5(x + 8) = 30 - (x - 4)6$

$\therefore 9x + 54 - 5x - 40 = 30 - 6x + 24$

$\therefore 10x = 40$

$\therefore x = 4$

j) $9r - (5r + 3) = 25$

$\therefore 9r - 5r - 3 = 25$

$\therefore 4r = 28$

$\therefore r = 7$

k) $(x + 2)^2 = (x + 1)(x - 3) + 2x + 4$

$\therefore x^2 + 4x + 4 = x^2 - 2x - 3 + 2x + 4$

$\therefore 4x = -3$

$\therefore x = \frac{-3}{4}$

l) $(b - 3)(b + 4) + 8b(b - 1) = b + (3b - 2)^2$

$\therefore b^2 + b - 12 + 8b^2 - 8b = b + 9b^2 - 12b + 4$

$\therefore 4b = 16$

$\therefore b = 4$

Exercise 2 number 1 is on page **265**

2)

a) $\frac{a}{2}=14-\frac{a}{5}$

$\therefore 5a=140-2a$

$\therefore 7a=140$

$\therefore a=20$

b) $\frac{5x+3}{12}-\frac{7x-6}{24}=1$

$\therefore 2(5x+3)-(7x-6)=24$

$\therefore 10x+6-7x+6=24$

$\therefore 3x=12$

$\therefore x=4$

c) $\frac{5x+10}{4}-\frac{5x-30}{3}=\frac{5}{2}\rightarrow \text{LCD}=12$

$\therefore 3(5x+10)-4(5x-30)=6(5)$

$\therefore 15x+30-20x+120=30$

$\therefore -5x=-120$

$\therefore x=24$

d) $\frac{5}{x}+5=\frac{11}{x}+7$, LCD $=x$ and $x\neq 0$

$\therefore 5+5x=11+7x$

$\therefore -2x=6$

$\therefore x=-3$

e) $10-\frac{4(y+4)}{y}=\frac{14}{y},\ y\neq 0$

$\therefore 10y-4(y+4)=14$

$\therefore 10y-4y-16=14$

$\therefore 6y=30$

$\therefore y=5$

f) $\frac{6x-3}{5}-3x=-15$

$\therefore 6x-3-5(3x)=5(-15)$

$\therefore 6x-3-15x=-75$

$\therefore -9x=-72$

$\therefore x=8$

g) $\frac{7}{5x}-\frac{4}{x}=\frac{4(x+2)}{5x}+2$

$\therefore 7-5(4)=4(x+2)+5x(2)$ and $x\neq 0$

$\therefore 7-20=4x+8+10x$

$\therefore -21=14x$

$\therefore x=-\frac{21}{14}$

$=-\frac{3}{2}$

h) $\frac{6}{x+5}=\frac{3}{x+2}$, $x\neq -5$ and $x\neq -2$

$\therefore 6(x+2)=3(x+5)$

$\therefore 6x+12=3x+15$

$\therefore 3x=3$

$\therefore x=1$

i) $\frac{4}{3y}-2=\frac{10}{3y}-\frac{2}{y}$

$\therefore 4-3y(2)=10-3(2)$ and $y\neq 0$

$\therefore 4-6y=10-6$

$\therefore -6y=0$

$\therefore y=0$ but $y\neq 0$

$\therefore$ no solution

j) $\frac{5}{x+2}+\frac{3}{x^2+3x+2}=\frac{1}{x+1}+\frac{2}{x+2}$

$\therefore \frac{5}{x+2}+\frac{3}{(x+2)(x+1)}=\frac{1}{x+1}+\frac{2}{x+2}$

$\therefore 5(x+1)+3=x+2+2(x+1)$; $x\neq -2$ and $x\neq -1$

$\therefore 5x+5+3=x+2+2x+2$

$\therefore 2x=-4$

$\therefore x=-2$ but $x\neq -2$

$\therefore$ No solution

Exercise 2 number 2 is on page **265**

3)

a) $2(3x+1)+4(x-1)=6x-2(1-2x)$

$\therefore 6x+2+4x-4=6x-2+4x$

$\therefore 10x-2=10x-2$

$\therefore 0x=0$

$\therefore x\in\mathbb{R}$

This means that this equation will be true for any value of x.

b) $(x+2)^2=x^2+4(x+5)$

$\therefore x^2+4x+4=x^2+4x+20$

$\therefore x^2+4x-x^2-4x=20-4$

$\therefore 0x=16$

$\therefore$ No solution

This means that there is no value for x that will make this equation true.

Exercise 2 number 3 is on page **265**

Exercise 3 Solutions:

a) $x^2-5x+6=0$

$\therefore (x-2)(x-3)=0$

$\therefore x=2 \quad or \quad x=3$

b) $x^2+4x+3=0$

$\therefore (x+3)(x+1)=0$

$\therefore x=-3 \quad or \quad x=-1$

Exercise 3 number 1 is on page **266**

c) $x^2 - 2x - 8 = 0$
$\therefore (x+2)(x-4) = 0$
$\therefore x = -2 \quad or \quad x = 4$

d) $x^2 - 6x - 16 = 0$
$\therefore (x+2)(x-8) = 0$
$\therefore x = -2 \quad or \quad x = 8$

e) $2x^2 - 2x = 4$
$\therefore 2x^2 - 2x - 4 = 0$
$\therefore x^2 - x - 2 = 0$
$\therefore (x+1)(x-2) = 0$
$\therefore x = -1 \quad or \quad x = 2$

f) $x^2 = 1$
$\therefore x^2 - 1 = 0$
$\therefore (x+1)(x-1) = 0$
$\therefore x = -1 \quad or \quad x = 1$

g) $x^2 = 4$
$\therefore x^2 - 4 = 0$
$\therefore (x+2)(x-2) = 0$
$\therefore x = -2 \quad or \quad x = 2$

h) $2x^2 = 8$
$\therefore 2x^2 - 8 = 0$
$\therefore x^2 - 4 = 0$
$\therefore (x+2)(x-2) = 0$
$\therefore x = -2 \quad or \quad x = 2$

i) $x^2 - 5x - 14 = 0$
$\therefore (x+2)(x-7) = 0$
$\therefore x = -2 \quad or \quad x = 7$

j) $x^2 + 4x + 4 = 0$
$\therefore (x+2)(x+2) = 0$
$\therefore x = -2$

k) $6x\left(x^2 - 36\right) = 0$
$\therefore x(x-6)(x+6) = 0$
$\therefore x = 0 \quad or \quad x = 6 \quad or \quad x = -6$

l) $(x-3)(x+9) = 0$
$\therefore x = 3 \quad or \quad x = -9$

m) $(x-15)(x+31) = 0$
$\therefore x = 15 \quad or \quad x = -31$

n) $(2x+3)(5x+1) = 0$
$\therefore x = -\frac{3}{2} \quad or \quad x = -\frac{1}{5}$

o) $(x+2)(x-3) = 50$
$\therefore x^2 - x - 6 = 50$
$\therefore x^2 - x - 56 = 0$
$\therefore (x+7)(x-8) = 0$
$\therefore x = -7 \quad or \quad x = 8$

p) $(x-5)(x-7) = 3$
$\therefore x^2 - 12x + 35 = 3$
$\therefore x^2 - 12x + 32 = 0$
$\therefore (x-8)(x-4) = 0$
$\therefore x = 8 \quad or \quad x = 4$

q) $x^3 - 3x^2 + 2x = 0$
$\therefore x\left(x^2 - 3x + 2\right) = 0$
$\therefore x(x-1)(x-2) = 0$
$\therefore x = 0 \text{ or } x = 1 \text{ or } x = 2$

r) $3x^3 - 2x^2 - 8x = 0$
$\therefore x\left(3x^2 - 2x - 8\right) = 0$
$\therefore x(3x+4)(x-2) = 0$
$\therefore x = 0 \text{ or } x = -\frac{4}{3} \text{ or } x = 2$

s) $6x^2 + 7x = -2$
$\therefore 6x^2 + 7x + 2 = 0$
$\therefore (2x+1)(3x+2) = 0$
$\therefore x = -\frac{1}{2} \quad or \quad x = -\frac{2}{3}$

t) $9x^2 + 24x + 15 = 0$
$\therefore 3\left(3x^2 + 8x + 5\right) = 0$
$\therefore (3x+5)(x+1) = 0$
$\therefore x = -\frac{5}{3} \quad or \quad x = -1$

u) $84x^3 + 50x^2 + 6x = 0$
$\therefore 2x\left(42x^2 + 25x + 3\right) = 0$
$\therefore x(7x+3)(6x+1) = 0$
$\therefore x = 0 \quad or \quad x = -\frac{3}{7} \quad or \quad x = -\frac{1}{6}$

v) $2(x+1)^2 = (2x+2)(2x+3)$
$\therefore 2\left(x^2 + 2x + 1\right) = 4x^2 + 10x + 6$
$\therefore 2x^2 + 4x + 2 = 4x^2 + 10x + 6$
$\therefore 0 = 2x^2 + 6x + 4$
$\therefore x^2 + 3x + 2 = 0$
$\therefore (x+2)(x+1) = 0$
$\therefore x = -2 \quad or \quad x = -1$

Exercise 3 number 1 is on page **266**

Exercise 4 Solutions:

1)

a) $V = l \times b \times h$

$\therefore h = \dfrac{V}{l \times b}$

b) $y = mx + c$

$\therefore y - c = mx$

$\therefore x = \dfrac{y - c}{m}$

c) $v = u + at$

$\therefore v - u = at$

$\therefore a = \dfrac{v - u}{t}$

d) $R = \dfrac{V}{I}$

$\therefore IR = V$

$\therefore I = \dfrac{V}{R}$

e) $E = mc^2$

$\therefore c^2 = \dfrac{E}{m}$

$\therefore c = \sqrt{\dfrac{E}{m}}$

f) $A = \dfrac{1}{2} b.h$

$\therefore 2A = b.h$

$\therefore h = \dfrac{2A}{b}$

g) $s = ut + \dfrac{1}{2} at^2$

$\therefore s - ut = \dfrac{1}{2} at^2$

$\therefore 2s - 2ut = at^2$

$\therefore a = \dfrac{2s - 2ut}{t^2}$

h) $x = \dfrac{pM}{m + M}$

$\therefore x(m + M) = pM$

$\therefore xm + xM = pM$

$\therefore xm = pM - xM$

$\therefore xm = M(p - x)$

$\therefore M = \dfrac{xm}{p - x}$

i) $s = \dfrac{v + u}{2} \times t$

$\therefore 2s = t(v + u)$

$\therefore 2s = tv + tu$

$\therefore tu = 2s - tv$

$\therefore u = \dfrac{2s - tv}{t}$

Exercise 4 number 1 is on page 266

2)

a) $\text{A}_{\text{shaded}} = A_{big\ cirlce} - A_{small\ circle}$

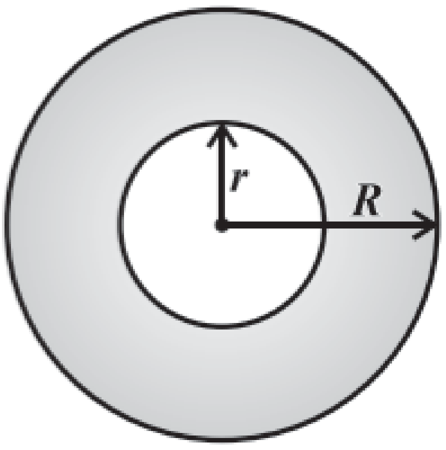
$\therefore A = \pi R^2 - \pi r^2$

b) $A = \pi R^2 - \pi r^2$

$\therefore A + \pi r^2 = \pi R^2$

$\therefore R^2 = \dfrac{A + \pi r^2}{\pi}$

$\therefore R = \sqrt{\dfrac{A + \pi r^2}{\pi}}$

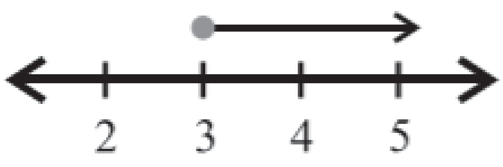


Exercise 4 number 2 is on page 266

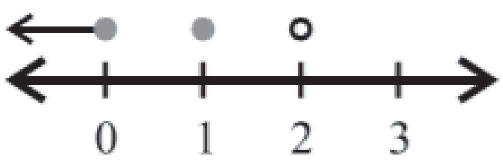
Exercise 5 Solutions:

1)

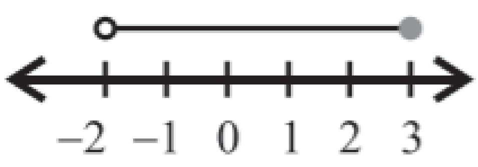
a)



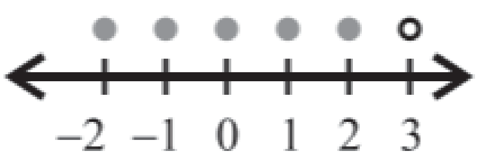
b)



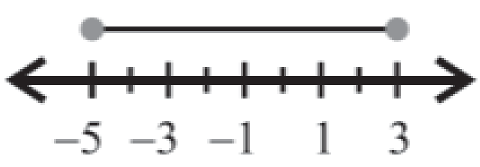
c)



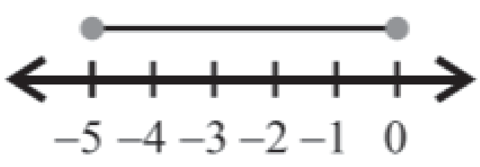
d)



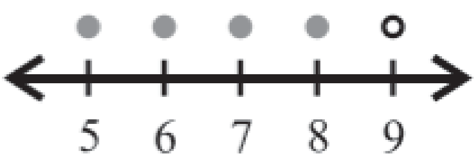
e)



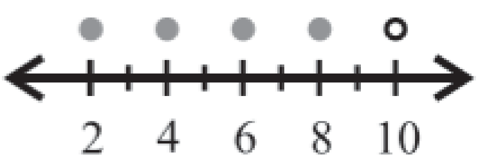
f)



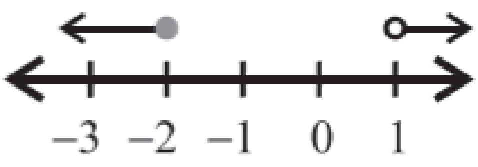
g)



h)



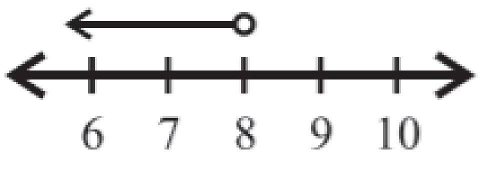
i)



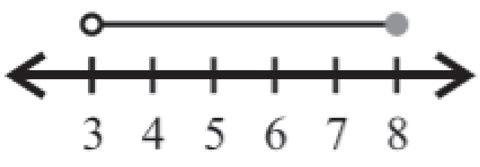
Exercise 5 number 1 is on page 266

2)

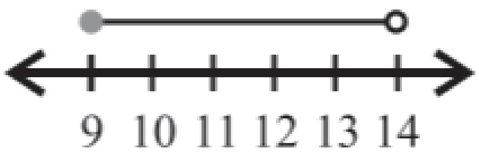
a)



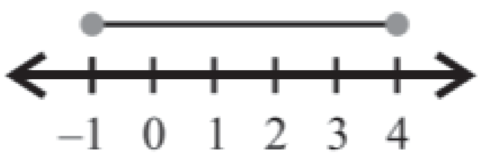
b)



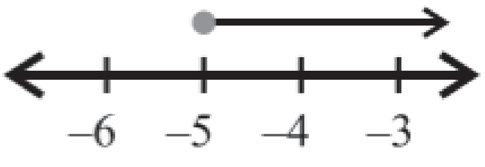
c)



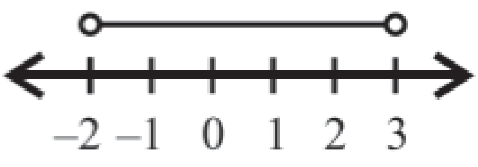
d)



e)



f)



Exercise 5 number 2 is on page 266

3)

a) Set Builder: $\{x : -3 < x \leq 3\}$

Interval: $(-3;3]$

b) Set Builder: $\{x : x \geq -1\}$

Interval: $[-1;\infty)$

c) Set Builder: $\{x : -2 \leq x \leq 7; x \in \mathbb{Z}\}$

Interval: No solution

d) Set Builder: $\{x : -3 < x < 10\}$

Interval: $(-3;10)$

e) Set Builder: $\{x : -4 < x \leq 1 \text{ or } 5 \leq x \leq 8\}$

Interval: $(-4;1] \cup [5;8] \rightarrow \cup$ means or

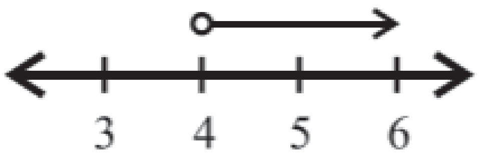
Exercise 5 number 2 is on page **266**

Exercise 6 Solutions:

a) $7x - 5 > 3x + 11$

$\therefore 4x > 16$

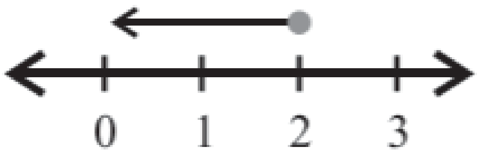
$\therefore x > 4$



b) $16x - 26 \leq 14 - 4x$

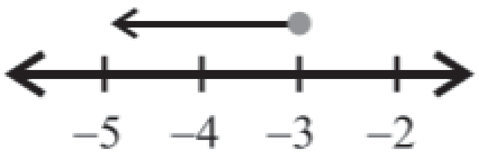
$\therefore 20x \leq 40$

$\therefore x \leq 2$



c) $-3x \geq 9$

$\therefore x \leq -3$



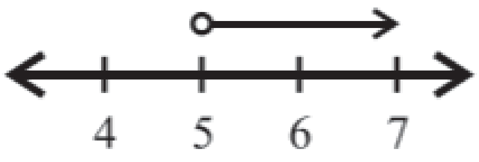
d) $\dfrac{x-5}{3} - 2x + 10 < \dfrac{x-5}{2}$

$\therefore 2(x-5) - 6(2x) + 6(10) < 3(x-5)$

$\therefore 2x - 10 - 12x + 60 < 3x - 15$

$\therefore -13x < -65$

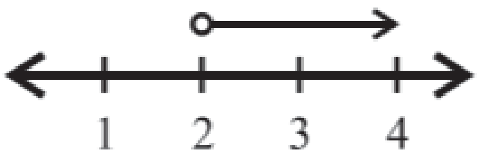
$\therefore x > 5$



e) $-5x + 30 < 20$

$\therefore -5x < -10$

$\therefore x > 2$

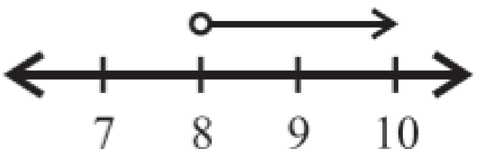


f) $\frac{1}{2}(x-8) < 2x - 16$

$\therefore x - 8 < 4x - 32$

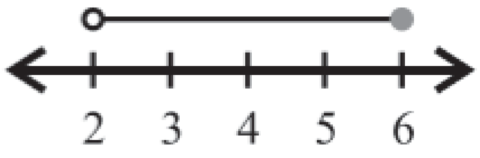
$\therefore -3x < -24$

$\therefore x > 8$



g) $5 < x + 3 \leq 9$

$\therefore 2 < x \leq 6$

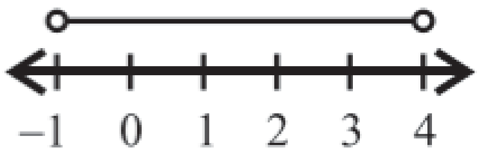


h) $-1 < 3 - x < 4$

$\therefore -4 < -x < 1$

$\therefore 4 > x > -1$

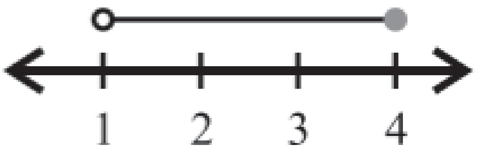
$\therefore -1 < x < 4$



i) $-1 < 2x - 3 \leq 5$

$\therefore 2 < 2x \leq 8$

$\therefore 1 < x \leq 4$



Exercise 6 is on page **267**

Exercise 7 Solutions:

1)

a) $8x + 3y = 28 \rightarrow \boxed{1}$

$7x - 3y = 2 \rightarrow \boxed{2}$

$\boxed{2} + \boxed{1}$:

$\therefore 7x - 3y + 8x + 3y = 2 + 28$

$\therefore 15x = 30$

$\therefore x = 2 \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{1}$:

$\therefore 8(2) + 3y = 28$

$\therefore 3y = 12$

$\therefore y = 4$

$\therefore x = 2$ and $y = 4$

b) $27x + 3y = 120 \rightarrow \boxed{1}$

$5x - 3y = 8 \rightarrow \boxed{2}$

$\boxed{2} + \boxed{1}$:

$\therefore 5x - 3y + 27x + 3y = 8 + 120$

$\therefore 32x = 128$

$\therefore x = 4 \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{2}$:

$\therefore 5(4) - 3y = 8$

$\therefore -3y = -12$

$\therefore y = 4$

$\therefore x = 4$ and $y = 4$

Exercise 7 number 1 is on page **267**

c) $3x+2y=22\rightarrow\boxed{1}$

$10x+2y=36\rightarrow\boxed{2}$

$\boxed{2}-\boxed{1}$:

$\therefore 10x+2y-(3x+2y)=36-22$

$\therefore 10x+2y-3x-2y=14$

$\therefore 7x=14$

$\therefore x=2\rightarrow\boxed{3}$

sub $\boxed{3}$ into $\boxed{2}$:

$\therefore 10(2)+2y=36$

$\therefore 2y=16$

$\therefore y=8$

$\therefore x=2$ and $y=8$

d) $5x+3y=36\rightarrow\boxed{1}$

$y=x-4\rightarrow\boxed{2}$

from $\boxed{2}$:

$x-y=4\rightarrow\boxed{3}$

$\boxed{3}\times 3$:

$\therefore 3x-3y=12\rightarrow\boxed{4}$

$\boxed{4}+\boxed{1}$:

$\therefore 3x-3y+5x+3y=12+36$

$\therefore 8x=48$

$\therefore x=6\rightarrow\boxed{5}$

sub $\boxed{5}$ into $\boxed{2}$:

$\therefore y=6-4$

$\therefore y=2$

$\therefore x=6$ and $y=2$

e) $3x-5y=9\rightarrow\boxed{1}$

$x-y=5\rightarrow\boxed{2}$

$\boxed{2}\times 5$:

$\therefore 5x-5y=25\rightarrow\boxed{3}$

$\boxed{3}-\boxed{1}$:

$\therefore 5x-5y-(3x-5y)=25-9$

$\therefore 5x-5y-3x+5y=16$

$\therefore 2x=16$

$\therefore x=8\rightarrow\boxed{4}$

sub $\boxed{4}$ into $\boxed{2}$:

$\therefore 8-y=5$

$\therefore y=3$

$\therefore x=8$ and $y=3$

f) $3x+4y=26\rightarrow\boxed{1}$

$x+2y=12\rightarrow\boxed{2}$

$\boxed{2}\times 2$:

$\therefore 2x+4y=24\rightarrow\boxed{3}$

$\boxed{3}-\boxed{1}$:

$\therefore 2x+4y-(3x+4y)=24-26$

$\therefore 2x+4y-3x-4y=-2$

$\therefore -x=-2$

$\therefore x=2\rightarrow\boxed{4}$

sub $\boxed{4}$ into $\boxed{2}$:

$\therefore 2+2y=12$

$\therefore 2y=10$

$\therefore y=5$

$\therefore x=2$ and $y=5$

Exercise 7 number 1 is on page **267**

2)

a) $2x+4y=20\rightarrow\boxed{1}$

$7x-3y=19\rightarrow\boxed{2}$

from $\boxed{1}$:

$2x=20-4y$

$\therefore x=10-2y\rightarrow\boxed{3}$

sub $\boxed{3}$ into $\boxed{2}$:

$\therefore 7(10-2y)-3y=19$

$\therefore 70-14y-3y=19$

$\therefore -17y=-51$

$\therefore y=3\rightarrow\boxed{4}$

sub $\boxed{4}$ into $\boxed{3}$:

$\therefore x=10-2(3)$

$\therefore x=4$

$\therefore x=4$ and $y=3$

b) $3x+y=2\rightarrow\boxed{1}$

$5x-3y=-34\rightarrow\boxed{2}$

from $\boxed{1}$:

$y=2-3x\rightarrow\boxed{3}$

sub $\boxed{3}$ into $\boxed{2}$:

$\therefore 5x-3(2-3x)=-34$

$\therefore 5x-6+9x=-34$

$\therefore 14x=-28$

$\therefore x=-2\rightarrow\boxed{4}$

sub $\boxed{4}$ into $\boxed{3}$:

$\therefore y=2-3(-2)$

$\therefore y=8$

$\therefore x=-2$ and $y=8$

Exercise 7 number 2 is on page **267**

c) $3(x+y)+y=-8 \rightarrow \boxed{1}$

$2x+y=-7 \rightarrow \boxed{2}$

from $\boxed{2}$:

$y=-7-2x \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{1}$:

$\therefore 3(x-7-2x)+(-7-2x)=-8$

$\therefore 3x-21-6x-7-2x=-8$

$\therefore -5x=20$

$\therefore x=-4 \rightarrow \boxed{4}$

sub $\boxed{4}$ into $\boxed{3}$:

$\therefore y=-7-2(-4)$

$\therefore y=1$

$\therefore x=-4$ and $y=1$

d) $2x+3y=12 \rightarrow \boxed{1}$

$2x+8y=62 \rightarrow \boxed{2}$

from $\boxed{2}$:

$2x=62-8y$

$\therefore x=31-4y \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{1}$:

$\therefore 2(31-4y)+3y=12$

$\therefore 62-8y+3y=12$

$\therefore -5y=-50$

$\therefore y=10 \rightarrow \boxed{4}$

sub $\boxed{4}$ into $\boxed{3}$:

$\therefore x=31-4(10)$

$\therefore x=-9$

$\therefore x=-9$ and $y=10$

e) $\frac{x}{2}+\frac{y}{3}=10 \rightarrow \boxed{1}$

$x+\frac{y}{2}=16 \rightarrow \boxed{2}$

from $\boxed{2}$:

$2x+y=32$

$\therefore y=32-2x \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{1}$:

$\therefore \frac{x}{2}+\frac{32-2x}{3}=10$

$\therefore 3x+2(32-2x)=6(10)$

$\therefore 3x+64-4x=60$

$\therefore x=4 \rightarrow \boxed{4}$

sub $\boxed{4}$ into $\boxed{3}$:

$\therefore y=32-2(4)$

$\therefore y=24$

$\therefore x=4$ and $y=24$

f) $2x+3y=15 \rightarrow \boxed{1}$

$5x-4y=-43 \rightarrow \boxed{2}$

from $\boxed{1}$:

$2x=15-3y$

$\therefore x=\frac{15}{2}-\frac{3y}{2} \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{2}$:

$\therefore 5\left(\frac{15}{2}-\frac{3y}{2}\right)-4y=-43$

$\therefore \frac{75}{2}-\frac{15y}{2}-4y=-43$

$\therefore 75-15y-8y=-86$

$\therefore -23y=-161$

$\therefore y=7 \rightarrow \boxed{4}$

sub $\boxed{4}$ into $\boxed{3}$:

$\therefore x=\frac{15}{2}-\frac{3(7)}{2}$

$\therefore x=-3$

$\therefore x=-3$ and $y=7$

Exercise 7 number 2 is on page **267**

Exercise 8 Solutions:

1) let $x=$ the number of sweets Dan has

$\therefore 2x=$ the number of sweets Roger has

$\therefore 2x-5=(x+5)+5$

$\therefore 2x-x=10+5$

$\therefore x=15$

$\therefore$ Dan has 15 sweets and Roger has 30 sweets

2) let $x=$ Price of a cheese sandwich

$\therefore x+2=$ Price of a beef sandwich

$3(x+2)+5x=78$

$\therefore 3x+6+5x=78$

$\therefore 8x=72$

$\therefore x=$ R9

$\therefore$ A beef sandwich costs R11

Exercise 8 is on page **267**

3) let $x =$ Danni's age

$\therefore 4x =$ Jenna's age

$\therefore 4x+10 = 2(x+10)$

$\therefore 4x+10 = 2x+20$

$\therefore 2x = 10$

$\therefore x = 5$

$\therefore$ Danni is 5 and Jenna is 20 years old

4) let $x =$ Tim's age

$\therefore x+20 =$ Ray's age

$\therefore (x+20)+6 = 3(x+6)$

$\therefore x+26 = 3x+18$

$\therefore -2x = -8$

$\therefore x = 4$

$\therefore$ Tim is 4 and Ray is 24 years old

5) let $x =$ the 1^{st} number

$\therefore x+1 =$ the 2^{nd} number

$x+(x+1) = 3(x+x+1)-22$

$\therefore 2x+1 = 6x+3-22$

$\therefore -4x = -20$

$\therefore x = 5$

$\therefore$ the numbers are 5 and 6

6) let $x =$ the 1^{st} number

$\therefore x+2 =$ the 2^{nd} number

$2(x+x+2) = 22+(x+2)-(x)$

$\therefore 4x+4 = 24$

$\therefore 4x = 20$

$\therefore x = 5$

$\therefore$ the numbers are 5 and 7

7) let $x =$ the number

$\therefore \frac{2x}{3} = 1+\frac{x}{2}$

$\therefore 2(2x) = 6+3x$

$\therefore 4x = 6+3x$

$\therefore x = 6$

$\therefore$ the number is 6

8) let $x =$ the number

$\therefore 8x+5 = x^2+20$

$\therefore x^2-8x+15 = 0$

$\therefore (x-5)(x-3) = 0$

$\therefore x = 5$ or $x = 3$

$\therefore$ the number is either 5 or 3

Exercise 8 is on page 267

Exercise 9 Solutions:

1)

Situation	Cost per T-Shirt	Number of T-shirts sold	Total Money Earned
Normal Price	80	x	$80x$
Sale Price	70	$60-x$	$70(60-x)$

$\therefore 80x+70(60-x) = 4\,500$

$\therefore 80x+4\,200-70x = 4\,500$

$\therefore 10x = 300$

$\therefore x = 30$ T-shirts

$\therefore$ 30 T-shirts were sold for R80 and 30 T-shirts were sold for R70

2)

Situation	Cost per meter	Number of meters	Total Cost
Plain	x	30	$30x$
Galvanised	$2x$	24	$2x(24)$

$\therefore 30x+2x(24) = 6\,240 \rightarrow$ total spent was R6 240

$\therefore 78x = 6\,240$

$\therefore x =$ R80 p/m

$\therefore$ Normal steel costs R80 p/m and galvanised steel costs R160 p/m

Exercise 9 is on page 268

3)

Situation	Cost Per brownie	Number of brownies	Total Cost
Brownies made	1,50	x	$1,5x$
Brownies sold	5,50	$x-5$	$5,5(x-5)$

$\therefore 5,5(x-5)-1,5x=120,5 \rightarrow$ profit = selling price – cost price

$\therefore 5,5x-27,5-1.5x=120,5$

$\therefore 4x=148$

$\therefore x=37$

$\therefore$ she made 37 brownies

4)

Situation	Cost Per kilo	Mass	Total Cost
Sour worms	99	x	$99x$
Gobstoppers	85	$1,2-x$	$85x(1,2-x)$

$\therefore 99x+85(1,2-x)=109 \rightarrow$ total cost was R109

$\therefore 99x+102-85x=109$

$\therefore 14x=7$

$\therefore x=0,5$

$\therefore$ he bought 500g of sour worms and 700g of gobstoppers

Exercise 9 is on page **268**

Exercise 10 Solutions:

1)

Situation	Speed	Distance	Time
Jeff	11	x	$\frac{x}{11}$
Tom	9	$40-x$	$\frac{40-x}{9}$

$\therefore \frac{x}{11}=\frac{40-x}{9} \rightarrow$ they have both cycled for the same amount of time

$\therefore 9x=11(40-x)$

$\therefore 9x=440-11x$

$\therefore 20x=440$

$\therefore x=22$

$\therefore t=\frac{22}{11}=2$ hours

$\therefore$ they will meet after 2 hours

2) 8 hours and 40 minutes $= 8\frac{2}{3}=\frac{26}{3}$ hours

Situation	Speed	Distance	Time
Running	8	x	$\frac{x}{8}$
Cycling	18	x	$\frac{x}{18}$

$\therefore \frac{x}{8}+\frac{x}{18}=\frac{26}{3} \rightarrow$ time to complete the race is the time spent cyclying and running

$\therefore 54x+24x=144(26)$

$\therefore 78x=3744$

$\therefore x=48$

$\therefore$ Ryan ran 48 km and cycled 48 km

$\therefore$ the race was 96 km long

Exercise 10 is on page **268**

3)

Situation	Speed	Distance	Time
Jake	2,5	x	$\frac{x}{2,5}$
Brother	2	x	$\frac{x}{2}$

$\therefore \frac{x}{2,5} = \frac{x}{2} - 15 \rightarrow$ Jake has been running for 15s less than his brother when he catches him

$\therefore \frac{2x}{5} = \frac{x}{2} - 15$

$\therefore 4x = 5x - 150$

$\therefore -x = -150$

$\therefore x = 150\text{m}$

$\therefore$ Jake's time $= 150 \div 2,5 = 60\text{s}$

$\therefore$ Jake will catch his brother after 1 minute

Exercise 10 is on page **268**

Exercise 11 Solutions:

1) Let the larger number $= x$

Let the smaller number $= y$

$x + y = 40 \rightarrow \boxed{1}$

$x - y = 8 \rightarrow \boxed{2}$

$\boxed{2} + \boxed{1}$

$\therefore 2x = 48$

$\therefore x = 24 \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{1}$

$\therefore 24 + y = 40$

$\therefore y = 16$

$\therefore$ the numbers are 16 and 24

2) Let the length $= x$

Let the width $= y$

$2x + 2y = 18 \rightarrow \boxed{1}$

$2(2x) + 2(y+3) = 36 \rightarrow \boxed{2}$

from $\boxed{1}$:

$\therefore x + y = 9$

$\therefore x = 9 - y \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{2}$

$\therefore 2(2)(9-y) + 2(y+3) = 36$

$\therefore 36 - 4y + 2y + 6 = 36$

$\therefore -2y = -6$

$\therefore y = 3 \rightarrow \boxed{4}$

sub $\boxed{4}$ into $\boxed{3}$

$\therefore x = 6$

$\therefore$ the dimensions were 6m × 3m

3) Let the floor seats $= x$ and balcony seats $= y$

$x + y = 1200 \rightarrow \boxed{1}$

$500x + 350y = 480\,000 \rightarrow \boxed{2}$

from $\boxed{1}$: $x = 1200 - y \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{2}$

$\therefore 500(1200 - y) + 350y = 480\,000$

$\therefore 600\,000 - 500y + 350y = 480\,000$

$\therefore -150y = -120\,000$

$\therefore y = 800 \rightarrow \boxed{4}$

sub $\boxed{4}$ into $\boxed{3}$

$\therefore x = 1200 - 800$

$\therefore x = 400$

$\therefore$ 400 floor seats and 800 balcony seats were sold

4) Let the tens digit $= x$ and let the units digit $= y$

$\therefore$ the number is: $10x + y$

$\therefore \frac{10x + y}{x} = 13 \rightarrow \boxed{1}$

If the digits are reversed the number is: $10y + x$

$\therefore 10y + x - (10x + y) = 36 \rightarrow \boxed{2}$

from $\boxed{2}$: $10y + x - 10x - y = 36$

$\therefore 9y - 9x = 36$

$\therefore y = 4 + x \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{1}$

$\therefore \frac{10x + 4 + x}{x} = 13$

$\therefore 10x + 4 + x = 13x$

$\therefore -2x = -4$

$\therefore x = 2 \rightarrow \boxed{4}$

sub $\boxed{4}$ into $\boxed{3}$

$\therefore y = 4 + 2 = 6$

$\therefore$ the number is 26

Exercise 11 is on page **268**

Mixed Exercise Solutions:

1)

a) $x(x+3)=0$

$\therefore x=0$ or $x=-3$

b) $5(x+1)=3(2x-4)$

$\therefore 5x+5=6x-12$

$\therefore -x=-17$

$\therefore x=17$

c) $3(x-2)-2(2x+1)=7-(3x+2)$

$\therefore 3x-6-4x-2=7-3x-2$

$\therefore 2x=13$

$\therefore x=\frac{13}{2}$

d) $(2x-5)(3x+2)=2\left(3x^2-4x+1\right)$

$\therefore 6x^2-11x-10=6x^2-8x+2$

$\therefore -3x=12$

$\therefore x=-4$

e) $2(x-3)-x=-3(x-2)$

$\therefore 2x-6-x=-3x+6$

$\therefore 4x=12$

$\therefore x=3$

f) $(x+1)(x-2)=4$

$\therefore x^2-x-2=4$

$\therefore x^2-x-6=0$

$\therefore (x-3)(x+2)=0$

$\therefore x=3$ *or* $x=-2$

Mixed Exercise number 1 is on page **268**

2)

a) $2x(x-1)=4$

$\therefore 2x^2-2x-4=0$

$\therefore x^2-x-2=0$

$\therefore (x-2)(x+1)=0$

$\therefore x=2$ or $x=-1$

b) $4x^2=16x$

$\therefore 4x^2-16x=0$

$\therefore 4x(x-4)=0$

$\therefore x=0$ or $x=4$

c) $3(x+1)-(3-3x)=3x-(10-3x)$

$\therefore 3x+3-3+3x=3x-10+3x$

$\therefore 0x=-10$

$\therefore$ No solution

d) $\frac{5x+8}{6}-\frac{x}{4}=\frac{2x-9}{3}$

$\therefore 2(5x+8)-3x=4(2x-9)$

$\therefore 10x+16-3x=8x-36$

$\therefore -x=-52$

$\therefore x=52$

e) $5+\frac{8}{x}=\frac{2(x+4)}{x}$

$\therefore 5x+8=2x+8 \quad ; \; x\neq 0$

$\therefore 3x=0$

$\therefore$ No solution

f) $(x-3)(x+4)-(3x-2)^2=x-8x(x-1)$

$\therefore x^2+x-12-\left(9x^2-12x+4\right)=x-8x^2+8x$

$\therefore x^2+x-12-9x^2+12x-4=x-8x^2+8x$

$\therefore 4x=16$

$\therefore x=4$

g) $4x^2-8x=0$

$\therefore 4x(x-2)=0$

$\therefore x=0$ *or* $x=2$

h) $x^2-8x=-12$

$\therefore x^2-8x+12=0$

$\therefore (x-6)(x-2)=0$

$\therefore x=6$ or $x=2$

i) $3x^2-18x+27=0$

$\therefore x^2-6x+9=0$

$\therefore (x-3)^2=0$

$\therefore x=3$

j) $x^4-13x^2+36=0$

$\therefore \left(x^2-9\right)\left(x^2-4\right)=0$

$\therefore (x-3)(x+3)(x-2)(x+2)=0$

$\therefore x=3$ or $x=-3$ or $x=2$ or $x=-2$

Mixed Exercise numbers 2a to 2f is on page **268** and 2g to 2j are on page **269** solutions are continued on the next page

k) $\frac{5x+2}{3}+\frac{x}{5}=\frac{3x-5}{15}+x$

$\therefore 5(5x+2)+3x=3x-5+15x$

$\therefore 25x+10+3x=3x-5+15x$

$\therefore 10x=-15$

$\therefore x=-\frac{15}{10}=-\frac{3}{2}$

l) $\frac{2(x-4)}{3}=x+1-\frac{x}{3}$

$\therefore 2x-8=3x+3-x$

$\therefore 0x=11$

$\therefore$ No solution

m) $(2x+1)(x+8)=27$

$\therefore 2x^2+17x+8=27$

$\therefore 2x^2+17x-19=0$

$\therefore (2x+19)(x-1)=0$

$\therefore x=-\frac{19}{2}$ or $x=1$

Mixed Exercise numbers 2k to 2m is on page **268**

3)

a) $\frac{x-p}{2q}=3$

$\therefore x-p=6q$

$\therefore x=6q+p$

b) $p^2+px=q^2+qx$

$\therefore px-qx=q^2-p^2$

$\therefore x(p-q)=-(p^2-q^2)$

$\therefore x=-\frac{(p-q)(p+q)}{(p-q)}$

$\therefore x=-p-q$

c) $(x-p)^2=(x-q)^2$

$\therefore x^2-2xp+p^2=x^2-2xq+q^2$

$\therefore 2xq-2xp=q^2-p^2$

$\therefore 2x(q-p)=(q-p)(q+p)$

$\therefore x=\frac{(q-p)(q+p)}{2(q-p)}=\frac{(q+p)}{2}$

Mixed Exercise numbers 3 is on page **269**

4)

a) $9C=5(F-32)$

$\therefore \frac{9C}{5}=F-32$

$\therefore F=\frac{9C}{5}+32$

b) $F=\frac{9(37)}{5}+32$

$\therefore F=98{,}6^\circ F$

Mixed Exercise number 4 is on page **269**

5)

a) $4(2x-1)>5x+2$

$\therefore 8x-4>5x+2$

$\therefore 3x>6$

$\therefore x>2$

b) $2x-7\geq 4x-10$

$\therefore -2x\geq -3$

$\therefore x\leq \frac{3}{2}$

c) $\frac{1-2x}{-3}\geq 2x+1$

$\therefore 1-2x\leq -3(2x+1)$

$\therefore 1-2x\leq -6x-3$

$\therefore 4x\leq -4$

$\therefore x\leq -1$

d) $3<-2x+1<7$

$\therefore 2<-2x<6$

$\therefore -1>x>-3$

Mixed Exercise number 5 is on page **269**

6)

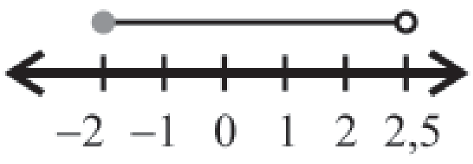
a) $-4<1-2p\leq 5$

$\therefore -5<-2p\leq 4$

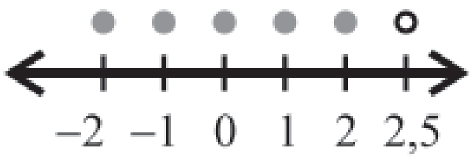
$\therefore \frac{5}{2}>p\geq -2$

$\therefore -2\leq p<\frac{5}{2}$

b)



c)



Mixed Exercise number 6 is on page **269**

7)

a) $7^x = 49$

$\therefore 7^x = 7^2$

$\therefore x = 2$

b) $9^{3r-1} = 27$

$\therefore \left(3^2\right)^{3r-1} = 3^3$

$\therefore 3^{6r-2} = 3^3$

$\therefore 6r - 2 = 3$

$\therefore r = \dfrac{5}{6}$

c) $9^{2x} = \dfrac{1}{81}$

$\therefore \left(3^2\right)^{2x} = 3^{-4}$

$\therefore 3^{4x} = 3^{-4}$

$\therefore 4x = -4$

$\therefore x = -1$

d) $4^{2m} = 16^{m-1}$

$\therefore \left(2^2\right)^{2m} = \left(2^4\right)^{m-1}$

$\therefore 2^{4m} = 2^{4m-4}$

$\therefore 4m = 4m - 4$

$\therefore 0m = -4$

$\therefore$ No solution

Mixed Exercise number 7 is on page **269**

8) $3^{x+1} = 3^x(3)$

$= 3p$

Mixed Exercise number 8 is on page **269**

9)

a) $y = 2x + 3 \rightarrow \boxed{1}$

$y = -3x - 7 \rightarrow \boxed{2}$

sub $\boxed{1}$ into $\boxed{2}$:

$\therefore 2x + 3 = -3x - 7$

$\therefore 5x = -10$

$\therefore x = -2 \rightarrow \boxed{4}$

sub $\boxed{4}$ into $\boxed{1}$:

$\therefore y = 2(-2) + 3$

$\therefore y = -1$

$\therefore x = -2$ and $y = -1$

b) $2x - 3y = 5 \rightarrow \boxed{1}$

$3x + 6y = 11 \rightarrow \boxed{2}$

$\boxed{1} \times 2$:

$\therefore 4x - 6y = 10 \rightarrow \boxed{3}$

$\boxed{3} + \boxed{2}$:

$\therefore 4x + 6y + 3x - 6y = 10 + 11$

$\therefore 7x = 21$

$\therefore x = 3 \rightarrow \boxed{4}$

sub $\boxed{4}$ into $\boxed{2}$:

$\therefore 3(3) + 6y = 11$

$\therefore 6y = 2$

$\therefore y = \dfrac{1}{3}$

$\therefore x = 3$ and $y = \dfrac{1}{3}$

Mixed Exercise number 9 is on page **269**

10) $(2p + q)^2 = 4p^2 + 4pq + q^2 = 21 + 4(2) = 29$

Mixed Exercise number 10 is on page **269**

11)

a) let $x =$ the breadth and let $x + 5 =$ the length

$\therefore 2x + 2(x + 5) = 70$

$\therefore 2x + 2x + 10 = 70$

$\therefore 4x = 60$

$\therefore x = 15\text{m}$

$\therefore$ The rectangle has a breadth of 15m and a length of 20m.

Mixed Exercise number 11 is on page **269** (solutions are continued on the next page)

b)

Situation	Speed	Distance	Time
Fast motorist	120	x	$\frac{x}{120}$
Slow	100	x	$\frac{x}{100}$

$\therefore \frac{x}{100} - \frac{x}{120} = 1,5 \rightarrow$ longer time – shorter time = 1,5 hours

$\therefore 120x - 100x = 1,5(100)(120)$

$\therefore 20x = 18\ 000$

$\therefore x = 900\,\text{km}$

$\therefore$ The slower motorist took 9 hours to complete his journey.

c)

Situation	Speed	Distance	Time
Aeroplane	$5x$	480	$\frac{480}{5x}$
Car	x	480	$\frac{480}{x}$

$\therefore \frac{480}{x} - \frac{480}{5x} = 4 \rightarrow$ longer time – shorter time = 4 hours ; $x \neq 0$

$\therefore 5(480) - 480 = 20x$

$\therefore 1920 = 20x$

$\therefore x = 96\,\text{km/h}$

$\therefore$ The car can travel at 96 km/h.

Mixed Exercise number 11 is on page **269**

12)

a) $120x + 80y = 600 \rightarrow \boxed{1}$

b) $x + y = 5,5 \rightarrow \boxed{2}$

c) from $\boxed{2}$: $x = 5,5 - y$

sub $\boxed{3}$ into $\boxed{1}$:

$\therefore 120(5,5 - y) + 80y = 600$

$\therefore 660 - 120y + 80y = 600$

$\therefore -40y = -60$

$\therefore y = 1,5$ hours $\rightarrow \boxed{4}$

sub $\boxed{4}$ into $\boxed{2}$:

$\therefore x = 5,5 - 1,5 = 4$ hours

$\therefore$ he drove at 80 km/h for 1,5 hours

$\therefore$ he a got puncture 120 km from Durban $\rightarrow 80 \times 1,5$

Mixed Exercise number 12 is on page **270**

13)

a) 9

b) $T_n = dn + c$

$\therefore T_n = 2n + c$

Subst $T_1 = 3$

$\therefore 3 = 2(1) + c$

$\therefore c = 1$

$\therefore T_n = 2n + 1$

c) $19 = 2n + 1$

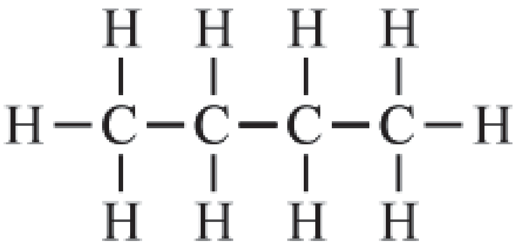
$\therefore 18 = 2n$

$\therefore n = 9$ triangles

Mixed Exercise number 13 is on page **270**

14)

a)



b) $T_n = dn + c$

$\therefore T_n = 2n + c$

Subst $T_1 = 4$

$\therefore 4 = 2(1) + c$

$\therefore c = 2$

$\therefore T_n = 2n + 2$

Mixed Exercise number 14 is on page **270**

15)

$$\frac{\mathbf{1+2}}{\mathbf{3}} = \mathbf{1} \qquad \frac{\mathbf{1+2+3}}{\mathbf{4}} = \frac{\mathbf{3}}{\mathbf{2}} \qquad \frac{\mathbf{1+2+3+4}}{\mathbf{5}} = \frac{\mathbf{4}}{\mathbf{2}} \qquad \frac{\mathbf{1+2+3+4+5}}{\mathbf{6}} = \frac{\mathbf{5}}{\mathbf{2}}$$

a) $\frac{6}{2} = 3$

b) $\frac{10}{2} = 5$

c) $\frac{40}{2} = 20$

d)

$1+2=3$ and $\frac{(2)(3)}{2} = \frac{6}{2} = 3$ and $1+2+3=6$ and $\frac{(3)(4)}{2} = \frac{12}{2} = 6$

If the pattern continues:

$1+2+3+4=10$ and $\frac{(4)(5)}{2} = 10$ and $1+2+3+4+5=15$ and $\frac{(5)(6)}{2} = 15$

$1+2+3+4+5+6=21$ and $\frac{(6)(7)}{2} = 21$ and $1+2+3+4+5+7=28$ and $\frac{(7)(8)}{2} = 28$

$\therefore S_n = \frac{n(n+1)}{2}$

Mixed Exercise number 15 is on page **270**

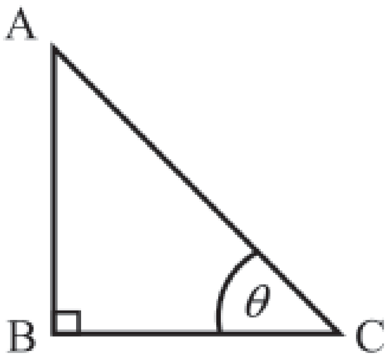
Chapter 5 – Trigonometry

Trigonometric Ratios

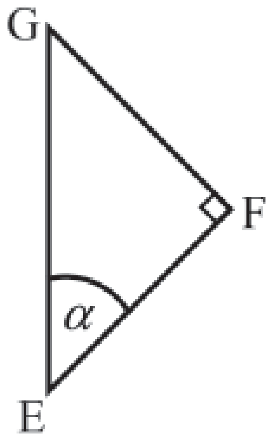
Exercise 1: (Theory on pages 77-80)

1) Identify the opposite, adjacent and hypotenuse with reference to the given angle:

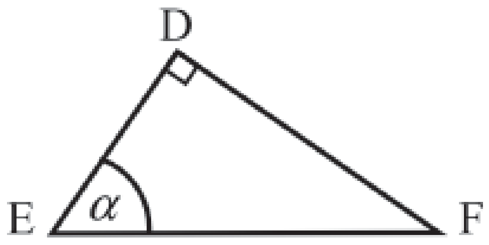
a)



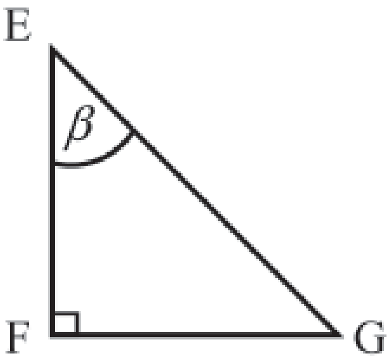
c)



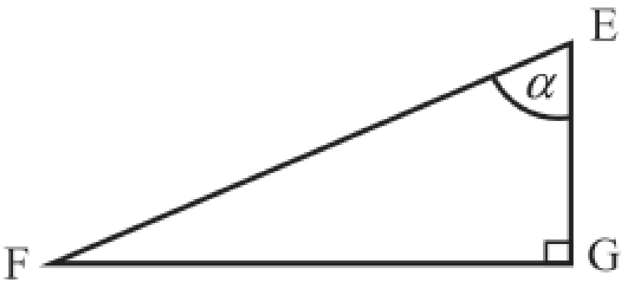
e)



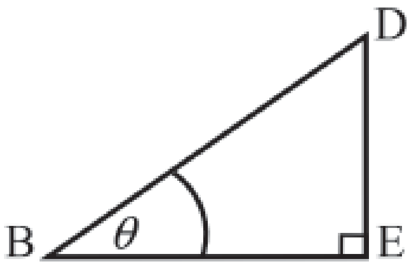
b)



d)



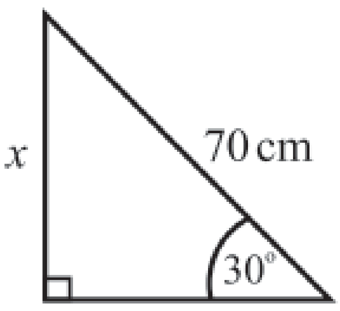
f)



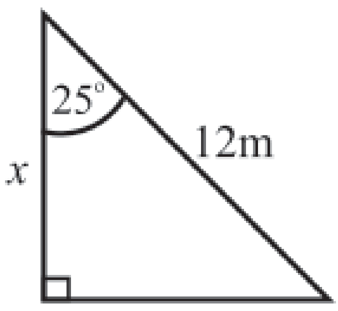
Solutions to Exercise 1 number 1 are on page **293**

2) Solve for the length of the unknown side in the following triangles correct to 2 decimal places:

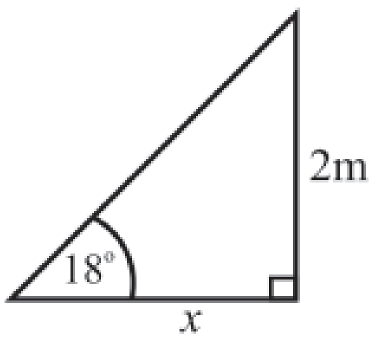
a)



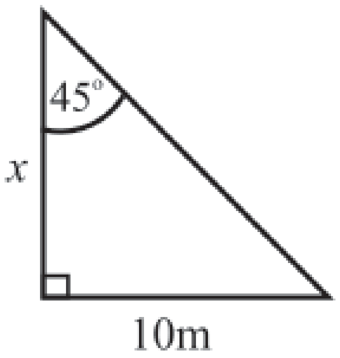
c)



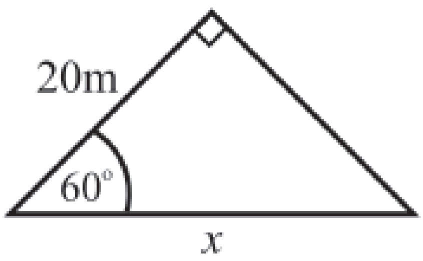
e)



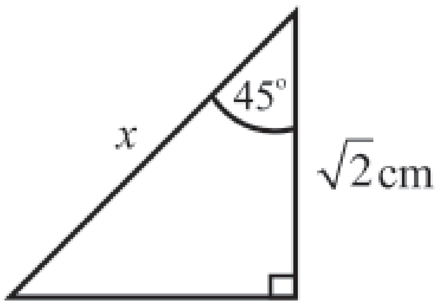
b)



d)



f)


x
45°
$\sqrt{2}$ cm

Solutions to Exercise 1 number 2 are on page **293**

3) If $A = 20^\circ$ and $B = 30^\circ$ determine the value of the following correct to 1 decimal place:

a) $2\sin B$

b) $\frac{\sqrt{3}}{2}\cos B$

c) $\cos(A+2B)$

d) $\sin(90^\circ - B)$

e) $\cos(90^\circ - B)$

f) $2\sin(2{,}5^\circ + A)\cos(2{,}5^\circ + A)$

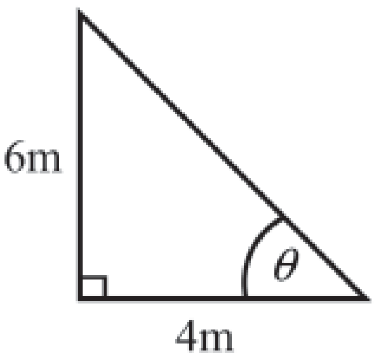
g) $\cos^2(A)+\sin^2(A)$

h) $\cos^2(B)-\sin^2(B)$

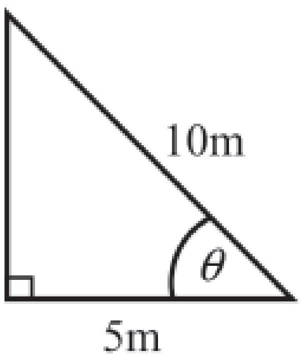
Solutions to Exercise 1 number 3 are on page **293**

4) Solve for θ in each of the following triangles if $\theta \in [0°;90°]$:

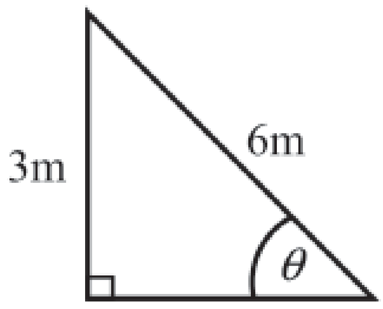
a)



b)

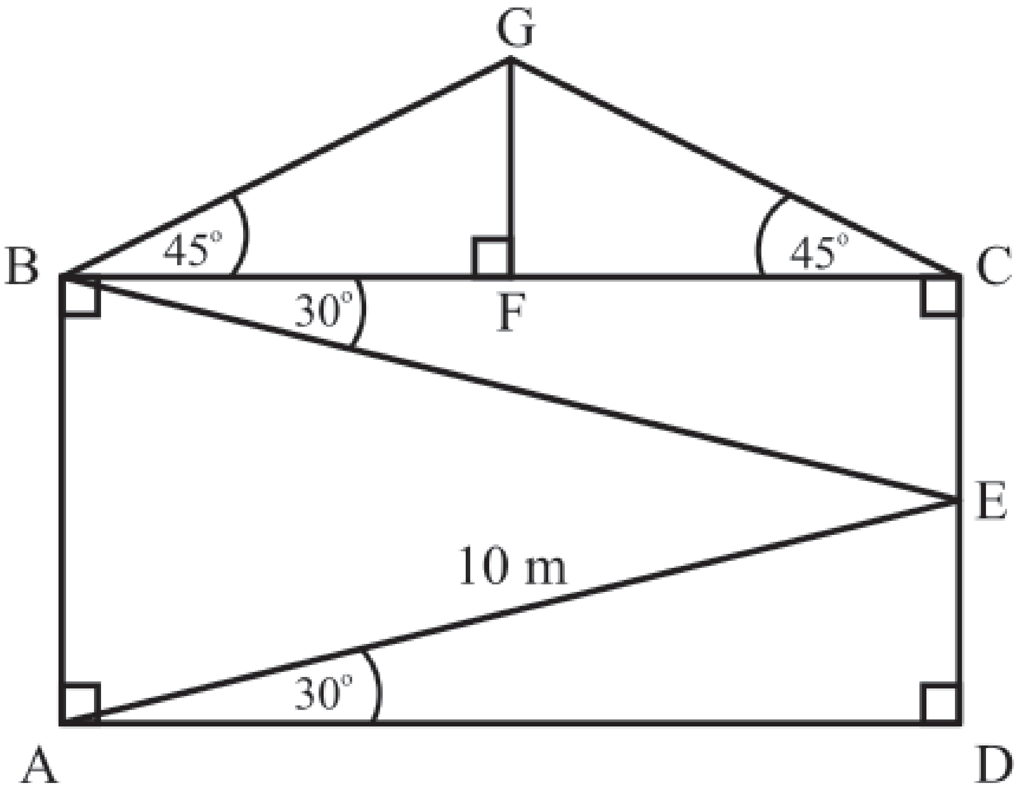

10m
θ
5m

c)



Solutions to Exercise 1 number 4 are on page **293**

5) A factory has a loft in its ceiling which it uses for additional storage space. It is accessed via 2 staircases, BE and AE, each 10m in length, as shown below. Determine:
a) BA, the length of the walls of the factory.
b) GF, the height of the loft given that BF=CF.



Solutions to Exercise 1 number 5 are on page **294**

6) Solve for θ in the following equations if $\theta \in \left[0^\circ;90^\circ\right]$:

a) $2\sin\theta+1=2$
b) $2\cos\theta-\sqrt{3}=0$
c) $2\tan\left(\theta+10^\circ\right)+3=5$
d) $3\sin\left(2\theta+12^\circ\right)-2=1$
e) $2\cos\theta+1=2$
f) $2\sin\theta=\sqrt{3}$
g) $2\sin\left(\theta-10^\circ\right)+3=3$
h) $3\cos\left(2\theta-12^\circ\right)-2=1$

Solutions to Exercise 1 number 6 are on page **294**

Reciprocals of Trigonometric Functions

Exercise 2: Evaluate the following trigonometric functions: (Theory on page 81)

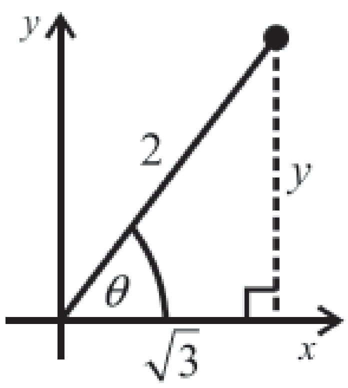
a) $\sec 30^\circ \cos 30^\circ$
b) $\sin 60^\circ + \cos 30^\circ$
c) $\operatorname{cosec}^2 60^\circ + \tan^2 45^\circ + \sec^2 30^\circ$
d) $1+\cot^2 30^\circ$
e) $\sec 30^\circ \cot 60^\circ$
f) $1+\operatorname{cosec}^2 60^\circ$

Solutions to Exercise 2 are on page **294**

Trigonometric Functions using Pythagoras

Exercise 3: (Theory on pages 81-83)

1) Using the diagram below and without the use of a calculator determine the value of:



a) $\sin\theta$
b) $\cos\theta$
c) $\tan\theta$
d) $\sin\theta\cos\theta$
e) $\tan\theta\cos\theta$
f) $2\sin\theta+3\cos\theta$

Solutions to Exercise 3 number 1 are on page **295**

2) Given that $\sin\theta=\dfrac{1}{\sqrt{2}}$ and that $0^\circ<\theta<90^\circ$, with aid of a diagram determine the value of:

a) $\cos\theta$
b) $\tan\theta$
c) $\sin^2\theta\tan\theta$
d) $\sin^2\theta+\cos^2\theta$
e) $1-\cos^2\theta$
f) $1-\sin^2\theta$

Solutions to Exercise 3 number 2 are on page **295**

3) Given that $\sin\theta = \frac{3}{5}$ and $0^\circ < \theta < 90^\circ$ solve for the following with aid of a diagram:

a) $\operatorname{cosec}\theta$

b) $\sec\theta$

c) $\cot\theta$

d) $\tan\theta\operatorname{cosec}\theta$

e) $\frac{\cos\theta}{\sec\theta}$

f) $\frac{\tan\theta}{\sec\theta}$

Solutions to Exercise 3 number 3 are on page **295**

Trigonometric Functions in the Cartesian Plane

Exercise 4: (Theory on pages 84-86)

1) Determine which quadrant the terminal arm lies in if:

a) $\sin\theta > 0$ and $\tan\theta > 0$

b) $\sin\theta < 0$ and $\cos\theta > 0$

c) $\tan\theta > 0$ and $\cos\theta < 0$

d) $\sin\theta < 0$ and $\theta \in \left[0^\circ;270^\circ\right]$

e) $\cos\theta > 0$ and $\theta \in \left[0^\circ;180^\circ\right]$

f) $\tan\theta < 0$ and $\theta \in \left[-90^\circ;90^\circ\right]$

g) $\sin\theta > 0$ and $90^\circ < \theta < 360^\circ$

h) $\tan\theta < 0$ and $0^\circ < \theta < 270^\circ$

i) $\cos\theta > 0$ and $90^\circ < \theta < 360^\circ$

j) $\cos\theta < 0$ and $\sin\theta < 0$

Solutions to Exercise 4 number 1 are on page **296**

2) If $5\tan\theta = 12$ and $90^\circ < \theta < 360^\circ$ use a diagram determine the value of:

a) $\sin\theta$

b) $\cos\theta$

c) $\cos^2\theta + \sin^2\theta$

Solutions to Exercise 4 number 2 are on page **296**

3) Given that $\cos\theta = \frac{1}{\sqrt{2}}$ and that $\tan\theta < 0$, with aid of a diagram determine the value of:

a) $\cos\theta$

b) $\tan\theta$

c) $\sin^2\theta\tan\theta$

d) $\sin^2\theta + \cos^2\theta$

e) $1 - \cos^2\theta$

f) $1 - \sin^2\theta$

Solutions to Exercise 4 number 3 are on page **296**

Special Angles (Theory on pages 87-89)

Exercise 5: Without the use of a calculator, determine the value of:

1)

a) $\cos 60^\circ \times \sin 60^\circ$

b) $\tan 45^\circ \times \sin 45^\circ$

c) $\sin^2 60^\circ + \cos^2 60^\circ$

d) $\sin^2 30^\circ + \cos^2 30^\circ$

e) $\sin 60^\circ \times \cos 30^\circ$

f) $\tan 60^\circ \times \tan 30^\circ + \cos 90^\circ$

Solutions to Exercise 5 number 1 are on page **296**

2)

a) $\sec 30^\circ \cos 30^\circ$

b) $\sin 60^\circ + \cos 30^\circ$

c) $\operatorname{cosec}^2 60^\circ + \tan^2 45^\circ + \sec^2 30^\circ$

d) $1 + \cot^2 30^\circ$

e) $\sec 30^\circ \cot 60^\circ$

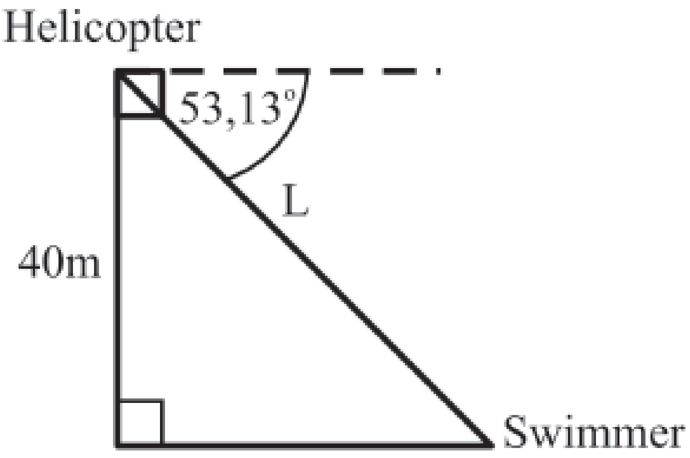
f) $1 + \operatorname{cosec}^2 60^\circ$

Solutions to Exercise 5 number 2 are on page **297**

Angles of Elevation and Depression

Exercise 6: (Theory on pages **90-93**)

1) A helicopter hovers 40m above the surface of the sea to rescue a swimmer in trouble. Once the swimmer is attached to the rescue line the current carries him such that the angle of depression from the helicopter to the swimmer is 53,13° as shown in the diagram below. Determine the length L of the rescue line.

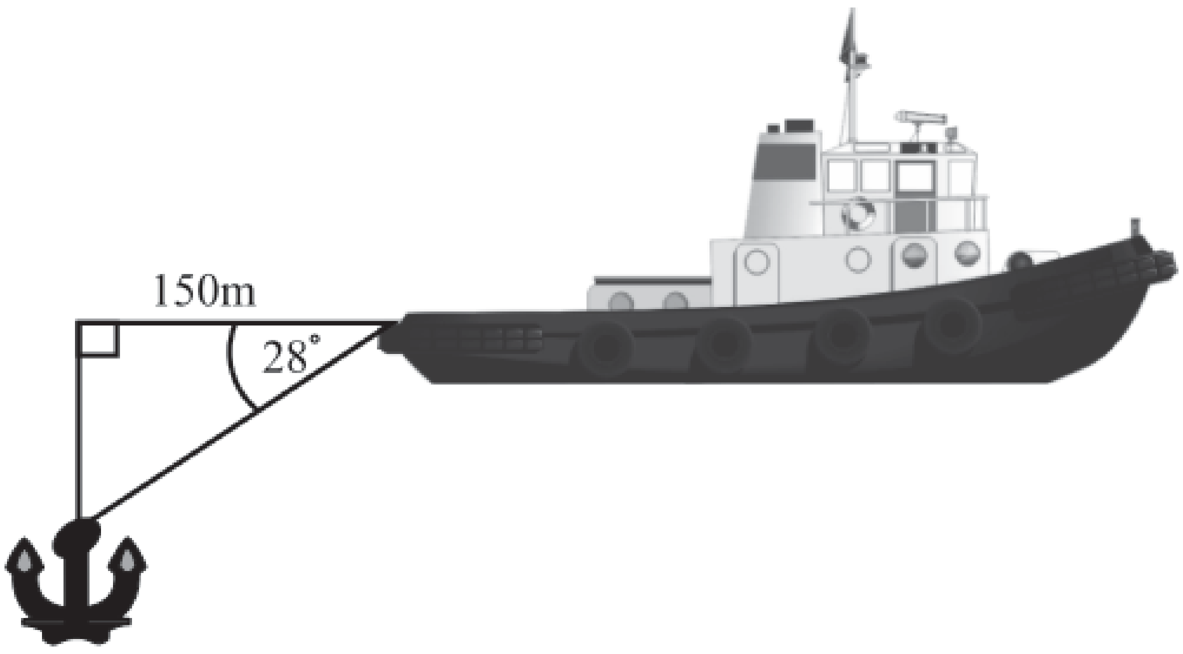


Solutions to Exercise 6 number 1 are on page **297**

2) A man places a ladder against a wall in order to clean a window 3m off the ground. If the man has a ladder that is 4m long, what angle must he place the ladder at and how far from the wall must he place the foot of the ladder in order to reach the window?
(Only use trigonometry in your solution.)

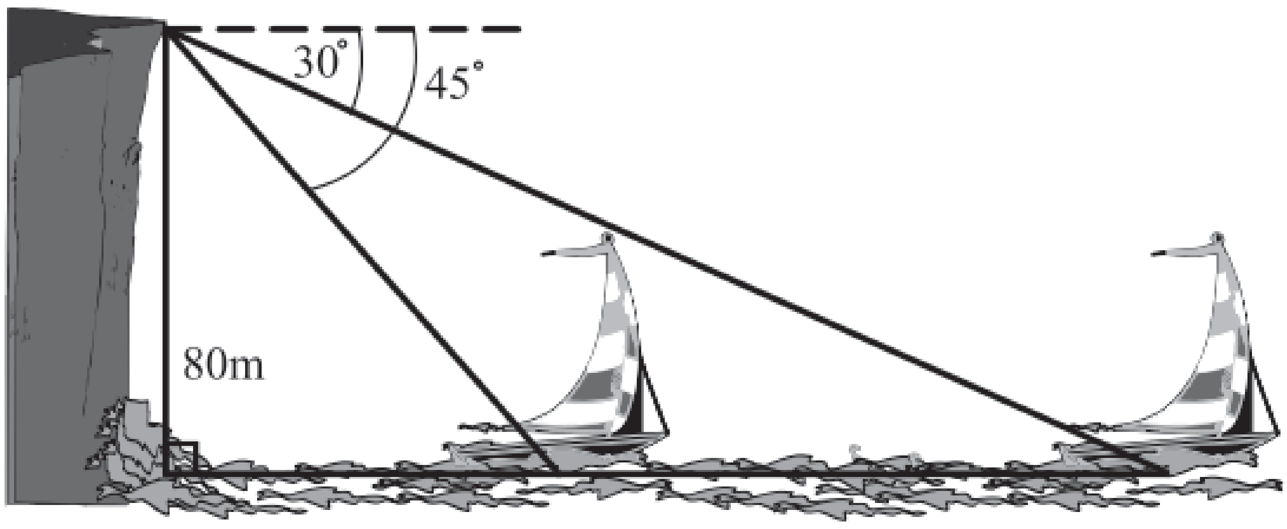
Solutions to Exercise 6 number 2 are on page **297**

3) A boat drops its anchor to the sea floor. The boat drifts 150m with the current until its anchor chain becomes taught. At this point the angle of depression from the boat to the anchor is 28°. How deep is the ocean at the point where the anchor was dropped? Give your answer to the nearest metre.



Solutions to Exercise 6 number 3 are on page **297**

4) A man stands on a cliff 80m high. He watches a boat from the cliff and notices that the angle of depression from the cliff to the boat changes from 45° to 30°. What distance did the boat cover while the man was watching it?



Solutions to Exercise 6 number 4 are on page **297**

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Mixed Exercise: (Chapter 5 is on pages 76-93)

1) If $\theta = 25{,}6^\circ$, determine the value of the following correct to 1 decimal place:

a) $2\sin\frac{\theta}{2}$ **b)** $\tan\left(2\theta - 20^\circ\right)$ **c)** $2\cos\left(\theta + 60^\circ\right)$

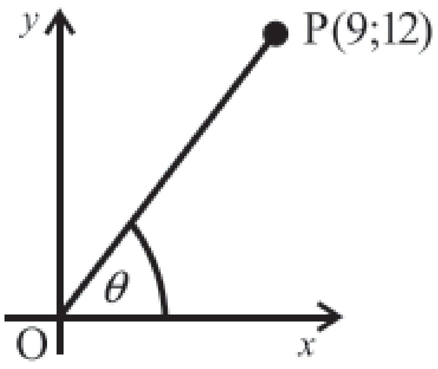
Solutions to the Mixed Exercise number 1 are on page **297**

2) Determine the value of θ correct to 1 decimal place if $0^\circ < \theta < 90^\circ$:

a) $\sin\theta = 0{,}63$ **b)** $\sin 2\theta = 0{,}5$ **c)** $\cos\left(\theta + 10^\circ\right) = 0{,}866$

Solutions to the Mixed Exercise number 2 are on page **297**

3) Using the diagram below, determine:



a) The length of OP **b)** The value of $\sin\theta$ **c)** The value of $2\cos\theta + 3\cot\theta$

Solutions to the Mixed Exercise number 3 are on page **298**

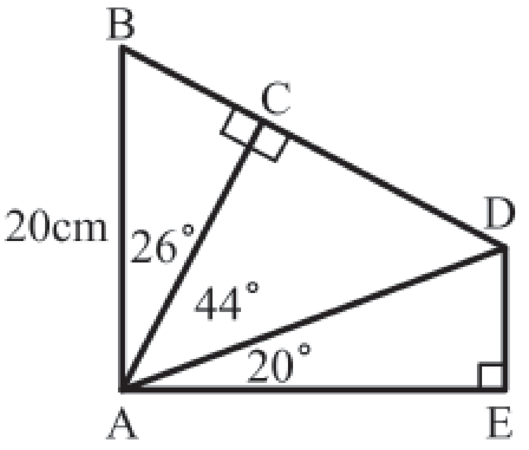
4) Given that $\sin\theta = \frac{1}{2}$ and that $\tan\theta < 0$, and that $\tan\beta = 1$ and $\sin\beta < 0$, determine:

a) $\cos^2\theta + \sin^2\beta$ **c)** $\cos^2\beta - \sin^2\beta$ **e)** $\cos^2\theta + \cos^2\beta$

b) $\cot\beta\sec\theta$ **d)** $\sin^2\theta - \cos^2\theta$ **f)** $1 - 2\sin^2\theta + \cos^2\theta$

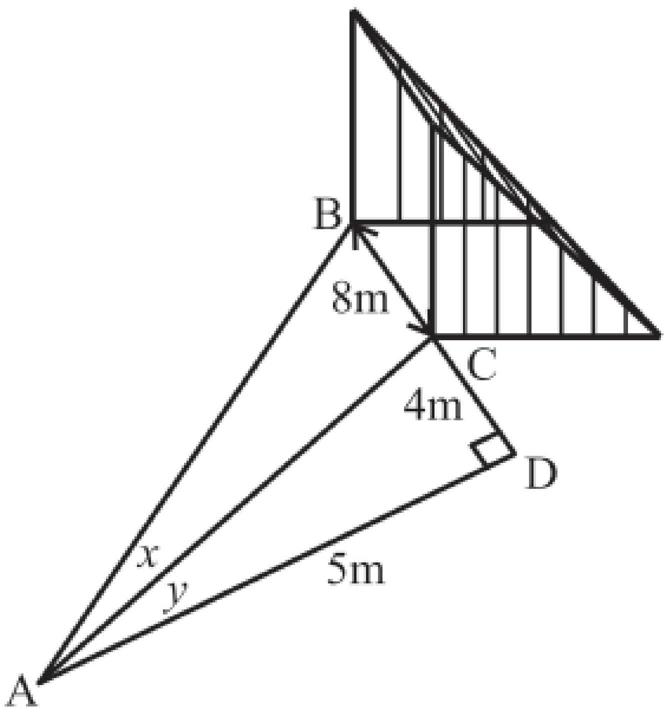
Solutions to the Mixed Exercise number 4 are on page **298**

5) Using the diagram below, calculate the length of DE:



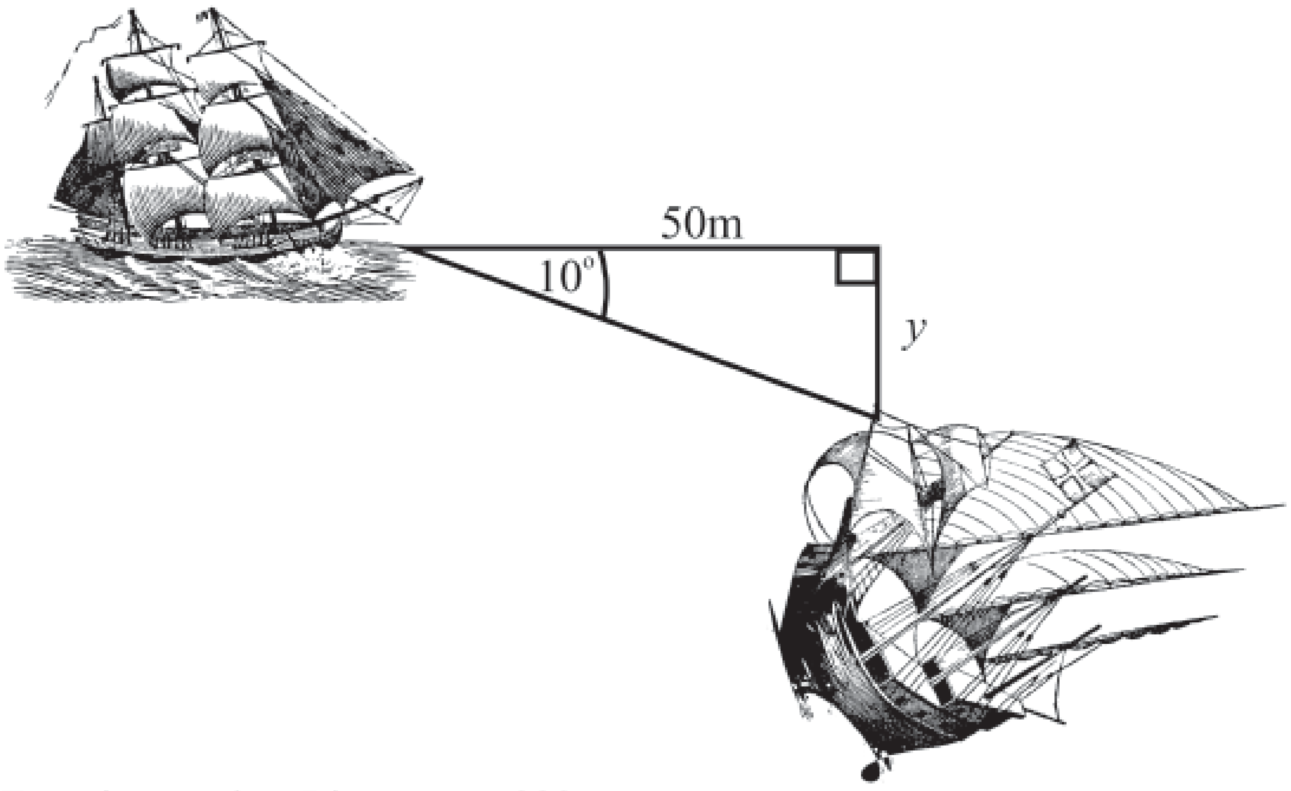
Solution to the Mixed Exercise number 5 is on page **298**

6) A soccer player standing at point A attempts to score a goal. Determine the values of *x* and *y*, the maximum and minimum angle respectively, that he can kick the ball at and still score.



Solution to the Mixed Exercise number 6 is on page **299**

7) A boat spots a wreckage at an angle of depression of 10°. If the hull of the boat extends 6m below the surface of the water and the wreckage is 50m from the boat, can the boat safely pass over the wreck?

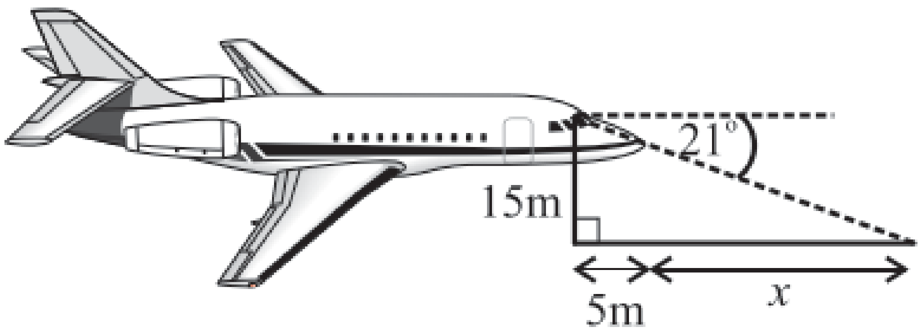


Solution to the Mixed Exercise number 7 is on page **299**

8) A painter places a 4m long ladder so that the angle of elevation from the foot of the ladder on the ground is 60º. When the painter is half way up the ladder, it slips and falls 1,46 m down the wall. What is the new angle of elevation that the ladder now makes with the wall?

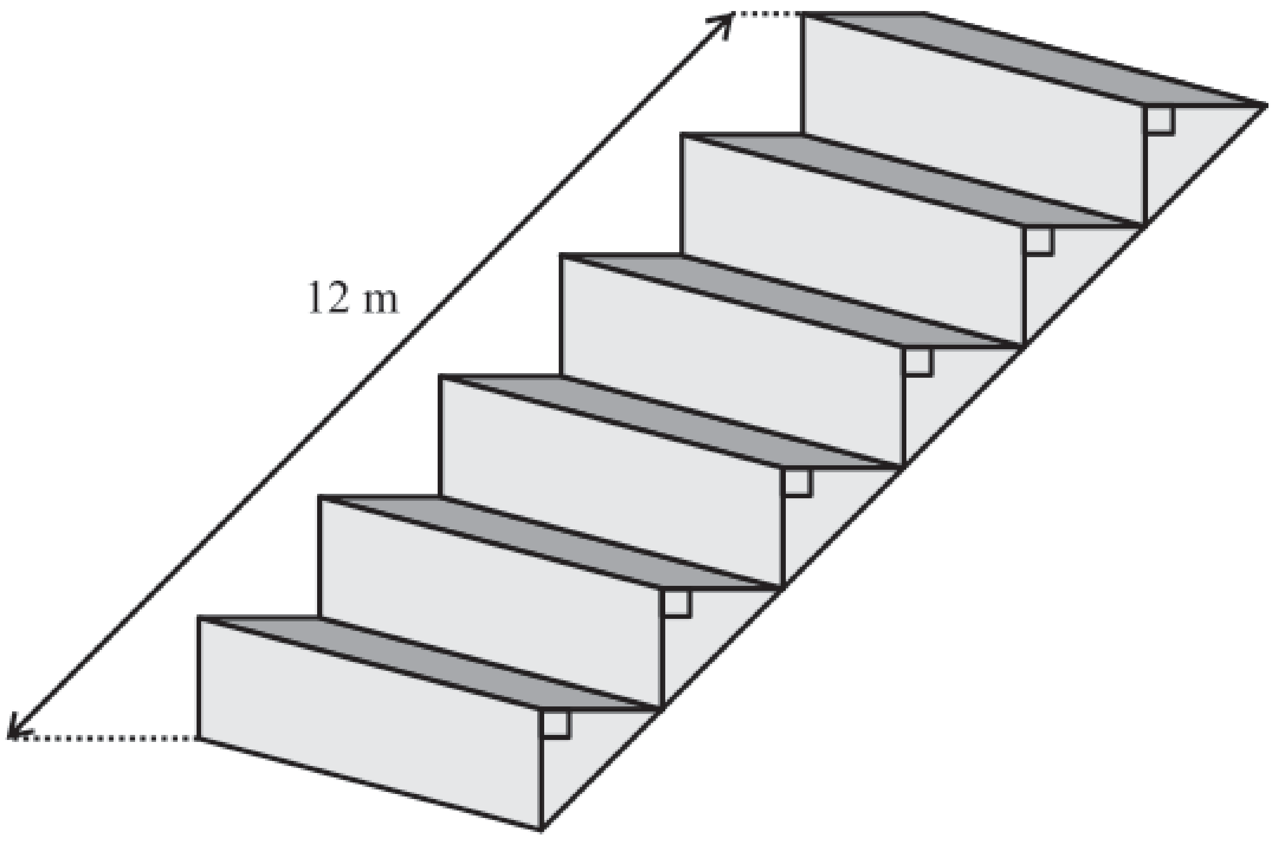
Solution to the Mixed Exercise number 8 is on page **299**

9) When on the ground, planes have a large blind distance in front of them depending on the length of the nose of the plane. A plane stands 15m off the ground and has a nose length of 5m. Determine *x*, the blind distance in front of the plane if the angle of depression from the plane to the ground is 21°.



Solution to the Mixed Exercise number 9 is on page **299**

10) A staircase which is 12 metres in length and connects two floors $6\sqrt{2}$ meters apart is shown below.



a) What angle must the staircase be placed at?
b) If the staircase contains 48 steps how high is each step?

Solution to the Mixed Exercise number 10 is on page **299**

Chapter 5 – Trigonometry (Solutions)

Exercise 1 Solutions:

1)

a) Opposite → AB
Adjacent → BC
Hypotenuse → AC

b) Opposite → FG
Adjacent → EF
Hypotenuse → EG

c) Opposite → FG
Adjacent → EF
Hypotenuse → EG

d) Opposite → FG
Adjacent → EG
Hypotenuse → EF

e) Opposite → DF
Adjacent → ED
Hypotenuse → EF

f) Opposite → ED
Adjacent → BE
Hypotenuse → BD

Exercise 1 number 1 is on page 287

2)

a) $\sin 30^\circ = \dfrac{x}{70}$
$\therefore x = 35\,\text{cm}$

b) $\tan 45^\circ = \dfrac{10}{x}$
$\therefore x = 10\,\text{m}$

c) $\cos 25^\circ = \dfrac{x}{12}$
$\therefore x = 10{,}88\,\text{m}$

d) $\cos 60^\circ = \dfrac{20}{x}$
$\therefore x = 40\,\text{m}$

e) $\tan 18^\circ = \dfrac{2}{x}$
$\therefore x = 6{,}16\,\text{m}$

f) $\cos 45^\circ = \dfrac{\sqrt{2}}{x}$
$\therefore x = 2\,\text{cm}$

Exercise 1 number 2 is on page 287

3)

a) $2\sin B$
$= 2\sin 30^\circ$
$= 1$

b) $\dfrac{\sqrt{3}}{2}\cos B$
$= \dfrac{\sqrt{3}}{2}\cos 30^\circ$
$= 0{,}8$

c) $\cos(A + 2B)$
$= \cos\left(20^\circ + 2 \times 30^\circ\right)$
$= 0{,}2$

d) $\sin\left(90^\circ - B\right)$
$= \sin\left(90^\circ - 30^\circ\right)$
$= 0{,}9$

e) $\cos\left(90^\circ - B\right)$
$= \cos\left(90^\circ - 30^\circ\right)$
$= 0{,}5$

f) $2\sin\left(2{,}5^\circ + A\right)\cos\left(2{,}5^\circ + A\right)$
$= 2\sin\left(2{,}5^\circ + 20^\circ\right)\cos\left(2{,}5^\circ + 20^\circ\right)$
$= 0{,}7$

g) $\cos^2(A) + \sin^2(A)$
$= \cos^2\left(20^\circ\right) + \sin^2\left(20^\circ\right)$
$= 1$

h) $\cos^2(B) - \sin^2(B)$
$= \cos^2\left(30^\circ\right) - \sin^2\left(30^\circ\right)$
$= 0{,}5$

Exercise 1 number 3 is on page 287

4)

a) $\tan^{-1}\left(\dfrac{6}{4}\right) = 56{,}31^\circ$
$\therefore \theta = 56{,}31^\circ$

b) $\cos^{-1}\left(\dfrac{5}{10}\right) = 60^\circ$
$\therefore \theta = 60^\circ$

c) $\sin^{-1}\left(\dfrac{3}{6}\right) = 30^\circ$
$\therefore \theta = 30^\circ$

Exercise 1 number 4 is on page 287

5)

a) $BA = CD$

$\sin 30^\circ = \dfrac{ED}{AE}$

$\therefore \sin 30^\circ = \dfrac{ED}{10}$

$\therefore 10\sin 30^\circ = ED$

$\therefore ED = 5\text{m}$

$\sin 30^\circ = \dfrac{CE}{BE}$

$\therefore \sin 30^\circ = \dfrac{CE}{10}$

$\therefore 10\sin 30^\circ = CE$

$\therefore CE = 5\text{m}$

$\therefore BA = CD = CE + ED = 10\,\text{m}$

b) $\cos 30^\circ = \dfrac{BC}{BE}$

$\therefore \cos 30^\circ = \dfrac{BC}{10}$

$\therefore 10\cos 30^\circ = BC$

$\therefore BC = 10\left(\dfrac{\sqrt{3}}{2}\right)$

$\therefore BF = \dfrac{5\sqrt{3}}{2}\,\text{m}$

$\tan 45^\circ = \dfrac{GF}{BF}$

$\therefore BF\tan 45^\circ = GF$

$\therefore GF = \dfrac{5\sqrt{3}}{2}(1) = \dfrac{5\sqrt{3}}{2}\ \text{m}$

Exercise 1 number 5 is on page **288**

6)

a) $2\sin\theta + 1 = 2$

$\therefore 2\sin\theta = 1$

$\therefore \sin\theta = \dfrac{1}{2}$

$\therefore \theta = 30^\circ$

b) $2\cos\theta - \sqrt{3} = 0$

$\therefore 2\cos\theta = \sqrt{3}$

$\therefore \cos\theta = \dfrac{\sqrt{3}}{2}$

$\therefore \theta = 30^\circ$

c) $2\tan\left(\theta + 10^\circ\right) + 3 = 5$

$\therefore 2\tan\left(\theta + 10^\circ\right) = 2$

$\therefore \tan\left(\theta + 10^\circ\right) = 1$

$\therefore \theta + 10^\circ = 45^\circ$

$\therefore \theta = 35^\circ$

d) $3\sin\left(2\theta + 12^\circ\right) - 2 = 1$

$\therefore 3\sin\left(2\theta + 12^\circ\right) = 3$

$\therefore \sin\left(2\theta + 12^\circ\right) = 1$

$\therefore 2\theta + 12^\circ = 90^\circ$

$\therefore 2\theta = 78^\circ$

$\therefore \theta = 39^\circ$

e) $2\cos\theta + 1 = 2$

$\therefore 2\cos\theta = 1$

$\therefore \cos\theta = \dfrac{1}{2}$

$\therefore \theta = 60^\circ$

f) $2\sin\theta = \sqrt{3}$

$\therefore \sin\theta = \dfrac{\sqrt{3}}{2}$

$\therefore \theta = 60^\circ$

g) $2\sin\left(\theta - 10^\circ\right) + 3 = 3$

$\therefore 2\sin\left(\theta - 10^\circ\right) = 0$

$\therefore \sin\left(\theta - 10^\circ\right) = 0$

$\therefore \theta - 10^\circ = 0^\circ$

$\therefore \theta = 10^\circ$

h) $3\cos\left(2\theta - 12^\circ\right) - 2 = 1$

$\therefore 3\cos\left(2\theta - 12^\circ\right) = 3$

$\therefore \cos\left(2\theta - 12^\circ\right) = 1$

$\therefore 2\theta - 12^\circ = 0^\circ$

$\therefore 2\theta = 12^\circ$

$\therefore \theta = 6^\circ$

Exercise 1 number 6 is on page **288**

Exercise 2 Solutions:

a) $\sec 30^\circ \cos 30^\circ$

$= \left(\dfrac{2}{\sqrt{3}}\right)\left(\dfrac{\sqrt{3}}{2}\right)$

$= 1$

b) $\sin 60^\circ + \cos 30^\circ$

$= \left(\dfrac{\sqrt{3}}{2}\right) + \left(\dfrac{\sqrt{3}}{2}\right)$

$= \sqrt{3}$

c) $\operatorname{cosec}^2 60^\circ + \tan^2 45^\circ + \sec^2 30^\circ$

$= \left(\dfrac{2}{\sqrt{3}}\right)^2 + (1)^2 + \left(\dfrac{2}{\sqrt{3}}\right)^2$

$= \dfrac{4}{3} + 1 + \dfrac{4}{3}$

$= \dfrac{11}{3}$

d) $1 + \cot^2 30^\circ$

$= 1 + \left(\sqrt{3}\right)^2$

$= 4$

e) $\sec 30^\circ \cot 60^\circ$

$= \left(\dfrac{2}{\sqrt{3}}\right)\left(\dfrac{1}{\sqrt{3}}\right)$

$= \dfrac{2}{3}$

f) $1 + \operatorname{cosec}^2 60^\circ$

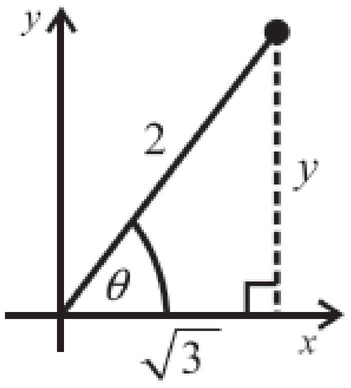
$= 1 + \left(\dfrac{2}{\sqrt{3}}\right)^2$

$= \dfrac{7}{3}$

Exercise 2 is on page **288**

Exercise 3 Solutions:

1)



Using Pythagoras: $r^2 = x^2 + y^2$

$$\therefore y^2 = (2)^2 - \left(\sqrt{3}\right)^2$$

$$\therefore y^2 = 4 - 3$$

$$\therefore y = 1$$

a) $\sin\theta = \frac{1}{2} \rightarrow \frac{\text{O}}{\text{H}}$ or $\frac{y}{r}$

b) $\cos\theta = \frac{\sqrt{3}}{2} \rightarrow \frac{\text{A}}{\text{H}}$ or $\frac{x}{r}$

c) $\tan\theta = \frac{1}{\sqrt{3}} \rightarrow \frac{\text{O}}{\text{A}}$ or $\frac{y}{x}$

d) $\sin\theta\cos\theta$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{4}$$

e) $\tan\theta\cos\theta$

$$= \left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{2}$$

f) $2\sin\theta + 3\cos\theta$

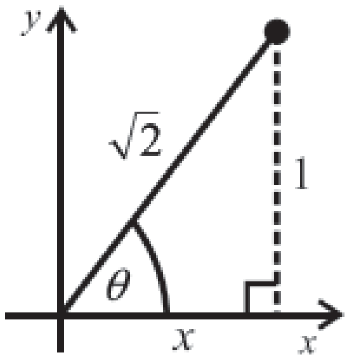
$$= 2\left(\frac{1}{2}\right) + 3\left(\frac{\sqrt{3}}{2}\right)$$

$$= 1 + \frac{3\sqrt{3}}{2}$$

$$= \frac{2 + 3\sqrt{3}}{2}$$

Exercise 3 number 1 is on page 288

2)



Using Pythagoras: $r^2 = x^2 + y^2$

$$\therefore x^2 = \left(\sqrt{2}\right)^2 - (1)^2$$

$$\therefore x^2 = 2 - 1$$

$$\therefore x = 1$$

a) $\cos\theta = \frac{1}{\sqrt{2}} \rightarrow \frac{\text{A}}{\text{H}}$ or $\frac{x}{r}$

b) $\tan\theta = \frac{1}{1} \rightarrow \frac{\text{O}}{\text{A}}$ or $\frac{y}{x}$

c) $\sin^2\theta\tan\theta$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 \times 1$$

$$= \frac{1}{2}$$

d) $\sin^2\theta + \cos^2\theta$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

e) $1 - \cos^2\theta$

$$= 1 - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2}$$

f) $1 - \sin^2\theta$

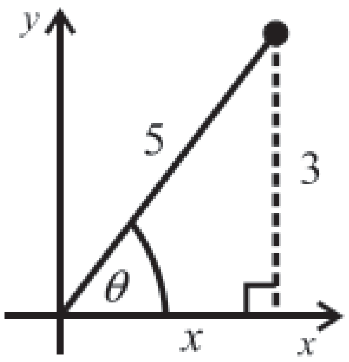
$$= 1 - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

Exercise 3 number 2 is on page 288

3)



Using Pythagoras: $r^2 = x^2 + y^2$

$$\therefore x^2 = (5)^2 - (3)^2$$

$$\therefore x^2 = 16$$

$$\therefore x = 4$$

a) $\operatorname{cosec}\theta = \frac{5}{3} \rightarrow \frac{\text{H}}{\text{O}}$ or $\frac{r}{y}$

b) $\sec\theta = \frac{5}{4} \rightarrow \frac{\text{H}}{\text{A}}$ or $\frac{r}{x}$

c) $\cot\theta = \frac{4}{3} \rightarrow \frac{\text{A}}{\text{O}}$ or $\frac{x}{y}$

d) $\tan\theta\operatorname{cosec}\theta = \left(\frac{3}{4}\right)\left(\frac{5}{3}\right) = \frac{5}{4}$

e) $\frac{\cos\theta}{\sec\theta} = \left(\frac{4}{5}\right) \div \left(\frac{5}{4}\right) = \frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$

f) $\frac{\tan\theta}{\sec\theta} = \left(\frac{3}{4}\right) \div \left(\frac{5}{4}\right) = \frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$

Exercise 3 number 3 is on page 289

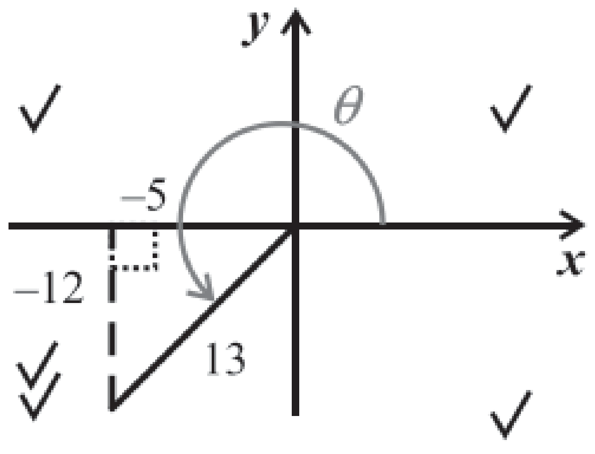
Exercise 4 Solutions:

1)

a) Quadrant I (1)
b) Quadrant IV (4)
c) Quadrant III (3)
d) Quadrant III (3)
e) Quadrant I (1)
f) Quadrant IV (4)
g) Quadrant II (2)
h) Quadrant II (2)
i) Quadrant IV (4)
j) Quadrant III (3)

Exercise 4 number 1 is on page **289**

2)



Using Pythagoras: $r^2 = x^2 + y^2$

$\therefore r^2 = (-5)^2 + (-12)^2$

$\therefore r^2 = 169$

$\therefore r = 13$

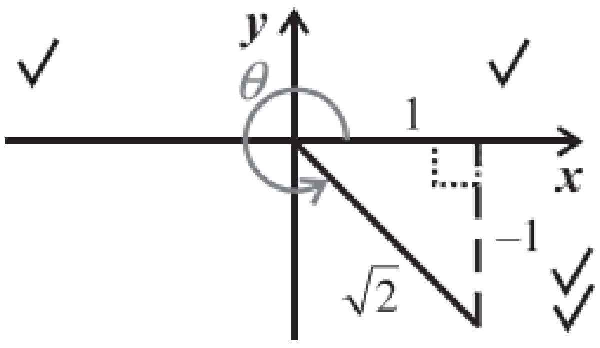
a) $\sin\theta = -\dfrac{12}{13}$

b) $\cos\theta = -\dfrac{5}{13}$

c) $\cos^2\theta + \sin^2\theta = \left(-\dfrac{5}{13}\right)^2 + \left(-\dfrac{12}{13}\right)^2 = 1$

Exercise 4 number 2 is on page **289**

3)



Using Pythagoras: $y^2 = r^2 - x^2$

$\therefore y^2 = \left(\sqrt{2}\right)^2 - (1)^2$

$\therefore y^2 = 2 - 1$

$\therefore y = -1 \rightarrow y$ is negative in the quadrant IV

a) $\cos\theta$

$= \dfrac{1}{\sqrt{2}}$

b) $\tan\theta$

$= -\dfrac{1}{1}$

$= -1$

c) $\sin^2\theta\tan\theta$

$= \left(-\dfrac{1}{\sqrt{2}}\right)^2\left(\dfrac{-1}{1}\right) = -\dfrac{1}{2}$

d) $\sin^2\theta + \cos^2\theta$

$= \left(-\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{1}{\sqrt{2}}\right)^2$

$= \dfrac{1}{2} + \dfrac{1}{2} = 1$

e) $1 - \cos^2\theta$

$= 1 - \left(\dfrac{1}{\sqrt{2}}\right)^2 = \dfrac{1}{2}$

f) $1 - \sin^2\theta$

$= 1 - \left(-\dfrac{1}{\sqrt{2}}\right)^2 = \dfrac{1}{2}$

Exercise 4 number 3 is on page **289**

Exercise 5 Solutions:

1)

a) $\cos 60^\circ \times \sin 60^\circ$

$= \left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)$

$= \dfrac{\sqrt{3}}{4}$

b) $\tan 45^\circ \times \sin 45^\circ$

$= (1)\left(\dfrac{\sqrt{2}}{2}\right)$

$= \dfrac{\sqrt{2}}{2}$

c) $\sin^2 60^\circ + \cos^2 60^\circ$

$= \left(\dfrac{\sqrt{3}}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2$

$= \dfrac{3}{4} + \dfrac{1}{4}$

$= 1$

d) $\sin^2 30^\circ + \cos^2 30^\circ$

$= \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2$

$= \dfrac{1}{4} + \dfrac{3}{4}$

$= 1$

e) $\sin 60^\circ \times \cos 30^\circ$

$= \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)$

$= \dfrac{3}{4}$

f) $\tan 60^\circ \times \tan 30^\circ + \cos 90^\circ$

$= \left(\sqrt{3}\right)\left(\dfrac{1}{\sqrt{3}}\right) + 0$

$= 1$

Exercise 5 number 1 is on page **289**

2)

a) $\sec 30^\circ \cos 30^\circ$

$$=\left(\frac{2}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$=1$$

b) $\sin 60^\circ + \cos 30^\circ$

$$=\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)$$

$$=\sqrt{3}$$

c) $\operatorname{co}\sec^2 60^\circ + \tan^2 45^\circ + \sec^2 30^\circ$

$$=\left(\frac{2}{\sqrt{3}}\right)^2+(1)^2+\left(\frac{2}{\sqrt{3}}\right)^2$$

$$=\frac{4}{3}+1+\frac{4}{3}=\frac{11}{3}$$

d) $1+\cot^2 30^\circ$

$$=1+\left(\sqrt{3}\right)^2$$

$$=1+3$$

$$=4$$

e) $\sec 30^\circ \cot 60^\circ$

$$=\left(\frac{2}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)$$

$$=\frac{2}{3}$$

f) $1+\operatorname{cosec}^2 60^\circ$

$$=1+\left(\frac{2}{\sqrt{3}}\right)^2$$

$$=1+\frac{4}{3}=\frac{7}{3}$$

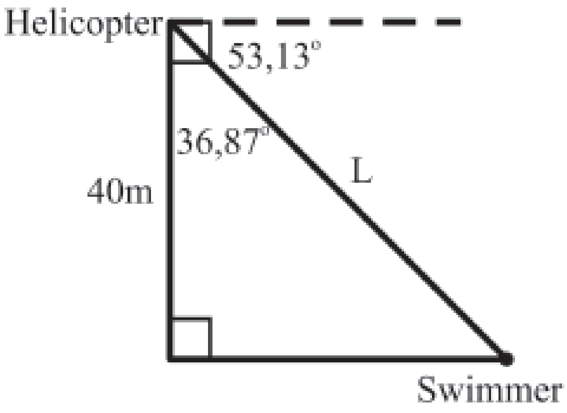
Exercise 5 number 2 is on page **289**

Exercise 6 Solutions:

1) $\cos 36{,}87^\circ = \dfrac{40}{\mathrm{L}}$

$$\therefore \mathrm{L}=\frac{40}{\cos 36{,}87^\circ}$$

$$\therefore \mathrm{L}=50\,\mathrm{m}$$



2) $\sin\theta=\dfrac{3}{4}$

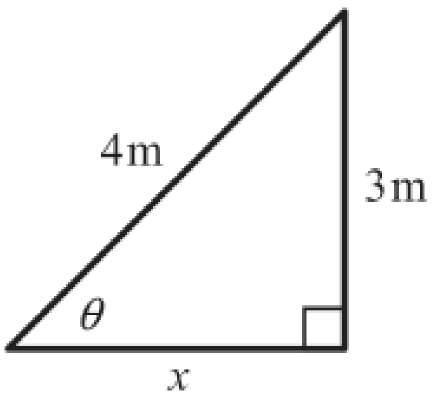
$$\therefore \theta = 48{,}59^\circ$$

$$\cos\theta=\frac{x}{4}$$

$$\therefore x = 4\times\cos 48{,}59^\circ$$

$$\therefore x = 2{,}65\,\mathrm{m}$$

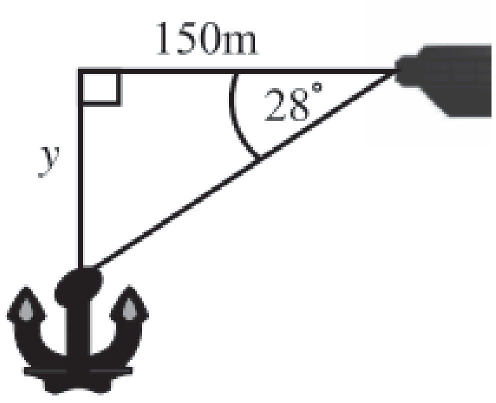
$\therefore$ he must place the ladder 2,65 m from the wall at an angle of $48{,}59^\circ$



3) $\tan 28^\circ = \dfrac{y}{150}$

$$\therefore y = 150\tan 28^\circ$$

$$\therefore y = 80\,\mathrm{m}$$



4) $\tan 45^\circ = \dfrac{x_1}{80}$

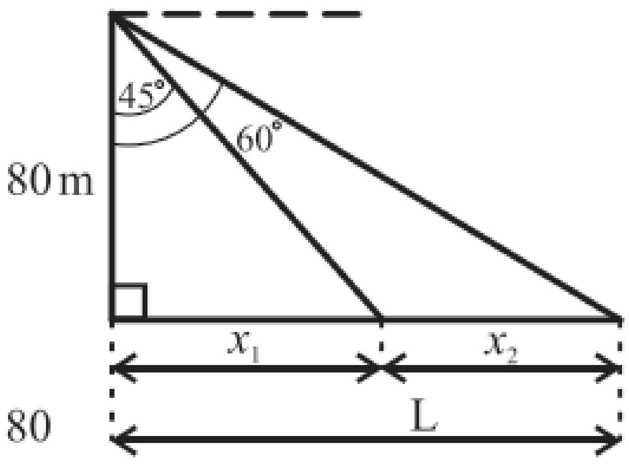
$$\therefore x_1 = 80\,\mathrm{m}$$

$$\tan 60^\circ = \frac{\mathrm{L}}{80}$$

$$\therefore \mathrm{L}=138{,}56\,\mathrm{m}$$

$$\therefore x_2 = 138{,}56-80$$

$$\therefore x_2 = 58{,}56\,\mathrm{m}$$



Exercise 6 is on page **290**

Mixed Exercise Solutions:

1)

a) $2\sin\dfrac{25{,}6^\circ}{2}=0{,}4$

b) $\tan\left[2\left(25{,}6^\circ\right)-20^\circ\right]=0{,}6$

c) $2\cos\left(25{,}6^\circ+60^\circ\right)=0{,}2$

The Mixed Exercise number 1 is on page **291**

2)

a) $\sin\theta = 0{,}63$

$$\therefore \theta = 39{,}1^\circ$$

b) $\sin 2\theta = 0{,}5$

$$\therefore 2\theta = 30^\circ$$

$$\therefore \theta = 15^\circ$$

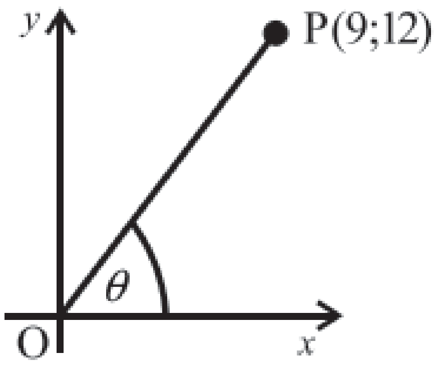
c) $\cos\left(\theta+10^\circ\right)=0{,}866$

$$\therefore \theta+10^\circ = 30^\circ$$

$$\therefore \theta = 20^\circ$$

The Mixed Exercise number 2 is on page **291**

3)



a) $r^2 = x^2 + y^2$

$\therefore r^2 = (9)^2 + (12)^2$

$\therefore r^2 = 225$

$\therefore OP = 15$

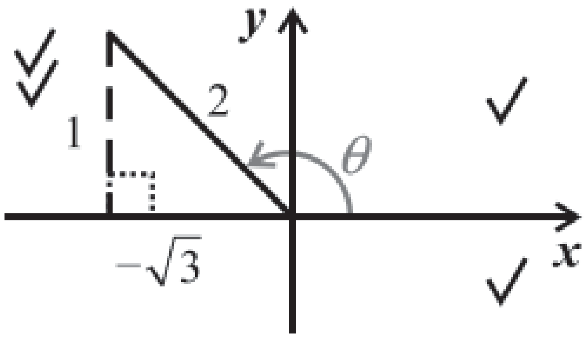
b) $\sin\theta = \dfrac{12}{15}$

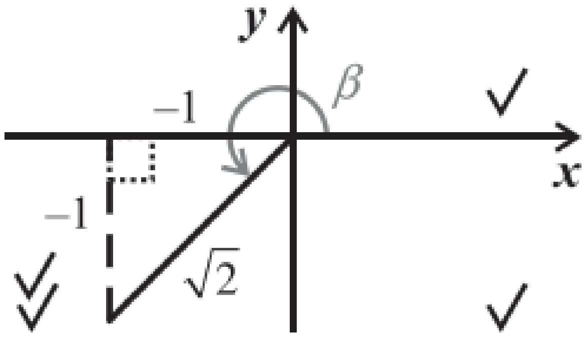
c) $2\left(\dfrac{9}{15}\right) + 3\left(\dfrac{9}{12}\right)$

$= \dfrac{69}{20}$

The Mixed Exercise number 3 is on page **291**

4)





a) $\cos^2\theta + \sin^2\beta$

$= \left(-\dfrac{\sqrt{3}}{2}\right)^2 + \left(\dfrac{-1}{\sqrt{2}}\right)^2$

$= \dfrac{3}{4} + \dfrac{1}{2}$

$= \dfrac{5}{4}$

b) $\cot\beta\sec\theta$

$= \left(\dfrac{-1}{-1}\right)\left(-\dfrac{2}{\sqrt{3}}\right)$

$= -\dfrac{2}{\sqrt{3}}$

c) $\cos^2\beta - \sin^2\beta$

$= \left(-\dfrac{1}{\sqrt{2}}\right)^2 - \left(-\dfrac{1}{\sqrt{2}}\right)^2$

$= \dfrac{1}{2} - \dfrac{1}{2}$

$= 0$

d) $\sin^2\theta - \cos^2\theta$

$= \left(\dfrac{1}{2}\right)^2 - \left(-\dfrac{\sqrt{3}}{2}\right)^2$

$= \dfrac{1}{4} - \dfrac{3}{4}$

$= -\dfrac{1}{2}$

e) $\cos^2\theta + \cos^2\beta$

$= \left(-\dfrac{\sqrt{3}}{2}\right)^2 + \left(-\dfrac{1}{\sqrt{2}}\right)^2$

$= \dfrac{3}{4} + \dfrac{1}{2}$

$= \dfrac{5}{4}$

f) $1 - 2\sin^2\theta + \cos^2\theta$

$= 1 - 2\left(\dfrac{1}{2}\right)^2 + \left(-\dfrac{\sqrt{3}}{2}\right)^2$

$= 1 - \dfrac{1}{2} + \dfrac{3}{4}$

$= \dfrac{5}{4}$

The Mixed Exercise number 4 is on page **291**

5)

$\cos 26^\circ = \dfrac{\text{AC}}{20}$

$\therefore \text{AC} = 17{,}98\,\text{cm}$

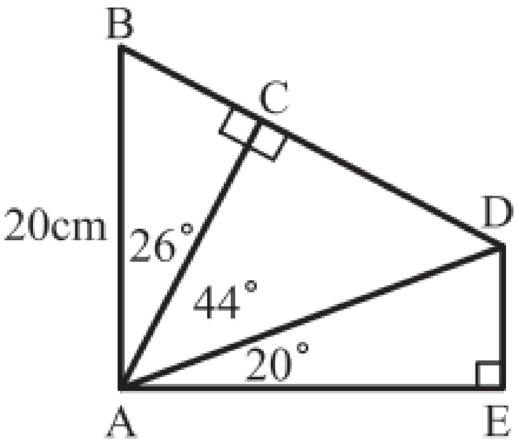
$\cos 44^\circ = \dfrac{\text{AC}}{\text{AD}}$

$\therefore \text{AD} = \dfrac{17{,}98}{\cos 44^\circ} = 25\,\text{cm}$

$\sin 20^\circ = \dfrac{\text{DE}}{\text{AD}}$

$\therefore \text{DE} = 25\sin 20^\circ$

$\therefore \text{DE} = 8{,}55\,\text{cm}$



The Mixed Exercise number 5 is on page **291**

6)

$\tan(y) = \frac{CD}{AD}$

$\therefore \tan(y) = \frac{4}{5}$

$\therefore y = 38{,}66^\circ$

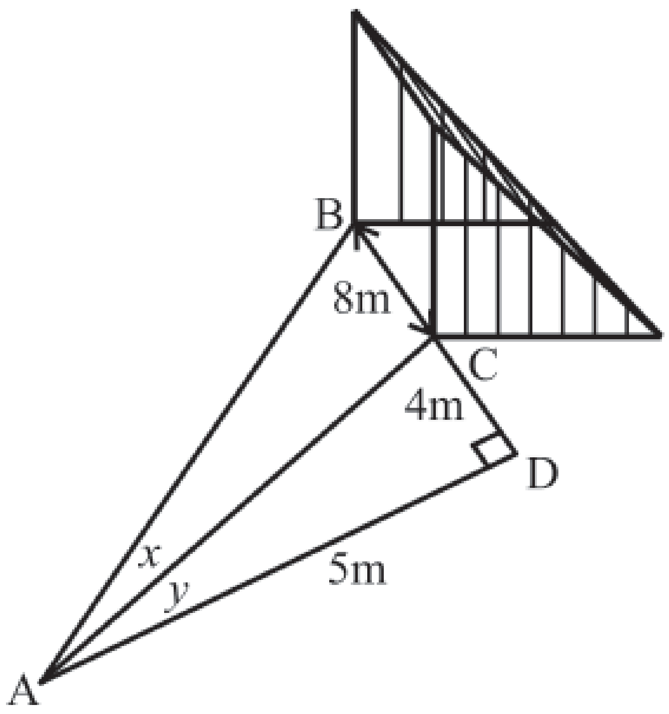
$\tan(x+y) = \frac{BD}{AD}$

$\therefore \tan(x+y) = \frac{12}{5}$

$\therefore x + y = 67{,}38^\circ$ and $y = 38{,}66^\circ$

$\therefore x = 67{,}38^\circ - 38{,}66^\circ$

$\therefore x = 28{,}72^\circ$



The Mixed Exercise number 6 is on page **291**

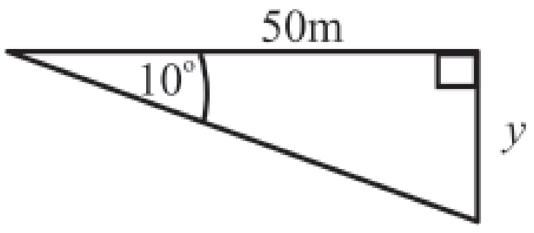
7)

$\tan 10^\circ = \frac{y}{50}$

$\therefore y = 50 \tan 10^\circ$

$\therefore y = 8{,}82\,\text{m}$

$\therefore$ The boat will clear the wreckage by 2,82 m



The Mixed Exercise number 7 is on page **292**

8)

$\sin 60^\circ = \frac{y}{4}$

$\therefore y = 3{,}46\,\text{m}$

After falling:

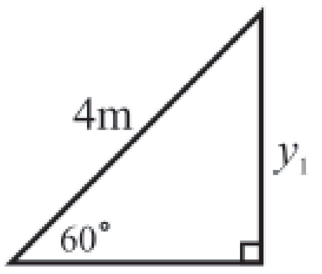
new height $(y_1) = 3{,}46 - 1{,}46 = 2\,\text{m}$

$\therefore \sin\theta = \frac{2}{4}$

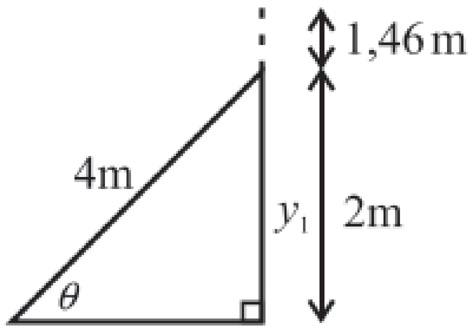
$\therefore \sin\theta = \frac{1}{2}$

$\therefore \theta = 30^\circ$

$\therefore$ the new angle of elevation is 30°



After falling:



The Mixed Exercise number 8 is on page **292**

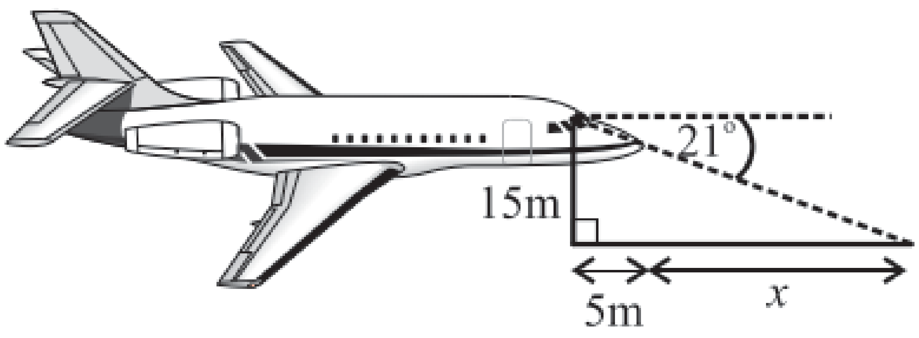
9)

$\tan 69^\circ = \frac{5+x}{15}$

$\therefore 5 + x = 15 \tan 69^\circ$

$x = 15 \tan 69^\circ - 5$

$\therefore x = 34{,}08\,\text{m}$

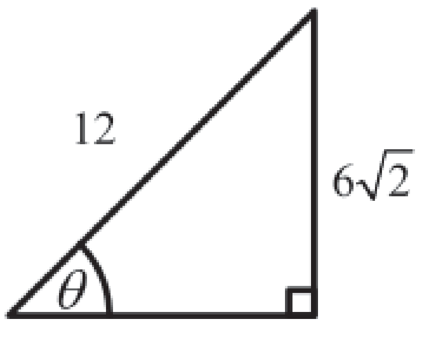


The Mixed Exercise number 9 is on page **292**

10)

a) $\sin\theta = \frac{6\sqrt{2}}{12}$

$\therefore \theta = 45^\circ$



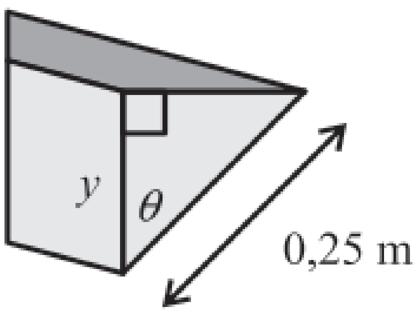
b) $\frac{12}{48} = 0{,}25\,\text{m}$

$\therefore \cos 45^\circ = \frac{y}{0{,}25}$

$\therefore y = 0{,}25 \cos 45^\circ$

$\therefore y = 0{,}18\,\text{m}$

$\therefore$ Each step is 0,18 m high



The Mixed Exercise number 10 is on page **292**

Chapter 6 – Functions

Working with Functions and their Notations

Exercise 1: (Theory on page 96)

1) Given the functions $f(x)=2x^2+3x-10$ and $h(x)=x^2-3x$ determine the value of:

a) $f(2)$ **c)** $f(-2)$ **e)** $h(-2)$

b) $h(2)$ **d)** $f(-5)$ **f)** $h(1)$

Solutions to Exercise 1 number 1 are on page **310**

2) Given the function $f:x\rightarrow x^2-4$ determine:

a) $f\left(\frac{1}{2}\right)$ **c)** $f(2x)$ **e)** $f(-x)$

b) $f(p)$ **d)** $f(x)+2$ **f)** $f(x+2)$

Solutions to Exercise 1 number 2 are on page **310**

Linear Functions

Exercise 2:

1) Sketch the following graphs using the table method: (theory on page 97)

a) $y=2x-3$ **c)** $y=x-1$

b) $f(x)=-x+2$ **d)** $f(x)=-4x+4$

Solutions to Exercise 2 number 1 are on page **310**

2) Sketch the following graphs using the dual intercept method: (theory on page 98)

a) $y=-x-2$ **c)** $y=4x-8$

b) $f(x)=-2x+4$ **d)** $f(x)=-x-4$

Solutions to Exercise 2 numbers 2a and 2b are on page **310** and numbers 2c and 2d are on page **311**

3) Sketch the following graphs using the gradient intercept method: (theory on page 99)

a) $y=3x-6$ **c)** $5x+2y=10$

b) $f(x)=-4x$ **d)** $3x=3y$

Solutions to Exercise 2 number 3 are on page **311**

4) Sketch the following graphs on the same set of axes:

a) $f(x)=x$ **c)** $p(x)=x-1$ **e)** $m(x)=x-2$

b) $h(x)=x+1$ **d)** $r(x)=x+2$ **f)** $v(x)=x+3$

Solutions to Exercise 2 number 4 are on page **311**

5) Using your graphs in question 4 state what increasing and decreasing the value of q does to the graph.

Solutions to Exercise 2 number 5 are on page **311**

6) Without sketching the graphs below, determine the gradient and y-intercept of the functions below:

a) $4y-8x=0$ **b)** $4x-2y=16$ **c)** $10x+5y=10$

Solutions to Exercise 2 number 6 are on page **311**

Exercise 3: (theory on page 100)

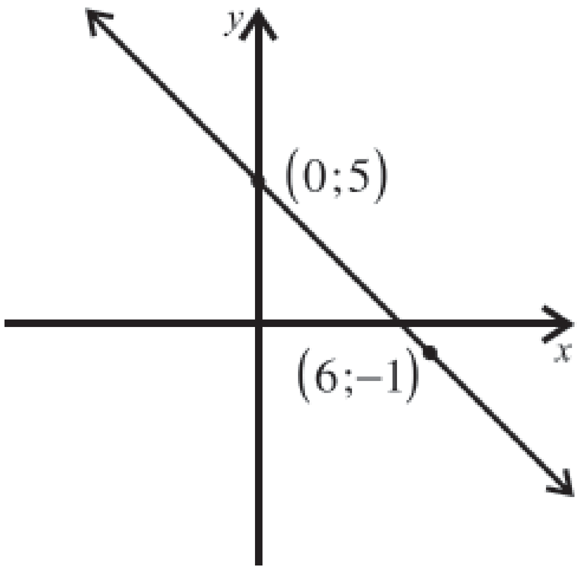
1) Determine the equations of the following linear functions if:

- **a)** The function has a gradient of –3 and a y-intercept of 10.
- **b)** The function has a gradient of –2 and passes through the point (8;38).
- **c)** The function has a y-intercept of 5 and passes through the point (3;–4).
- **d)** The function passes through the point (5;22) and is parallel to the line $y=4x-12$.
- **e)** The function passes through the points (1;4) and (8;25).
- **f)** The function passes through the points (2;4) and (12;–1).

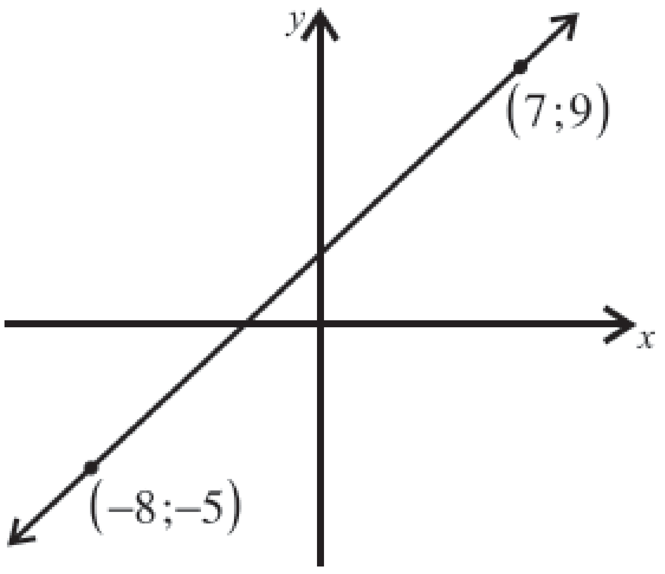
Solutions to Exercise 3 number 1 are on page **312**

2) Determine the equation of the functions shown below: (theory on page 100)

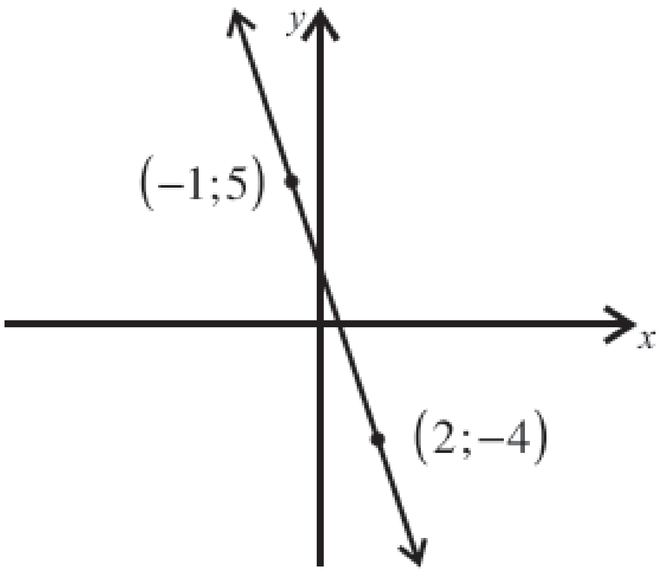
a)



b)



c)



Solutions to Exercise 3 number 2 are on page **312**

3) Given the line $g(x)=\frac{1}{2}x+3$, determine the equation of:

- **a) A line parallel to *g* which passes through the point (2;6).**
- **b) A line which has the same *y*-intercept as *g* which passes through the point (3;–12)**
- **c) A horizontal line which has the same *y*-intercept as *g*.**
- **d) A vertical line which has the same *x*-intercept as *g*.**

Solutions to Exercise 3 number 2 are on page **312**

Quadratic Functions

Exercise 4: (theory on pages 101-103)

1) Determine the shape (concave up or down) , turning point and *x*-intercepts of the functions below:

a) $f(x)=x^2$

b) $y=2x^2-2$

c) $10x^2+5y=-20$

d) $f(x)=-4x^2+16$

e) $f(x)=4-x^2$

f) $2y-8x^2=2$

Solutions to Exercise 4 numbers 1a to 1c are on page **312** and numbers 1d to 1f are on page **313**

2) Sketch the graphs of the following functions showing all intercepts with the axes and turning points:

a) $f(x)=x^2$

b) $f(x)=2x^2$

c) $f(x)=x^2-1$

d) $f(x)=x^2-4$

e) $f(x)=-2x^2+2$

f) $f(x)=2x^2-8$

Solutions to Exercise 4 numbers 2a and 2d are on page **313** and 2e and 2f are on page **314**

Exercise 5: (theory on pages 104-105)

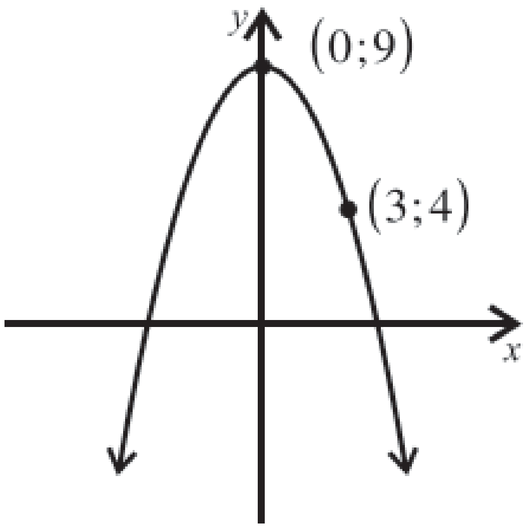
1) Determine the equations of the following parabolas if:

- **a)** The turning point is $(0;4)$ and it passes through the point $(-1;3)$.
- **b)** The turning point is $(0;-8)$ and it passes through the point $(2;0)$.
- **c)** The parabola passes through the points $(1;-4)$ and $(2;-7)$.
- **d)** The parabola passes through the points $(3;0)$ and $(2;15)$.
- **e)** The parabola passes through the points $(1;15)$ and $(3;-25)$.

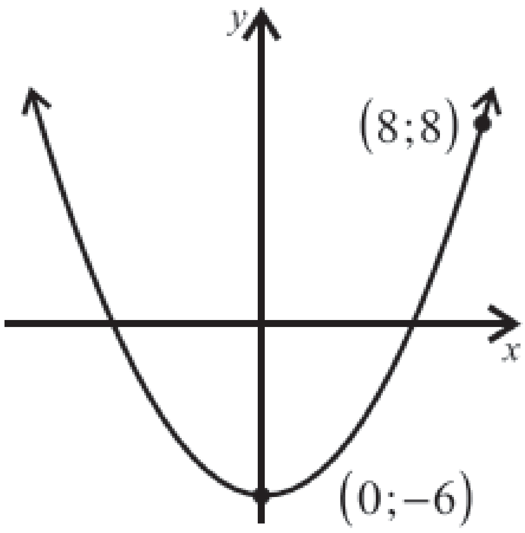
Solutions to Exercise 5 number 1 are on page **314**

2)

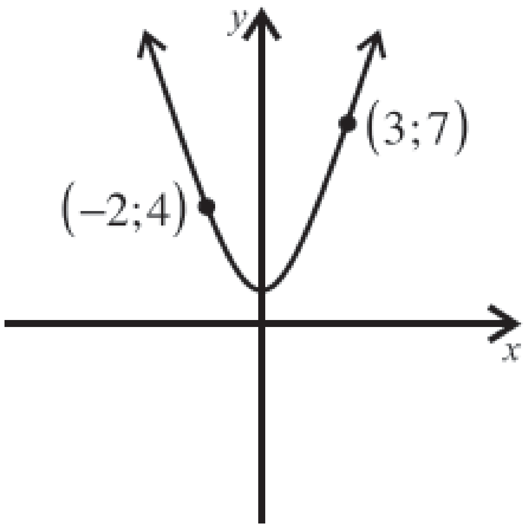
a)



b)



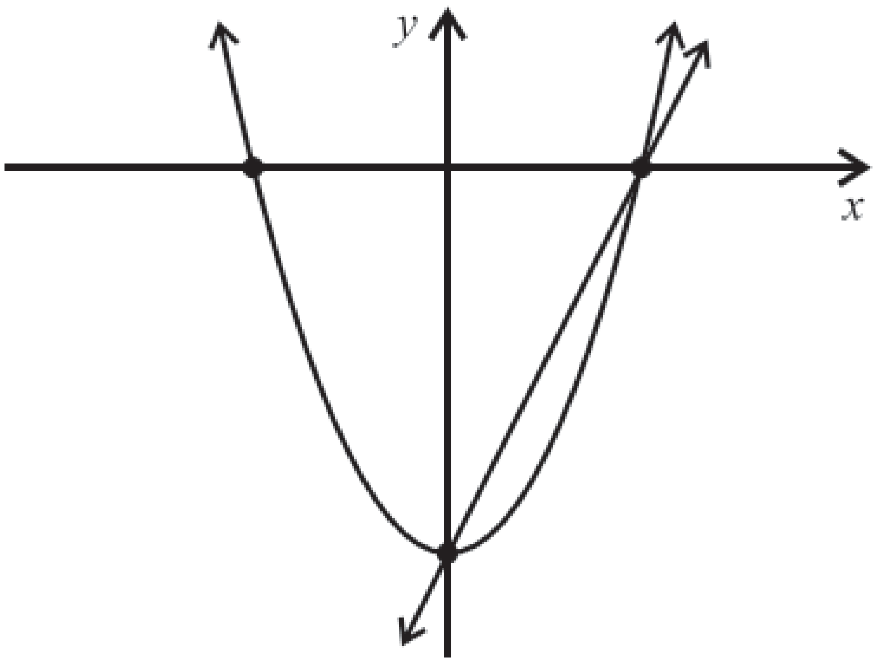
c)



Solutions to Exercise 5 number 2 are on page **315**

3) The graphs of $y-2x+4=0$ and $g(x)=ax^2+q$ are shown below. The graphs share a common *x*, and *y*-intercept. Determine:

- **a) The gradient of the straight line.**
- **b) The *x*-intercepts and turning point of *g*.**
- **c) The equation of *g*.**



Solutions to Exercise 5 number 3 are on page **315**

Hyperbolic Functions

Exercise 6: (theory on pages 106-108)

1) Determine the equations of the asymptotes, lines of symmetry and their points of intersections with the following hyperbolas as well as all intercepts with the axes:

a) $y=\frac{4}{x}$

b) $f(x)=\frac{4}{x}+1$

c) $y=\frac{4}{x}-1$

d) $f(x)=\frac{9}{x}+4$

e) $f(x)=-\frac{4}{x}+4$

f) $f(x)=-\frac{9}{x}-2$

Solutions to Exercise 6 numbers 2a to 2b are on page **315** and numbers 2c to 2f are on page **316**

2) Sketch the graphs in question 1 above. Show all intersections with the axes as well as the axes of symmetry and their points of intersections with the graphs.

Solutions to Exercise 6 numbers 2a to 2c are on page **316** and numbers 2e to 2f are on page **317**

Exercise 7: (theory on pages 109-110)

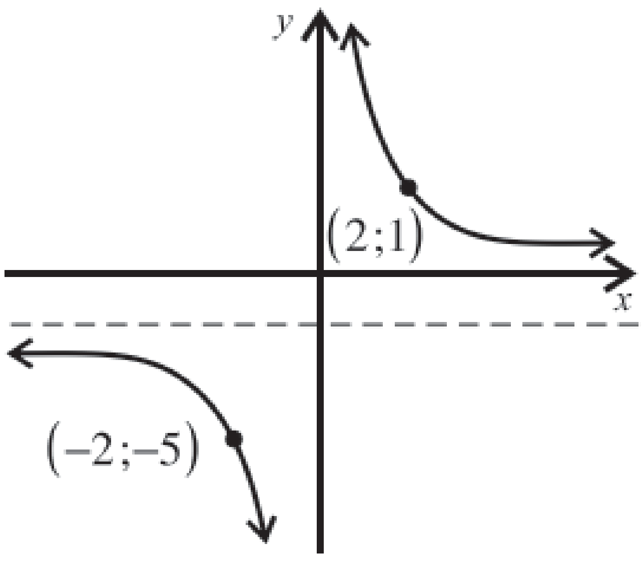
1) Determine the value of *a* and *q* in the graph of $y=\frac{a}{x}+q$ if:

- **a)** The function passes through the point $(2;-1)$ and has a horizontal asymptote at $y=-3$.
- **b)** The function passes through the point $(3;7)$ and has a horizontal asymptote at $y=5$.
- **c)** The function passes through the points $(4;-2)$ and $(6;-1)$.

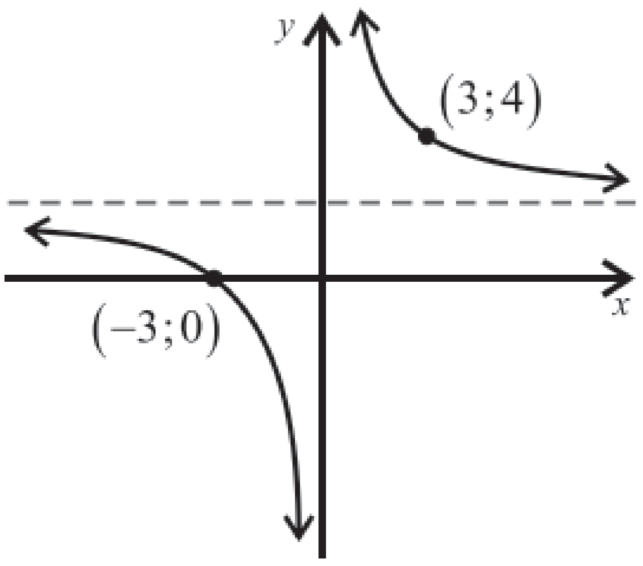
Solutions to Exercise 7 number 1 are on page **317**

2) The function $p(x)=\frac{a}{x}+q$ is shown below. In each case determine the equation of *p*.

a)



b)



Solutions to Exercise 7 number 2 are on page **317**

The Exponential Function

Exercise 8: (theory on pages 111-115)

1) **Determine the equations of the asymptotes and all intercepts with the axes of the following exponential functions. In addition, determine whether the graph is increasing or decreasing and state whether the graph lies above or below the asymptote:**

a) $y = 2^x$ **c)** $y = 2^x + 1$ **e)** $f(x) = -2(2)^x + 2$

b) $y = 2^x - 1$ **d)** $f(x) = -2^{-x} + 4$ **f)** $f(x) = 3\left(\frac{1}{3}\right)^x + 2$

Solutions to Exercise 8 number 1 are on page **318**

2) **Sketch the graphs in question 1 above showing all intercepts with the axes, asymptotes and at least one other point on the graph.**

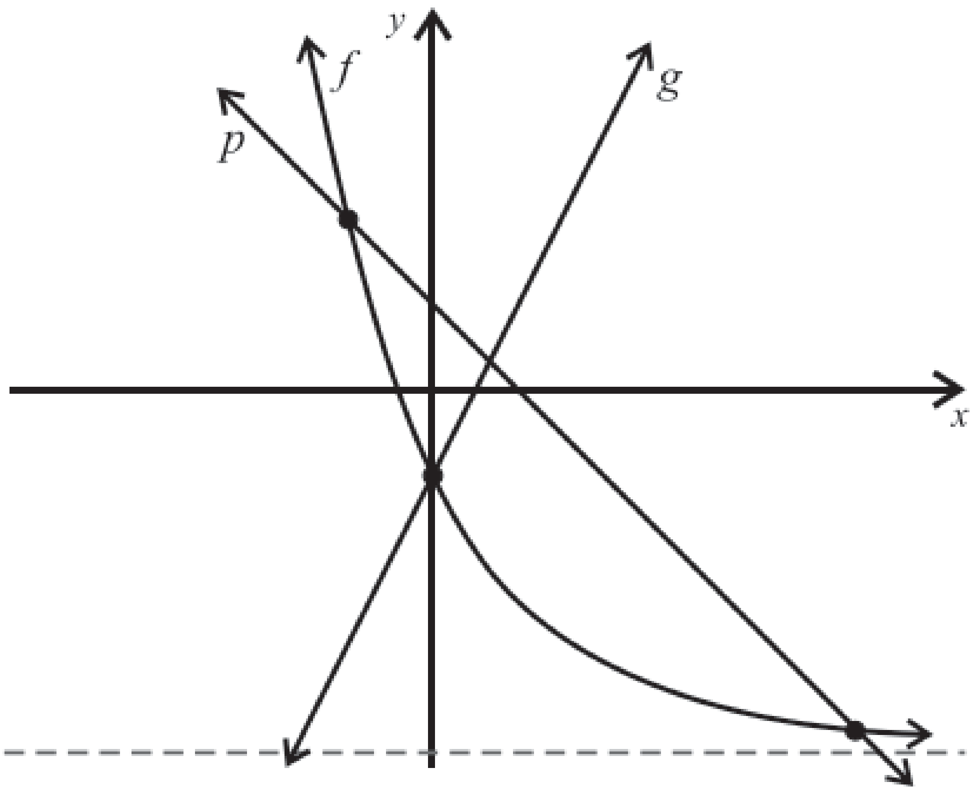
Solutions to Exercise 8 number 2 are on page **319**

Exercise 9: (theory on pages 116-118)

1) **Determine the values of *a*, *b*, and *q* in the following functions if:**
   - **a)** The function $y = b^x + q$ passes through the point $(3;19)$ and it has a horizontal asymptote at $y = -8$.
   - **b)** The function $y = ab^x + q$ passes through the points $(2;13)$ and $(1;1)$ and has a horizontal asymptote at $y = -5$.

Solutions to Exercise 9 number 1 are on page **319**

2) **The graphs of $f(x) = ab^x - 4$, $g(x) = 2x - 1$ and $p(x) = -x + 1$ are shown below. The graphs of *f* and *g* have a common *y*-intercept and *f* and *p* intersect when $x = -1$. Determine the values of *a* and *b*.**

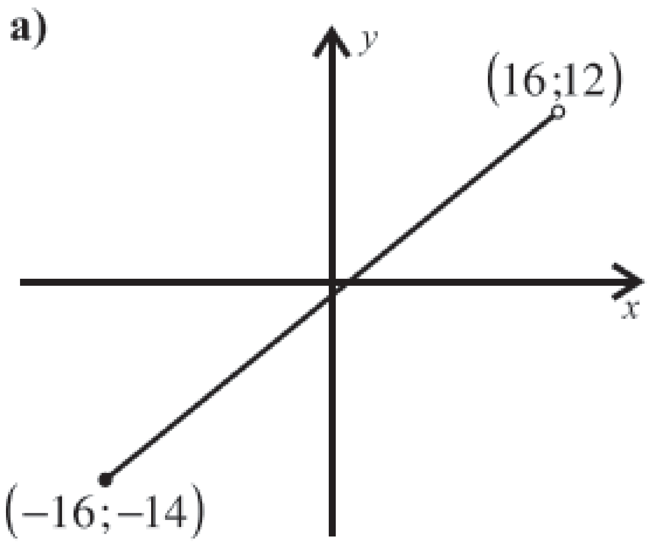


Solutions to Exercise 9 number 2 are on page **320**

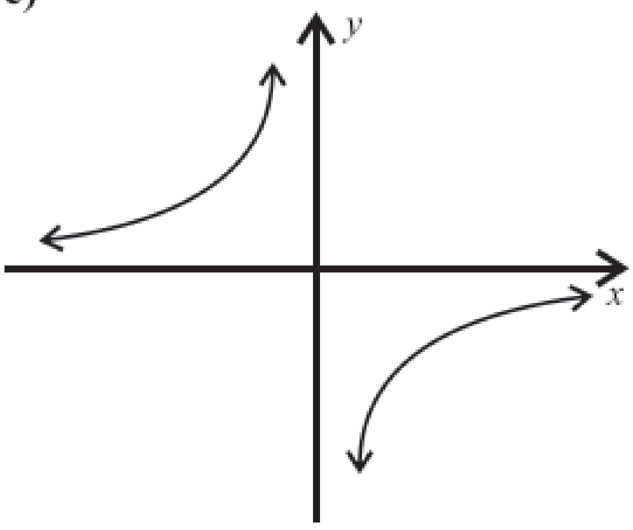
Domain and Range

Exercise 10: Determine the Domain and range of the functions shown below: (theory on page 119)

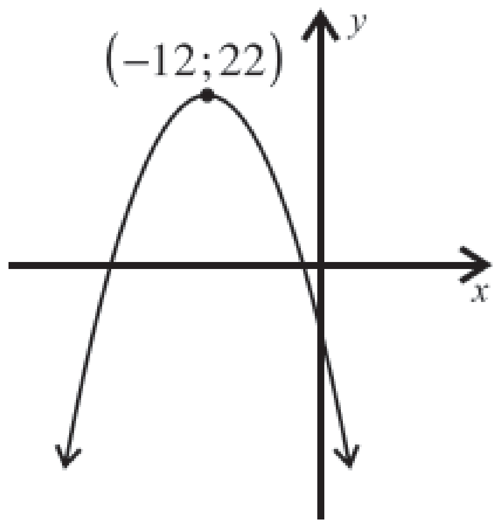
a)


y
(16;12)
x
(−16;−14)

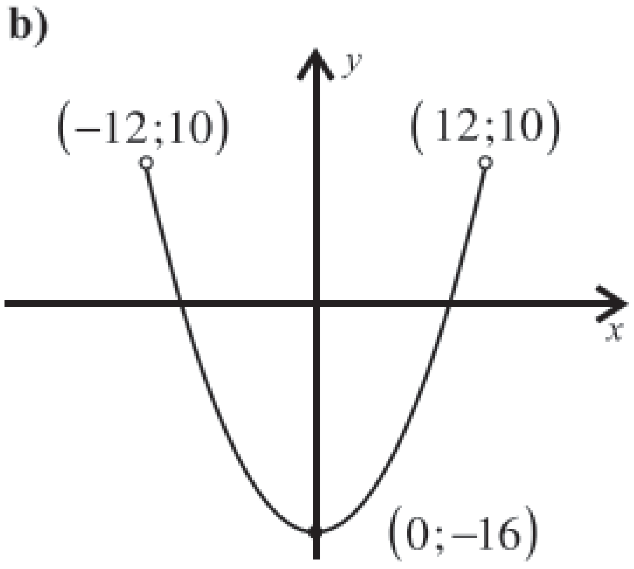
c)



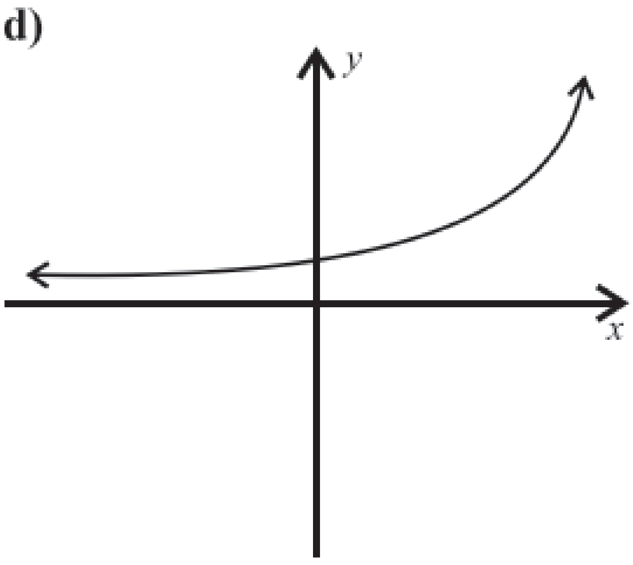
e)



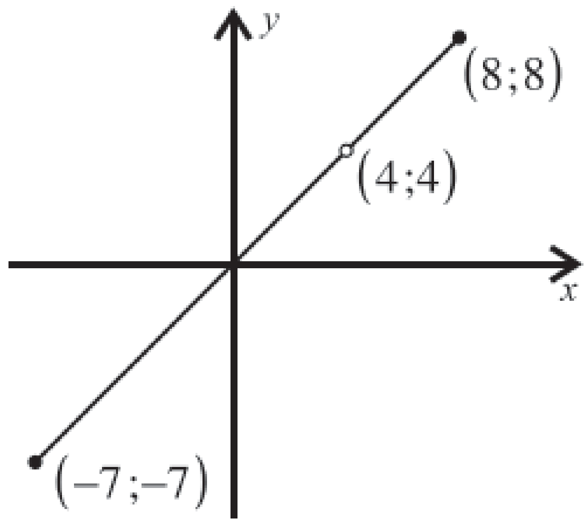
b)


y
(−12;10)
(12;10)
x
(0;−16)

d)



f)



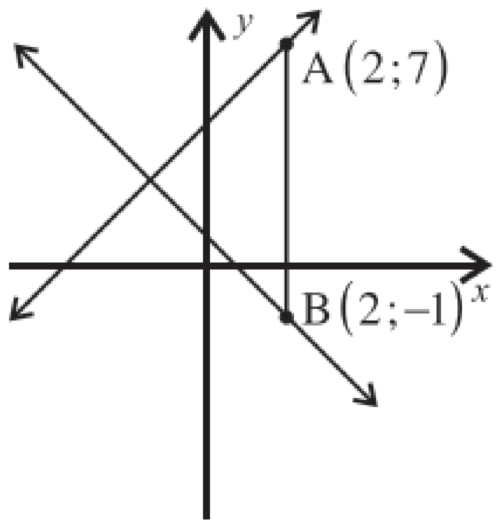
Solutions to Exercise 10 are on page **320**

Lengths of Lines

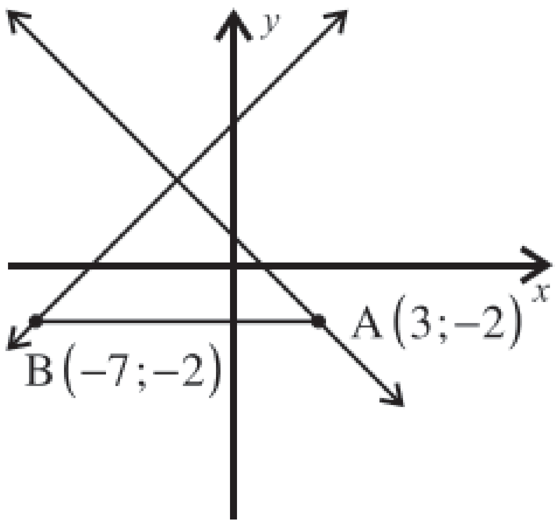
Exercise 11: (theory on pages 127-128)

1) Determine the lengths of the lines AB shown below:

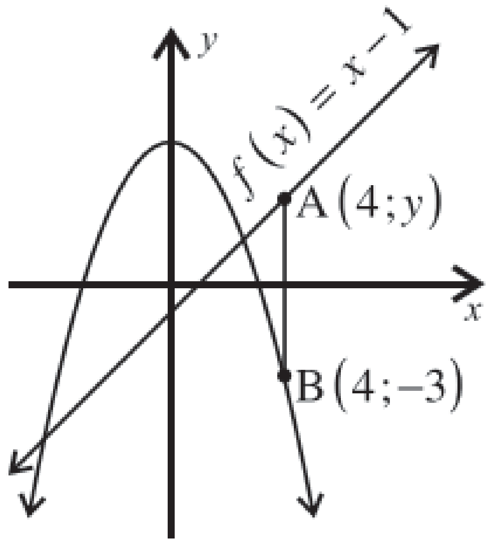
a)



b)



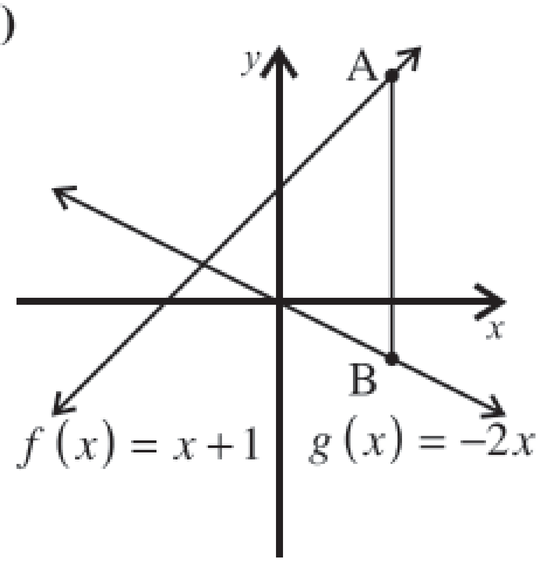
c)


y
f(x) = x − 1
A(4;y)
x
B(4;−3)

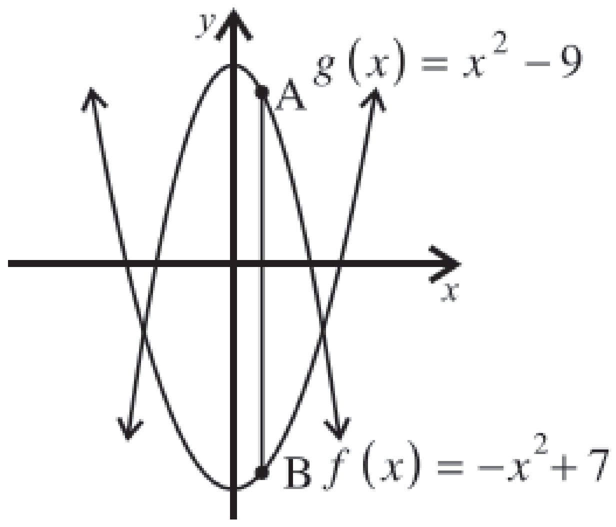
Solutions to Exercise 11 number 1 are on page **320**

2) Determine the lengths of the vertical lines AB in terms of x in the following diagrams:

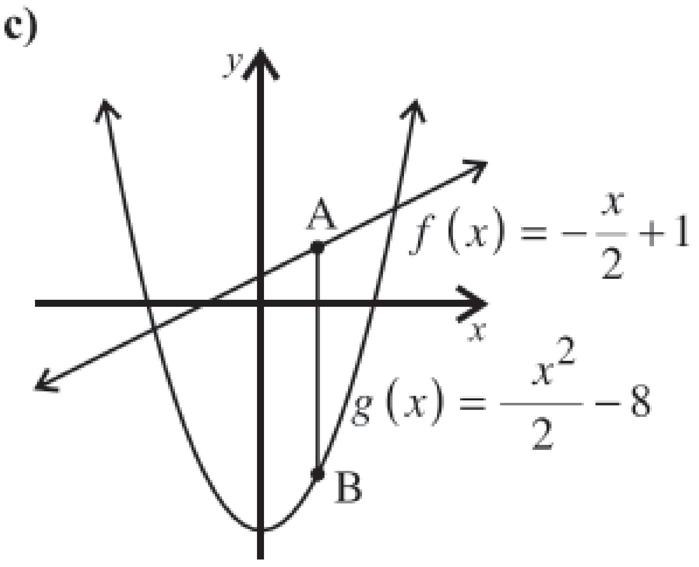
a)


y
A
x
B
f(x) = x + 1
g(x) = −2x

b)


y
g(x) = x² − 9
A
x
B
f(x) = −x² + 7

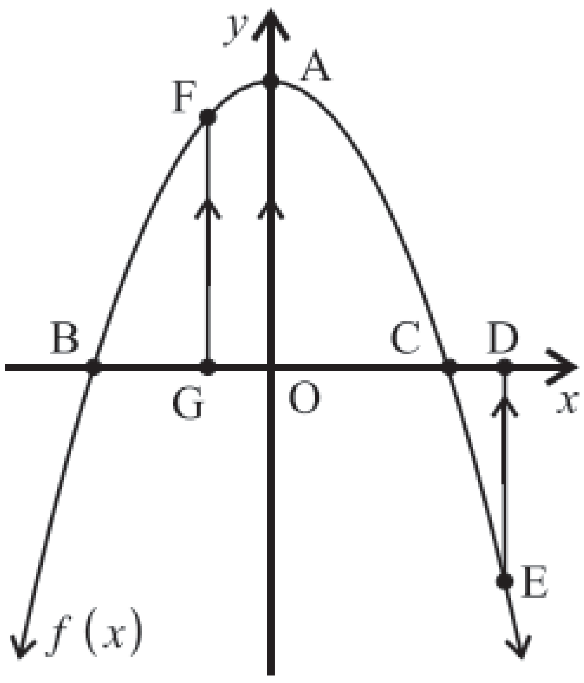
c)


y
A
f(x) = −x/2 + 1
x
g(x) = x²/2 − 8
B

Solutions to Exercise 11 number 2 are on page **320**

Exercise 12: (theory on pages 129-134)

1) The graph of $f(x) = -x^2 + 4$ is shown alongside:



a) Determine the length of OA and BC.

b) Determine the length of FG if OG is 1unit in length.

c) Determine the length of OD if ED is 5 units in length.

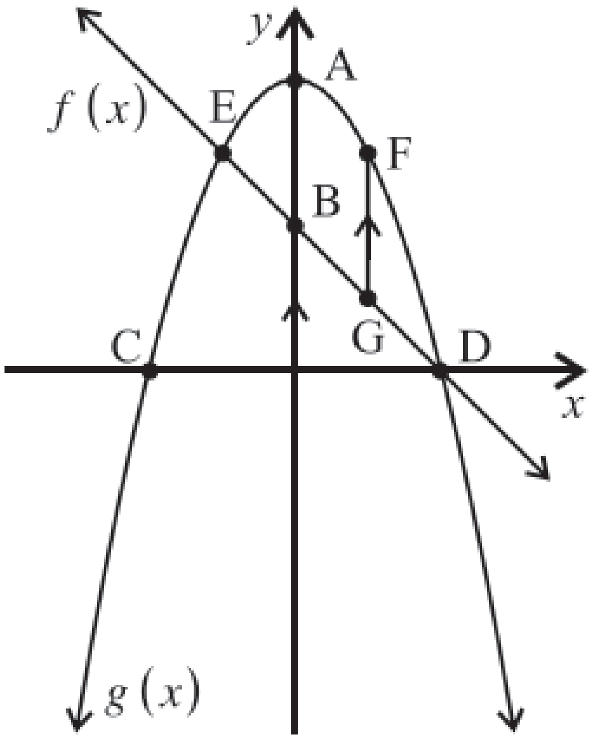
d) For what values of x is f increasing?

e) For what values of x is $f(x) > 0$?

f) For what values of x is $f(x) \le 0$?

Solutions to Exercise 12 number 1 are on page **320**

2) The graphs of $f(x) = -2x + 4$ and $g(x) = -2x^2 + 8$ are shown alongside:



a) Determine the coordinates of A, B, C and D.

b) Determine the coordinates of E, a point of intersection of f and g.

c) Determine the length of line CD.

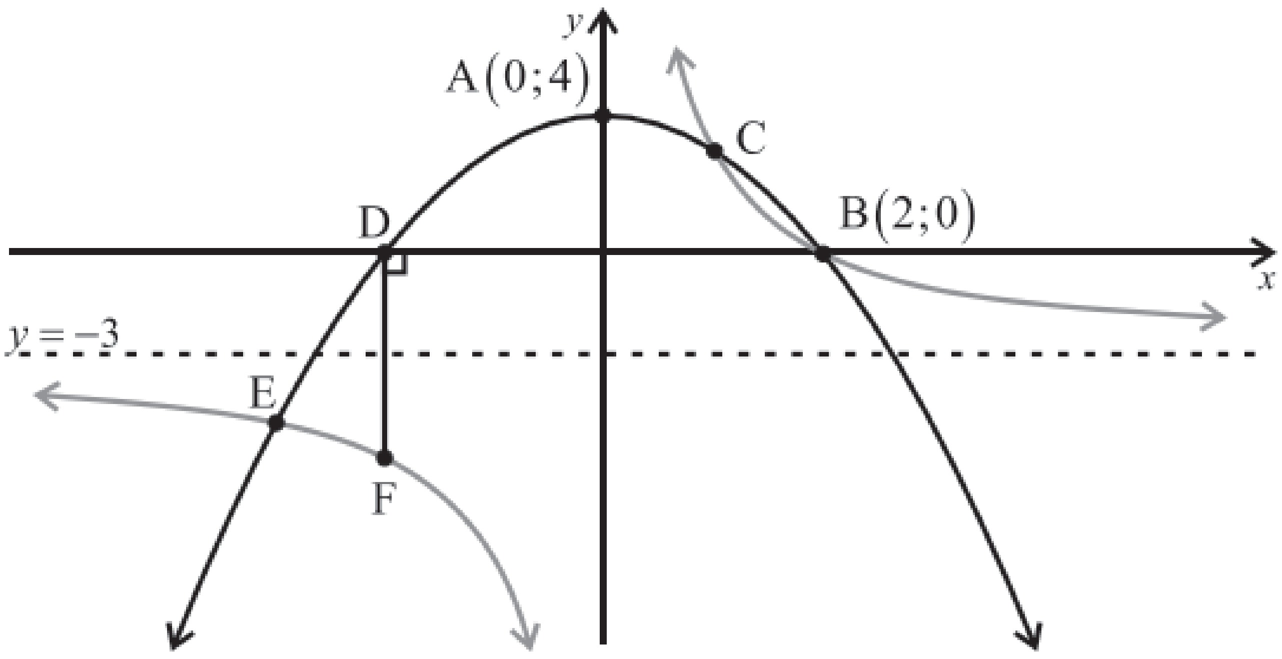
d) Determine the length of FG if G has an x-coordinate of 1.

e) For what values of x is $f(x) \le g(x)$?

f) For what values of x is $f(x).g(x) \ge 0$?

Solutions to Exercise 12 number 2 are on page **321**

3) The graphs of $f(x) = ax^2 + q$ and $g(x) = \frac{k}{x} + c$ are sketched below. A is the y-intercept of f and the graphs intersect at B, C and E. D and B are both x-intercepts.



a) Determine the values of a, q, k and c.

b) If $x^3 - 7x + 6 = (x+3)(x-2)(x-1)$, determine the coordinates of C, and E.

c) Determine the coordinates of D.

d) Determine the length of DF if DF is perpendicular to the x-axis.

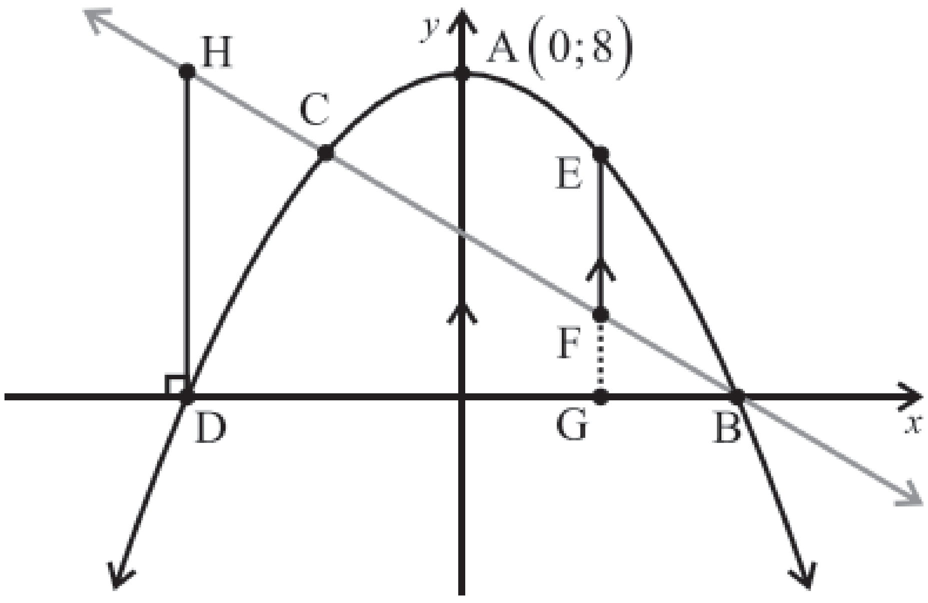
e) For which values of x is $g(x) > f(x)$?

f) For which values of x is $f(x) \times g(x) > 0$?

g) Determine the domain and range of f and g.

Solutions to Exercise 12 number 3 are on page **321**

4) The graphs of $f(x)=ax^2+q$ and $g(x)=-x+4$ are sketched below. A is the *y*-intercept of *f* and B is an *x*-intercept of both *f* and *g*.



a) Determine the coordinates of B.

b) Determine the coordinates of D, the other *x*-intercept of *f*.

c) Determine the values of *a* and *q* in the graph of $f(x)=ax^2+q$.

d) Hence, determine the coordinates of C, a point of intersection of *f* and *g*.

e) Determine the length of DH if DH perpendicular to the *x*-axis.

f) Determine the length of DG if EF is 4 units long and is parallel to the *y*-axis.

g) For what values of *x* is $g(x)<f(x)$?

h) Determine the domain and range of *f* and *g*.

Solutions to Exercise 12 number 4 are on page **322**

Trigonometric Functions

Exercise 13: (theory on pages 135-137)

1) Sketch the graphs of the following trigonometric functions if $x\in[0°;360°]$:

a) $y=\sin x$

b) $y=\cos x$

c) $y=\tan x$

d) $y=2\sin x$

e) $y=0,5\cos x$

f) $y=3\tan x$

Solutions to Exercise 13 number 1 are on page **322**

2) Sketch the graphs of the following trigonometric functions if $x\in[0°;360°]$:

a) $y=-2\sin x$

b) $y=-2\cos x$

c) $y=-\tan x$

Solutions to Exercise 13 number 2 are on page **322**

3) Sketch the graphs of the following trigonometric functions if $x\in[0°;360°]$:

a) $y=2\sin x+1$

b) $y=2\cos x-2$

c) $y=\tan x+1$

d) $y=2\sin x-1$

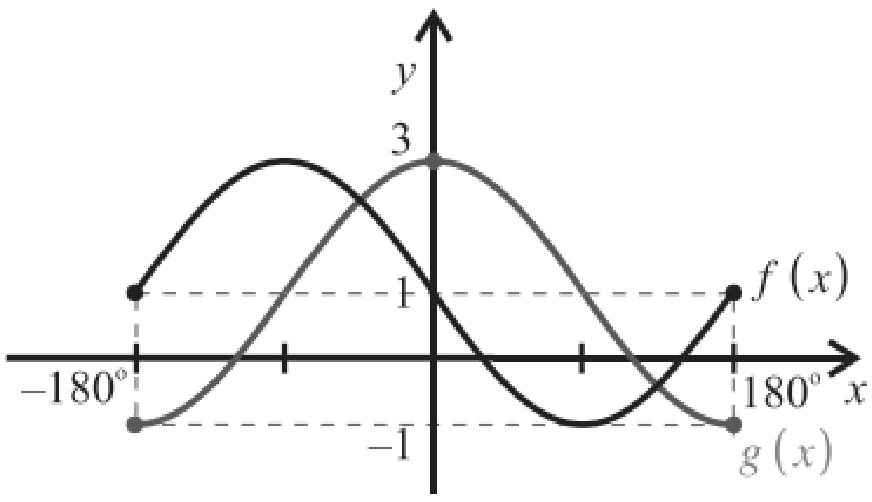
e) $y=-2\cos x+1$

f) $y=-\tan x+1$

Solutions to Exercise 13 number 3 are on page **323**

Exercise 14: (theory on pages 138-139)

1) The graphs of $f(x)=a\sin x+p$ and $g(x)=q\cos x+r$ are shown below:



a) Determine the values of *a*, *q*, *p* and *r*.

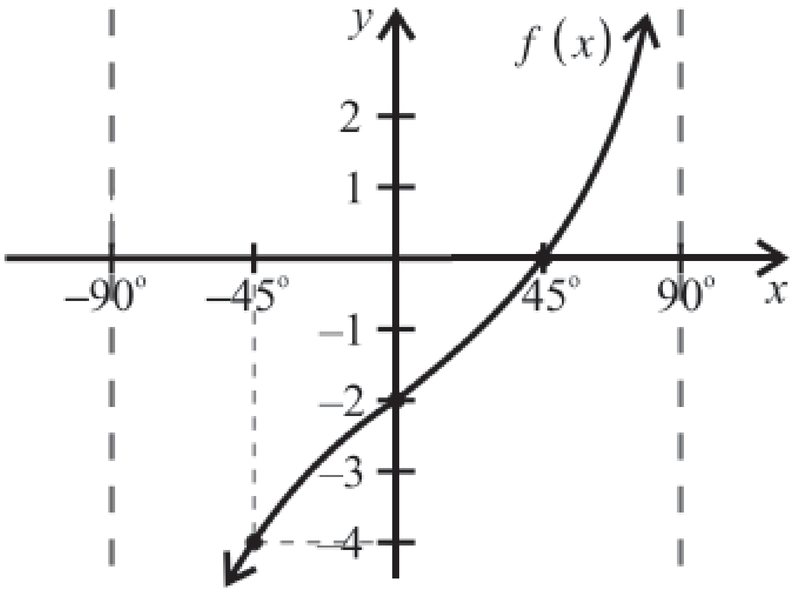
b) State the domain and range of *f*.

c) What is the period of *g*?

d) What is the amplitude of *f*?

Solutions to Exercise 14 number 1 are on page **323**

2) The graph of $f(x)=a\tan x+q$ is shown below:



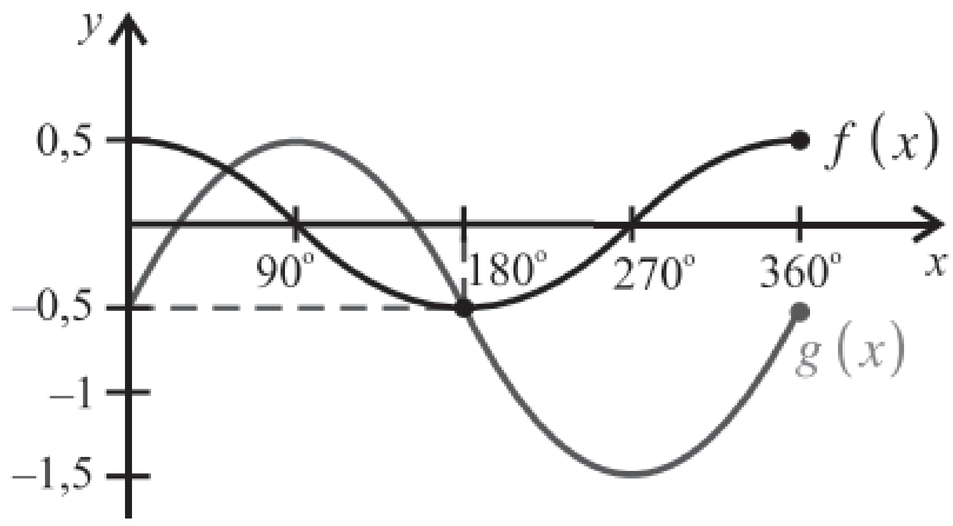
a) Determine the values of *a* and *q*.

b) State the domain and range of *f*.

c) What is the period of *f*?

Solutions to Exercise 14 number 2 are on page **323**

3) The graphs of $f(x)=a\cos x$ and $g(x)=\sin x+q$ are shown below:



a) Determine the values of *a* and *q*.

b) What is the range of g(*x*)?

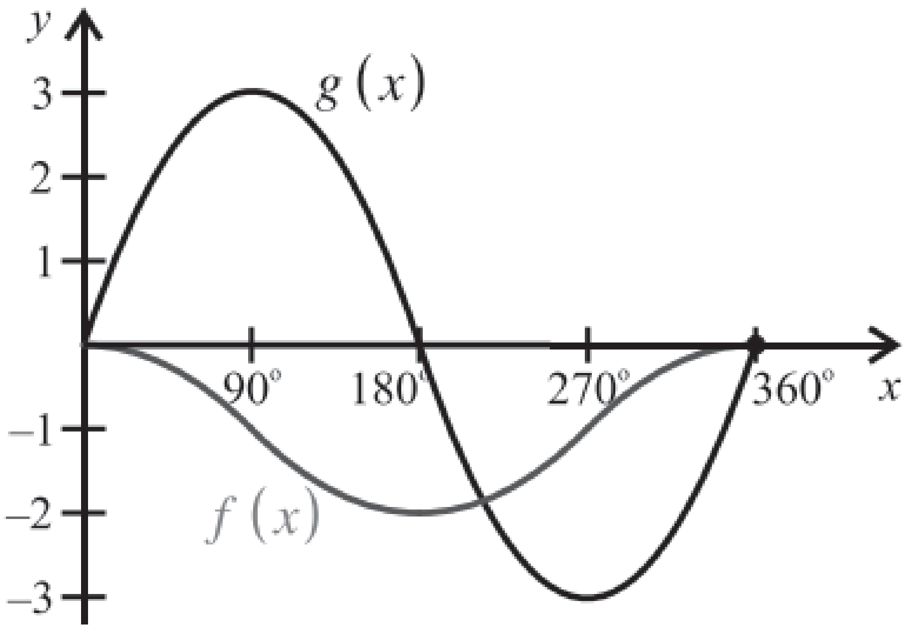
c) What is the period of *f*?

d) Give one value of *x* for which $g(x)=f(x)$.

e) For which value/s of *x* is $g(x)-f(x)=-1$?

Solutions to Exercise 14 number 3 are on page **323**

4) The graphs of $f(x)=\cos x+q$ and $g(x)=a\sin x$ are shown below:



a) Determine the values of *a* and *q*.

b) What is the range of *g*?

c) What is the period of *g*?

d) For what value/s of *x* is $f(x)=0$?

Solutions to Exercise 14 number 4 are on page **323**

Mixed Exercise: (chapter 6 is on pages 94-139)

1)

a) Sketch the graphs of $f(x)=-2x+8$ and $x=4$ on the same set of axes showing all intersections with the axes.

b) Use your graphs to determine the points of intersection of the two graphs.

c) Write down the domain and range of each graph.

Solutions to the Mixed Exercise number 1 are on page **323**

2)

a) Sketch the graphs of $f(x)=-2x^2+8$ and $g(x)=-2x+4$ on the same set of axes showing all intersections with the axes.

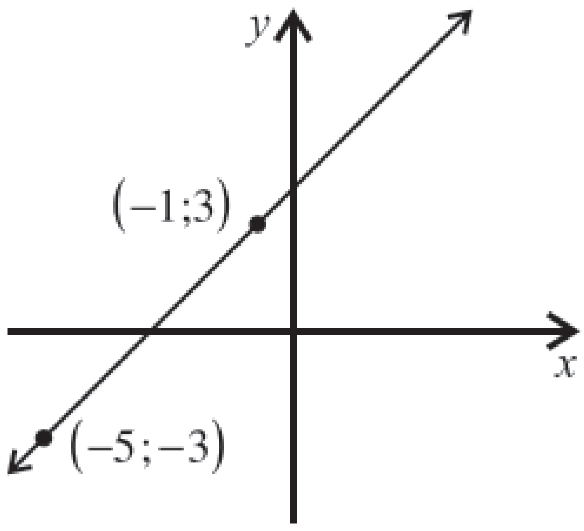
b) Write down the domain and range of *f*.

c) State the domain on which the graph of *f* is increasing.

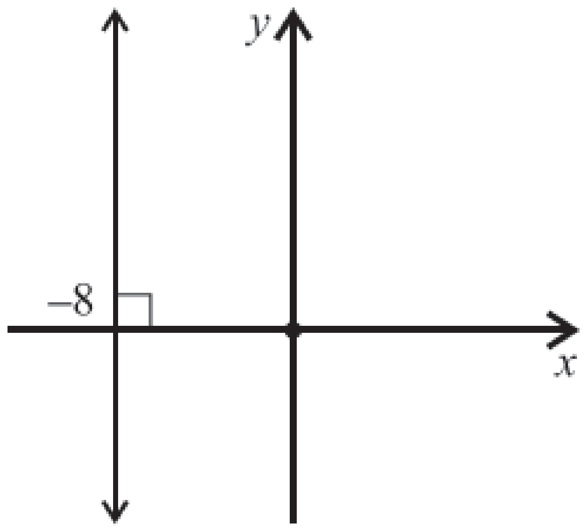
Solutions to the Mixed Exercise number 2 are on page **323**

3) Determine the equations of each of the functions sketched below:

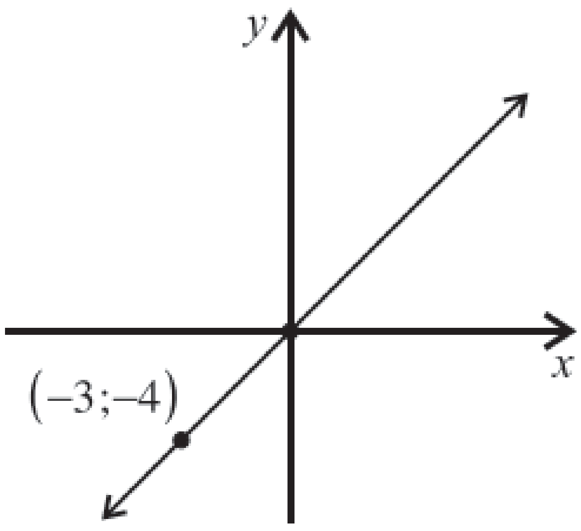
a)



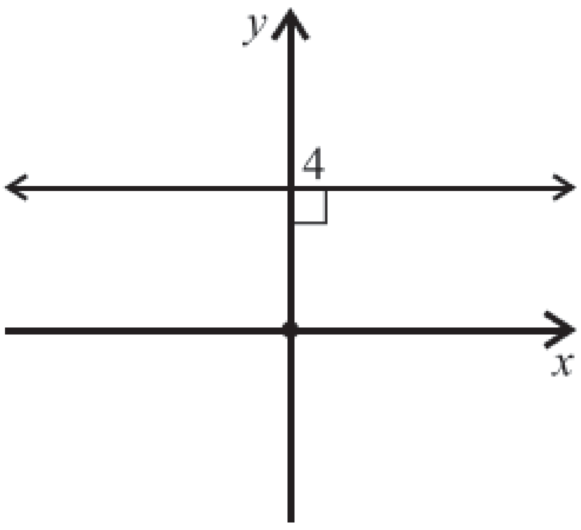
b)



c)



d)

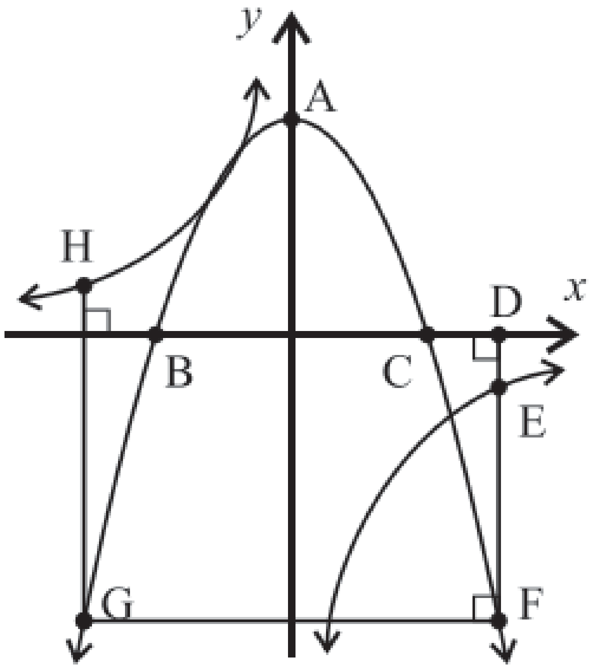


Solutions to the Mixed Exercise number 3 are on page **324**

4) Determine the coordinate of P, the point of intersection of the graphs of $3x-y=4$ and $y=2x-5$.

Solutions to the Mixed Exercise number 4 are on page **324**

5) The graphs of $f(x) = -\dfrac{3}{x}$ and $g(x) = -x^2 + 4$ are shown alongside:



- **a)** Determine the coordinates of A, B and C.
- **b)** Determine the length of EF if DE is 1 unit in length.
- **c)** The coordinates of G.
- **d)** Determine the length of FG.
- **e)** Determine the coordinates of H.

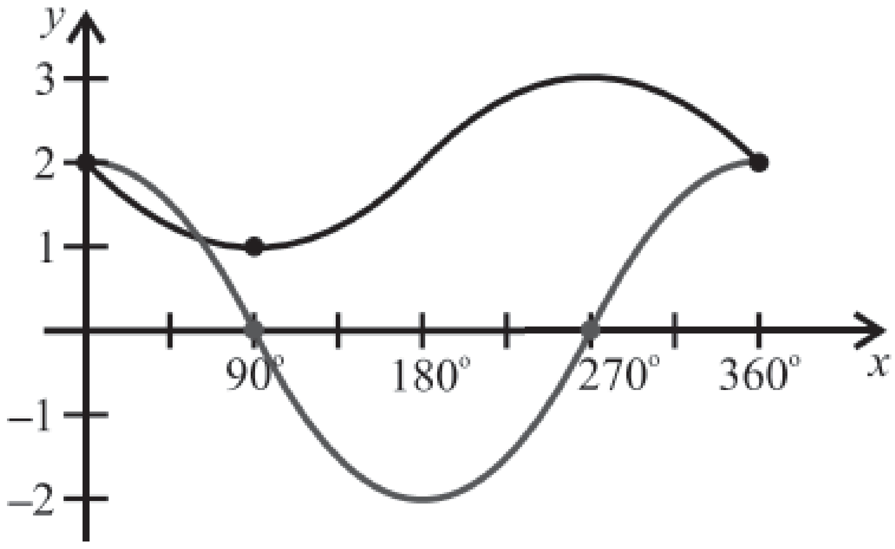
Solutions to the Mixed Exercise number 5 are on page **324**

6)

- **a)** Sketch the graphs of $f(x) = 2\sin x$ and $g(x) = \cos x$ if $x \in [0^\circ; 360^\circ]$.
- **b)** Write down the range of *g*.
- **c)** What is the period of *f*.

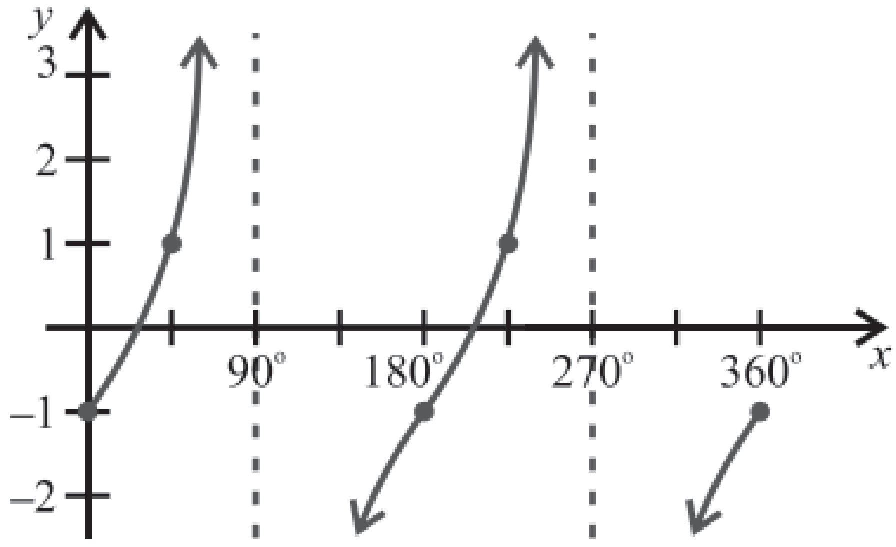
Solutions to the Mixed Exercise number 6 are on page **324**

7) The graphs of $f(x) = a\sin x + q$ and $g(x) = b\cos x + p$ are shown below. Determine the values of *a*, *b*, *p* and *q*.



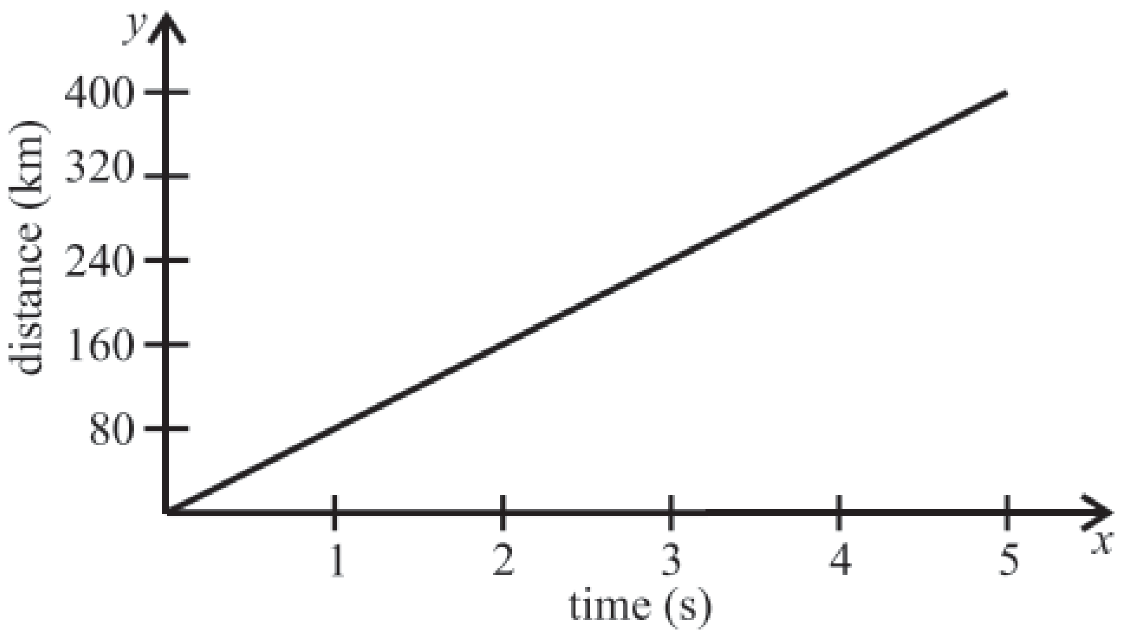
Solutions to the Mixed Exercise number 7 are on page **324**

8) The graph of $f(x) = a\tan x + q$ is shown below. Determine the value of *a* and *q* and write down the equations of the asymptotes.



Solutions to the Mixed Exercise number 8 are on page **324**

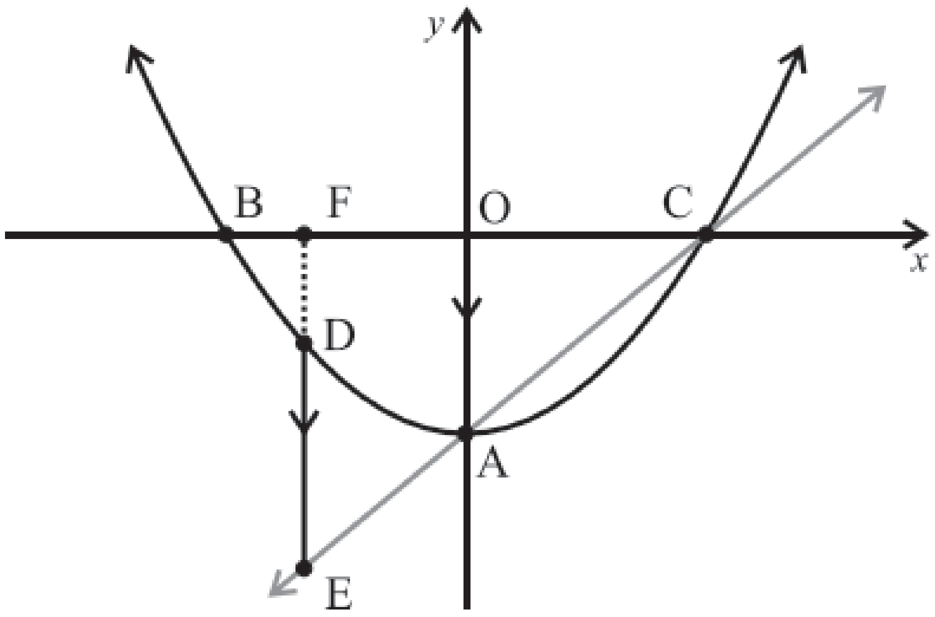
9) The graph below shows the distance travelled by a car as a function of time.



- **a)** How far did the car travel in the first 2 hours?
- **b)** How long did it take the car to travel to 320km?
- **c)** Use the graph to determine the speed of the car in km/h.
  (hint: speed is the rate of change of distance against time)

Solutions to the Mixed Exercise number 9 are on page **324**

10) The graphs of $f(x)=ax^2+q$ and $g(x)=6x-18$ are sketched below. A is the *y*-intercept of *f* and *g*. C is the *x*-intercept of *g*. The graphs intersect at A and C.



a) Determine the coordinates of A and C.

b) Determine the coordinates of B, the other *x*-intercept of *f*.

c) Determine the values of *a* and *q* in the graph of $f(x)=ax^2+q$.

d) Determine the length of OF if DE is 20 units in length.

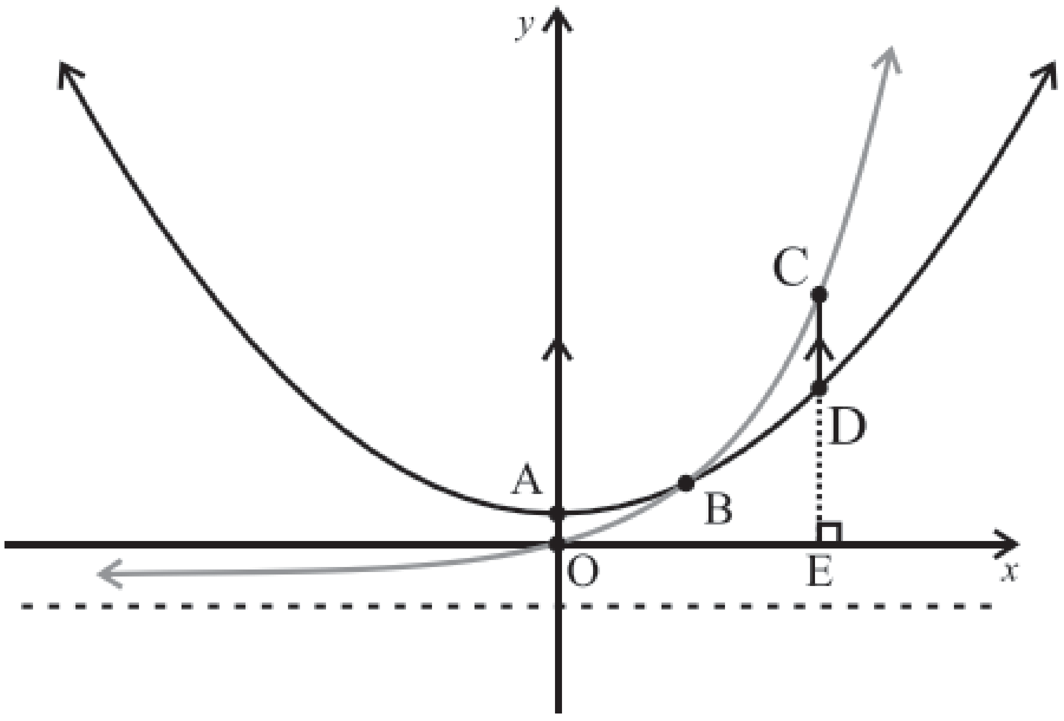
e) For which values of *x* is $f(x)\le g(x)$?

f) For which values of *x* is $\frac{f(x)}{g(x)}<0$?

g) Determine the domain and range of *f* and *g*.

Solutions to the Mixed Exercise number 10 are on page **325**

11) The graphs of $f(x)=ax^2+q$ and $g(x)=b^x+c$ are sketched below. $A(0;1)$ is the *y*-intercept of *f* and $B(1;2)$ is the point of intersection of *f* and *g*. The graph of *g* passes through the origin.



a) Determine the values of *a* and *q* in $f(x)=ax^2+q$.

b) Determine the values of *b* and *c* in $g(x)=b^x+c$.

c) Given that OE is 2 units in length determine the length of CD.

d) For which values of *x* is $f(x).g(x)<0$?

e) Determine the domain and range of *f* and *g*.

Solutions to the Mixed Exercise number 11 are on page **325**

Chapter 6 – Functions (Solutions)

Exercise 1 Solutions:

1)

a) $f(2) = 2(2)^2 + 3(2) - 10$
$= 4$

b) $h(2) = (2)^2 - 3(2)$
$= -2$

c) $f(-2) = 2(-2)^2 + 3(-2) - 10$
$= -8$

d) $f(-5) = 2(-5)^2 + 3(-5) - 10$
$= 25$

e) $h(-2) = (-2)^2 - 3(-2)$
$= 10$

f) $h(1) = (1)^2 - 3(1)$
$= -2$

Exercise 1 number 1 is on page 300

2)

a) $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 4$
$= \frac{1}{4} - 4$
$= -\frac{15}{4}$

b) $f(p) = p^2 - 4$

c) $f(2x) = (2x)^2 - 4$
$= 4x^2 - 4$

d) $f(x) + 2 = x^2 - 4 + 2$
$= x^2 - 2$

e) $f(-x) = (-x)^2 - 4$
$= x^2 - 4$

f) $f(x+2) = (x+2)^2 - 4$
$= x^2 + 4x + 4 - 4$
$= x^2 + 4x$

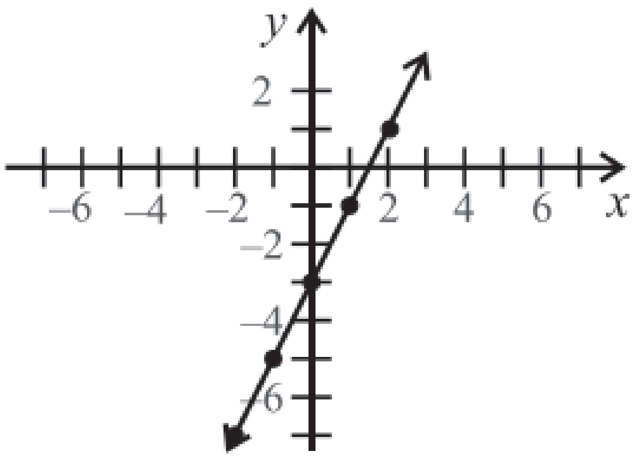
Exercise 1 number 2 is on page 300

Exercise 2 Solutions:

1)

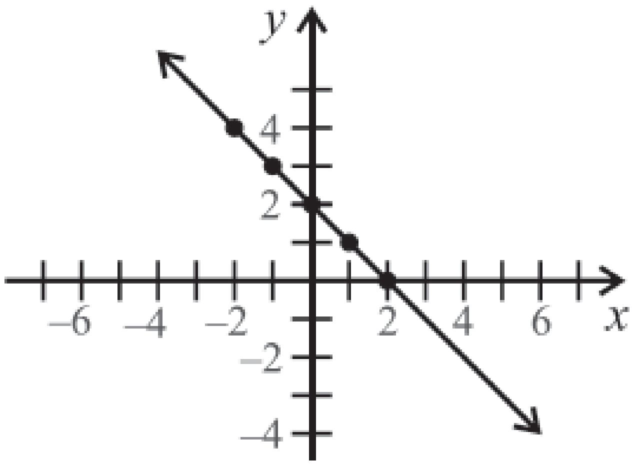
a) $y = 2x - 3$

x	-2	-1	0	1	2
y	-7	-5	-3	-1	1



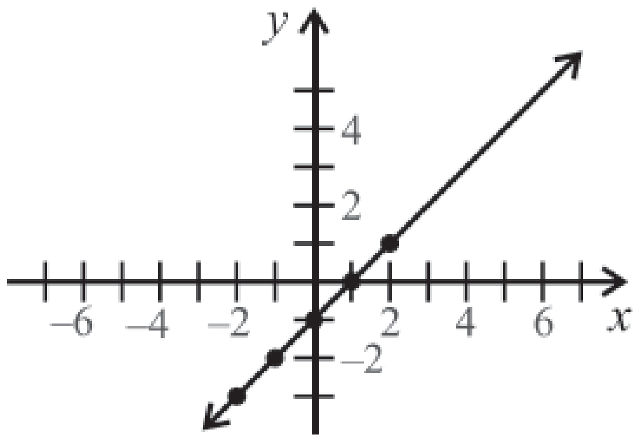
b) $f(x) = -x + 2$

x	-2	-1	0	1	2
y	4	3	2	1	0



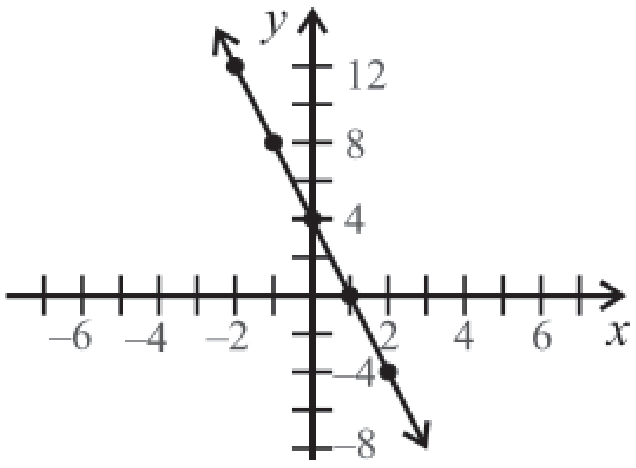
c) $y = x - 1$

x	-2	-1	0	1	2
y	-3	-2	-1	0	1



d) $f(x) = -4x + 4$

x	-2	-1	0	1	2
y	12	8	4	0	-4



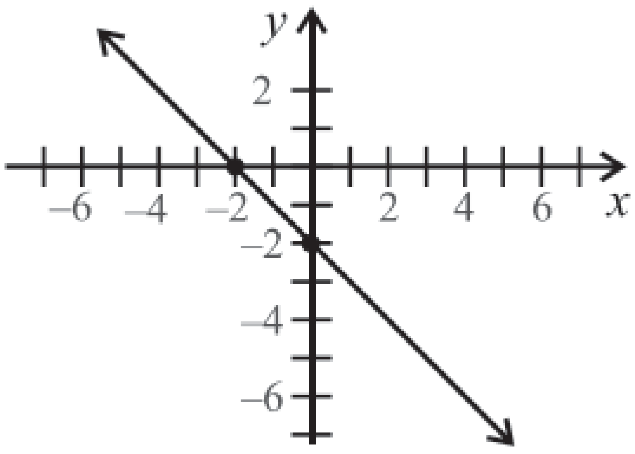
Exercise 2 number 1 is on page 300

2)

a) $y = -x - 2$

x-int: let $y = 0$
$0 = -x - 2$
$\therefore x = -2$

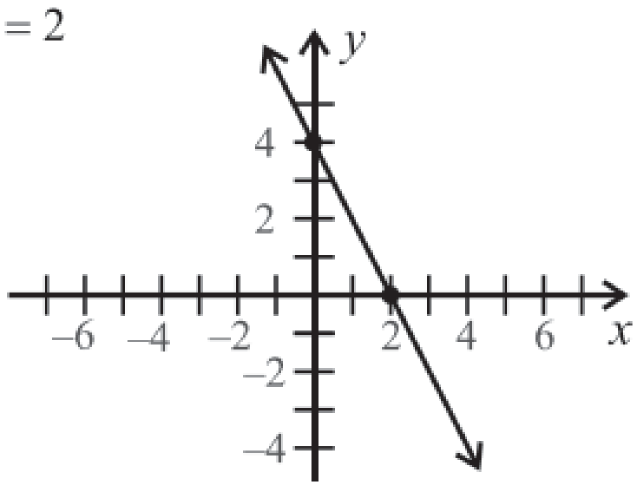
y-int: let $x = 0$
$\therefore y = -2$



b) $f(x) = -2x + 4$

x-int: let $y = 0$
$0 = -2x + 4$
$\therefore x = 2$

y-int: let $x = 0$
$\therefore y = 4$



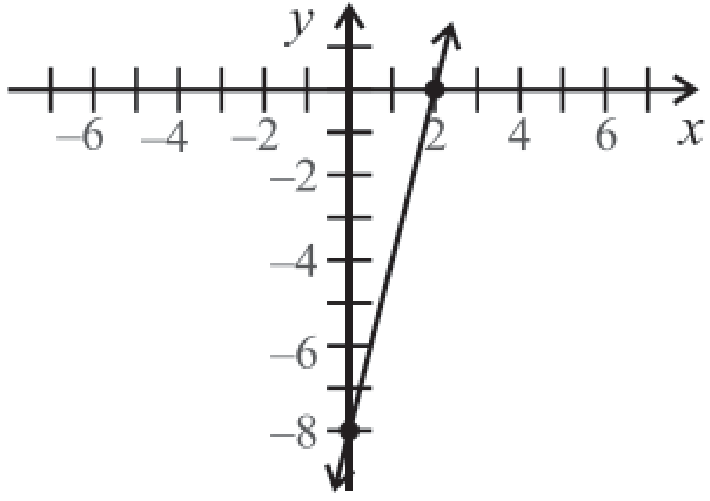
Exercise 2 number 2 is on page 300 (solutions are continued on the next page)

c) $y = 4x - 8$

x-int: let $y = 0$ $\quad$ y-int: let $x = 0$

$0 = 4x - 8$ $\quad$ $\therefore y = -8$

$\therefore x = 2$

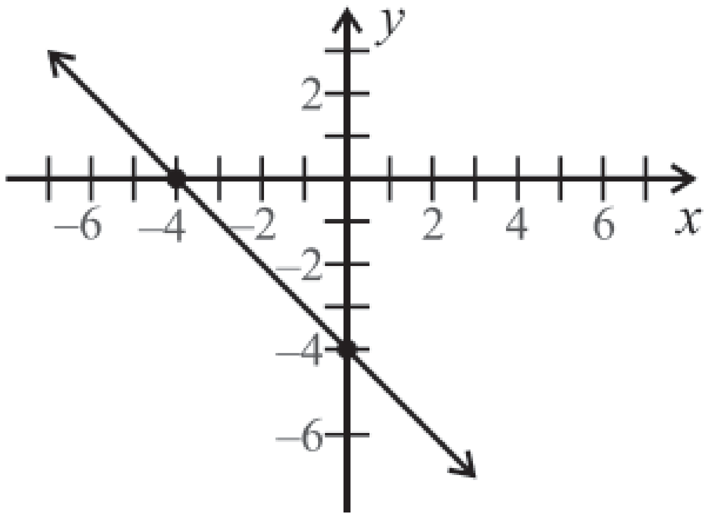


d) $f(x) = -x - 4$

x-int: let $y = 0$ $\quad$ y-int: let $x = 0$

$0 = -x - 4$ $\quad$ $\therefore y = -4$

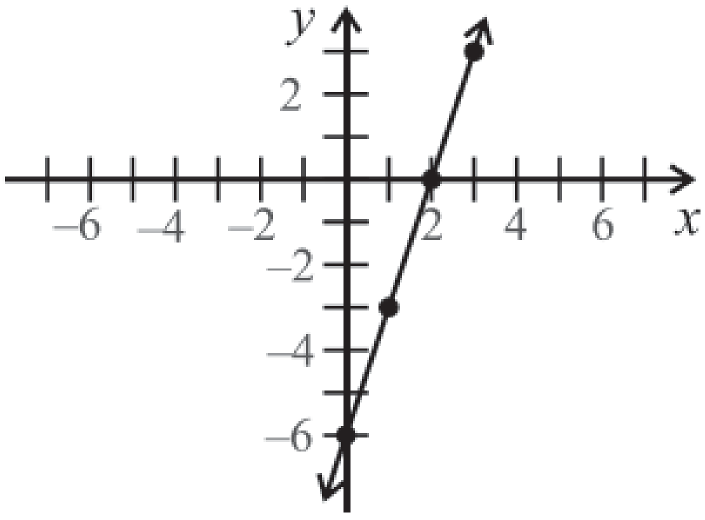
$\therefore x = -4$



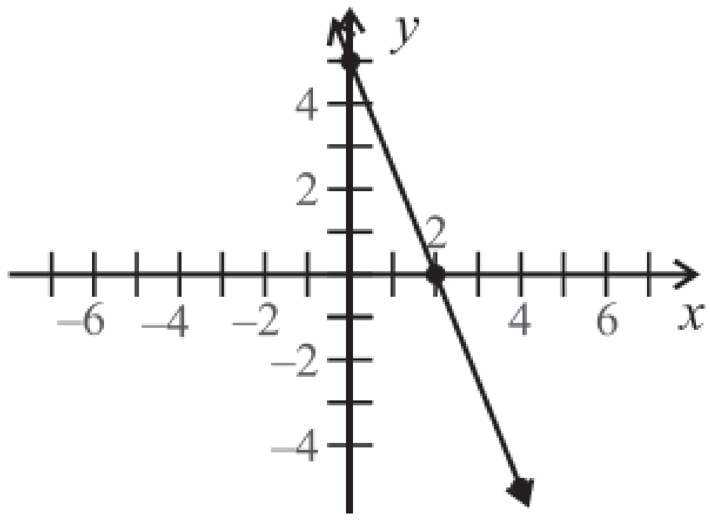
Exercise 2 number 2 is on page **300**

3)

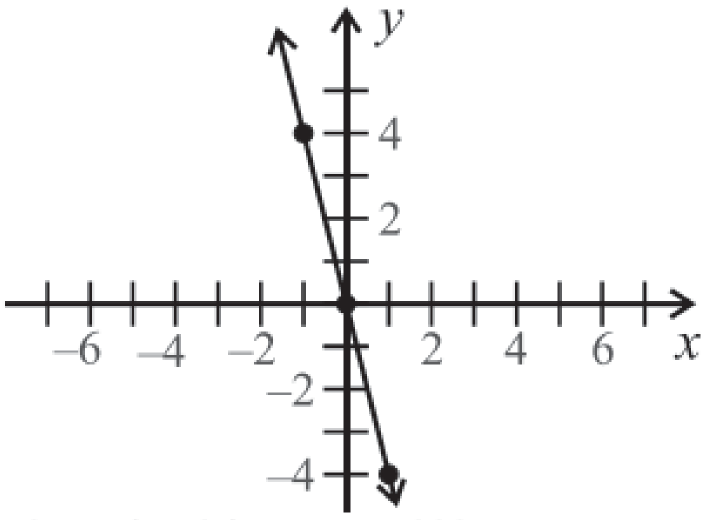
a) $y = 3x - 6$



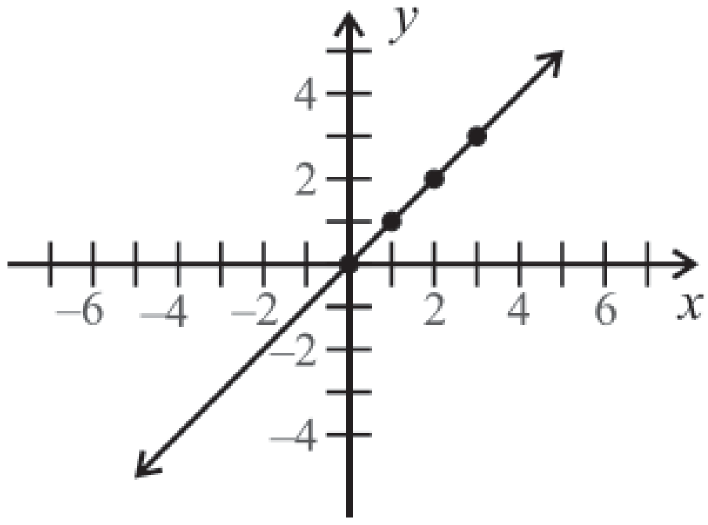
c) $5x + 2y = 10$



b) $f(x) = -4x$

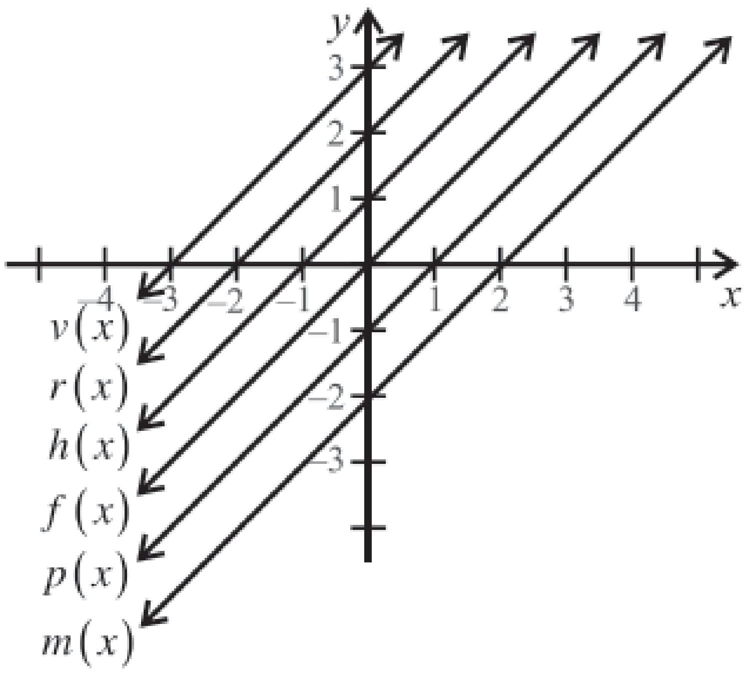


d) $3x = 3y$



Exercise 2 number 3 is on page **300**

4)



Exercise 3 number 4 is on page **300**

5) Increasing the value of q shifts the graph up while decreasing the value of q shifts the graph down.

Exercise 2 number 5 is on page **300**

6)

a) $4y - 8x = 0$
$\therefore 4y = 8x$
$\therefore y = 2x$
$\therefore m = 2$
y − int at $(0;0)$

b) $4x - 2y = 16$
$\therefore -2y = -4x + 16$
$\therefore y = 2x - 8$
$\therefore m = 2$
y − int at $(0;-8)$

c) $10x + 5y = 10$
$\therefore 5y = -10x + 10$
$\therefore y = -2x + 2$
$\therefore m = -2$
y − int at $(0;2)$

Exercise 2 number 6 is on page **300**

Exercise 3 Solutions:

1)

a) $y=-3x+10$

b) $y=-2x+c \rightarrow \text{sub}\left(8;38\right)$
$\therefore 38=-2\left(8\right)+c$
$\therefore c=54$
$\therefore y=-2x+54$

c) $y=mx+5 \rightarrow \text{sub}\left(3;-4\right)$
$\therefore -4=m\left(3\right)+5$
$\therefore 3m=-9$
$\therefore m=-3$
$\therefore y=-3x+5$

d) $y=4x+c \rightarrow \text{sub}\left(5;22\right)$
$\therefore 22=4\left(5\right)+c$
$\therefore c=2$
$\therefore y=4x+2$

e) $y=mx+c$
$m=\frac{y_2-y_1}{x_2-x_1}=\frac{25-4}{8-1}=3$
$\therefore y=3x+c \rightarrow \text{sub}\left(1;4\right)$
$\therefore 4=3\left(1\right)+c$
$\therefore c=1$
$\therefore y=3x+1$

f) $y=mx+c$
$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-1-4}{12-2}=-\frac{1}{2}$
$\therefore y=-\frac{1}{2}x+c \rightarrow \text{sub}\left(2;4\right)$
$\therefore 4=-\frac{1}{2}\left(2\right)+c$
$\therefore c=5$
$\therefore y=-\frac{1}{2}x+5$

Exercise 3 number 1 is on page **300**

2)

a) $y=mx+c$
$\therefore y=mx+5$
$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-1-5}{6-0}=-1$
$\therefore y=-x+5$

b) $y=mx+c$
$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-5-9}{-8-7}=\frac{14}{15}$
$\therefore y=\frac{14}{15}x+c \rightarrow \text{sub}\left(7;9\right)$
$\therefore 9=\frac{14}{15}\left(7\right)+c$
$\therefore c=\frac{37}{15}$
$\therefore y=\frac{14}{15}x+\frac{37}{15}$

c) $y=mx+c$
$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-4-5}{2+1}=-3$
$\therefore y=-3x+c \rightarrow \text{sub}\left(-1;5\right)$
$\therefore 5=-3\left(-1\right)+c$
$\therefore c=2$
$\therefore y=-3x+2$

Exercise 3 number 2 is on page **301**

3)

a) $y=\frac{1}{2}x+c$
sub $\left(2;6\right)$
$\therefore 6=\frac{1}{2}\left(2\right)+c$
$\therefore c=5$
$\therefore y=\frac{1}{2}x+5$

b) $y=mx+3$
sub $\left(3;-12\right)$
$\therefore -12=m\left(3\right)+3$
$\therefore -15=3m$
$\therefore m=-5$
$\therefore y=-5x+3$

c) y-int: let $x=0$
$\therefore y=3 \rightarrow$ horizontal line

d) x-int: let $y=0$
$0=\frac{1}{2}x+3$
$\therefore -3=\frac{1}{2}x$
$\therefore x=-6 \rightarrow$ vertical line

Exercise 3 number 3 is on page **301**

Exercise 4 Solutions:

1)

a) $f\left(x\right)=x^2$
$a>0$: Concave up (Happy)
TP: $\left(0;0\right)$
x-int: Let $y=0$
$\therefore x^2=0$
$\therefore x=0$
$\therefore x$-int at $\left(0;0\right)$

b) $y=2x^2-2$
$a>0$: Concave up (Happy)
TP: $\left(0;-2\right)$
x-int: Let $y=0$
$\therefore 2x^2-2=0$
$\therefore 2\left(x^2-1\right)=0$
$\therefore \left(x+1\right)\left(x-1\right)=0$
$\therefore x$-int's at $\left(-1;0\right)$ and $\left(1;0\right)$

c) $5y=-10x^2-20$
$\therefore y=-2x^2-4$
$a<0$: Concave down (Sad)
TP: $\left(0;-4\right)$
x-int: Let $y=0$
$\therefore -10x^2-20=0$
$\therefore -10\left(x^2+2\right)=0$
$\therefore$ No solution
$\therefore$ There are no x-intercepts

Exercise 4 number 1 is on page **301** (solutions are continued on the next page)

d) $f(x) = -4x^2 + 16$

$a < 0$:

$\therefore$ Concave down (Sad)

TP: $(0;16)$

x-int: Let $y = 0$

$\therefore -4x^2 + 16 = 0$

$\therefore -4(x^2 - 4) = 0$

$\therefore (x+2)(x-2) = 0$

$\therefore$ x-int's at $(-2;0)$ and $(2;0)$

e) $f(x) = 4 - x^2$

$a < 0$:

$\therefore$ Concave down (Sad)

TP: $(0;4)$

x-int: Let $y = 0$

$\therefore -x^2 + 4 = 0$

$\therefore -(x^2 - 4) = 0$

$\therefore (x+2)(x-2) = 0$

$\therefore$ x-int's at $(-2;0)$ and $(2;0)$

f) $2y - 8x^2 = 2$

$\therefore 2y = 8x^2 + 2$

$\therefore y = 4x^2 + 1$

$a > 0$:

$\therefore$ Concave up (Happy)

TP: $(0;1)$

x-int: Let $y = 0$

$\therefore 4x^2 + 1 = 0$

$\therefore$ No solution

$\therefore$ there are no x-intercepts

Exercise 4 number 1 is on page 301

2)

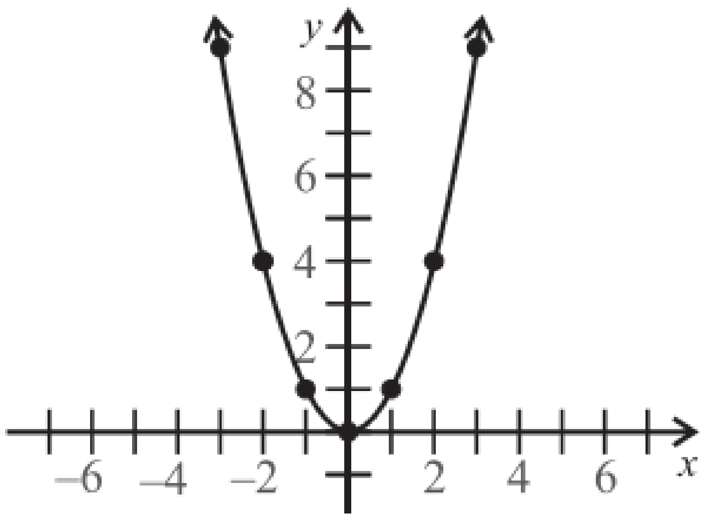
a) $f(x) = x^2$

shape: $a > 0$ (happy)

y-int: $(0;0)$

x-int: $(0;0)$

x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9



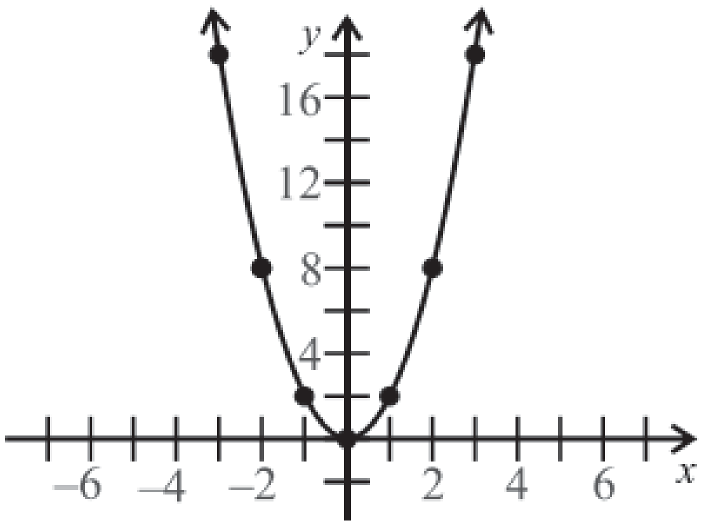
b) $f(x) = 2x^2$

shape: $a > 0$ (happy)

y-int: $(0;0)$

x-int: $(0;0)$

x	–3	–2	–1	0	1	2	3
y	18	8	2	0	2	8	18



c) $f(x) = x^2 - 1$

shape: $a > 0$ (happy)

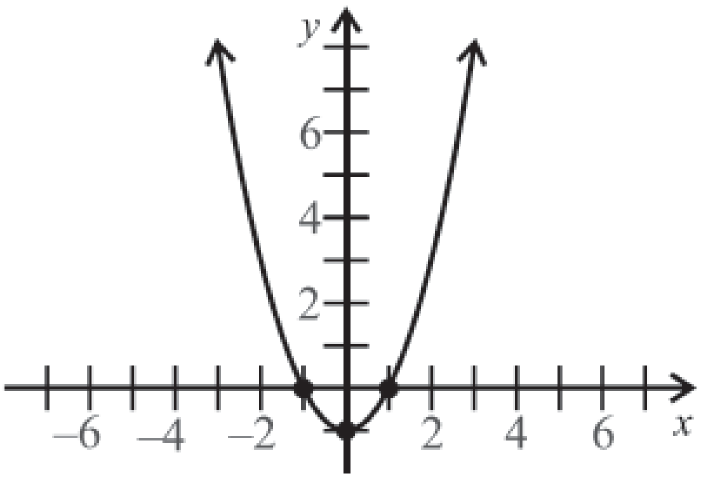
y-int: $(0;-1)$

x-int: let $y = 0$

$\therefore x^2 - 1 = 0$

$\therefore (x+1)(x-1) = 0$

$\therefore$ x-int at $(-1;0)$ and $(1;0)$



d) $f(x) = x^2 - 4$

shape: $a > 0$ (happy)

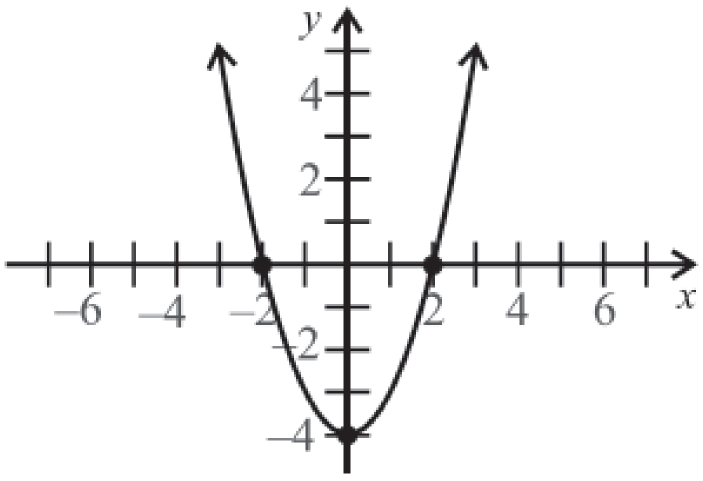
y-int: $(0;-4)$

x-int: let $y = 0$

$\therefore x^2 - 4 = 0$

$\therefore (x+2)(x-2) = 0$

$\therefore$ x-int at $(0;-2)$ and $(0;2)$



Exercise 4 number 2 is on page 301 (solutions are continued on the next page)

e) $f(x) = -2x^2 + 2$

shape: $a < 0$(sad)

y-int: $(0;2)$

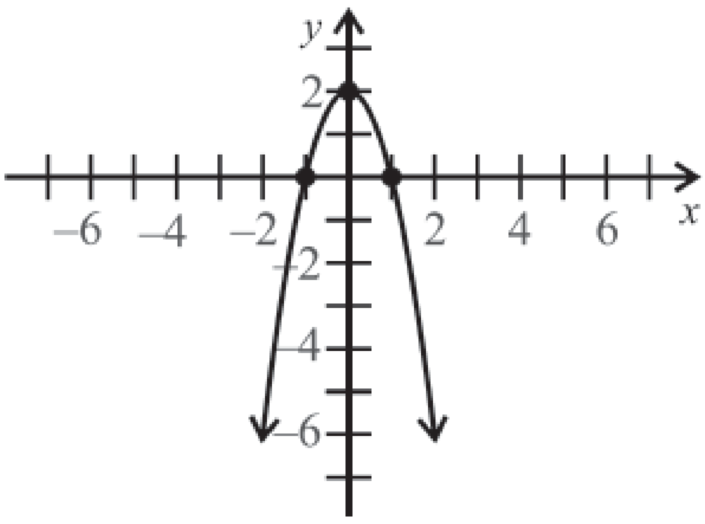
x-int: let $y = 0$

$\therefore -2x^2 + 2 = 0$

$\therefore -2(x^2 - 1) = 0$

$\therefore (x+1)(x-1) = 0$

$\therefore x$-int at $(-1;0)$ and $(1;0)$



Exercise 4 number 2 is on page **301**

f) $f(x) = 2x^2 - 8$

shape: $a > 0$(happy)

y-int: $(0;-8)$

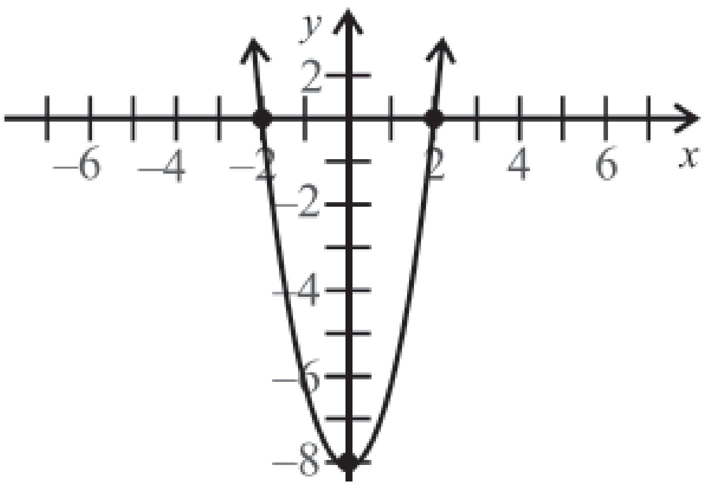
x-int: let $y = 0$

$\therefore 2x^2 - 8 = 0$

$\therefore 2(x^2 - 4) = 0$

$\therefore (x+2)(x-2) = 0$

$\therefore x$-int at $(-2;0)$ and $(2;0)$



Exercise 5 Solutions:

1)

a) $y = ax^2 + 4$

sub $(-1;3)$

$\therefore 3 = a + 4$

$\therefore a = -1$

$\therefore y = -x^2 + 4$

b) $y = ax^2 - 8$

sub $(2;0)$

$\therefore 0 = 4a - 8$

$\therefore a = 2$

$\therefore y = 2x^2 - 8$

c) $y = ax^2 + q$

sub $(1;-4)$

$\therefore -4 = a + q \rightarrow \boxed{1}$

sub $(2;-7)$

$\therefore -7 = 4a + q \rightarrow \boxed{2}$

$\boxed{2} - \boxed{1}$:

$\therefore -3 = 3a$

$\therefore a = -1 \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{1}$:

$\therefore -4 = -1 + q$

$\therefore q = -3$

$\therefore y = -x^2 - 3$

Exercise 5 number 1 is on page **301**

d) $y = ax^2 + q$

sub $(3;0)$

$\therefore 0 = 9a + q \rightarrow \boxed{1}$

sub $(2;15)$

$\therefore 15 = 4a + q \rightarrow \boxed{2}$

$\boxed{2} - \boxed{1}$:

$\therefore 15 = -5a$

$\therefore a = -3 \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{1}$:

$\therefore 0 = -27 + q$

$\therefore q = 27$

$\therefore y = -3x^2 + 27$

e) $y = ax^2 + q$

sub $(1;15)$

$\therefore 15 = a + q \rightarrow \boxed{1}$

sub $(3;-25)$

$\therefore -25 = 9a + q \rightarrow \boxed{2}$

$\boxed{2} - \boxed{1}$:

$\therefore -40 = 8a$

$\therefore a = -5 \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{1}$:

$\therefore 15 = -5 + q$

$\therefore q = 20$

$\therefore y = -5x^2 + 20$

2)

a) $y = ax^2 + 9$

sub $(3;4)$

$\therefore 4 = 9a + 9$

$\therefore -5 = 9a$

$\therefore a = -\dfrac{5}{9}$

$\therefore y = -\dfrac{5}{9}x^2 + 9$

b) $y = ax^2 - 6$

sub $(8;8)$

$\therefore 8 = 64a - 6$

$\therefore 14 = 64a$

$\therefore a = \dfrac{7}{32}$

$\therefore y = \dfrac{7}{32}x^2 - 6$

c) $y = ax^2 + q$

sub $(-2;4)$

$\therefore 4 = 4a + q \rightarrow \boxed{1}$

sub $(3;7)$

$\therefore 7 = 9a + q \rightarrow \boxed{2}$

$\boxed{2} - \boxed{1}$:

$\therefore 3 = 5a$

$\therefore a = \dfrac{3}{5} \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{1}$:

$\therefore 4 = 4\left(\dfrac{3}{5}\right) + q$

$\therefore 4 = \dfrac{12}{5} + q$

$\therefore 20 = 12 + 5q$

$\therefore 8 = 5q$

$\therefore q = \dfrac{8}{5}$

$\therefore y = \dfrac{3}{5}x^2 + \dfrac{8}{5}$

Exercise 5 number 2 is on page **301**

3)

a) $y - 2x + 4 = 0$

$\therefore y = 2x - 4$

$\therefore m = 2$

b) x-int: let $y = 0$

$\therefore 0 = 2x - 4$

$\therefore 2x = 4$

$\therefore x = 2$

$\therefore$ x-ints: $(-2;0)$ and $(2;0)$

y-int: let $x = 0$

$\therefore y = -4$

$\therefore$ Turning point is $(0;-4)$

c) $g(x) = ax^2 - 4$

sub $(2;0)$

$\therefore 0 = a(2)^2 - 4$

$\therefore 4 = 4a$

$\therefore a = 1$

$\therefore g(x) = x^2 - 4$

Exercise 5 number 3 is on page **302**

Exercise 6 Solutions:

1)

a) $y = \dfrac{4}{x}$

asymptotes:

$y = 0$ and $x = 0$

lines of symmetery

$y = x + 0$ and $y = -x + 0$

x-int: none

y-int: none

points of intersection:

$\left(\sqrt{4};\sqrt{4}\right)$ and $\left(-\sqrt{4};-\sqrt{4}\right)$

$= (2;2)$ and $(-2;-2)$

b) $f(x) = \dfrac{4}{x} + 1$

asymptotes:

$y = 1$ and $x = 0$

lines of symmetery: $y = x + 1$ and $y = -x + 1$

x-int: Let $y = 0$

$\therefore 0 = \dfrac{4}{x} + 1$

$\therefore -\dfrac{4}{x} = 1$

$\therefore x = -4$

$\therefore$ x-int at $(-4;0)$

y-int: none

points of intersection:

$\left(\sqrt{4};\sqrt{4}+1\right)$ and $\left(-\sqrt{4};-\sqrt{4}+1\right) = (2;3)$ and $(-2;-1)$

Exercise 6 number 1 is on page **302** (solutions are continued on the next page)

c) $y=\frac{4}{x}-1$
asymptotes:
$y=-1$ and $x=0$
lines of symmetery
$y=x-1$ and $y=-x-1$
x-int: Let $y=0$
$\therefore 0=\frac{4}{x}-1$
$\therefore -\frac{4}{x}=-1$
$\therefore x=4$
$\therefore x\text{-int at } (4;0)$
y-int: none
points of intersection:
$\left(\sqrt{4};\sqrt{4}-1\right)$ and $\left(-\sqrt{4};-\sqrt{4}-1\right)$
$=(2;1)$ and $(-2;-3)$

d) $f(x)=\frac{9}{x}+4$
asymptotes:
$y=4$ and $x=0$
lines of symmetery
$y=x+4$ and $y=-x+4$
x-int: Let $y=0$
$\therefore 0=\frac{9}{x}+4$
$\therefore -\frac{9}{x}=4$
$\therefore x=-\frac{9}{4}$
$\therefore x\text{-int at } \left(-\frac{9}{4};0\right)$
y-int: none
points of intersection:
$\left(\sqrt{9};\sqrt{9}+4\right)$ and $\left(-\sqrt{9};-\sqrt{9}+4\right)$
$=(3;7)$ and $(-3;1)$

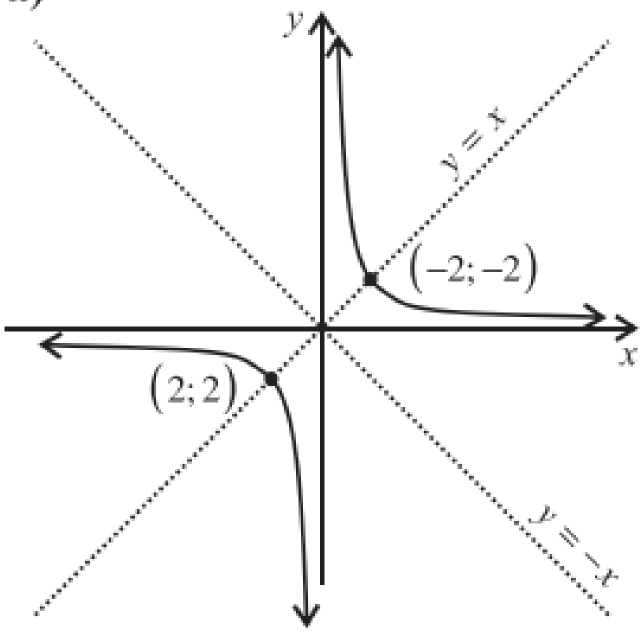
e) $f(x)=-\frac{4}{x}+4$
asymptotes:
$y=4$ and $x=0$
lines of symmetery
$y=x+4$ and $y=-x+4$
x-int: Let $y=0$
$\therefore 0=-\frac{4}{x}+4$
$\therefore \frac{4}{x}=4$
$\therefore x=1$
$\therefore x\text{-int at } (1;0)$
y-int: none
points of intersection:
$\left(-\sqrt{4};\sqrt{4}+4\right)$ and $\left(\sqrt{4};-\sqrt{4}+4\right)$
$=(-2;6)$ and $(2;2)$

f) $f(x)=-\frac{9}{x}-2$
asymptotes:
$y=-2$ and $x=0$
lines of symmetery
$y=x-2$ and $y=-x-2$
x-int: Let $y=0$
$\therefore 0=-\frac{9}{x}-2$
$\therefore \frac{9}{x}=-2$
$\therefore x=-\frac{9}{2}$
$\therefore x\text{-int at } \left(-\frac{9}{2};0\right)$
y-int: none
points of intersection:
$\left(\sqrt{9};-\sqrt{9}-2\right)$ and $\left(-\sqrt{9};\sqrt{9}-2\right)$
$=(3;-5)$ and $(-3;1)$

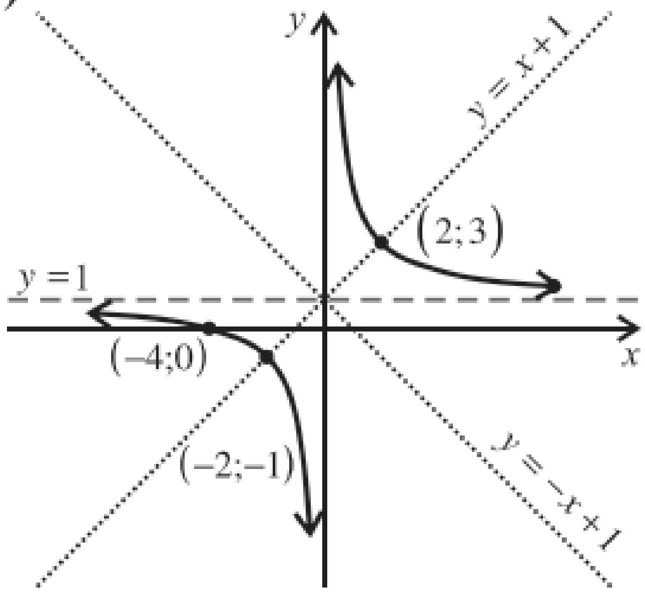
Exercise 6 number 1 is on page **302**

2)

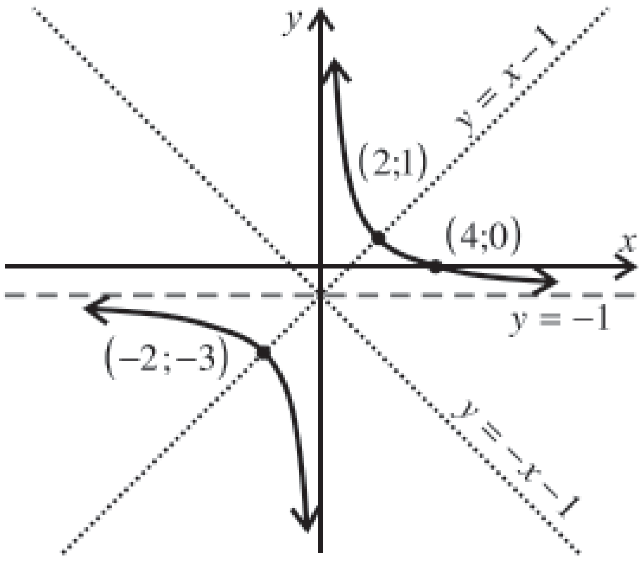
a)



b)

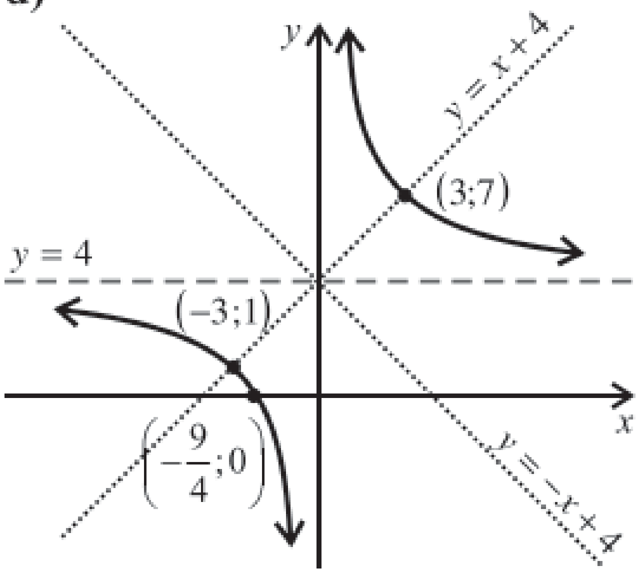


c)

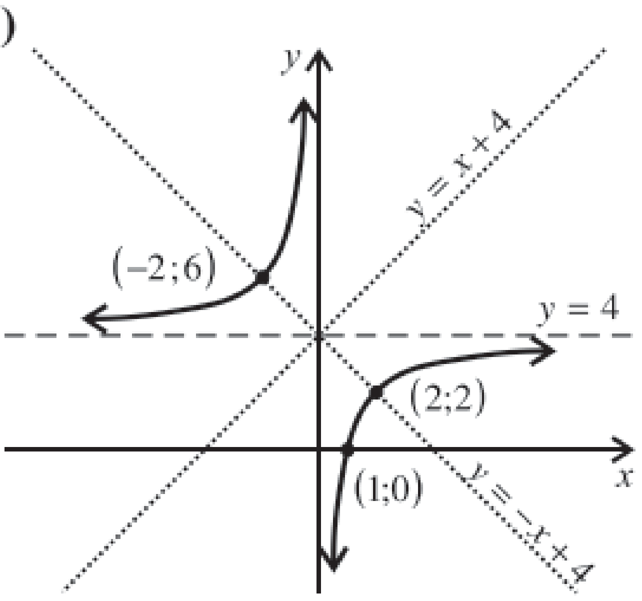


Exercise 6 number 2 is on page **302** (solutions are continued on the next page)

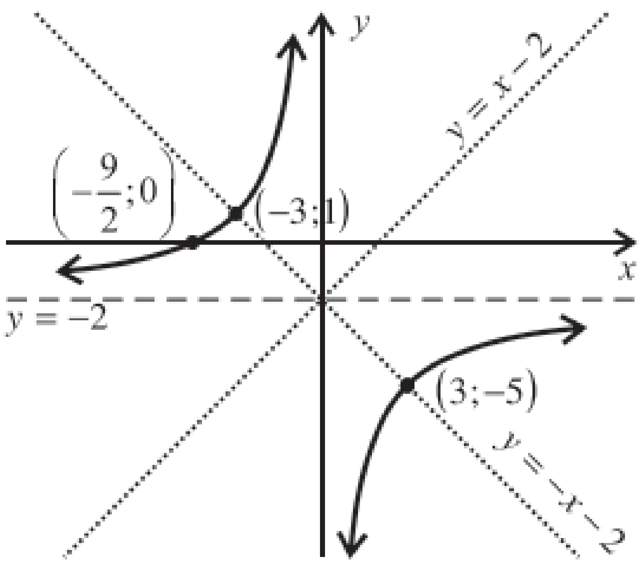
d)



e)



f)



Exercise 6 number 2 is on page **302**

Exercise 7 Solutions:

1)

a) $y=\frac{a}{x}-3 \rightarrow \text{sub}\left(2;-1\right)$

$\therefore -1=\frac{a}{2}-3$

$\therefore -2=a-6$

$\therefore a=4$

$\therefore y=\frac{4}{x}-3$

b) $y=\frac{a}{x}+5 \rightarrow \text{sub}\left(3;7\right)$

$\therefore 7=\frac{a}{3}+5$

$\therefore 21=a+15$

$\therefore a=6$

$\therefore y=\frac{6}{x}+5$

c) $y=\frac{a}{x}+q \rightarrow \text{sub}\left(4;-2\right)$

$\therefore -2=\frac{a}{4}+q$

$\therefore -8=a+4q \rightarrow \boxed{1}$

sub $\left(6;-1\right)$

$\therefore -1=\frac{a}{6}+q$

$\therefore -6=a+6q \rightarrow \boxed{2}$

$\boxed{2}-\boxed{1}$:

$\therefore 2=2q$

$\therefore q=1 \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{1}$:

$\therefore -8=a+4$

$\therefore a=-12$

$\therefore y=-\frac{12}{x}+1$

Exercise 7 number 1 is on page **302**

2)

a) $y=\frac{a}{x}+q \rightarrow \text{sub}\left(-2;-5\right)$

$\therefore -5=\frac{a}{-2}+q$

$\therefore 10=a-2q \rightarrow \boxed{1}$

sub $\left(2;1\right)$

$\therefore 1=\frac{a}{2}+q$

$\therefore 2=a+2q \rightarrow \boxed{2}$

$\boxed{2}+\boxed{1}$:

$\therefore 12=2a$

$\therefore a=6 \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{2}$:

$\therefore 2=6+2q$

$\therefore q=-2$

$\therefore p\left(x\right)=\frac{6}{x}-2$

b) $y=\frac{a}{x}+q \rightarrow \text{sub}\left(-3;0\right)$

$\therefore 0=\frac{a}{-3}+q$

$\therefore 0=a-3q \rightarrow \boxed{1}$

sub $\left(3;4\right)$

$\therefore 4=\frac{a}{3}+q$

$\therefore 12=a+3q \rightarrow \boxed{2}$

$\boxed{2}+\boxed{1}$:

$\therefore 12=2a$

$\therefore a=6 \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{2}$:

$\therefore 12=6+3q$

$\therefore q=2$

$\therefore p\left(x\right)=\frac{6}{x}+2$

Exercise 7 number 2 is on page **302**

Exercise 8 Solutions:

1)

a) $y = 2^x$

$a > 0$ and $b > 0$:

$\therefore$ increasing and above the asymptote

asymptote : $y = 0$

x-int: none

y-int: let $x = 0$

$\therefore y = 2^0$

$\therefore y = 1$

$\therefore$ y-int at $(0;1)$

b) $y = 2^x - 1$

$a > 0$ and $b > 0$:

$\therefore$ increasing and above the asymptote

asymptote : $y = -1$

x-int: let $y = 0$

$\therefore 0 = 2^x - 1$

$\therefore 2^x = 1$

$\therefore x = 0$

y-int: let $x = 0$

$\therefore y = 2^0 - 1$

$\therefore y = 0$

$\therefore$ y-int at $(0;0)$ and x-int at $(0;0)$

c) $y = 2^x + 1$

$a > 0$ and $b > 0$:

$\therefore$ increasing and above the asymptote

asymptote : $y = 1$

x-int: none

y-int: let $x = 0$

$\therefore y = 2^0 + 1$

$\therefore y = 2$

$\therefore$ y-int at $(0;2)$

d) $f(x) = -2^{-x} + 4$

$f(x) = -\left(\frac{1}{2}\right)^x + 4$

$a < 0$ and $0 < b < 1$:

$\therefore$ increasing and below the asymptote

asymptote : $y = 4$

x-int: let $y = 0$

$\therefore 0 = -\left(\frac{1}{2}\right)^x + 4$

$\therefore 2^{-x} = 2^2$

$\therefore x = -2$

y-int: let $x = 0$

$\therefore y = -\left(\frac{1}{2}\right)^0 + 4$

$\therefore y = 3$

$\therefore$ y-int at $(0;3)$ and x-int at $(-2;0)$

e) $f(x) = -2(2)^x + 2$

$a < 0$ and $b > 1$:

$\therefore$ decreasing and below the asymptote

asymptote : $y = 2$

x-int: let $y = 0$

$\therefore 0 = -2(2)^x + 2$

$\therefore 2(2)^x = 2$

$\therefore 2^x = 1$

$\therefore x = 0$

y-int: let $x = 0$

$\therefore y = -2(2)^0 + 2$

$\therefore y = 0$

$\therefore$ y-int at $(0;0)$ and x-int at $(0;0)$

f) $f(x) = 3\left(\frac{1}{3}\right)^x + 2$

$a > 0$ and $0 < b < 1$:

$\therefore$ decreasing and above the asymptote

asymptote : $y = 2$

x-int: none

y-int: let $x = 0$

$\therefore y = 3\left(\frac{1}{3}\right)^0 + 2$

$\therefore y = 5$

$\therefore$ y-int at $(0;5)$

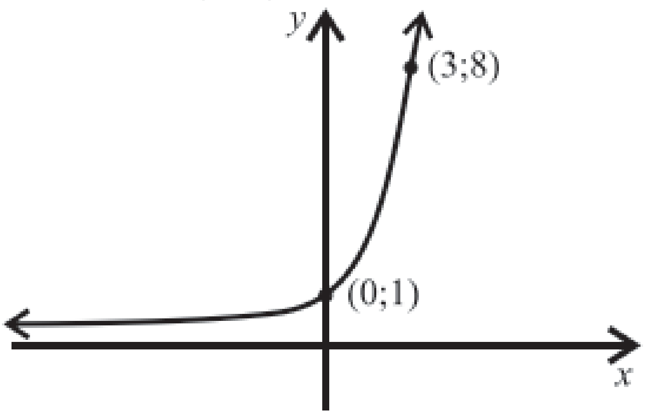
Exercise 8 number 1 is on page **303**

2)

a) $y = 2^x$

sub $x = 3$

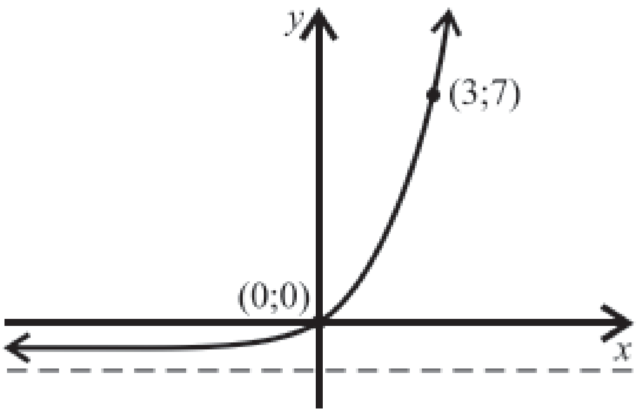
$\therefore$ extra point $(3;8)$



b) $y = 2^x - 1$

sub $x = 3$

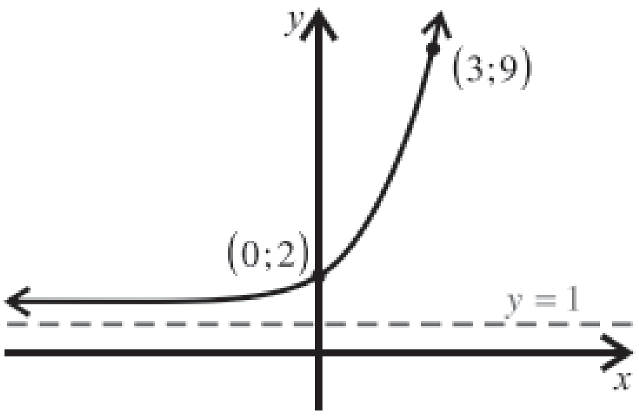
$\therefore$ extra point $(3;7)$



c) $y = 2^x + 1$

sub $x = 3$

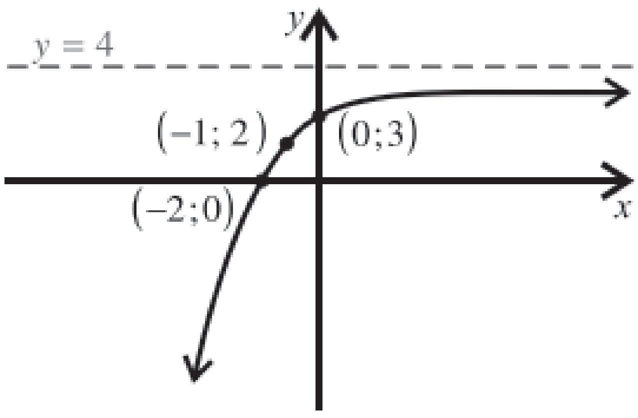
$\therefore$ extra point $(3;9)$



d) $f(x) = -2^{-x} + 4$

sub $x = -1$

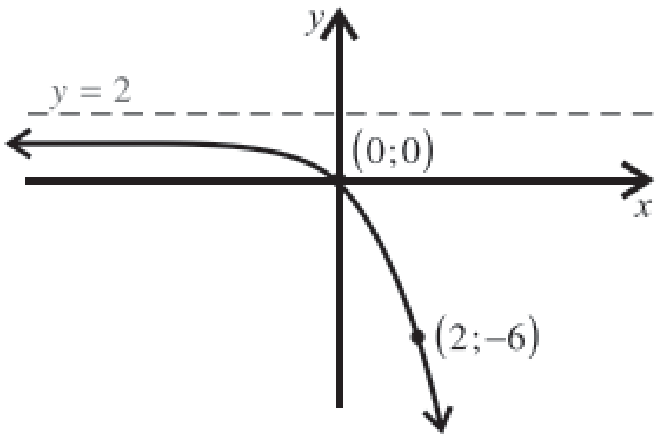
$\therefore$ extra point $(-1;2)$



e) $f(x) = -2(2)^x + 2$

sub $x = 2$

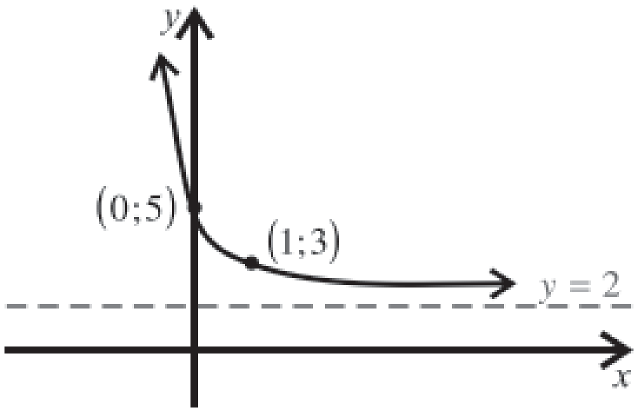
$\therefore$ extra point $(2;-6)$



f) $f(x) = 3\left(\frac{1}{3}\right)^x + 2$

sub $x = 1$

$\therefore$ extra point $(1;3)$



Exercise 8 number 2 is on page **303**

Exercise 9 Solutions:

1)

a) $y = b^x - 8$

sub $(3;19)$

$\therefore 19 = b^3 - 8$

$\therefore 27 = b^3$

$\therefore b = 3$

$\therefore b = 3$ and $q = -8$

b) $y = ab^x - 5$

sub $(1;1)$

$\therefore 1 = ab^1 - 5$

$\therefore 6 = ab$

$\therefore a = \frac{6}{b} \rightarrow \boxed{1}$

sub $(2;13)$

$\therefore 13 = ab^2 - 5 \rightarrow \boxed{2}$

sub $\boxed{1}$ into $\boxed{2}$:

$\therefore 13 = \left(\frac{6}{b}\right)b^2 - 5$

$\therefore 13 = 6b - 5$

$\therefore 18 = 6b$

$\therefore b = 3 \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{1}$:

$\therefore a = 2;\ b = 3$ and $q = -5$

Exercise 9 number 1 is on page **303**

2) $p(-1) = -(-1) + 1 = 2$

Sub $(-1;2)$ into $f(x)$:

$2 = ab^{-1} - 4 \rightarrow \boxed{1}$

sub $\boxed{2}$ into $\boxed{1}$:

$\therefore 2 = 3(b)^{-1} - 4$

$\therefore 6 = 3b^{-1}$

$\therefore 2 = b^{-1}$

$\therefore b = \frac{1}{2}$ and $a = 3$

The y-intercept of f and g is the point $(0;-1)$

Sub $(0;-1)$ into $f(x)$:

$\therefore -1 = ab^{0} - 4$

$\therefore a = 3 \rightarrow \boxed{2}$

Exercise 9 number 2 is on page **303**

Exercise 10 Solutions:

a) Domain: $x \in [-16;16)$

Range: $y \in [-14;12)$

b) Domain: $x \in (-12;12)$

Range: $y \in [-16;10)$

c) Domain: $x \in \mathbb{R}\ ;\ x \neq 0$

Range: $y \in \mathbb{R}\ ;\ y \neq 0$

d) Domain: $x \in \mathbb{R}$

Range: $y \in (0;\infty)$

e) Domain: $x \in \mathbb{R}$

Range: $y \in (-\infty;22]$

f) Domain: $x \in [-7;8]\ ;\ x \neq 4$

Range: $y \in [-7;8]\ ;\ y \neq 4$

Exercise 10 is on page **304**

Exercise 11 Solutions:

1)

a) $\text{AB} = y_{top} - y_{bot}$

$= 7 - (-1)$

$= 8$ units

b) $\text{AB} = x_{right} - x_{left}$

$= 3 - (-7)$

$= 10$ units

c) $\text{AB} = y_{top} - y_{bot}$

$= (4-1) - (-3)$

$= 6$ units

Exercise 11 number 1 is on page **304**

2)

a) $\text{AB} = f(x) - g(x)$

$= x + 1 - (-2x)$

$= 3x + 1$ units

b) $\text{AB} = f(x) - g(x)$

$= -x^2 + 7 - (x^2 - 9)$

$= -2x^2 + 16$ units

c) $\text{AB} = f(x) - g(x)$

$= -\frac{x}{2} + 1 - \left(\frac{x^2}{2} - 8\right)$

$= -\frac{x^2}{2} - \frac{x}{2} + 9$

Exercise 11 number 2 is on page **304**

Exercise 12 Solutions:

1)

a) $\text{OA} = 4$ units

BC: x-int: let $y = 0$

$\therefore 0 = -x^2 + 4$

$\therefore x^2 - 4 = 0$

$\therefore (x-2)(x+2) = 0$

$\therefore x = 2$ or $x = -2$

$\text{BC} = x_{right} - x_{left}$

$= 2 - (-2)$

$= 4$ units

b) $f(-1) = -(-1)^2 + 4$

$= 3$

$\therefore$ FG = 3 units

c) ED: $-5 = -x^2 + 4$

$\therefore x^2 - 9 = 0$

$\therefore (x-3)(x+3) = 0$

$\therefore x = 3$ or $x = -3$

$\therefore$ OD = 3 units

d) $x < 0$

e) $-2 < x < 2$

f) $x \leq -2$ or $x \geq 2$

Exercise 12 number 1 is on page **305**

2)

a) $A:(0;8)$

$B:(0;4)$

x-int: let $y=0$

$\therefore -2x^2+8=0$

$\therefore x^2-4=0$

$\therefore (x-2)(x+2)=0$

$\therefore x=2$ or $x=-2$

$\therefore C(-2;0)$ and $D(2;0)$

b) $f(x)=g(x)$

$\therefore -2x+4=-2x^2+8$

$\therefore 2x^2-2x-4=0$

$\therefore x^2-x-2=0$

$\therefore (x+1)(x-2)=0$

$\therefore x=-1$ *or* $x=2$

$f(-1)=6$

$\therefore E(-1;6)$

c) $CD=x_{right}-x_{left}$

$=2-(-2)$

$=4$ units

d) $FG=g(x)-f(x)$

$=-2x^2+8-(-2x+4)$

$=-2x^2+2x+4$

sub $x=1$ into FG

$\therefore FG=-2(1)^2+2(1)+4$

$=4$ units

e) $-1\le x\le 2$

f) $x\ge -2$

Exercise 12 number 2 is on page **305**

3)

a) $f(x)=ax^2+4$

Sub $(2;0)$

$\therefore 0=a(2)^2+4$

$\therefore -4=4a$

$\therefore a=-1$

$g(x)=\dfrac{k}{x}-3$

Sub $(2;0)$

$\therefore 0=\dfrac{k}{2}-3$

$\therefore 3=\dfrac{k}{2}$

$\therefore k=6$

$\therefore a=-1$, $q=4$, $k=6$ and $c=-3$

b) C and E: $f(x)=g(x)$

$\therefore -x^2+4=\dfrac{6}{x}-3$

$\therefore -x^3+4x=6-3x$

$\therefore x^3-7x+6=0$

$\therefore (x+3)(x-2)(x-1)=0$

$\therefore x=-3$ or $x=2$ or $x=1$

$f(-3)=-(-3)^2+4=-5$

$f(1)=-(1)^2+4=3$

$\therefore$ E is the point $(-3;-5)$

and C is the point $(1;3)$

c) By symmetry: $D(-2;0)$

d) $g(-2)=\dfrac{6}{-2}-3=-6$

$\therefore$ point F is $(-2;-6)$

$\therefore DF=6$ units

e) $x<-3$ or $0<x<1$ or $x>2$

this can also be written as:

$x\in(-\infty;-3)$ or $(0;1)$ or $(2;\infty)$

f) $x<-2$ or $x>0$; $x\ne 2$

this can also be written as:

$x\in(-\infty;-2)$ or $(0;\infty)$; $x\ne 2$

g) Domain: $f(x): x\in\mathbb{R}$

$g(x): x\in\mathbb{R}\,;\, x\ne 0$

Range: $f(x): y\le 4 \rightarrow$ also written as $y\in(-\infty;4]$

$g(x): y\in\mathbb{R}\,;\, y\ne -3$

Exercise 12 number 3 is on page **305**

4)

a) x-int : let $y=0$

$\therefore -x+4=0$

$\therefore x=4$

$\therefore$ B is the point $(4;0)$

b) By symmetry: $\text{D}(-4;0)$

c) $f(x)=ax^2+8$

sub $\text{B}(4;0)$

$\therefore 0=a(4)^2+8$

$\therefore -16a=8$

$\therefore a=-\dfrac{1}{2}$ and $q=8$

d) $\text{C}: f(x)=g(x)$

$\therefore -\dfrac{1}{2}x^2+8=-x+4$

$\therefore -x^2+16=-2x+8$

$\therefore x^2-2x-8=0$

$\therefore (x+2)(x-4)=0$

$\therefore x=-2$ or $x=4$

$g(-2)=-(-2)+4=6$

$\therefore$ C is the point $(-2;6)$

e) $g(-4)=-(-4)+4=8$

$\therefore$ DH $=8$ units

f) $\text{EF}=f(x)-g(x)$

$4=-\dfrac{1}{2}x^2+8-(-x+4)$

$\therefore 4=-\dfrac{1}{2}x^2+8+x-4$

$\therefore 8=-x^2+16+2x-8$

$\therefore x^2-2x=0$

$\therefore x(x-2)=0$

$\therefore x=0$ or $x=2$

$\text{DG}=x_{right}-x_{left}$

$\therefore \text{DG}=2-(-4)$

$\therefore$ DG $=6$ units

g) $-2<x<4 \rightarrow$ also written as $x\in(-2;4)$

h) Domain: $f(x): x\in\mathbb{R}$
$g(x): x\in\mathbb{R}$

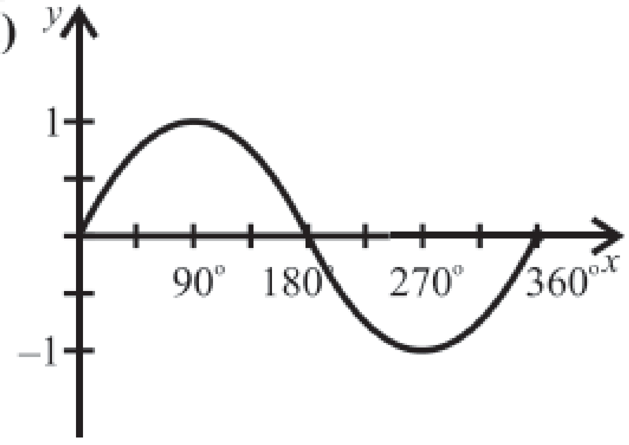
Range: $f(x): \text{y}\leq 8 \rightarrow$ also written as $y\in(-\infty;8]$
$g(x): y\in\mathbb{R}$

Exercise 12 number 4 is on page 306

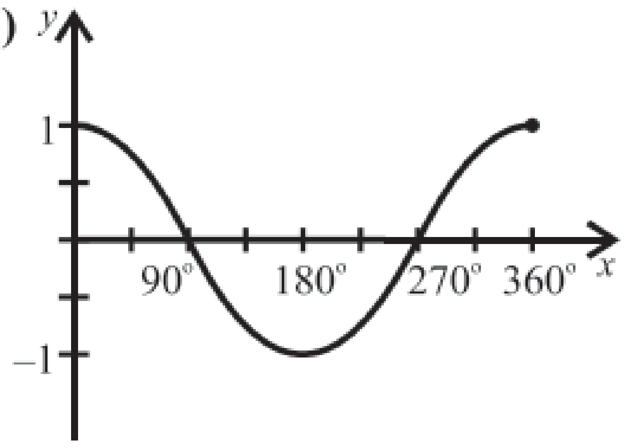
Exercise 13 Solutions:

1)

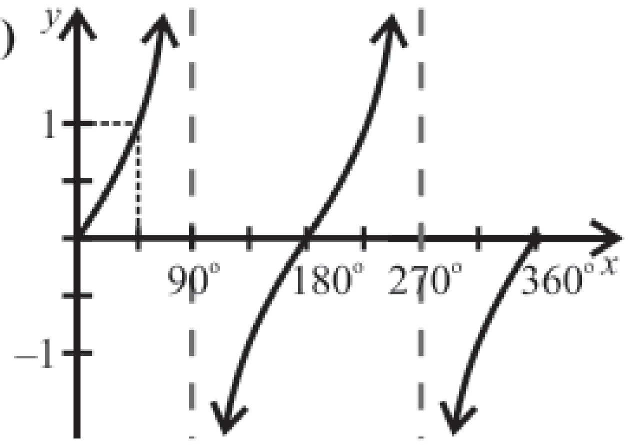
a)



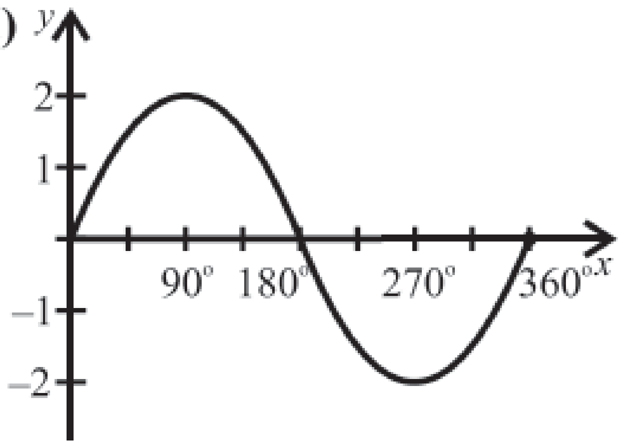
b)



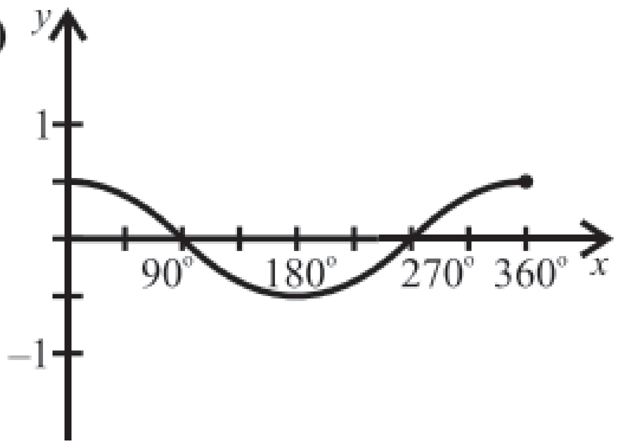
c)



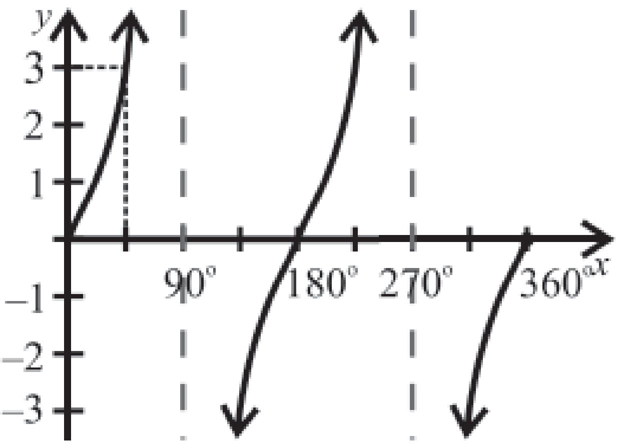
d)



e)



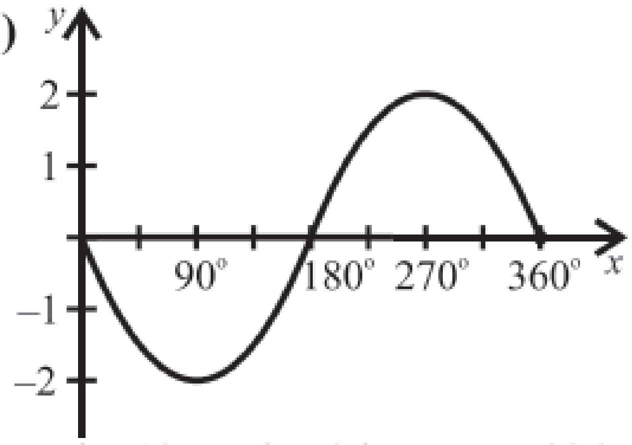
f)



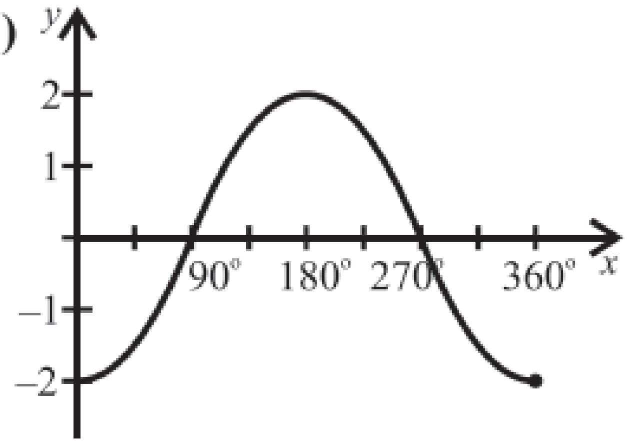
Exercise 13 number 1 is on page 306

2)

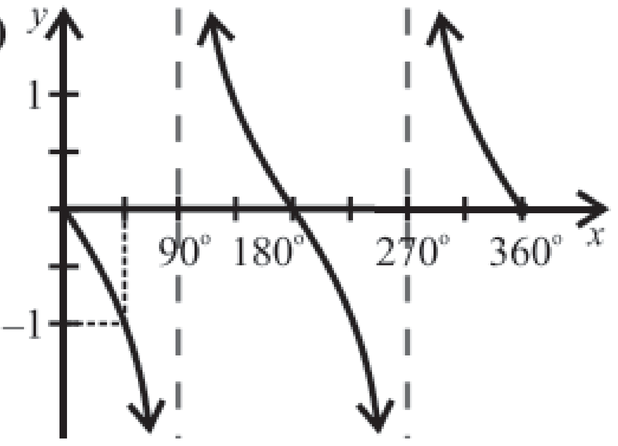
a)



b)



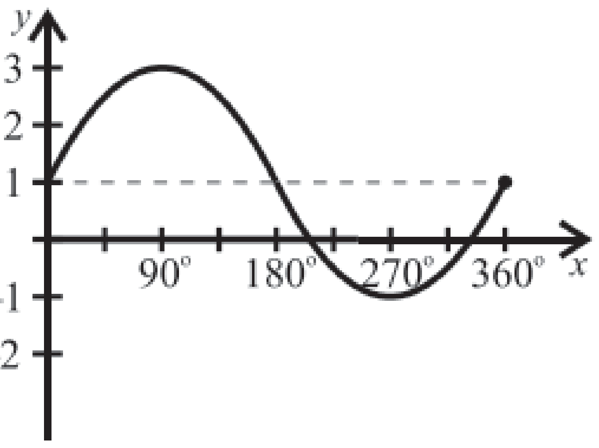
c)



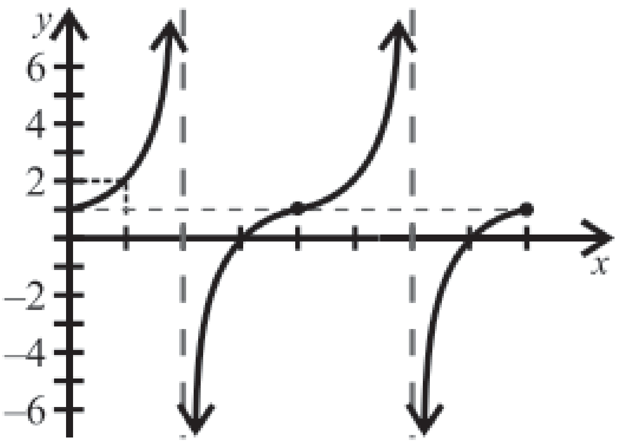
Exercise 13 number 2 is on page 306

3)

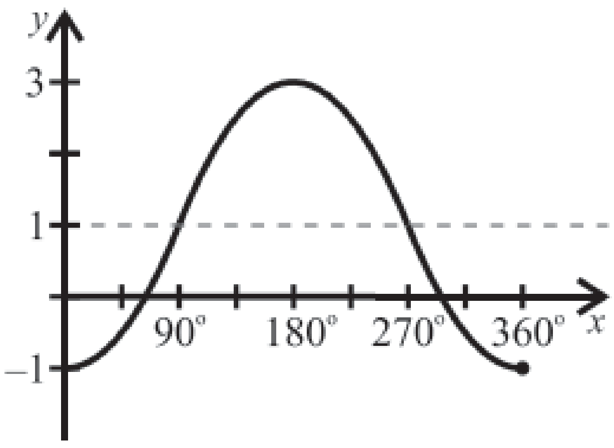
a)



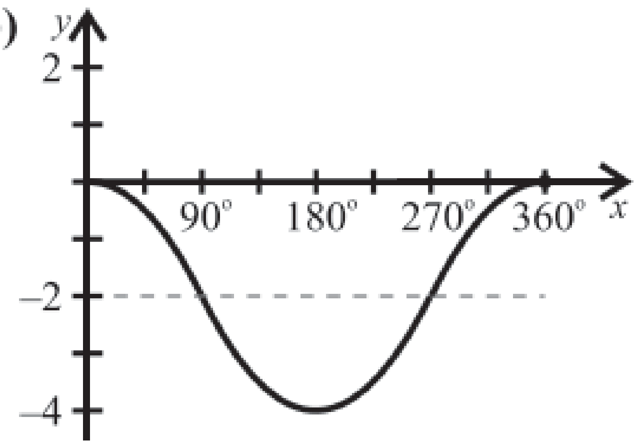
c)



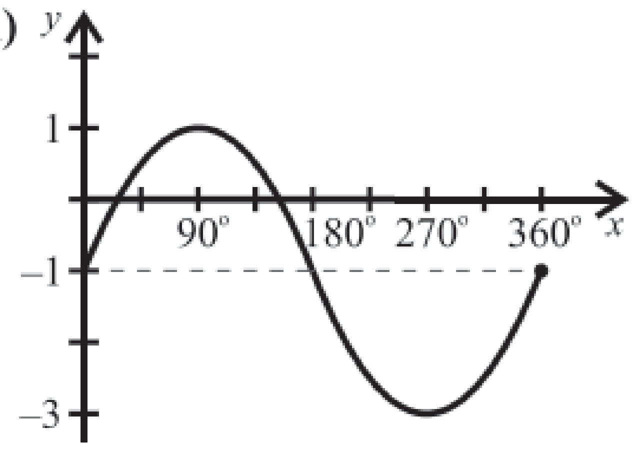
e)



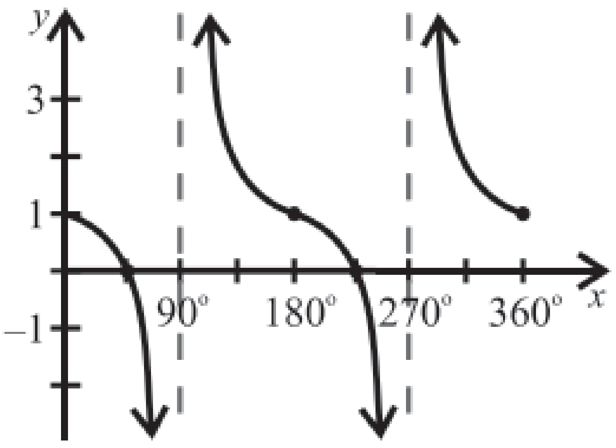
b)



d)



f)



Exercise 13 number 3 is on page **306**

Exercise 14 Solutions:

1)

a) $a=-2\,;\,p=1\,;\,q=2\,;\,r=1$

b) Domain: $x\in\left[-180^\circ\,;180^\circ\right]$

Range: $y\in[-1;3]$

c) The period of g is 360°

d) $a=\dfrac{y_{\max}-y_{\min}}{2}=2$

Exercise 14 number 1 is on page **306**

2)

a) $a=2\,;\,q=-2$

b) Domain $x\in\left(-90^\circ\,;90^\circ\right)$

Range: $y\in\mathbb{R}$

c) The period of g is 180°

Exercise 14 number 2 is on page **306**

3)

a) $a=\dfrac{1}{2}\,;\,q=-\dfrac{1}{2}$

b) Range of $g(x)$: $y\in[-1{,}5;0{,}5]$

c) The period of f is 360°

d) $x=180^\circ$

e) $x=0^\circ$ and $x=360^\circ$

Exercise 14 number 3 is on page **307**

4)

a) $a=3\,;\,q=-1$

b) Range of $f(x)$: $y\in[-3;3]$

c) The period of g is 360°

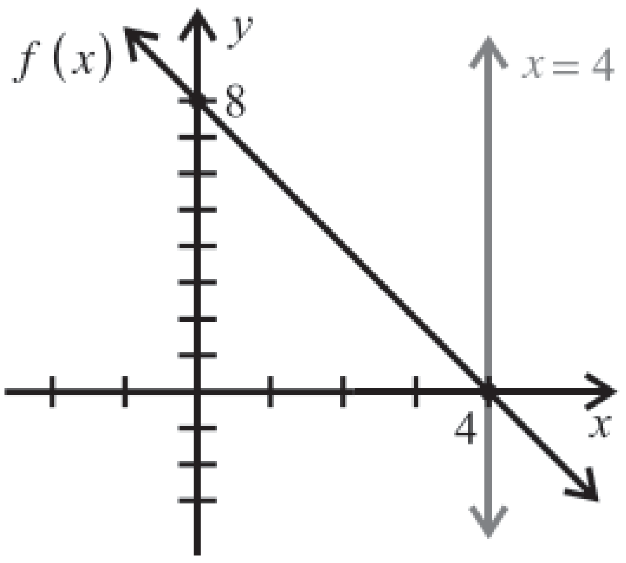
d) $x=0^\circ$ and $x=180^\circ$ and $x=360^\circ$

Exercise 14 number 4 is on page **307**

Mixed Exercise Solutions:

1)

a) $f(x)$: x-int: $(4;0)$ and y-int: $(0;8)$



b) $(4;0)$

c) $f(x)$:

Domain: $x\in\mathbb{R}$

Range: $y\in\mathbb{R}$

$x=4$:

Domain: $x=4$

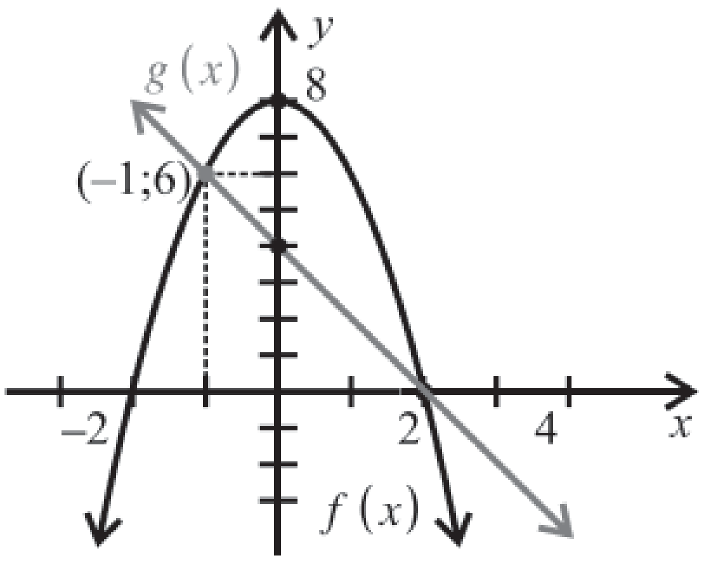
Range: $y\in\mathbb{R}$

Mixed Exercise number 1 is on page **307**

2)

a) $f(x)$: x-int: $(\pm2;0)$ and y-int: $(0;8)$

$g(x)$: x-int: $(2;0)$ and y-int: $(0;4)$



b) Domain: $x\in\mathbb{R}$

Range: $y\in(-\infty;8]\rightarrow$ also written as $x\le8$

c) $x\in(-\infty;0)\rightarrow$ also written as $x<0$

Mixed Exercise number 2 is on page **307**

3)

a) $y = mx + c$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-3)}{-1-(-5)} = \frac{3}{2}$

$\therefore y = \frac{3}{2}x + c$

sub $(-1;3)$

$\therefore 3 = \frac{3}{2}(-1) + c$

$\therefore c = \frac{9}{2}$

$\therefore y = \frac{3}{2}x + \frac{9}{2}$

b) $x = -8$

c) $y = mx + c$ and $c = 0$

$\therefore y = mx$

sub $(-3;-4)$

$\therefore -4 = -3m$

$\therefore m = \frac{4}{3}$

$\therefore y = \frac{4}{3}x$

d) $y = 4$

Mixed Exercise number 3 is on page **307**

4) $3x - y = 4$

$\therefore y = 3x - 4$

$\therefore 3x - 4 = 2x - 5$

$\therefore x = -1$

$\therefore y = -7$

Mixed Exercise number 4 is on page **307**

5)

a) $\mathrm{A}:(0;4)$

x-int: let $y = 0$

$\therefore -x^2 + 4 = 0$

$\therefore x^2 - 4 = 0$

$\therefore (x-2)(x+2) = 0$

$\therefore x = 2$ or $x = -2$

$\therefore \mathrm{B}(-2;0)$ and $\mathrm{C}(2;0)$

b) $f(x) = -1$

$\therefore -1 = -\frac{3}{x}$

$\therefore -x = -3$

$\therefore x = 3$

$\therefore$ E is the point $(3;-1)$

$g(3) = -5$

$\therefore \mathrm{EF} = -1-(-5)$

$= 4$ units

c) $\mathrm{G}(-3;-5)$ by symmetry

d) $\mathrm{FG} = x_{right} - x_{left}$

$= 3-(-3)$

$= 6$ units

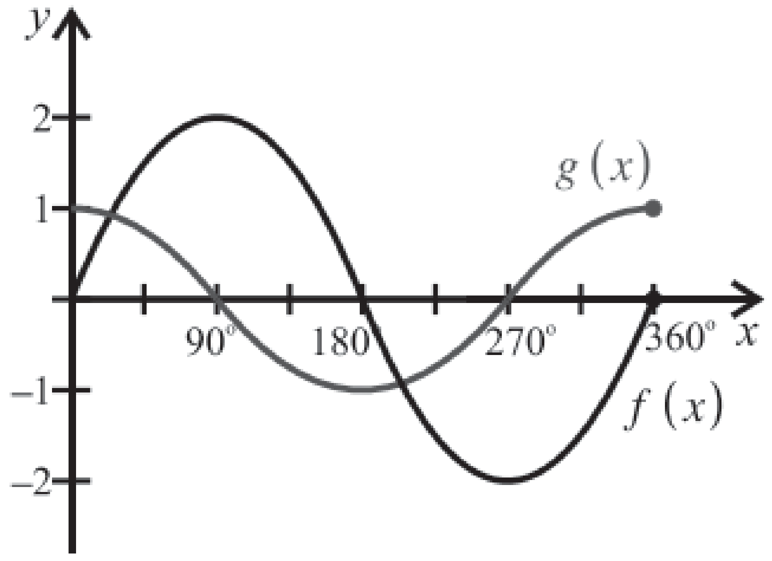
e) $g(-3) = -\frac{3}{-3} = 1$

$\therefore$ H is the point $(-3;1)$

Mixed Exercise number 5 is on page **308**

6)

a)



b) Range: $y \in [-1;1]$

c) 360°

Mixed Exercise number 6 is on page **308**

7) $a = -1$; $q = 2$; $b = 2$; $p = 0$

Mixed Exercise number 7 is on page **308**

8) $a = 2$; $q = -1$ with asymptotes at: $x = 90^\circ$ and $x = 270^\circ$

Mixed Exercise number 8 is on page **308**

9)

a) 160 km

b) 4 hours

c) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{400-0}{5-0} = 80$

$\therefore$ the car was travelling at 80 km/h

Mixed Exercise number 9 is on page **308**

10)

a) x-int : let $y = 0$

$\therefore 6x - 18 = 0$

$\therefore 6x = 18$

$\therefore x = 3$

$\therefore$ C is the point $(3;0)$

y-int : let $x = 0$

$\therefore y = -18$

$\therefore$ A is the point $(0;-18)$

b) By symmetry: $B(-3;0)$

c) $f(x) = ax^2 - 18$

sub $C(3;0)$

$\therefore 0 = a(3)^2 - 18$

$\therefore 9a = 18$

$\therefore a = 2$ and $q = -18$

d) $DE = f(x) - g(x)$

$\therefore 20 = 2x^2 - 18 - (6x - 18)$

$\therefore 0 = 2x^2 - 6x - 20$

$\therefore 0 = 2(x^2 - 3x - 10)$

$\therefore 0 = (x+2)(x-5)$

$\therefore x = -2$ or $x = 5$

OF = 2 units

e) $0 \le x \le 3 \rightarrow$ also written as $x \in [0;3]$

f) $x < -3 \rightarrow$ also written as $x \in (-3;\infty)$

g) Domain: $f(x) : x \in \mathbb{R}$
$g(x) : x \in \mathbb{R}$

Range: $f(x) : y \ge -18 \rightarrow$ also written as $y \in [-18;\infty)$
$g(x) : y \in \mathbb{R}$

Mixed Exercise number 10 is on page **309**

11)

a) $f(x) = ax^2 + 1$

sub $B(1;2)$

$\therefore 2 = a(1)^2 + 1$

$\therefore a = 1$ and $q = 1$

b) $g(x) = b^x + c$

Sub $(0;0)$

$\therefore 0 = b^0 + c$

$\therefore c = -1$

$\therefore g(x) = b^x - 1$

Sub $B(1;2)$

$\therefore 2 = b^1 - 1$

$\therefore b = 3$ and $c = -1$

c) $CD = g(x) - f(x)$

$= 3^x - 1 - (x^2 + 1)$

$= 3^x - x^2 - 2$

Sub $x = 2$

$\therefore CD = 3^2 - 2^2 - 2$

$= 3$ units

d) $x < 0 \rightarrow$ also written as $x \in (-\infty;0)$

h) Domain: $f(x) : x \in \mathbb{R}$
$g(x) : x \in \mathbb{R}$

e) Range: $f(x) : y \ge 1 \rightarrow$ also written as $y \in [1;\infty)$
$g(x) : y \ge -1 \rightarrow$ also written as $y \in [-1;\infty)$

Mixed Exercise number 11 is on page **309**

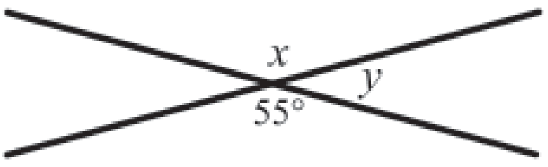
Chapter 7 – Geometry

Revision of Lines, Angles and Triangles

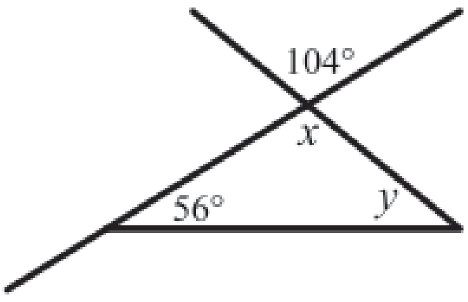
Exercise 1: (Theory on pages 141-142)

1) Solve for x, y and z in the following diagrams:

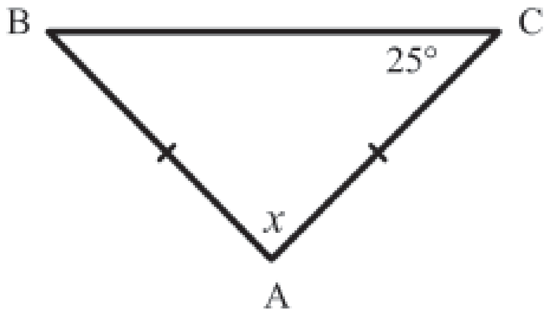
a)



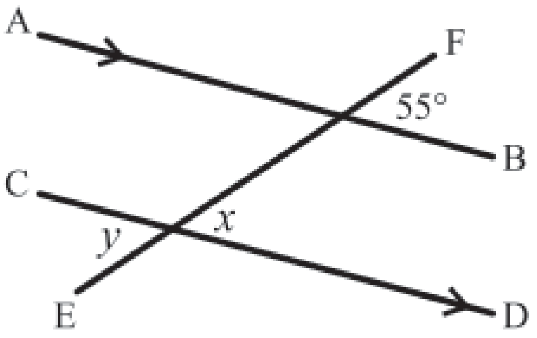
c)



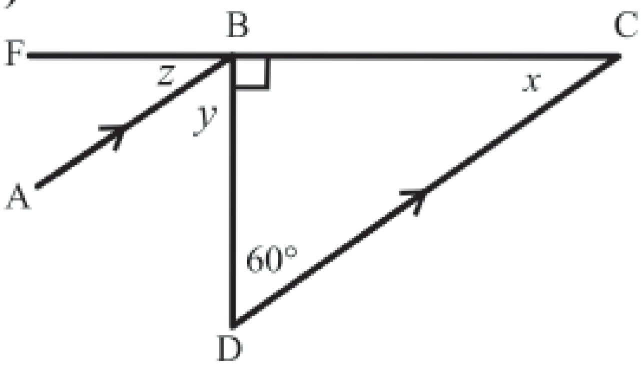
e)



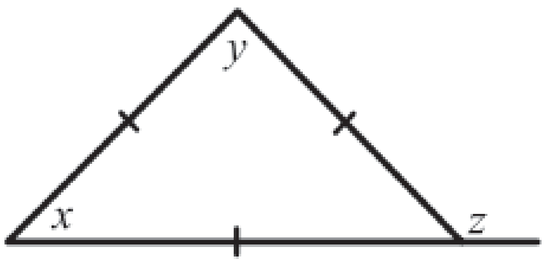
b)


A
F
55°
B
C
x
y
E
D

d)


F
B
C
z
x
y
A
60°
D

f)



Solutions to Exercise 1 number 1 are on page **334**

2) Using Pythagoras' theorem, determine if triangles with the following lengths are right angled or not:

a) 3 : 4 : 5

b) 5 : 12 : 15

c) 8 : 15 : 17

d) 7 : 22 : 25

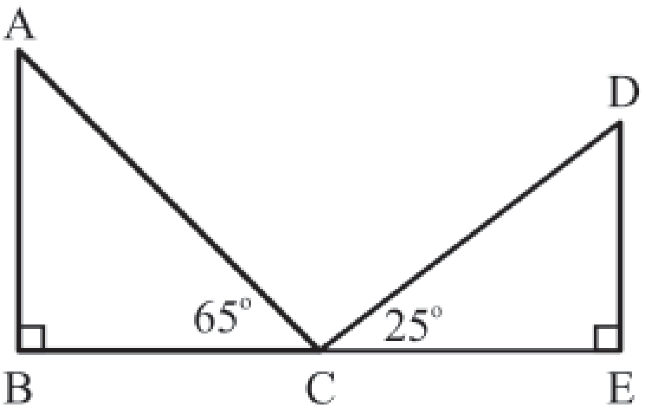
Solutions to Exercise 1 number 2 are on page **334**

Similar Triangles

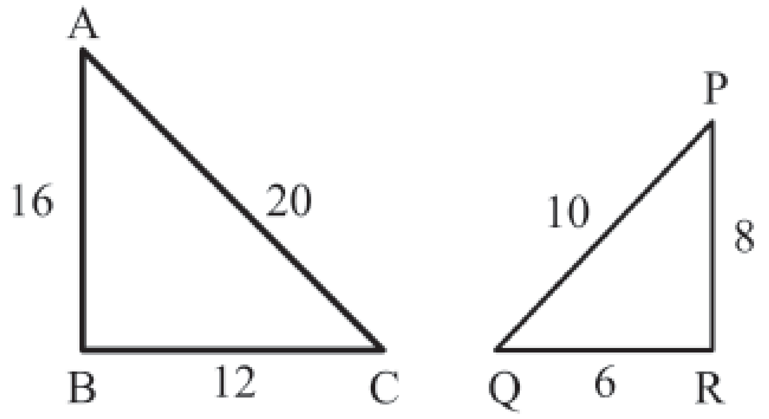
Exercise 2: (Theory on pages 142-143)

1) State with reason whether or not the following triangles are similar.

a)



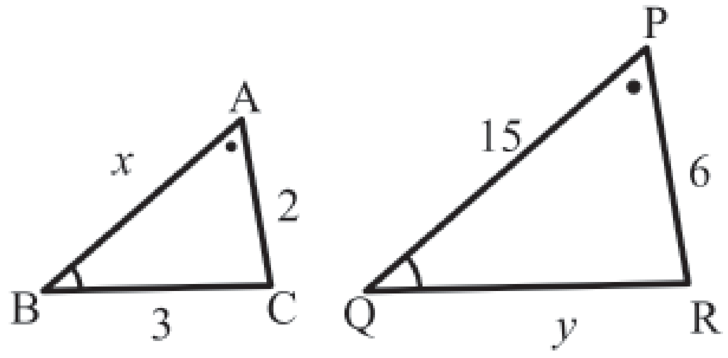
b)


A
P
16
20
10
8
B
12
C
Q
6
R

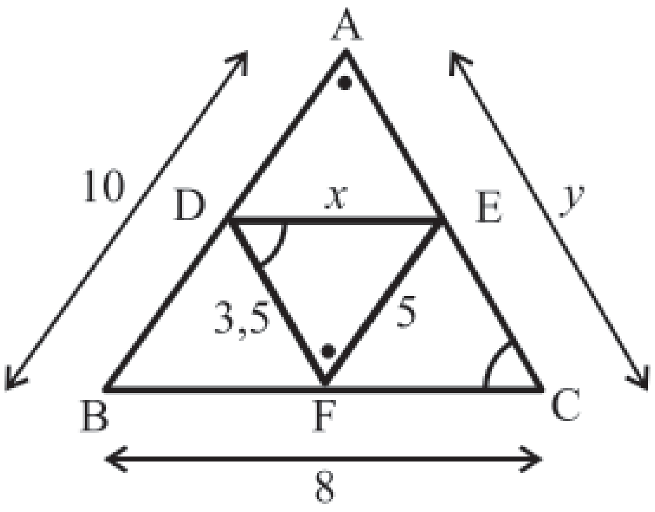
Solutions to Exercise 2 number 1 are on page **334**

2) Given that the following triangles are similar, determine the values of x and y.

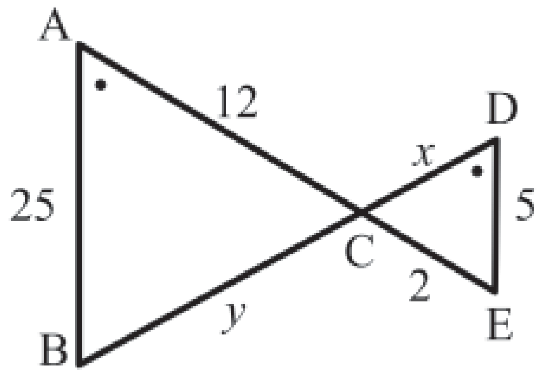
a)


P
A
x
15
2
6
B
3
C
Q
y
R

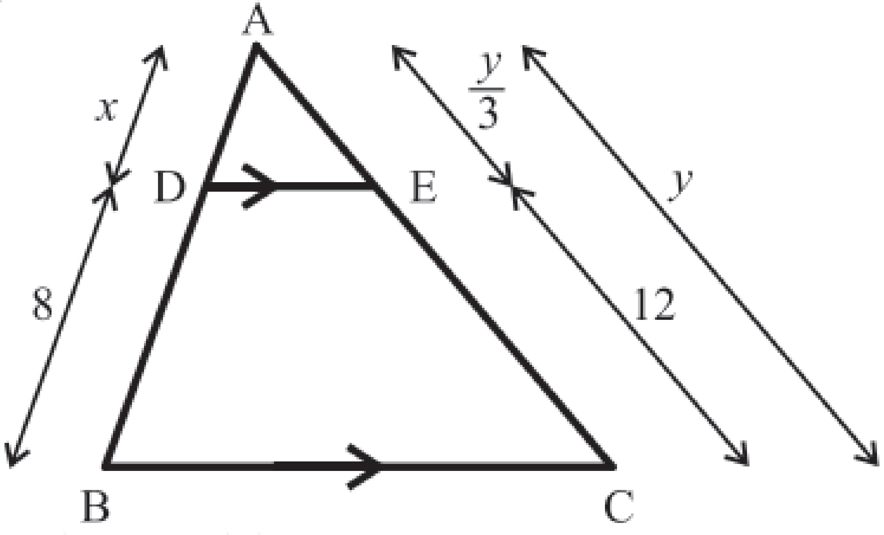
c)


A
10
D
x
E
y
3,5
5
B
F
C
8

b)


A
12
D
x
25
5
C
2
y
B
E

d)

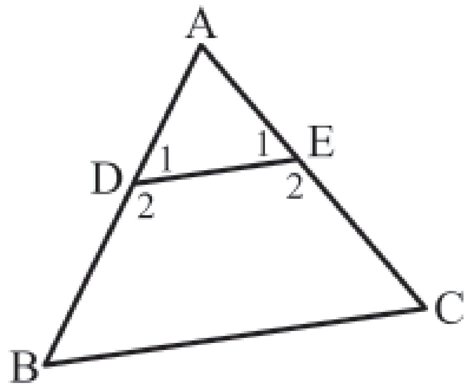

A
x
y/3
D
E
y
8
12
B
C

Solutions to Exercise 2 numbers 2a and b are on page **334** and numbers c and d are on page **335**

3) In the diagram below $\hat{D}_1 = \hat{B}$.

a) Prove that $\Delta ABC /// \Delta ADE$.

b) Hence find the length of DB if AD = 10mm, AE = 6mm and EC = 12mm.

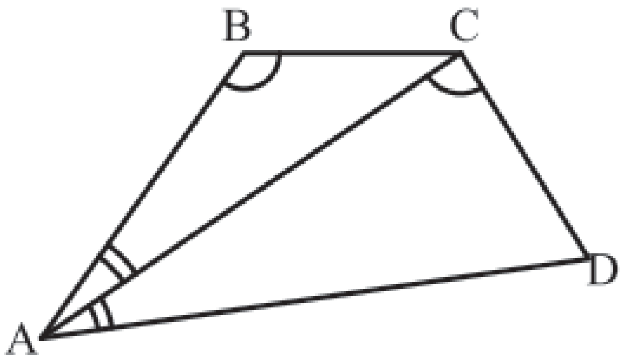


Solutions to Exercise 2 number 3 are on page **335**.

4) In the diagram below there are 2 similar triangles:

a) State, with reason, the triangles that are similar. Remember that the order the triangles are written in is important.

b) Given that AB = 10m and AC = 12m, determine the length of AD.

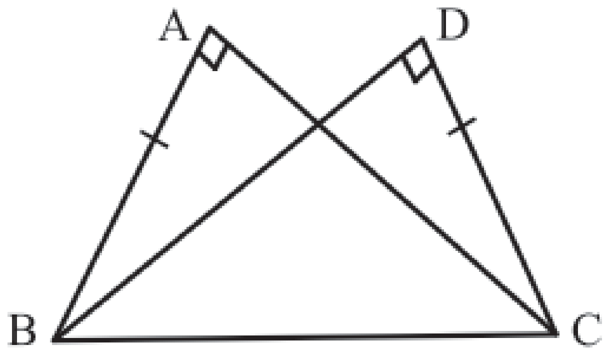


Solutions to Exercise 2 number 4 are on page **335**

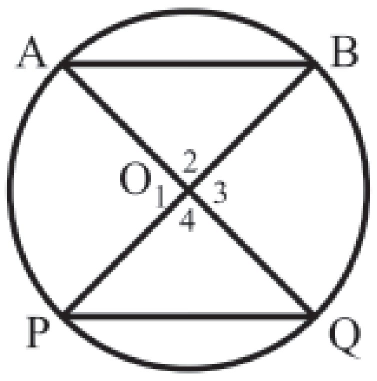
Congruency

Exercise 3: (Theory on pages 144-146)

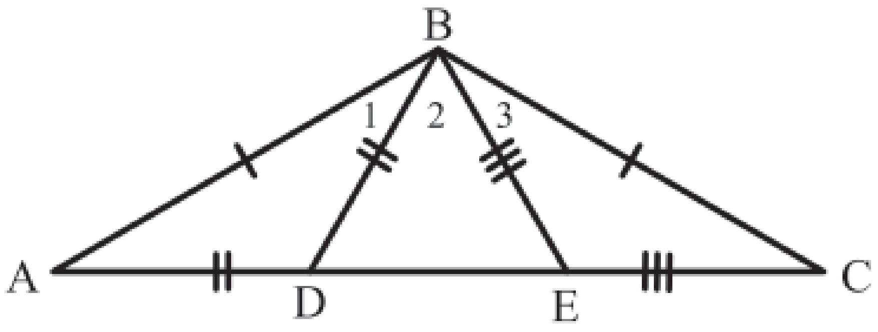
1) Prove that $\Delta ABC \equiv \Delta DCB$:



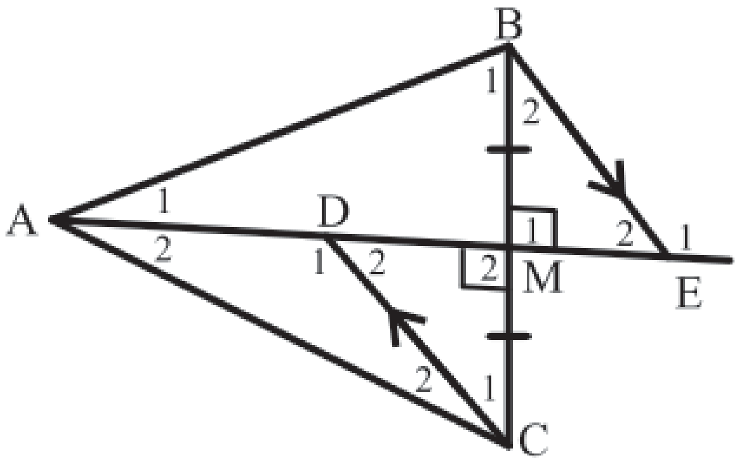
2) Given O is the circle's centre, prove that $\Delta ABO \equiv \Delta PQO$:



3) Prove that $\Delta ABD \equiv \Delta CBE$:



4) Prove that $\Delta MBE \equiv \Delta MCD$:

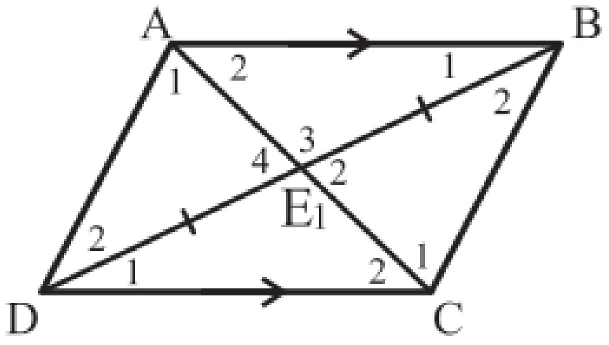


Solutions to Exercise 3 numbers 1 and 2 are on page **335** and number 3 and 4 are on page **336**

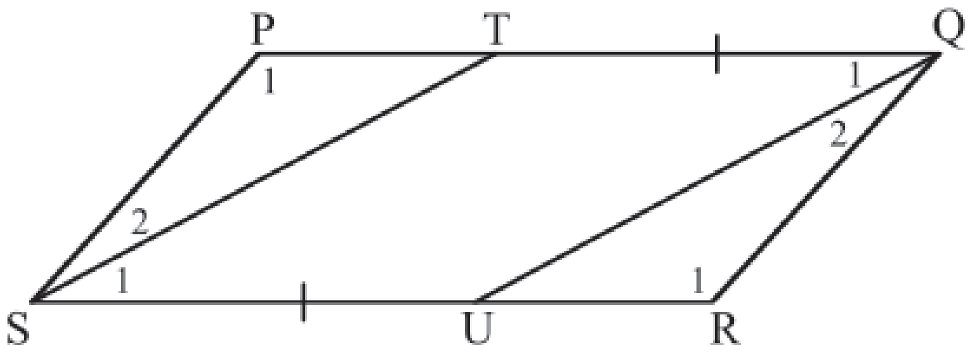
Quadrilaterals

Exercise 4: (Theory on pages 147-150)

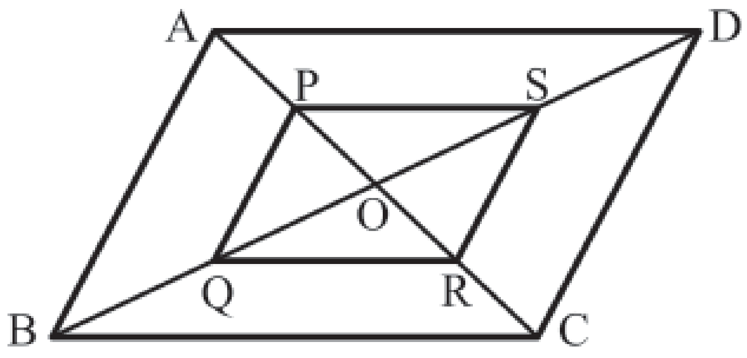
1) Prove that quadrilateral ABCD below is a parallelogram:



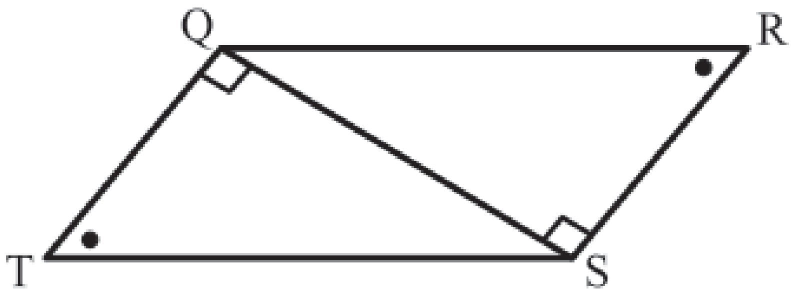
2) Prove that quadrilateral TQUS below is a parallelogram given that PQRS is a parallelogram and that TQ = SU



3) ABCD below is a parallelogram. PQRS is a quadrilateral formed by joining the midpoints of AO, BO, CO and DO respectively. Prove that PQRS is a parallelogram.

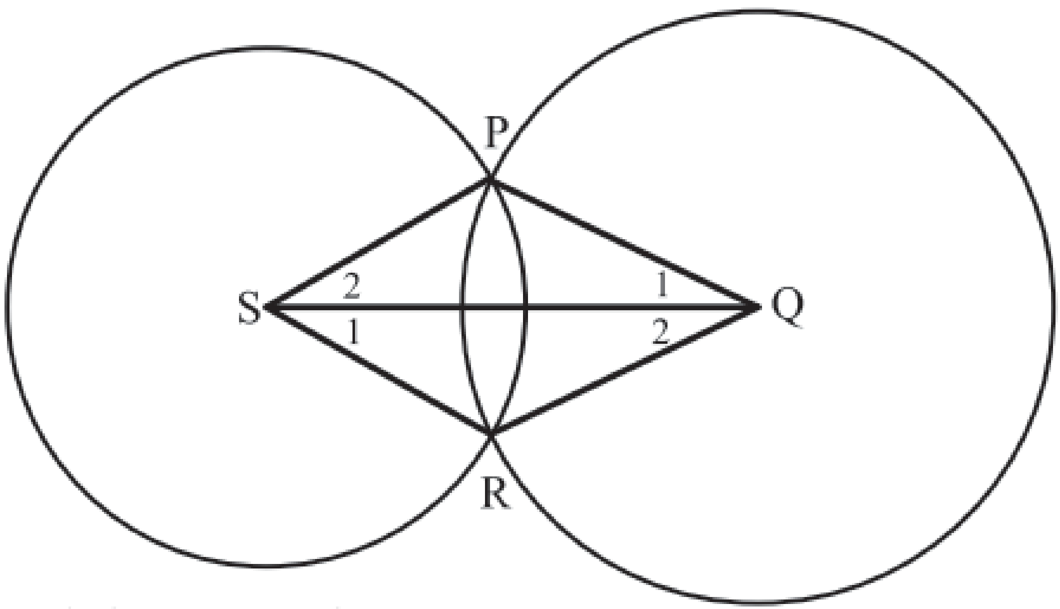


4) Prove that quadrilateral QRST below is a parallelogram given that $\hat{T} = \hat{R}$:

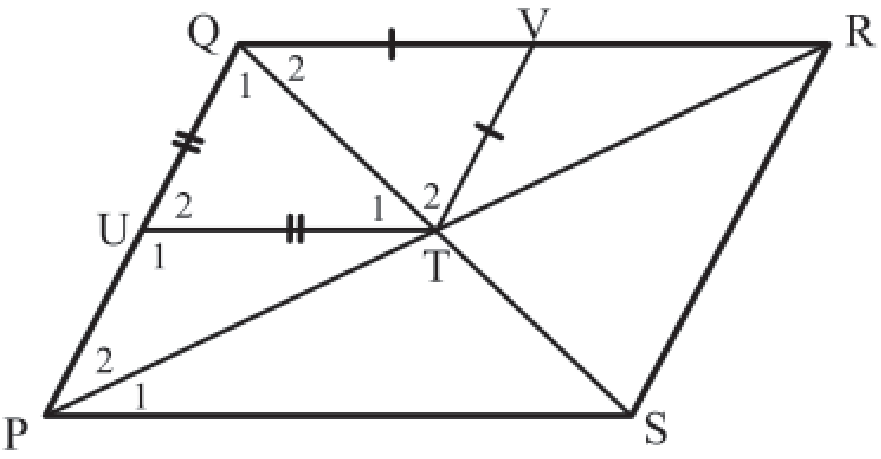


Solutions to Exercise 4 are on page **336**

5) **Given that S and Q are the centres of their respective circles, prove that PQRS is a kite.**



6) **PQRS is a rhombus with its diagonals intersecting at T. Prove that TUQV is a rhombus given that TV = QV and TU = QU.**

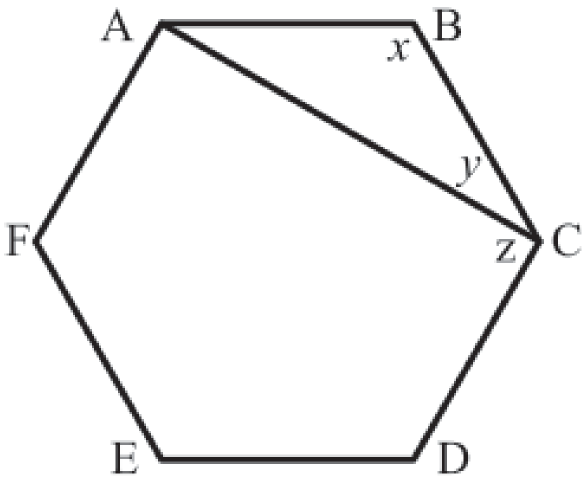


Solutions to Exercise 4 are on page **336**

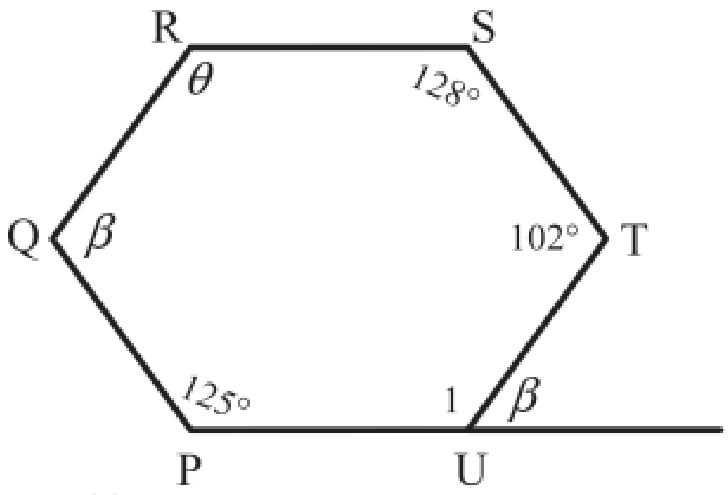
Angles in a Polygon

Exercise 5: (Theory on pages 151 and 152)

1) **Quadrilateral ABCDEF is a regular hexagon. Calculate the values of x, y and z.**



2) **Determine the value of θ in the quadrilateral below:**

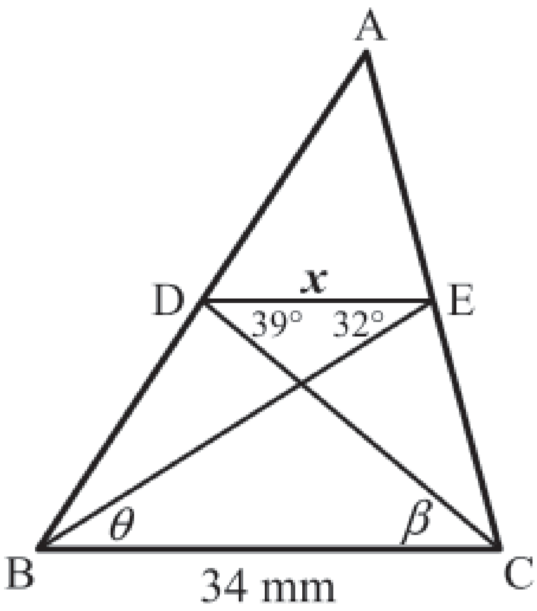


Solutions to Exercise 5 number 1 is on page **336** and number 2 is on page **337**

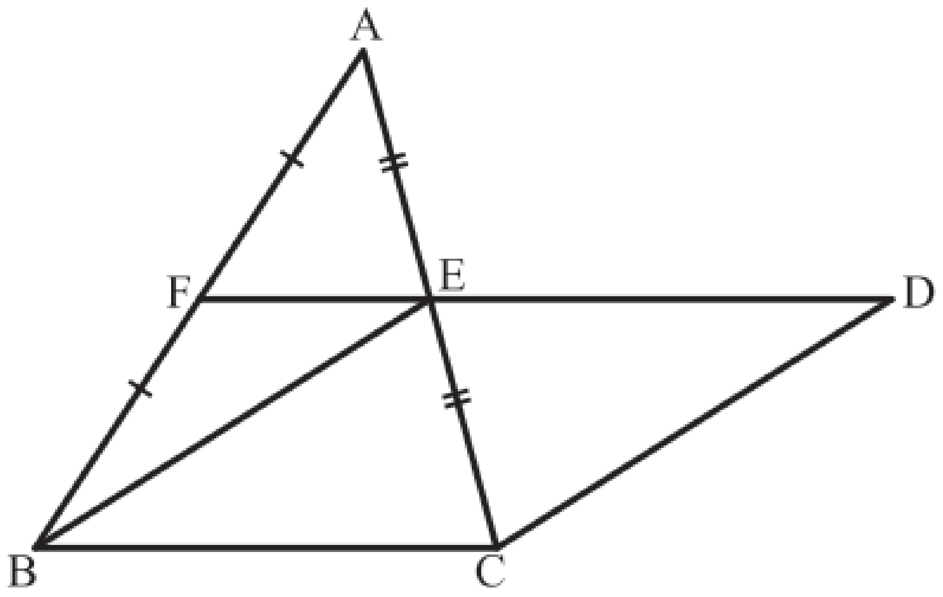
Midpoint Theorem

Exercise 6: (Theory on pages 153-154)

1) **Determine the values of x, θ and β in ΔABC below, given that D and E are the midpoints of AB and AC respectively.**

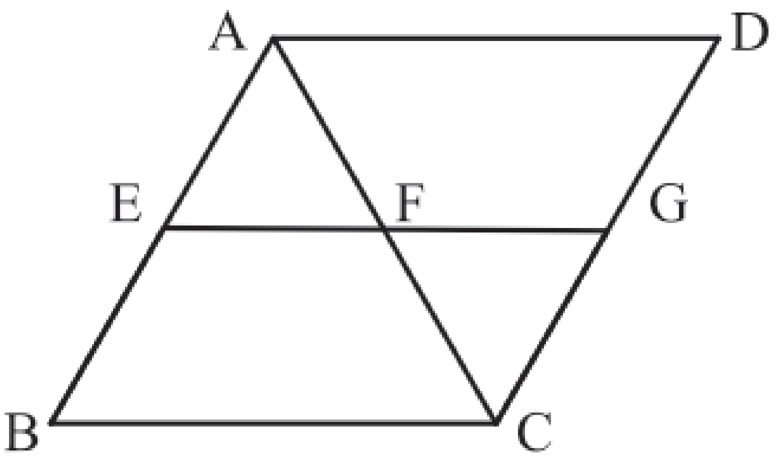


2) **In ΔABC below, F and E are the midpoints of AB and AC respectively and ED = 2FE. Prove that BCDE is a parallelogram.**

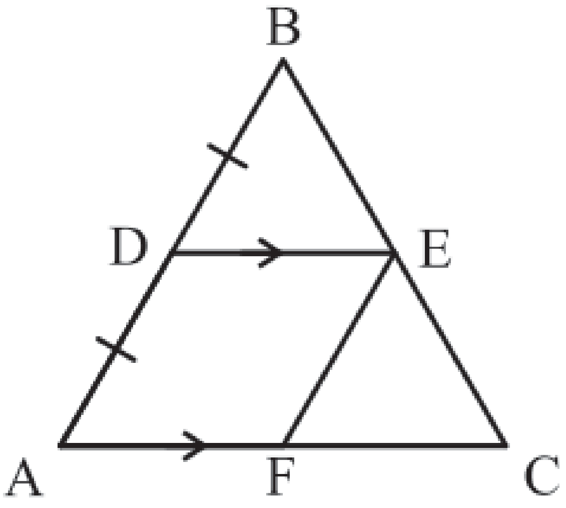


3) **In the diagram below AE = EB, EF = FG and AF = FC.**

Prove that BEGC is parallelogram.

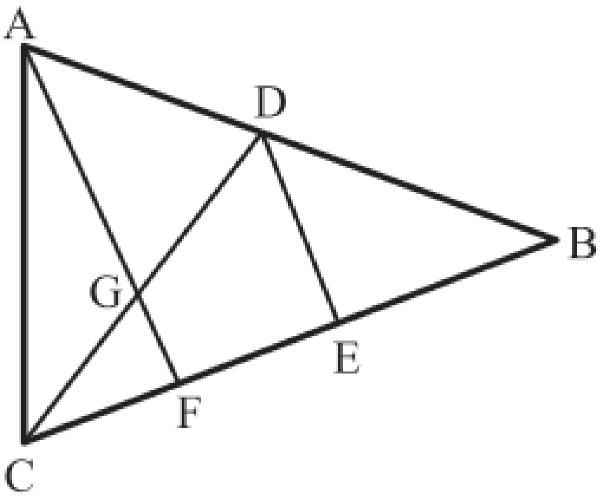


4) **In the diagram below AD = DB and AF//DE. If EF bisects AC prove that ADEF is a parm.**

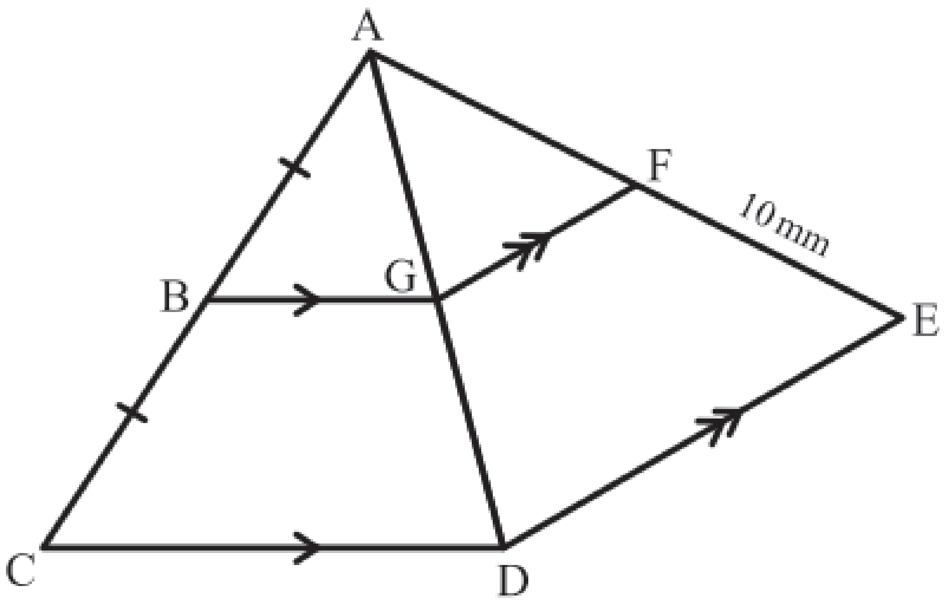


Solutions to Exercise 6 are on page **337**

5) Determine the length of GF if:
AF = 10cm
AD = DB
CF = FE = EB



6) In ΔACD below, $AB = BC$ and $BG // CD$. Determine the length of AF if GF // DE.



Solutions to Exercise 6 are on page **337**

Distance, Midpoint and Average Gradient Formula

Exercise 7: (Theory on pages 155- 162)

1) Calculate the lengths of the lines between the following points, leave your answer in simplest surd form where necessary:

a) $A(5;3)$ and $B(-2;6)$

c) $P(3;5)$ and $Q(-2;-7)$

b) $A(-2;-1)$ and $B(3;2)$

d) $S(8;0)$ and $R(2;-3)$

Solutions to Exercise 7 number 1 are on page **337**

2) Determine the midpoint of line AB if A and B are the following points:

a) $A(6;1)$ and $B(-2;5)$

c) $A(-3;-2)$ and $B(-3;2)$

b) $A(5;3)$ and $B(3;1)$

d) $A(2;3)$ and $B(-2;5)$

Solutions to Exercise 7 number 2 are on page **338**

3) Determine the coordinates of B if M is the midpoint of line AB and M and A are the following points:

a) $A(5;-1)$ and $M(4;-3)$

c) $A(-3;5)$ and $M(1;1)$

b) $A(-9;4)$ and $M(-6;1)$

d) $A(10;20)$ and $M(-1;5)$

Solutions to Exercise 7 number 3 are on page **338**

4) Determine the average gradient of line segment AB if A and B are the points:

a) $A(4;5)$ and $B(2;3)$ **b)** $A(8;-2)$ and $B(2;3)$ **c)** $A(-8;-5)$ and $B(-2;-2)$

Solutions to Exercise 7 number 4 are on page **338**

5) Determine if AB and CD are parallel, perpendicular or neither given the points:

a) $A(-8;-4)$ and $B(2;4)$ and $C(-4;0)$ and $D(-20;20)$

b) $A(5;6)$ and $B(4;2)$ and $C(8;24)$ and $D(4;8)$

c) $A(5;2)$ and $B(2;-3)$ and $C(10;9)$ and $D(-5;18)$

d) $A(2;-5)$ and $B(-1;-8)$ and $C(-1;-6)$ and $D(1;-10)$

Solutions to Exercise 7 number 5 are on page **338**

6) Prove that the following points are collinear:

a) $A(-6;-10), B(-2;-8), C(0;-7)$

b) $A(0;10), B(9;4), C(6;6)$

Solutions to Exercise 7 number 6 are on page **338**

7) Given the points $P(x;5), Q(-3;2), R(2;5), S(-1;4)$ determine the value of x if:

a) PQ//RS

b) P, Q and S are collinear

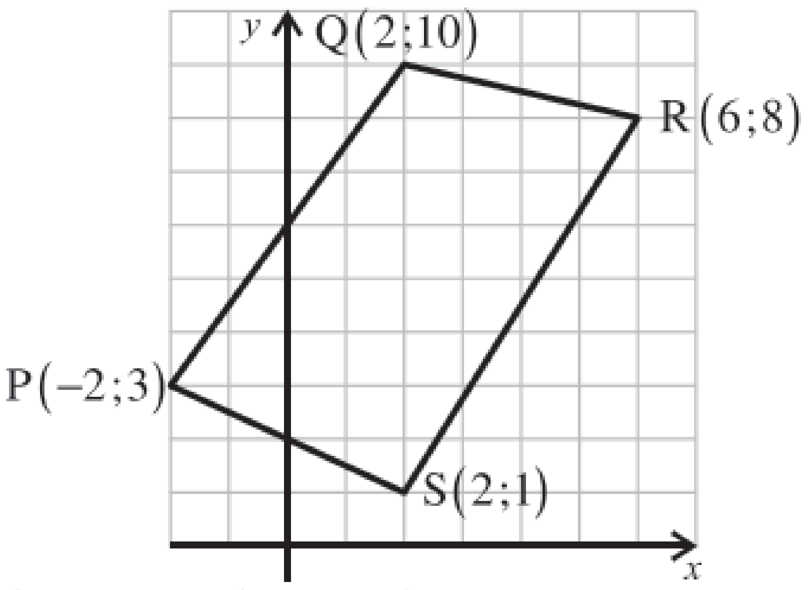
c) PQ⊥RS

Solutions to Exercise 7 number 7 are on page **339**

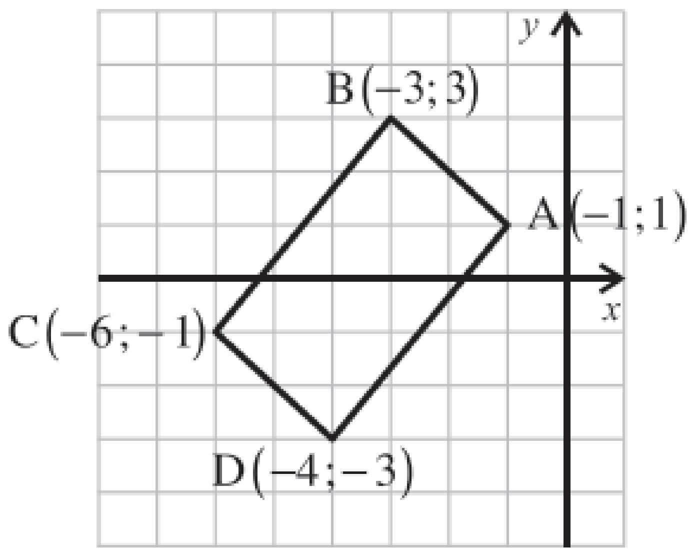
Exercise 8: (Theory on pages 163 - 164)

1) Prove that the following quadrilaterals are parallelograms using analytical methods.

a)


y
Q(2;10)
R(6;8)
P(−2;3)
S(2;1)
x

b)

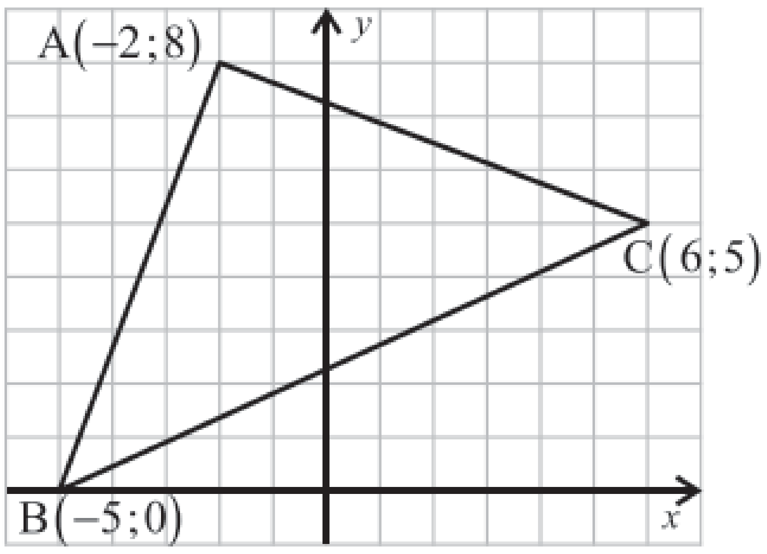

y
B(−3;3)
A(−1;1)
C(−6;−1)
x
D(−4;−3)

Solutions to Exercise 8 number 1 are on page **339**

2) Show that triangle ABC below is right angled at $\hat{A}$ using:

a) The theorem of Pythagoras.

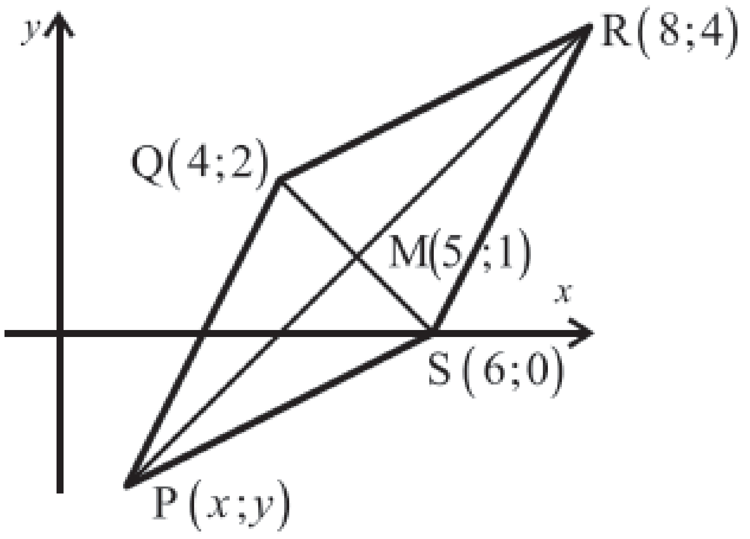
b) The properties of perpendicular lines.


A(−2;8)
y
C(6;5)
B(−5;0)
x

3) Parallelogram PQRS with vertices $P(x;y), Q(4;2), R(8;4), S(6;0)$ is drawn on the Cartesian plane below. Diagonals intersect at point $M(5;1)$.

a) Determine the coordinates of point P.

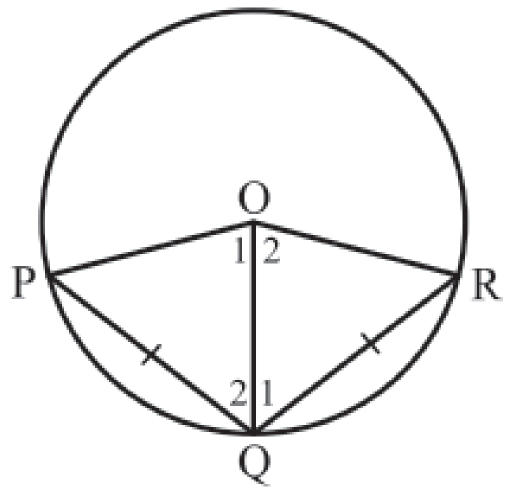
b) Prove that PQRS is a rhombus.


y
R(8;4)
Q(4;2)
M(5;1)
x
S(6;0)
P(x;y)

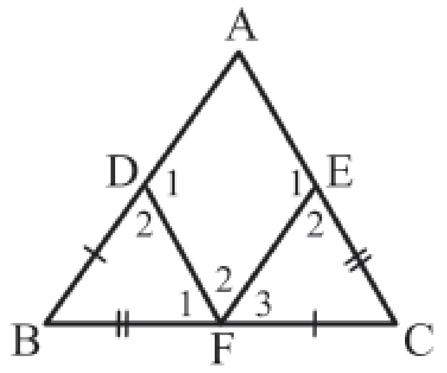
Solutions to Exercise 8 number 2 and 3 are on page **340**

Mixed Exercise: (chapter 7 is on pages 140 - 165)

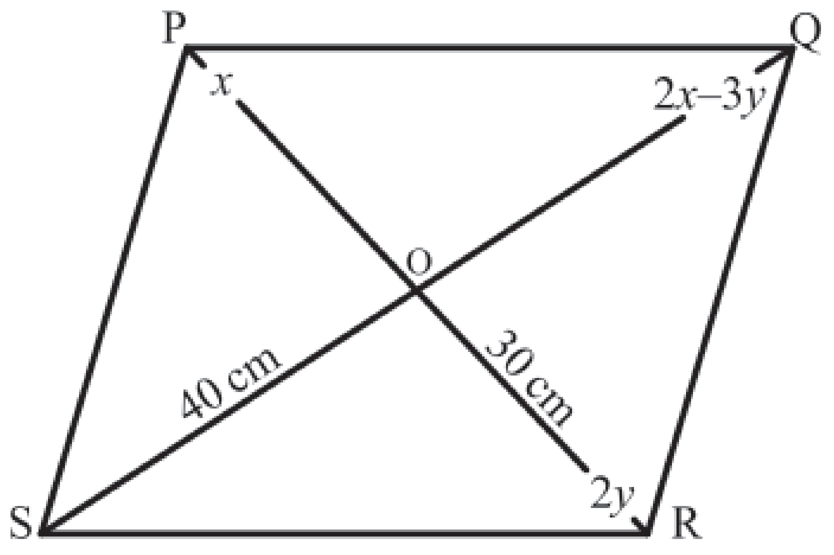
1) The circle below has centre O and PQ = QR. Prove that $\Delta PQO \equiv \Delta RQO$.



2) In ΔABC, AB = AC, DB = FC and BF = EC. Prove that DF = FE.


A
D
1
2
E
1
2
2
1
3
B
F
C

3) The diagram below shows quadrilateral PQRS. If PQRS is a rhombus:

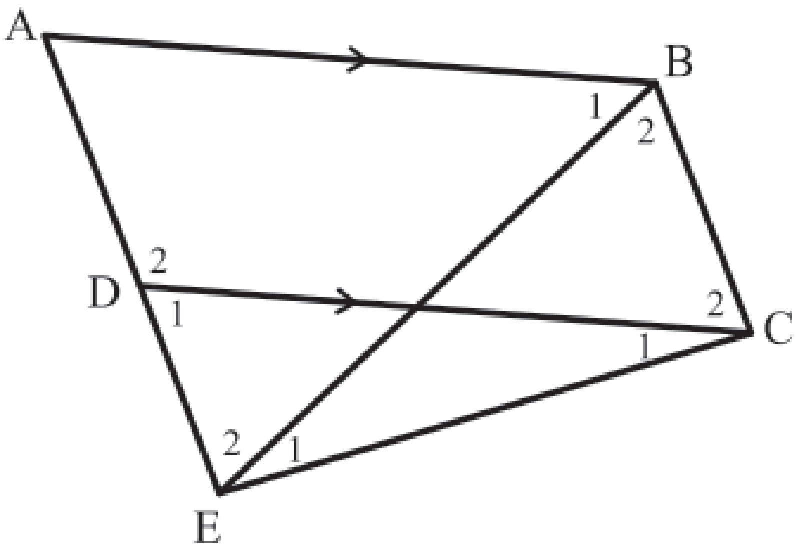

P
Q
x
2x–3y
O
40 cm
30 cm
2y
S
R

a) Determine the values of x and y stating all reasons where necessary.

b) Determine the length of PS stating all reasons where necessary.

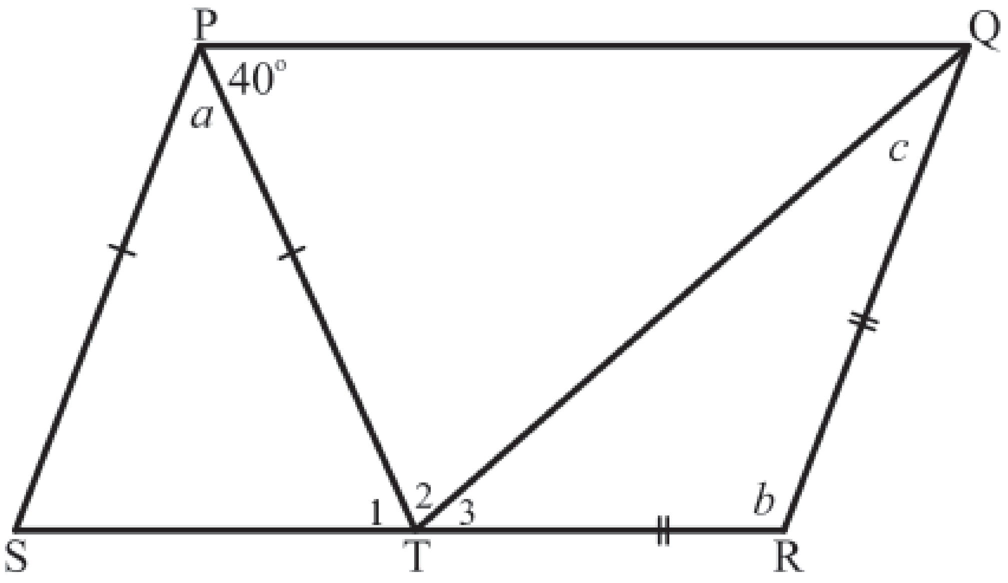
4) Prove that $\Delta BAE \equiv \Delta EDC$ given that: $AB \parallel DC$ and $\hat{B}_1 = \hat{E}_1 + \hat{E}_2$ and $BE = EC$.

(Note that the diagram is not drawn to scale.)

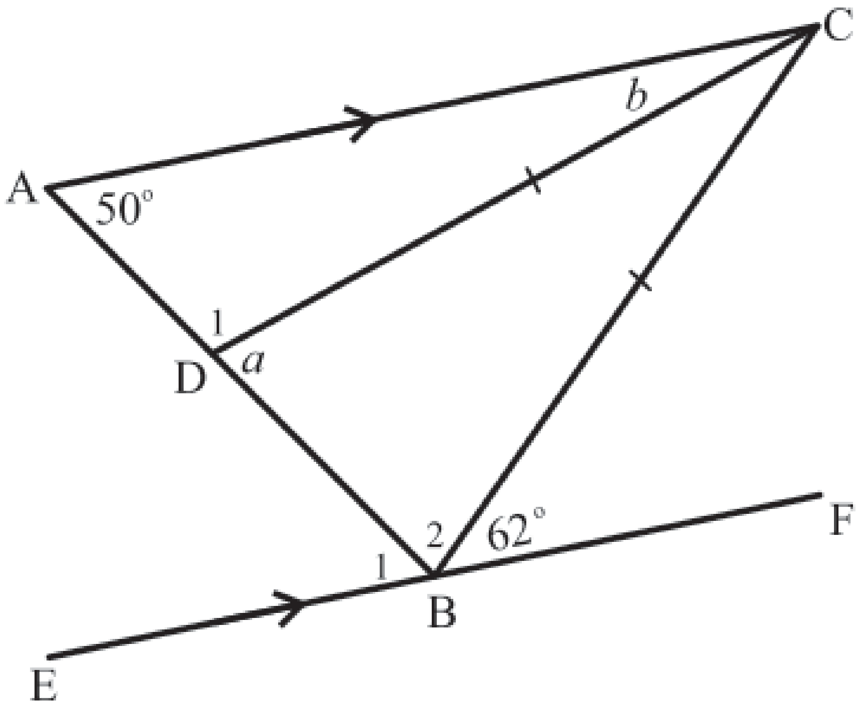

A
B
1
2
2
D
1
2
C
1
2
1
E

Solutions to the Mixed Exercise numbers 1 and 2 are on page **340** and numbers 3 and 4 are on page **341**

5) The diagram below shows parallelogram PQRS. If PS = PT and TR = RQ determine the values of *a*, *b* and *c*.

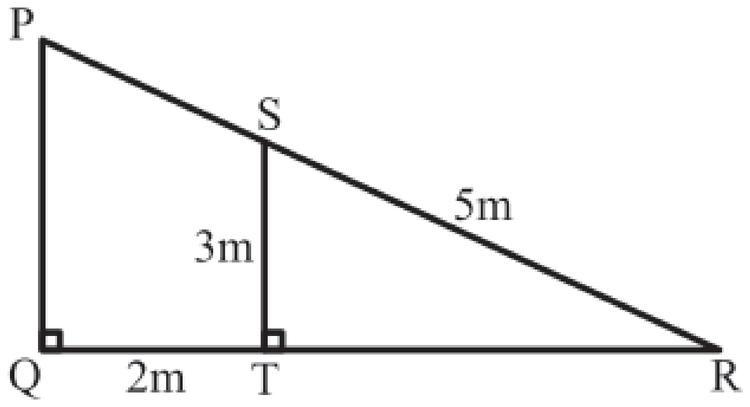


6) In the diagram below AC//EF and CD = CB. Vertex B lies on the line EF as shown. Calculate the values of *a* and *b* stating all reasons where necessary.



Solutions to the Mixed Exercise numbers 5 and 6 are on page **341**

7) If ΔPQR /// ΔSTR, determine the lengths of TR, PQ and PS.



Solutions to the Mixed Exercise number 7 are on page **341**

8) Triangle PQR has vertices at $\mathrm{P}(0;0)$, $\mathrm{Q}(2;5)$ and $\mathrm{R}(4;0)$:

a) Calculate the lengths of PQ, QR and PR, leaving your answers in surd form where necessary.

b) Determine what type of triangle PQR is and give a reason for your answer.

Solutions to the Mixed Exercise number 8 are on page **342**

9) Determine the equations of the following lines:

a) The line passes through the point $\mathrm{P}(0;7)$ and parallel to the line $y=-2x+3$.

b) The line parallel to the *x*-axis passing through the point $\mathrm{P}(-2;-5)$.

c) The line parallel to the *y*-axis passing through the point $\mathrm{P}(6;2)$.

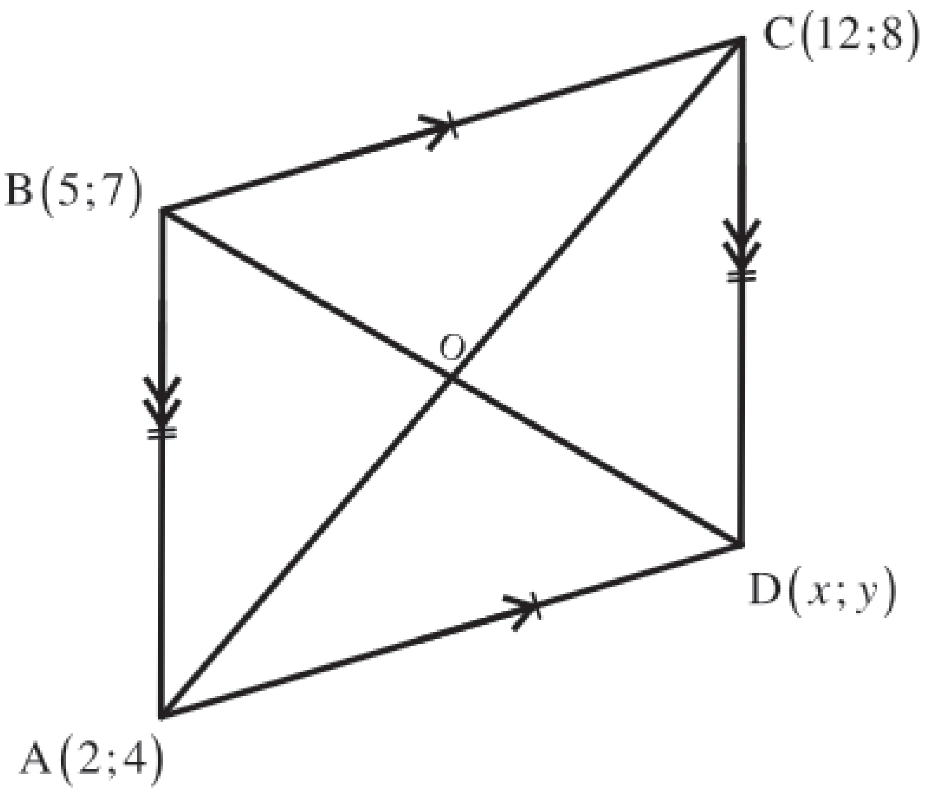
d) The line perpendicular to the line $y=-\frac{1}{2}x-3$ and passing through the point $\mathrm{P}(2;3)$.

Solutions to the Mixed Exercise number 9 are on page **342**

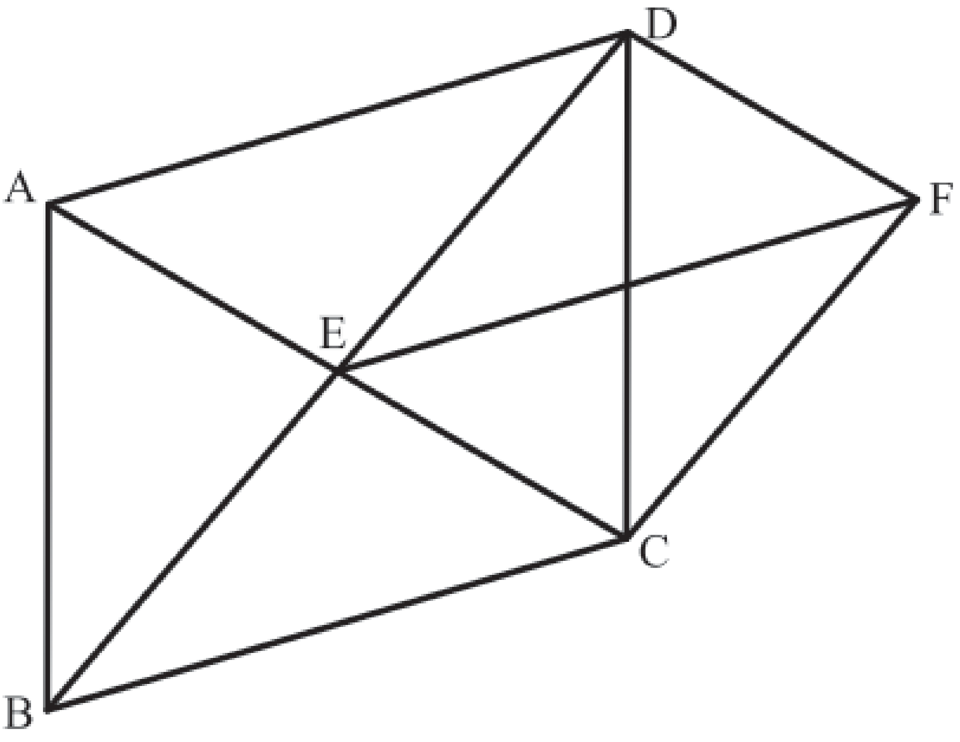
10) Parallelogram ABCD is shown below:

a) Determine the coordinates of D.

b) Prove that points D, O and B are collinear.

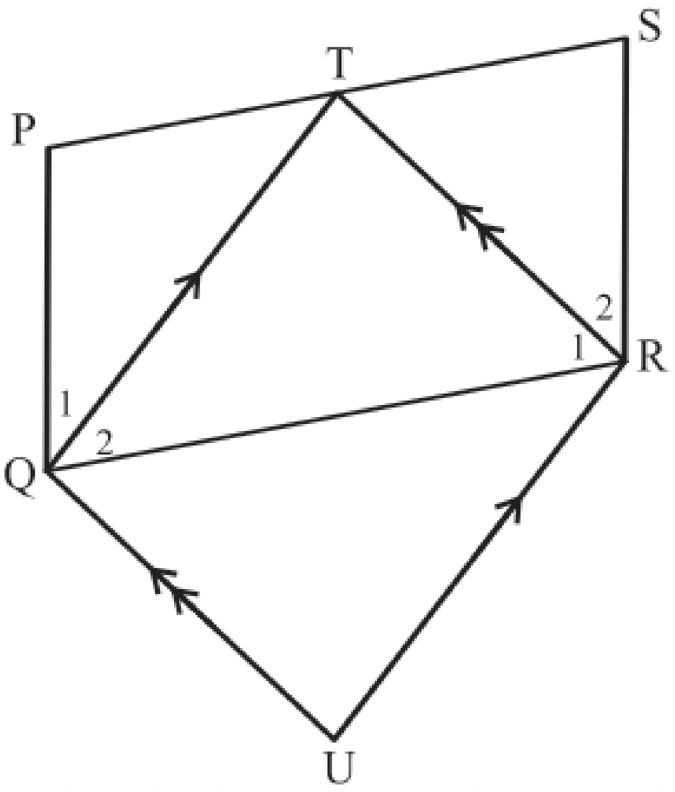


11) ABCD and EBCF below are parallelograms. Prove that EDFC is also a parallelogram.

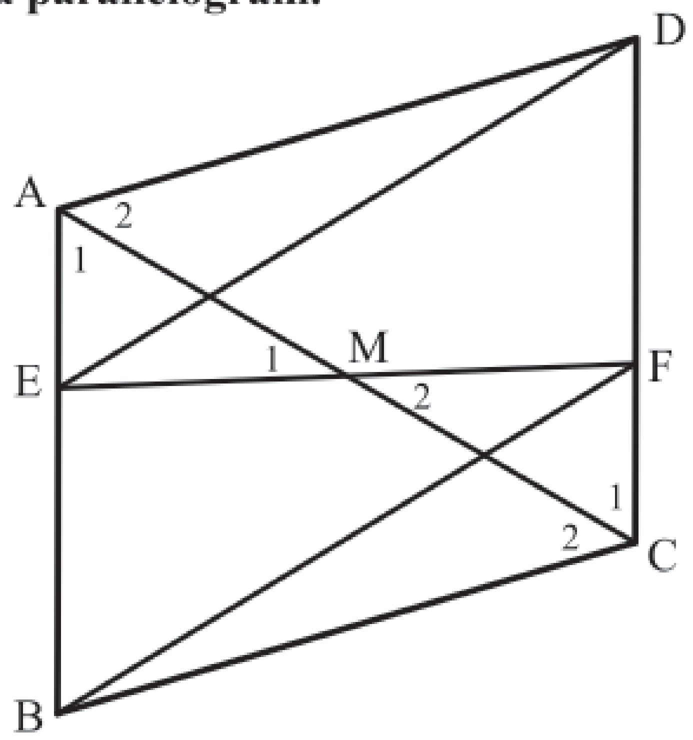


Solutions to the Mixed Exercise numbers 10 and 11 are on page **342**

12) In parallelogram PQRS $Q_1 = Q_2$ and $R_1 = R_2$. Prove that QTRU is a rectangle.

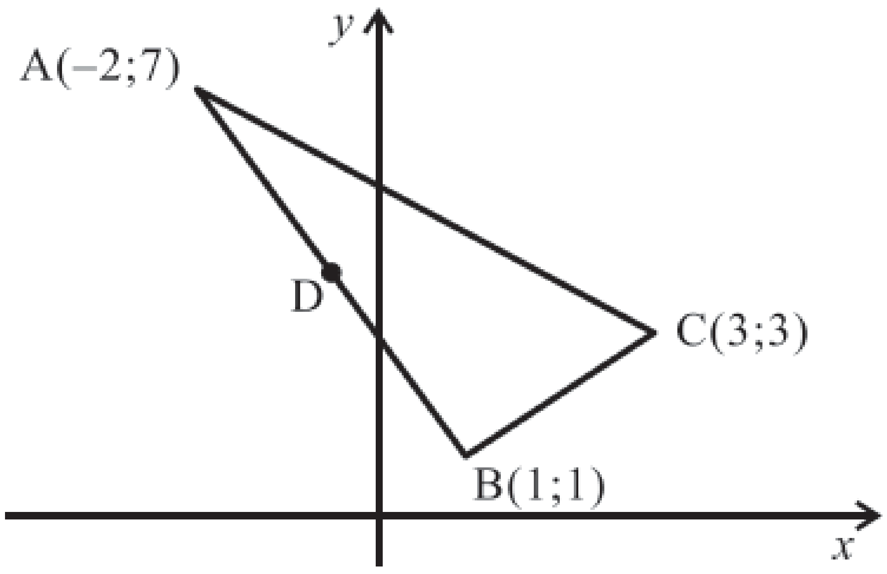


13) In parallelogram ABCD M is the midpoint of diagonal AC. Prove that quadrilateral DEBF is a parallelogram.



Solutions to the Mixed Exercise numbers 12 and 13 are on page **342**

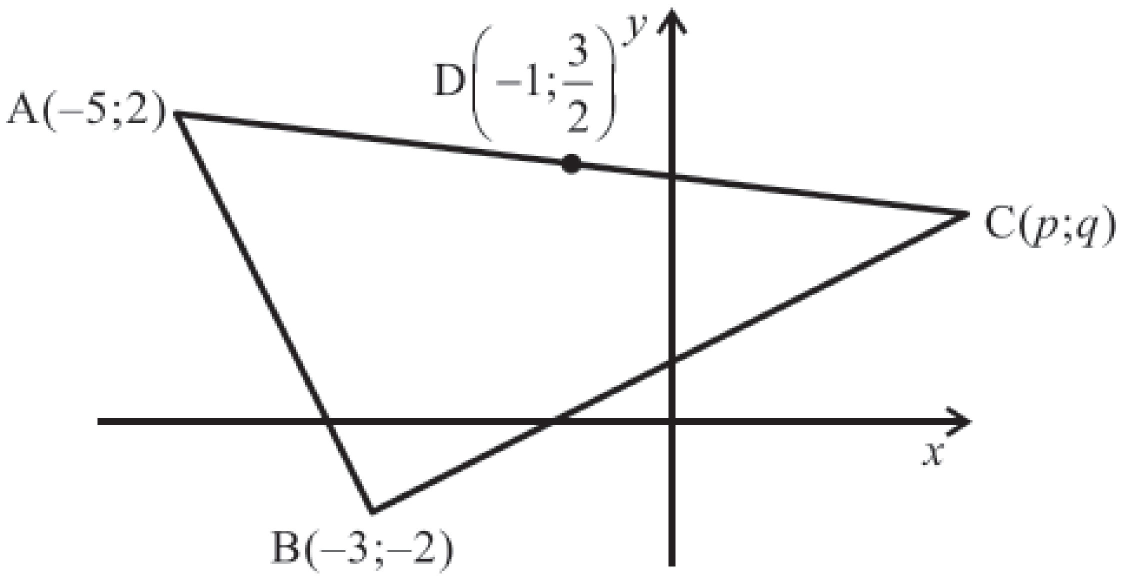
14) In the Diagram below D is the midpoint of AB.



a) Determine the coordinates of D, the midpoint of AB.
b) Determine the equation of AB.
c) Calculate the gradient of BC and AC.
d) Hence, state with reason whether or not triangle ABC is a right-angled triangle.
e) Determine the equation of the perpendicular bisector of AB.
Hint: A perpendicular bisector cuts a line in half and is perpendicular to that line.

Solutions to the Mixed Exercise number 14 are on page **343**

15) In the diagram below D is the midpoint of AC.



a) Determine the coordinates of C.
b) Determine the length of AC. Leave your answer in simplest surd form.
c) Determine the gradient of AB and BC.
d) Hence, state with reason whether or not triangle ABC is a right-angled triangle.
e) Determine the equation of the perpendicular bisector of AC.
Hint: A perpendicular bisector cuts a line in half and is perpendicular to that line.

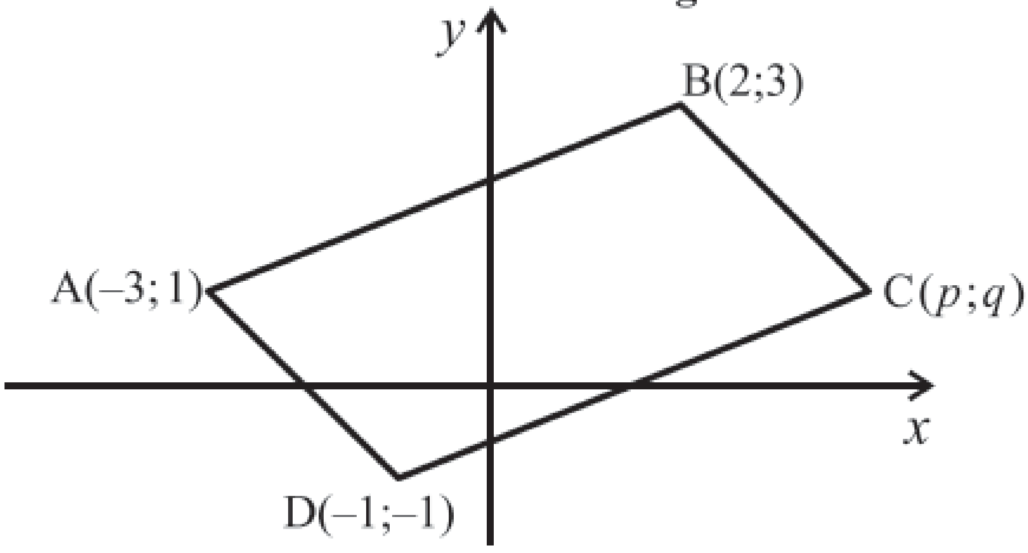
Solutions to the Mixed Exercise number 15 are on page **343**

16) In the diagram below ABCD is a parallelogram with vertices as shown on the diagram.

a) Determine the length of DC. Leave your answer in simplest surd form.

b) Determine the coordinates of point C.

c) E is the point $(3;-4)$. F is the point $(x;8)$ and lies in the second quadrant. Determine the value of x if EF is $6\sqrt{5}$ units in length.



Solutions to the Mixed Exercise number 16 are on page **344**

17) Use the diagram alongside to answer the questions below.

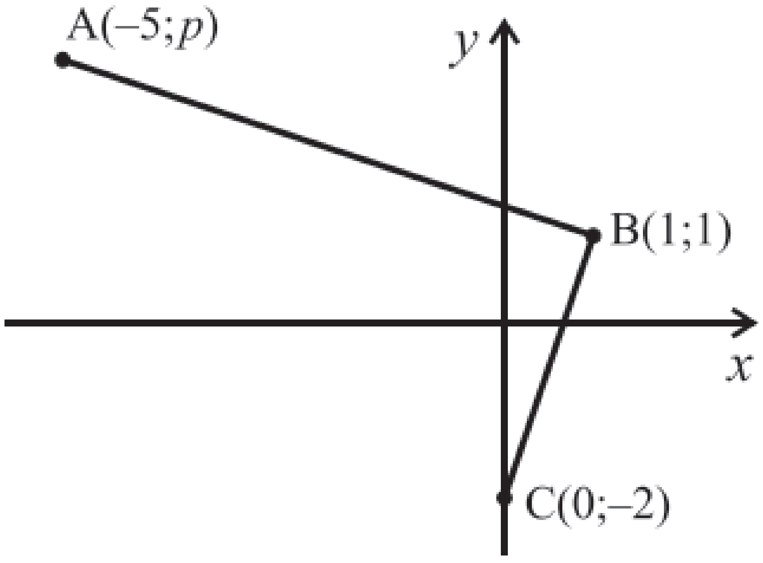
a) Calculate the length of BC.

b) Determine the gradient of BC.

c) Hence, determine the value of p if AB⊥BC.

d) If $p = 3$, determine the coordinates of M the midpoint of AC.

e) Hence, determine the coordinates of D so that quadrilateral ABCD is a parallelogram.



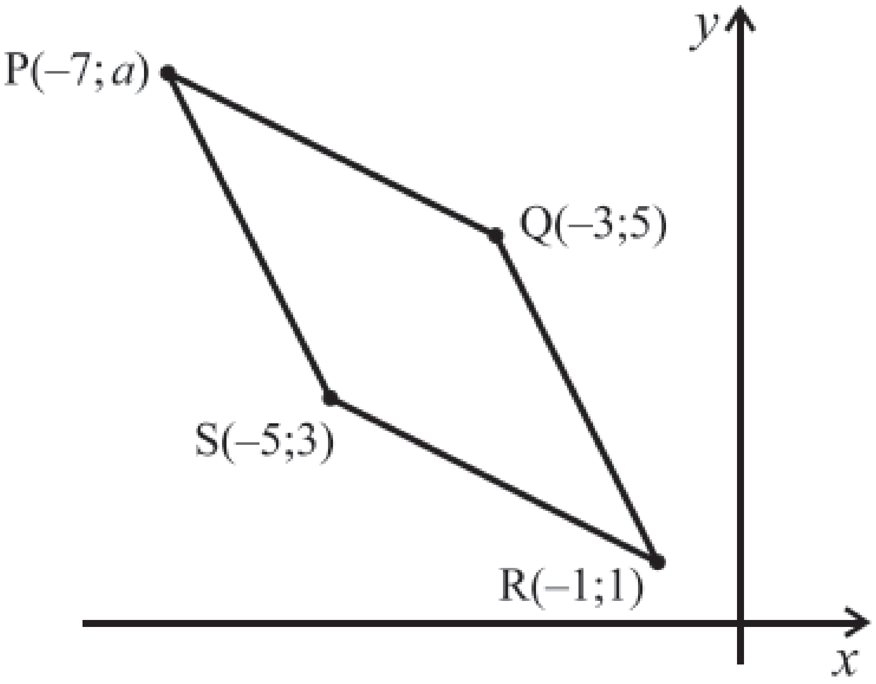
Solutions to the Mixed Exercise number 17 are on page **344**

18) In the diagram below P, Q, R and S are vertices of a rhombus.

a) Calculate the gradient of QS.

b) Hence determine the equation of PR.

c) Hence, determine the value a.



Solutions to the Mixed Exercise number 18 are on page **344**

19) The points A(1;2), B(–5; q), C(p ;7) and D(–3;5) lie in the Cartesian plane.

- **a)** Determine the value of q if A, B and D are collinear.
- **b)** Determine the values of p and q if A is the midpoint of BC.

Solutions to the Mixed Exercise number 19 are on page **345**

20) Given that the equation of a line is $2ky - 3y - 3x = 5$.
Determine the value of k if the above line is:

- **a)** If the above line is perpendicular to the line $y = -3x + 10$.
- **b)** If the above line is has a y-intercept of 5.
- **c)** The point $(1;1)$ lies on the above line.

Solutions to the Mixed Exercise number 20 are on page **345**

Chapter 7 – Geometry (Solutions)

Exercise 1 Solutions:

1)

a)

Statement	Reason
$x = 55^\circ$	Vert. opp. $\angle$'s
$y = 180^\circ - x$	Adj. $\angle$'s on a str. line
$\therefore y = 180^\circ - 55^\circ$	
$\therefore y = 125^\circ$	

b)

Statement	Reason
$x = 55^\circ$	Corr. $\angle$'s AB//CD
$y = x = 55^\circ$	Vert. opp. $\angle$'s

c)

Statement	Reason
$x = 104^\circ$	Vert. opp. $\angle$'s
$y = 180^\circ - 56^\circ - 104^\circ$	$\angle$'s in a Δ
$\therefore y = 20^\circ$	

d)

Statement	Reason
$y = 60^\circ$	Alt. $\angle$'s AB//CD
$\therefore x = 180^\circ - 60^\circ - 90^\circ$	$\angle$'s in a Δ
$\therefore x = 30^\circ$	
$\therefore z = 180^\circ - y - 90^\circ$	Adj. $\angle$'s on a str. line
$\therefore z = 180^\circ - 60^\circ - 90^\circ$	
$\therefore z = 30^\circ$	

e)

Statement	Reason
$\hat{B} = \hat{C} = 25^\circ$	$\angle$'s opp. = sides
$\therefore x = 180^\circ - 25^\circ - 25^\circ$	$\angle$'s in a Δ
$\therefore x = 130^\circ$	

f)

Statement	Reason
$\therefore x = y = 60^\circ$	Equilateral Δ
$\therefore z = x + y = 120^\circ$	Ext. $\angle$'s of a Δ

Exercise 1 number 1 is on page 326

2) Check using Pythagoras. The longest side of a right angled triangle is the hypotenuse.

a) $3^2 + 4^2 = 25$

$5^2 = 25$

$\therefore$ Triangle is right angled

b) $5^2 + 12^2 = 169$

$15^2 = 225$

$\therefore$ Triangle is not right angled

c) $8^2 + 15^2 = 289$

$17^2 = 289$

$\therefore$ Triangle is right angled

d) $7^2 + 22^2 = 533$

$25^2 = 625$

$\therefore$ Triangle is not right angled

Exercise 1 number 2 is on page 326

Exercise 2 Solutions:

1)

a)

Statement	Reason
$\hat{A} = 25^\circ$	$\angle$'s in a Δ
$\hat{D} = 65^\circ$	$\angle$'s in a Δ
ΔABC ///ΔCED	AAA

b)

Statement	Reason
$\frac{AB}{PR} = \frac{AC}{PQ} = \frac{BC}{QR} = \frac{1}{2}$	Given
ΔABC ///ΔPRQ	Sides in proportion

Exercise 2 number 1 is on page 326

2)

a)

Statement	Reason
$\frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC} = 3$	Similar Δ's
$\therefore \frac{y}{3} = 3$	
$\therefore y = 9$ units	
$\therefore x = \frac{15}{3} = 5$ units	

b)

Statement	Reason
$\frac{AB}{DE} = \frac{BC}{EC} = \frac{AC}{DC} = 5$	Similar Δ's
$\therefore \frac{y}{2} = 5$	
$\therefore y = 10$ units	
$\therefore \frac{x}{12} = \frac{1}{5}$	
$\therefore x = \frac{12}{5} = 2,4$ units	

Exercise 2 number 2 is on page 326

c)

Statement	Reason
$\frac{BC}{DE}=\frac{AB}{FE}=\frac{8}{10}$	Similar Δ's
$\therefore \frac{8}{x}=\frac{10}{5}$	
$\therefore x=4$ units	
$\frac{AC}{DF}=\frac{AB}{EF}=\frac{10}{5}=2$	Similar Δ's
$\therefore \frac{y}{3,5}=2$ units	
$\therefore y=7$ units	

d)

Statement	Reason
$\frac{y}{3}+12=y$	Similar Δ's
$\therefore y=18$	
$\frac{AB}{AD}=\frac{AC}{AE}=\frac{18}{6}=3$	Similar Δ's
$\frac{x+8}{x}=3$	
$x+8=3x$	
$\therefore x=4$ units	

Exercise 2 number 2 is on page 326

3)

a)

Statement	Reason
In ΔABC and ΔADE	
$\hat{A}=\hat{A}$	Common ∠
$\hat{B}=\hat{D}_1$	Given
$\hat{C}=\hat{E}_1$	3rd angle of a Δ
$\therefore \Delta ABC /// \Delta ADE$	AAA
$\therefore \frac{AB}{AD}=\frac{BC}{DE}=\frac{AC}{AE}$	Similar Δ's

b)

Statement	Reason
$\therefore \frac{AB}{AD}=\frac{AC}{AE}$ $\therefore \frac{AB}{10}=\frac{18}{6}$ $\therefore AB=30$ $DB=AB-AD$ $=30-10$ $=20\,\text{mm}$	Similar Δ's

Exercise 2 number 3 is on page 327

4)

a)

Statement	Reason
In ΔABC and ΔACD	
$\hat{A}=\hat{A}$	Given
$\hat{B}=\hat{C}$	Given
$\hat{C}=\hat{D}$	3rd angle of a Δ
$\therefore \Delta ABC /// \Delta ACD$	AAA
$\therefore \frac{AB}{AC}=\frac{BC}{CD}=\frac{AC}{AD}$	Similar Δ's

b)

Statement	Reason
$\frac{AB}{AC}=\frac{AC}{AD}$ $\therefore \frac{10}{12}=\frac{12}{AD}$ $\therefore \frac{12}{10}=\frac{AD}{12}$ $\therefore AD=\frac{12}{10}\times 12$ $=14,4\,\text{m}$	Similar Δ's

Exercise 2 number 4 is on page 327

Exercise 3 Solutions:

1)

Statement	Reason
In ΔABC and ΔDCB	
$AB=DC$	Given
$\hat{A}=\hat{D}=90^\circ$	Given
$BC=BC$	Common side
$\therefore \Delta ABC \equiv \Delta DCB$	RHS

2)

Statement	Reason
In ΔABO and ΔPQO	
$AO=PO$	Radii
$BO=QO$	Radii
$\hat{O}_2=\hat{O}_4$	Vert. opp. ∠'s
$\therefore \Delta ABO \equiv \Delta PQO$	SAS

Exercise 3 is on page 327. Solutions are continued on the next page.

3)

Statement	Reason
In ΔABD and ΔCBE	
$AB = BC$	Given
$\hat{A} = \hat{C}$	$\angle$'s opp. = sides
$AD = DB$	Given
$\hat{A} = \hat{B}_1$	$\angle$'s opp. = sides
$BE = EC$	Given
$\hat{B}_3 = \hat{C}$	$\angle$'s opp. = sides
$\therefore \hat{A} = \hat{B}_1 = \hat{B}_3 = \hat{C}$	
$\therefore \Delta ABD \equiv \Delta CBE$	AAS

4)

Statement	Reason
In ΔMBE and ΔMCD	
$\hat{M}_1 = \hat{M}_2 = 90^\circ$	Given
$BM = MC$	Given
$\hat{B}_2 = \hat{C}_1$	Alt. $\angle$'s, DC//BE
$\therefore \Delta MBE \equiv \Delta MCD$	ASA

Exercise 3 is on page 327

Exercise 4 Solutions:

1)

Statement	Reason
In ΔABE and ΔCDE	
$\hat{E}_1 = \hat{E}_3$	Vert. opp. $\angle$'s
$\hat{B}_1 = \hat{D}_1$	Alt. $\angle$'s, AB//DC
$DE = BE$	Given
$\therefore \Delta ABE \equiv \Delta CDE$	ASA
$\therefore AB = CD$	Congruent Δ's
$\therefore$ ABCD is a parm.	One pair of opp. sides equal and parallel

2)

Statement	Reason
TQ // SU	Opp. sides of a parm. //
$TQ = SU$	Given
$\therefore$ TQUS is a parm.	One pair of opp. sides equal and parallel

3)

Statement	Reason
$AO = OC$	Diags of a parm.
$BO = DO$	Diags of a parm.
$PO = RO$	Given
$QO = SO$	Given
$\therefore PO = QR$ and $QO = OS$	
$\therefore$ PQRS is a parm.	Daigs of a parm. Bisect each other

4)

Statement	Reason
In ΔQRS and ΔSTQ	
$\hat{Q} = \hat{S} = 90^\circ$	Given
$\hat{R} = \hat{T}$	Given
$QS = QS$	Common side
$\therefore \Delta QRS \equiv \Delta STQ$	AAS
$\therefore RS = TQ$ and $QR = ST$	
$\therefore$ QRST is a parm.	two pairs of opp. sides equal

5)

Statement	Reason
$SP = SR$	Radii
$PQ = RQ$	Radii
$\therefore$ PQRS is a kite.	2 pairs of adj. sides =

6)

Statement	Reason
$\hat{Q}_1 = \hat{Q}_2$	Diags of a rhombus.
$\hat{Q}_2 = \hat{T}_2$ and $\hat{Q}_1 = \hat{T}_1$	$\angle$'s opp. = sides
$\therefore \hat{Q}_2 = \hat{T}_1$ and $\hat{Q}_1 = \hat{T}_2$	
$\therefore$ QV//UT and QU//VT	Alt. $\angle$'s, DC//BE
$\therefore$ QUTV is a parm.	2 pairs of opp. sides //
$\therefore$ QUTV is a rhombus	Parm. With adj. sides =

Exercise 4 numbers 1 – 4 are on page 327 and numbers 5 and 6 are on page 328

Exercise 5 Solutions:

1)

Statement	Reason
sum of $\angle$'s $= 180^\circ(4) = 720^\circ$	Sum of $\angle$'s in a polygon
$\therefore x = \frac{720^\circ}{6} = 120^\circ$	ABCDEF is a regular hexagon
$\therefore y = \frac{60^\circ}{2} = 30^\circ$	$\angle$'s opp. = sides
$\therefore z = 120^\circ - 30^\circ$ $= 90^\circ$	ABCDEF is a regular hexagon

Exercise 5 number 1 is on page 328

2)

Statement	Reason
sum of $\angle$'s $= 180^\circ(4) = 720^\circ$	Sum of $\angle$'s in a polygon
$\therefore \hat{U}_1 = 180^\circ - \beta$	Adj. $\angle$'s on a str. line
$\therefore \theta + \beta + 125^\circ + 180^\circ - \beta + 102^\circ + 128^\circ = 720^\circ$	$\angle$'s opp. = sides
$\therefore \theta + 535^\circ = 720^\circ$	
$\therefore \theta = 185^\circ$	

Exercise 5 number 2 is on page **328**

Exercise 6 Solutions:

1)

Statement	Reason
AD = DB and AE = EC	Given
∴ DE//BC and BC = 2DE	Mipt. Thm.
$\therefore \theta = 32^\circ$ and $\beta = 39^\circ$	Alt. $\angle$'s, DE//BC
$\therefore x = 17$ mm	

2)

Statement	Reason
AF = FB and AE = EC	Given
∴ FE//BC and BC = 2FE	Mipt. Thm.
ED = 2FE	Given
∴ ED//BC and ED = BC	
∴ BCDE is parm.	1 pair of opp. sides = and //

3)

Statement	Reason
AE = EB and AF = FC	Given
∴ BC = 2EF and EF//BC	Mipt. Thm.
But EF = FG	Given
∴ EG = BC	
∴ BEGC is a parm.	One pair of opp. sides = and //

4)

Statement	Reason
AD = DB and AF//DE	Given
∴ AC = 2DE and CE = BE	Mipt. Thm.
But AF = FC	Given
∴ DE = AF	
∴ ADEF is a parm.	One pair of opp. sides = and //

5)

Statement	Reason
AD = DB and FE = BE	Given
∴ DE//AF and AF=DE	Midpt. Thm.
∴ DE = 5cm	Given AF = 10cm
DE//GF	Proved above
CF = FE	Given
∴ DE = 2GF	Conv. Midpt Thm.
$\therefore \text{GF} = \frac{1}{2}(\text{DE}) = \frac{1}{2}(5) = 2{,}5\,\text{cm}$	

6)

Statement	Reason
BG//CD and AB = BC	
∴ AG= GD and CD=2BG	
GF//DE	Given
∴ AF = EF and DE = 2GF	Midpt. Thm.
∴ AF = 10 mm	

Exercise 6 numbers 1 to 4 are on page **328** and numbers 5 and 6 are on page **329**

Exercise 7 Solutions:

1)

a) $\text{AB} = \sqrt{(-2-5)^2 + (6-3)^2}$

$\therefore \text{AB} = \sqrt{58}$ units

b) $\text{AB} = \sqrt{(-2-3)^2 + (-1-2)^2}$

$\therefore \text{AB} = \sqrt{34}$ units

c) $\text{PQ} = \sqrt{(3+2)^2 + (5+7)^2}$

$\therefore \text{PQ} = 13$ units

d) $\text{SR} = \sqrt{(8-2)^2 + (0+3)^2}$

$\therefore \text{SR} = 3\sqrt{5}$ units

Exercise 7 number 1 is on page **329**

2)

a) $M_{AB} = \left(\frac{6-2}{2};\frac{1+5}{2}\right) = (2;3)$

b) $M_{AB} = \left(\frac{5+3}{2};\frac{3+1}{2}\right) = (4;2)$

c) $M_{AB} = \left(\frac{-3-3}{2};\frac{-2+2}{2}\right) = (-3;0)$

d) $M_{AB} = \left(\frac{2-2}{2};\frac{3+5}{2}\right) = (0;4)$

Exercise 7 number 2 is on page **329**

3)

a) $\frac{5+x_B}{2} = 4$ and $\frac{-1+y_B}{2} = -3$

$\therefore x_B = 3$ and $y_B = -5$

b) $\frac{-9+x_B}{2} = -6$ and $\frac{4+y_B}{2} = 1$

$\therefore x_B = -3$ and $y_B = -2$

c) $\frac{-3+x_B}{2} = 1$ and $\frac{5+y_B}{2} = 1$

$\therefore x_B = 5$ and $y_B = -3$

d) $\frac{10+x_B}{2} = -1$ and $\frac{20+y_B}{2} = 5$

$\therefore x_B = -12$ and $y_B = -10$

Exercise 7 number 3 is on page **329**

4)

a) $m = \frac{y_2-y_1}{x_2-x_1} = \frac{5-3}{4-2} = 1$

b) $m = \frac{y_2-y_1}{x_2-x_1} = \frac{-2-3}{8-2} = -\frac{5}{6}$

c) $m = \frac{y_2-y_1}{x_2-x_1} = \frac{-5+2}{-8+2} = \frac{1}{2}$

Exercise 7 number 4 is on page **329**

5)

a) $A(-8;-4)$ and $B(2;4)$ and $C(-4;0)$ and $D(-20;20)$

$m_{AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{4+4}{2+8} = \frac{4}{5}$

$m_{CD} = \frac{y_2-y_1}{x_2-x_1} = \frac{20-0}{-20+4} = -\frac{5}{4}$

$\therefore m_{AB} \times m_{CD} = -1$

$\therefore AB \perp CD$

Exercise 7 number 5a is on page **329**

b) $A(5;6)$ and $B(4;2)$ and $C(8;24)$ and $D(4;8)$

$m_{AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{2-6}{4-5} = 4$

$m_{CD} = \frac{y_2-y_1}{x_2-x_1} = \frac{8-24}{4-8} = 4$

$\therefore m_{AB} = m_{CD}$

$\therefore AB \parallel CD$

Exercise 7 number 5b is on page **329**

c) $A(5;6)$ and $B(4;2)$ and $C(8;24)$ and $D(4;8)$

$m_{AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{-3-2}{2-5} = \frac{5}{3}$

$m_{CD} = \frac{y_2-y_1}{x_2-x_1} = \frac{18-9}{-5-10} = -\frac{3}{5}$

$\therefore m_{AB} \times m_{CD} = -1$

$\therefore AB \perp CD$

Exercise 7 number 5c is on page **329**

d) $A(2;-5)$ and $B(-1;-8)$ and $C(-1;-6)$ and $D(1;-10)$

$m_{AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{-8+5}{-1-2} = 1$

$m_{CD} = \frac{y_2-y_1}{x_2-x_1} = \frac{-10+6}{1+1} = -2$

$\therefore m_{AB} \times m_{CD} \neq -1$ and $m_{AB} \neq m_{CD}$

$\therefore$ AB is not parallel or perpendicular to CD

Exercise 7 number 5d is on page **329**

6)

a) $m_{AB} = \frac{y_2-y_1}{x_2-x_1}$ $\qquad m_{BC} = \frac{y_2-y_1}{x_2-x_1}$

$\therefore m_{AB} = \frac{-8+10}{-2+6}$ $\qquad \therefore m_{BC} = \frac{-7+8}{0+2}$

$\therefore m_{AB} = \frac{1}{2}$ $\qquad \therefore m_{BC} = \frac{1}{2}$

$\therefore$ A, B and C are collinear

b) $m_{AB} = \frac{y_2-y_1}{x_2-x_1}$ $\qquad m_{BC} = \frac{y_2-y_1}{x_2-x_1}$

$\therefore m_{AB} = \frac{4-10}{9-0}$ $\qquad \therefore m_{BC} = \frac{6-4}{6-9}$

$\therefore m_{AB} = -\frac{2}{3}$ $\qquad \therefore m_{BC} = -\frac{2}{3}$

$\therefore$ A, B and C are collinear

Exercise 7 number 6 is on page **329**

7)

a) $\mathrm{m_{PQ}} = \mathrm{m_{RS}}$

$$\therefore \frac{5-2}{x+3} = \frac{4-5}{-1-2}$$

$$\therefore \frac{3}{x+3} = \frac{-1}{-3}$$

$$\therefore -9 = -(x+3)$$

$$\therefore 9 = x+3$$

$$\therefore x = 6$$

b) $\mathrm{m_{PQ}} = \mathrm{m_{QS}}$

$$\therefore \frac{5-2}{x+3} = \frac{4-2}{-1+3}$$

$$\therefore \frac{3}{x+3} = \frac{2}{2}$$

$$\therefore 3 = x+3$$

$$\therefore x = 0$$

c) $\mathrm{m_{PQ}} \times \mathrm{m_{RS}} = -1$

$$\therefore \frac{5-2}{x+3} \times \frac{4-5}{-1-2} = -1$$

$$\therefore \frac{3}{x+3} \times \frac{1}{3} = -1$$

$$\therefore \frac{3}{3x+9} = -1$$

$$\therefore 3 = -3x-9$$

$$\therefore 12 = -3x$$

$$\therefore x = -4$$

Exercise 7 number 7 is on page **329**

Exercise 8 Solutions:

1) The easiest way of proving a quadrilateral is a parm is to prove that their diagonals bisect each other:

a) $\mathrm{M_{PR}}\left(\frac{x_{\mathrm{R}}+x_{\mathrm{P}}}{2};\frac{y_{\mathrm{R}}+y_{\mathrm{P}}}{2}\right)$

$$=\left(\frac{6-2}{2};\frac{8+3}{2}\right)$$

$$=\left(2;\frac{11}{2}\right)$$

$\mathrm{M_{QS}}\left(\frac{x_{\mathrm{R}}+x_{\mathrm{P}}}{2};\frac{y_{\mathrm{R}}+y_{\mathrm{P}}}{2}\right)$

$$=\left(\frac{2+2}{2};\frac{10+1}{2}\right)$$

$$=\left(2;\frac{11}{2}\right)$$

$\therefore \mathrm{M_{PR}} = \mathrm{M_{QS}}$

$\therefore$ PQRS is a parm (diagonals of a parm bisect each other)

b) $\mathrm{M_{AC}}\left(\frac{x_{\mathrm{R}}+x_{\mathrm{P}}}{2};\frac{y_{\mathrm{R}}+y_{\mathrm{P}}}{2}\right)$

$$=\left(\frac{-1-6}{2};\frac{1-1}{2}\right)$$

$$=\left(-\frac{7}{2};0\right)$$

$\mathrm{M_{BD}}\left(\frac{x_{\mathrm{R}}+x_{\mathrm{P}}}{2};\frac{y_{\mathrm{R}}+y_{\mathrm{P}}}{2}\right)$

$$=\left(\frac{-3-4}{2};\frac{3-3}{2}\right)$$

$$=\left(-\frac{7}{2};0\right)$$

$\therefore \mathrm{M_{AC}} = \mathrm{M_{DB}}$

$\therefore$ ABCD is a parm (diagonals of a parm bisect each other)

Exercise 8 number 1 is on page **330**

2)

a)

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(-2+5)^2+(8-0)^2}$$
$$=\sqrt{73}\text{ units}$$

$$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(-2-6)^2+(8-5)^2}$$
$$=\sqrt{73}\text{ units}$$

$$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(-5-6)^2+(5-0)^2}$$
$$=\sqrt{146}\text{ units}$$

$AB^2+AC^2=73+73=146$

$BC^2=146$

$\therefore BC^2=AB^2+AC^2$

$\therefore$ ABC is right angled at $\hat{A}$

b)

$$m_{AB}=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{8-0}{-2+5}$$
$$=\frac{8}{3}$$

$$m_{AC}=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{8-5}{-2-6}$$
$$=-\frac{3}{8}$$

$\therefore m_{AB}\times m_{AC}=-1$

$\therefore AB\perp AC$

Exercise 8 number 2 is on page **330**

3)

a)

$x_M=\frac{x_R+x_P}{2}$ $\qquad$ $y_M=\frac{y_R+y_P}{2}$

$\therefore 5=\frac{8+x_P}{2}$ $\qquad$ $1=\frac{4+y_P}{2}$

$\therefore 10=8+x_P$ $\qquad$ $2=4+y_P$

$\therefore x_P=2$ $\qquad$ $y_P=-2$

$\therefore$ P is the point $(2\,;-2)$

b) Prove that diagonals bisect at 90°:

$m_{QS}=\frac{y_2-y_1}{x_2-x_1}$ $\qquad$ $m_{PR}=\frac{y_2-y_1}{x_2-x_1}$

$\therefore m_{QS}=\frac{2-0}{4-6}$ $\qquad$ $m_{PR}=\frac{-2-4}{2-8}$

$\therefore m_{QS}=-1$ $\qquad$ $m_{PR}=1$

$\therefore m_{QS}\times m_{PR}=-1$

$\therefore QS\perp PR$

$\therefore$ PQRS is rhombus (diagonals bisect at 90°)

Exercise 8 number 3 is on page **330**

The Mixed Exercise Solutions:

1)

Statement	Reason
In ΔPQO and ΔRQO	
OP = OR	Radii
PQ = QR	Given
OQ = OQ	Common side
∴ ΔPQO ≡ ΔRQO	SSS

2)

Statement	Reason
In ΔBDF and ΔCFE	
BF = EC	Given
DB = FC	Given
$\hat{B}=\hat{C}$	∠'s opp. = sides
∴ ΔBDF ≡ ΔCFE	SAS
∴ DF = FE	Cong. Δ's

The Mixed Exercise number 1 and 2 is on page **330**

3)

a) $x = 2y \rightarrow \boxed{1}$ (opp. $\angle$'s of a rhombus are $=$)

$x + 2x - 3y = 180^\circ \rightarrow \boxed{2}$ (co-int. $\angle$'s , opp sides of a rhombus are //)

sub $\boxed{1}$ into $\boxed{2}$:

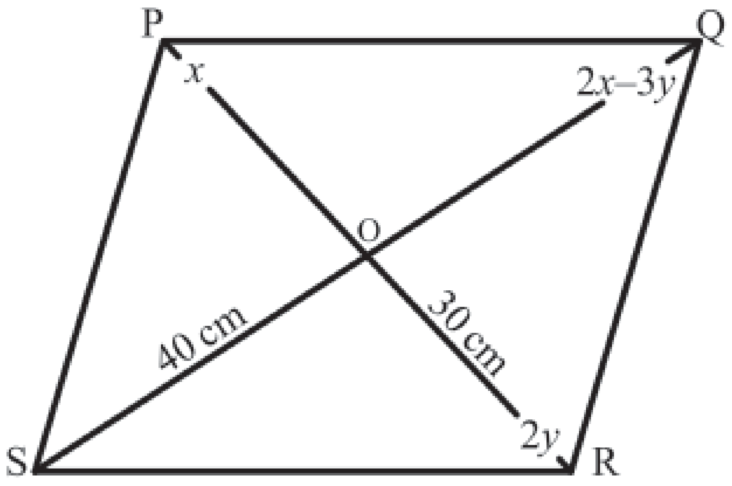
$\therefore 2y + 2(2y) - 3y = 180^\circ$

$\therefore 3y = 180^\circ$

$\therefore y = 60^\circ \rightarrow \boxed{3}$

sub $\boxed{3}$ into $\boxed{1}$:

$\therefore x = 120^\circ$



b) $\text{PO} = \text{OR} = 30\,\text{cm}$ (Diags. of a rhombus bisect each other)

$\text{P}\hat{\text{O}}\text{S} = 90^\circ$ (Diags. of a rhombus bisect each other at 90°)

$\therefore \text{PS}^2 = \text{PO}^2 + \text{SO}^2$ (Pythag)

$\therefore \text{PS}^2 = (30)^2 + (40)^2$

$\therefore \text{PS} = 50\,\text{cm}$

The Mixed Exercise number 3 is on page 330

4)

Statement	Reason
In ΔBAE and ΔEDC	
$\hat{\text{B}}_1 = \hat{\text{E}}$	Given
BE = EC	Given
$\hat{\text{A}} = \hat{\text{D}}_1$	Corr. $\angle$'s AB//CD
$\therefore \Delta\text{BAE} \equiv \Delta\text{EDC}$	AAS

The Mixed Exercise number 4 is on page 330

5)

$\hat{\text{T}}_1 = 40^\circ$	Alt. $\angle$'s PQ//SR
$\hat{\text{T}}_1 = \hat{\text{S}}$ $\therefore a = 100^\circ$	$\angle$'s opp. = sides $\angle$'s in a Δ
$\therefore \hat{\text{P}} = b = 140^\circ$	Opp $\angle$'s of a parm. =
$\therefore \hat{\text{T}}_3 + c = 40^\circ$	$\angle$'s in a Δ
$\hat{\text{T}}_3 = c$	$\angle$'s opp. = sides
$\therefore c = 20^\circ$	

The Mixed Exercise number 5 is on page 331

6)

Statement	Reason
$\hat{\text{B}}_1 = 50^\circ$	Alt. $\angle$'s AC//EF
$\hat{\text{B}}_2 = 180^\circ - 50^\circ - 62^\circ$ $= 68^\circ$	Adj. $\angle$'s on str. line
$\hat{\text{B}}_2 = a = 68^\circ$	$\angle$'s opp. = sides
$\hat{\text{D}}_1 = 180^\circ - 68^\circ = 112^\circ$	Adj. $\angle$'s on str. line
$b = 180^\circ - 112^\circ - 50^\circ$ $\therefore b = 18^\circ$	$\angle$'s in a Δ

The Mixed Exercise number 6 is on page 331

7)

$\text{TR}^2 = \text{SR}^2 - \text{ST}^2$	Pythag.
$\therefore \text{TR} = 4\,\text{m}$	
$\therefore \text{QR} = 6\,\text{m}$	
$\therefore \frac{\text{QR}}{\text{TR}} = \frac{6}{4} = \frac{3}{2}$	
$\therefore \frac{\text{PQ}}{\text{ST}} = \frac{\text{PR}}{\text{SR}} = \frac{\text{QR}}{\text{TR}} = \frac{3}{2}$	Similar Δ's
$\therefore \frac{\text{PQ}}{3} = \frac{3}{2}$	
$\therefore \text{PQ} = 4{,}5\,\text{cm}$	
$\therefore \frac{\text{PR}}{5} = \frac{3}{2}$	
$\therefore \text{PR} = 7{,}5\,\text{cm}$	
$\therefore \text{PS} = 7{,}5 - 5 = 2{,}5\,\text{cm}$	

The Mixed Exercise number 7 is on page 331

8)

a) $\text{PQ} = \sqrt{(2-0)^2 + (5-0)^2}$

$\therefore \text{PQ} = \sqrt{29}$ units

$\text{QR} = \sqrt{(4-2)^2 + (0-5)^2}$

$\therefore \text{QR} = \sqrt{29}$ units

$\text{PR} = \sqrt{(4-0)^2 + (0-0)^2}$

$\therefore \text{PR} = 4$ units

b) PQR is an isosceles triangle as PQ = PR. (Two sides are equal in length.)

The Mixed Exercise number 8 is on page 331

9)

a) $y = -2x + 7$

b) $y = -5$

c) $x = 6$

d) **Method 1:** $y = mx + c$

$m = 2$

$\therefore y = 2x + c \rightarrow \text{sub } (2;3)$

$3 = 2(2) + c$

$\therefore c = -1$

$\therefore y = 2x - 1$

Method 2: $y - y_1 = m(x - x_1)$

$m = 2$

$\therefore y - y_1 = 2(x - x_1) \rightarrow \text{sub } (2;3)$

$y - 3 = 2(x - 2)$

$\therefore y - 3 = 2x - 4$

$\therefore y = 2x - 1$

The Mixed Exercise number 9 is on page 331

10)

a) $\text{M}_{\text{AC}} = \text{O} = \left(\frac{2+12}{2}; \frac{4+8}{2}\right)$

$\therefore$ O is the point $(7;6) \rightarrow$ Diags of a parm bisect each other

$\therefore \frac{5 + x_D}{2} = 7$ and $\frac{7 + y_D}{2} = 6$

$\therefore x_D = 9$ and $y_D = 5$

$\therefore$ D is the point $(9;5)$

b) $\text{m}_{\text{DO}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-6}{9-7} = -\frac{1}{2}$

$\text{m}_{\text{OB}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-6}{5-7} = -\frac{1}{2}$

$\therefore \text{m}_{\text{DO}} = \text{m}_{\text{OB}}$

$\therefore$ Points B, O and D are collinear

The Mixed Exercise number 10 is on page 331

11)

Statement	Reason
FC = EB	Opp sides of a parm. =
DE = EB	Diags of a parm. Bisect each other
$\therefore$ DE = FC	
BE // FC	Opp sides of a parm =
$\therefore$ DE // FC	
$\therefore$ EDFC is a parm	One pair of opp. sides = and //

12)

Statement	Reason
$\hat{Q} + \hat{R} = 180^\circ$	
$\hat{Q}_2 = \frac{1}{2}(\hat{Q}_1 + \hat{Q}_2)$	
$\hat{R}_1 = \frac{1}{2}(\hat{R}_1 + \hat{R}_2)$	
$\therefore \hat{Q}_2 + \hat{R}_1 = \frac{1}{2}(\hat{Q} + \hat{R})$	
$\therefore \hat{Q}_2 + \hat{R}_1 = 90^\circ$	
$\therefore \hat{T} = 90^\circ$	
QTRU is a parm.	Given opp. sides //
$\therefore$ QTRU is a rectangle	Parm. With a right $\angle$

The Mixed Exercise number 11 is on page 331 and number 12 is on page 332

13)

Statement	Reason
In ΔAEM and ΔCFM	
$\hat{\text{A}}_1 = \hat{\text{C}}_1$	Alt. $\angle$'s AB//DC
$\hat{\text{M}}_1 = \hat{\text{M}}_2$	Vert. opp. $\angle$'s
AM = MC	Diags of a parm. bisect each other
$\therefore \Delta\text{AEM} \equiv \Delta\text{CFM}$	AAS
$\therefore$ AE = FC	
$\therefore$ BE = DF	Opp. Sides of a parm. =
AB // FC	Opp sides of a parm. //
$\therefore$ BEDF is a parm.	One pair of opp sides = and //

The Mixed Exercise number 13 is on page 332.

14)

a) $D=\left(\dfrac{1-2}{2};\dfrac{1+7}{2}\right)=\left(-\dfrac{1}{2};4\right)$

b) **Method 1:** $y=mx+c$

$m=\dfrac{7-1}{-2-1}=\dfrac{6}{-3}=-2$

$\therefore y=-2x+c\rightarrow \text{sub B}(1;1)$

$\therefore 1=-2(1)+c$

$\therefore c=3$

$\therefore y=-2x+3$

Method 2: $y-y_1=m(x-x_1)$

$m=\dfrac{7-1}{-2-1}=\dfrac{6}{-3}=-2$

$\therefore y-y_1=-2(x-x_1)\rightarrow \text{sub B}(1;1)$

$\therefore y-1=-2(x-1)$

$\therefore y=-2x+2+1$

$\therefore y=-2x+3$

c) $m_{BC}=\dfrac{3-1}{3-1}=1$ $\qquad m_{AC}=\dfrac{7-3}{-2-3}=-\dfrac{4}{5}$

d) ABC is not right angled. For perpendicular lines $m_1\times m_2=-1$.

This is not the case for any of the gradients in triangle ABC.

e) **Method 1:** $y=mx+c$

$m=\dfrac{1}{2}$

$\therefore y=\dfrac{1}{2}x+c\rightarrow \text{sub D}\left(-\dfrac{1}{2};4\right)$

$\therefore 4=\dfrac{1}{2}\left(-\dfrac{1}{2}\right)+c$

$\therefore c=\dfrac{17}{4}$

$\therefore y=\dfrac{1}{2}x+\dfrac{17}{4}$

Method 2: $y-y_1=m(x-x_1)$

$m=\dfrac{1}{2}$

$\therefore y-y_1=\dfrac{1}{2}(x-x_1)\rightarrow \text{sub D}\left(-\dfrac{1}{2};4\right)$

$\therefore y-4=\dfrac{1}{2}\left(x+\dfrac{1}{2}\right)$

$\therefore y=\dfrac{1}{2}x+\dfrac{1}{4}+4$

$\therefore y=\dfrac{1}{2}x+\dfrac{17}{4}$

The Mixed Exercise number 14 is on page **332**

15)

a) $\dfrac{p-5}{2}=-1$ $\qquad \dfrac{q+2}{2}=\dfrac{3}{2}$

$\therefore p-5=-2$ $\qquad \therefore q+2=3$

$\therefore p=3$ $\qquad \therefore q=1$

$\therefore$ C is the point $(3;1)$

b) AC $=\sqrt{(3+5)^2+(1-2)^2}=\sqrt{65}$ units

c) $m_{AB}=\dfrac{2+2}{-5+3}=-\dfrac{4}{2}=-2$ and $m_{BC}=\dfrac{1+2}{3+3}=\dfrac{3}{6}=\dfrac{1}{2}$

d) $m_{AB}\times m_{BC}=-1$. Therefore, AB $\perp$ BC and ΔABC is right angled at B.

e) **Method 1:** $y=mx+c$

$m_{AC}=\dfrac{1-2}{3+5}=-\dfrac{1}{8}$

$\therefore m_{\perp\text{bisector}}=8$

$\therefore y=8x+c\rightarrow \text{sub D}\left(-1;\dfrac{3}{2}\right)$

$\therefore \dfrac{3}{2}=8(-1)+c$

$\therefore c=\dfrac{19}{2}$

$\therefore y=8x+\dfrac{19}{2}$

Method 2: $y-y_1=m(x-x_1)$

$m_{AC}=\dfrac{1-2}{3+5}=-\dfrac{1}{8}$

$\therefore m_{\perp\text{bisector}}=8$

$\therefore y-y_1=8(x-x_1)\rightarrow \text{sub D}\left(-1;\dfrac{3}{2}\right)$

$\therefore y-\dfrac{3}{2}=8(x+1)$

$\therefore y=8x+8+\dfrac{3}{2}$

$\therefore y=8x+\dfrac{19}{2}$

The Mixed Exercise number 15 is on page **332**

16)

a) $\text{DC} = \text{AB} = \sqrt{(2+3)^2 + (3-1)^2} = \sqrt{29}$ units

b) **Method 1:** Use the property that diagonals of a parallelogram bisect each other.

$\text{M}_{\text{DB}} = \left(\frac{2-1}{2}; \frac{3-1}{2}\right) = \left(\frac{1}{2}; 1\right)$

$\therefore \frac{p-3}{2} = \frac{1}{2}$ and $\frac{q+1}{2} = 1$

$\therefore p = 4$ and $q = 1$

$\therefore$ C is the point $(4;1)$

Method 2: inspection:
To go from point A to B you move across 5 (right) and up 2. Therefore to go from point D to C you move across (right) 5 and up 2.
$\therefore$ C is the point $(4;1)$.

c) **Method 1:**

$\text{EF}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\therefore 180 = (x-3)^2 + (8+4)^2$

$\therefore 36 = (x-3)^2$

$\therefore \pm 6 = x - 3$

$\therefore x = -3$ or $x \neq 9$

$\therefore x = -3$

x cannot equal 9 as F is in quadrant II.

Method 2:

$\text{EF}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\therefore 180 = (x-3)^2 + (8+4)^2$

$\therefore 0 = x^2 - 6x + 9 + 144 - 180$

$\therefore 0 = x^2 - 6x - 27$

$\therefore 0 = (x+3)(x-9)$

$\therefore x = -3$ or $x \neq 9$

$\therefore x = -3$

The Mixed Exercise number 16 is on page 333

17)

a) $\text{BC} = \sqrt{(0-1)^2 + (-2-1)^2}$

$= \sqrt{10}$ units

b) $\text{m}_{\text{BC}} = \frac{-2-1}{0-1} = 3$

c) $\text{m}_{\text{AB}} = -\frac{1}{3}$

$\therefore \frac{p-1}{-5-1} = -\frac{1}{3}$

$\therefore p = 3$

d) $\text{M}_{\text{AB}} = \left(\frac{-5+0}{2}; \frac{3-2}{2}\right)$

$= \left(-\frac{5}{2}; \frac{1}{2}\right)$

e) $\text{M}_{\text{DB}} = \left(-\frac{5}{2}; \frac{1}{2}\right)$

$\therefore \frac{x+1}{2} = -\frac{5}{2}$

$\therefore x = -6$

$\therefore \frac{y+1}{2} = \frac{1}{2}$

$\therefore y = 0$

$\therefore$ D is the point $(-6;0)$

The Mixed Exercise number 17 is on page 333

18)

a) $\text{m}_{\text{QS}} = \frac{5-3}{-3+5} = 1$

b) **Method 1:** $y = mx + c$.

$\text{m}_{\text{PR}} = -1$

$\therefore y = -1x + c \rightarrow$ sub $\text{R}(-1;1)$

$\therefore 1 = -1(-1) + c$

$\therefore c = 0$

$\therefore y = -x$

Method 2: $y - y_1 = m(x - x_1)$

$\text{m}_{\text{PR}} = -1$

$\therefore y - y_1 = -(x - x_1) \rightarrow$ sub $\text{B}(-1;1)$

$\therefore y - 1 = -(x+1)$

$\therefore y = -x - 1 + 1$

$\therefore y = -x$

c) $y = -(-7) = 7 \rightarrow$ substitute $x = -7$ into $y = -x$

$\therefore a = 7$

The Mixed Exercise number 18 is on page 333

19) A(1;2), B(–5; q), C(p ;7) and D(–3;5).

a) $\mathrm{m}_{\mathrm{AB}} = \dfrac{q-2}{-5-1}$ $\qquad$ $\mathrm{m}_{\mathrm{BD}} = \dfrac{q-5}{-5+3}$

$\therefore \dfrac{q-2}{-6} = \dfrac{q-5}{-2}$

$\therefore -2q+4 = -6q+30$

$\therefore 4q = 26$

$\therefore q = \dfrac{13}{2}$

b) $\dfrac{p-5}{2} = 1$ $\qquad$ $\dfrac{q+7}{2} = 2$

$\therefore p-5 = 2$ $\qquad$ $\therefore q+7 = 4$

$\therefore p = 7$ $\qquad$ $\therefore q = -3$

The Mixed Exercise number 19 is on page **333**

20) Before answering the questions you have to write the equation in the form $y = mx + c$.

$\therefore 2ky - 3y - 3x = 5$

$\therefore 2ky - 3y = 3x + 5$

$\therefore y(2k-3) = 3x + 5$

$\therefore y = \dfrac{3}{2k-3}x + \dfrac{5}{2k-3}$

a) $\dfrac{1}{3} = \dfrac{3}{2k-3}$

$\therefore 2k - 3 = 9$

$\therefore 2k = 12$

$\therefore k = 6$

b) $5 = \dfrac{5}{2k-3}$

$\therefore 10k - 15 = 5$

$\therefore 10k = 20$

$\therefore k = 2$

c) $1 = \dfrac{3}{2k-3}(1) + \dfrac{5}{2k-3}$

$\therefore 2k - 3 = 8$

$\therefore 2k = 11$

$\therefore k = \dfrac{11}{2}$

The Mixed Exercise number 20 is on page **333**

Chapter 8 – Financial Mathematics

Simple Interest (Theory on pages 167 - 168)

Exercise 1: Assume that simple interest is used for all calculations in this exercise.

1) Mpho invests R8 000 in an account that offers an interest rate of 9% p.a. Calculate:
   a) The amount he will have after 8 years.
   b) The interest received after each year.

Solutions to Exercise 1 number 1 are on page **350**

2) Sam wants to go to England in 5 years time. He invests his money into an account that offers an interest rate of 12% p.a. Calculate:
   a) The amount he must invest if he needs R50 000 for his holiday.
   b) The amount of interest earned each year.

Solutions to Exercise 1 number 2 are on page **350**

3) Determine how long it would take for an investment to double at interest rate of 20% p.a.?

Solutions to Exercise 1 number 3 are on page **350**

4) Tebogo invests R3 000 into a savings account. After 5 years he has R4 500 in his account. What interest rate did Tebogo receive?

Solutions to Exercise 1 number 4 are on page **350**

5) Percy invests R1 000 into an account offering an interest rate of 15% p.a. At the same time, Jane invests R1 300 into an account that offers an interest rate of 10% p.a. How long will it take for Percy and Jane to have the same amount of money in their accounts?

Solutions to Exercise 1 number 5 are on page **350**

Hire Purchase

Exercise 2: (Theory on pages 169 - 170)

1) Sarah buys a microwave for R2 500 and pays for it using a hire purchase loan. The interest rate is 18% per annum and the loan is to paid back over 2 years. Calculate:
   a) Sarah's monthly repayments.
   b) The interest that Sarah pays over 2 years.

Solutions to Exercise 2 number 1 are on page **350**

2) Mahesh buys a laptop that costs R6 000. He pays a 10% deposit and takes out a 24 month hire purchase loan on the balance. Interest is charged at 12,5% and a monthly insurance premium of R50 per month is required. Calculate:
   a) The amount that Mahesh actually paid for the laptop.
   b) Mahesh's monthly repayments.

Solutions to Exercise 2 number 2 are on page **350**

3) An advertisement offers a desk and chair set for R290 per month with 36 months to pay and no deposit necessary. If the company bases their hire purchase loans on an interest rate of 15% per annum, what is the cash price of desk and chair set?

Solutions to Exercise 2 number 3 are on page **350**

4) Tony would like a radio for his car. He finds an offer where he has to pay a R400 deposit and then make 6 payments of R185 at the end of every month for 6 months. Calculate the interest rate that Tony received if the cash price of the radio is R1 400.

Solutions to Exercise 2 number 4 are on page **350**

Compound interest and inflation

Exercise 3: (Theory on pages 171 - 172)

1) R8 000 is deposited into a savings account at interest rate of 8,5% per annum, compounded annually. Calculate how much money will be in the account after 16 years.

Solutions to Exercise 3 number 1 are on page **351**

2) R12 000 is invested for 7 years at an interest rate of 11% p.a:
   a) What is the value of the investment if the interest is calculated as a simple interest rate?
   b) What is the value of the investment if the interest is calculated using compound interest?

Solutions to Exercise 3 number 2 are on page **351**

3) At the beginning of Grade 8 Sipho invests R6 000 into a savings account to pay for textbooks for his first year at university. At the end of Matric Sipho has R7 000 for his textbooks. If interest is compounded annually what interest rate did Sipho receive?

Solutions to Exercise 3 number 3 are on page **351**

4) **A loaf of brown bread costs R9,50 and the average inflation rate over the past 7 years has been 9%:**
   - **a)** Determine the cost of a loaf of brown bread in 8 years' time if the inflation rate continues at the same level.
   - **b)** How much did a loaf of brown bread cost 7 years ago?

Solutions to Exercise 3 number 4 are on page **351**

5) **Lara is offered 12% p.a. compounded annually or 13% p.a. simple interest on an investment for 5 years. Which offer should Lara choose?**

Solutions to Exercise 3 number 5 are on page **351**

6) **Rob decides to play the stock market and purchases R5 000 worth of shares. After 6 years he sells his shares for R9 200. Determine:**
   - **a)** The compound interest rate that would yield the same return on investment.
   - **b)** The simple interest rate that would yield the same return on investment.

Solutions to Exercise 3 number 6 are on page **351**

7) **What interest rate is required for an investment to double in 5 years if:**
   - **a)** Simple interest is used.
   - **b)** Compound interest is used.

Solutions to Exercise 3 number 7 are on page **351**

8) **Mr Wolf borrows R12 000 from a friend in order to carpet his house. The terms of the loan are that it gets repaid in full at the end of 4 years at an interest rate of 16% p.a. compounded annually. How much will Mr Wolf have to repay at the end of 4 years?**

Solutions to Exercise 3 number 8 are on page **351**

Time Lines (Theory on pages 173 - 176)

Exercise 4: Assume that interest is compounded annually in this exercise.

1) **Jamie deposits R6 000 into a savings account that offers an interest rate of 12% p.a. Four years later he deposits another R2 000 into the account. How much will Jamie have in his account after 6 years?**

Solutions to Exercise 4 number 1 are on page **352**

2) **R8 000 is deposited into a savings account that offers an interest rate of 8% p.a. After 2 years R1 500 is withdrawn from the account and 1 year later the interest rate drops to 6,5% p.a. Determine how much money will be in the account after 5 years.**

Solutions to Exercise 4 number 2 are on page **352**

3) **Jenna deposits R5 000 into unit trusts in order to pay for an overseas holiday in 3 years' time. After the first year the interest rate decreases from 10% p.a. to 6,5% p.a. She also runs into financial difficulty and has to withdraw R1 000 after the first year. How much will she have for holiday?**

Solutions to Exercise 4 number 3 are on page **352**

4) **Kelvin starts to save for his son's university education when he is in grade 8. At the end of January he deposits R13 000 into a savings account that offers 13,5% interest p.a. He estimates that university will cost R9 500 payable at the end of January in the first year and will escalate at 10% p.a. Determine:**
   - **a)** The fees for the second and third year.
   - **b)** The additional amount that Kelvin will have to pay to cover his son's third year in university.

Solutions to Exercise 4 number 4 are on page **352**

5) **When Mr Hertz purchased his car, he deposited R20 000 into a savings account in order to pay for maintenance of the car. The account offered an interest rate of 14,5% p.a. for the first two years and then dropped to 13% p.a. If he spends R6 000 per year to maintain the car, how many years will he be able to use his savings to pay for the full maintenance of the car?**

Solutions to Exercise 4 number 5 are on page **352**

6) **A savings account is opened up by depositing x Rand into the account. Four years later $2x$ Rand is deposited into the account. After 7 years There is R21 166,24 in the account. Determine the initial amount invested if the account earned an interest rate of 8% p.a. for the entire period.**

Solutions to Exercise 4 number 6 are on page **353**

Exchange Rates (Theory on pages 177 - 178)

Exercise 5:

1) **Tarik goes on holiday to London with R10 000 spending money. The Rand/Pound exchange rate is R11,23 to the Pound.**
   - **a)** How many Pounds did Tarik have to spend in London?
   - **b)** Tarik bought a watch for £150 in London, how much did it cost him in Rands?
   - **c)** Tarik Returned to South Africa with £200 and the exchange rate dropped to R9,80 to the Pound by the time he returned, how many Rands will he receive for the money that he returned with?

Solutions to Exercise 5 number 1 are on page 353

2) **If one Japanese Yen costs R0,32, how many Yen would you receive for R6 400?**

Solutions to Exercise 5 number 2 are on page 353

3) **James is travelling to London and Frankfurt and plans to purchase a Laptop while overseas. He researches prices on the internet and finds that the laptop he wants costs £220 in London and €310 in Frankfurt. If the exchange rate is R12,50 to the Pound and R9,80 to the Euro, where should James purchase the Laptop?**

Solutions to Exercise 5 number 3 are on page 353

4) **Nomtha purchases $7 000 for an overseas trip when the exchange rate is R8,72 to the Dollar. She is charged a commission of 1,3% on the transaction. When Nomtha returned to South Africa she had $1 200 left from her trip and the exchange rate had changed to R9,20 to the dollar.**

a) What commission was Nomtha charged to purchase her dollars (give your answer in Rands)?

b) How much did Nomtha pay in total for her Dollars (give your answer in Rands)?

c) When Nomtha changed her Dollars back to Rands, how many Rands did she receive assuming no commission was charged?

Solutions to Exercise 5 number 4 are on page 353

Mixed Exercise: (Chapter 8 is on pages 165 - 177)

1) **Kim invests R5 000 at a simple interest rate of 15% p.a. for 5 years. Stephen invests the same amount at a compound interest rate of 12% p.a. for the same time. Whose bank offered the better deal?**

Solutions to the Mixed Exercise number 1 are on page 353

2) **In 5 years from now Natasha and Jessica want to go on an overseas holiday together. They estimate that they will need a R150 000 in total for the trip. How much money must they invest into a joint savings account if the bank offers them a compound interest rate of 11,85% p.a.?**

Solutions to the Mixed Exercise number 2 are on page 353

3) **Denise invests Rx into a savings account and finds that it has doubled in 6,5 years. What compound interest rate did she receive from her bank?**

Solutions to the Mixed Exercise number 3 are on page 353

4) **Ndumi buys car that costs R25 000. He pays a 10% deposit and pays the balance using a hire purchase agreement over 3 years at an interest rate of 13,8% p.a. There is also a mandatory insurance cost of R63,50 per month. Calculate Ndumi's monthly instalments.**

Solutions to the Mixed Exercise number 4 are on page 353

5) **Alexa buys a furniture set that costs R12 400. She pays for it using a hire purchase agreement with monthly payments of R450 over 3 years. If there is no deposit or insurance cost what interest rate did she receive?**

Solutions to the Mixed Exercise number 5 are on page 353

6) **In 2012 Sango replaced his 2002 motor bike with a new one costing R185 300. If the average rate of inflation is 6,1% p.a. how much did he originally pay for his motorbike in 2002?**

Solutions to the Mixed Exercise number 6 are on page 353

7) **The exchange rate is R6,8455 to the Dollar and the price of Brent crude oil is $56 per barrel:**
   - **a)** Determine the price of 7 500 barrels of Brent crude oil in Rands.
   - **b)** If the price of Brent crude oil increases to $60 per barrel and the rand weakens to R7,2531 to the dollar, determine the new price of 7 500 barrels of Brent Crude oil in Rands.
   - **c)** Determine the percentage increase in price correct to two decimal places.

Solutions to the Mixed Exercise number 7 are on page 354

8) R100 000 worth of Dollars is purchased when the exchange rate is R6,78 to the Dollar:

- **a)** How many Dollars were purchased?
- **b)** If \$30000 is exchanged back to Rands when R1 is worth \$0,14285, how many rand would you receive?

Solutions to the Mixed Exercise number 8 are on page **354**

9) If 100 Japanese Yen can buy R6,2287 and 1 Australian Dollar can buy R5,1094 determine the exchange rate between the Australian Dollar and the Japanese Yen.

Solutions to the Mixed Exercise number 9 are on page **354**

10) At the beginning of Grade 8 Justin's grandfather gives him R5 000 and continues to do so at the beginning of each year of high school. Justin invests his money into an account that initially offers an interest rate of 12,1% p.a. compounded annually. After two years the interest rate increases to 12,8% p.a. compounded annually. How much will he have in his account at the end of matric?

Solutions to the Mixed Exercise number 10 are on page **354**

11) R500 is deposited into a savings account which offers an interest rate of 10% p.a. compounded annually. 3 years later the interest rate increases by 2%. 2 years after the initial deposit R150 is deposited into the account. 4 years after the initial deposit R50 is withdrawn from the account. How much will be in the account after 10 years?

Solutions to the Mixed Exercise number 11 are on page **354**

12) Jade wishes to save money for a holiday in 8 years' time. She does so by depositing R1 000 into a savings account which offers an interest rate of 11% p.a. compounded annually. 3 years later she deposits an additional R2 000. After 7 years she withdraws R500 from the account. If the interest rate changed to 10% p.a. 4 years after Jade made her initial deposit, how much money will Jade have for her holiday?

Solutions to the Mixed Exercise number 12 are on page **354**

13) Lerato wishes to buy a new cell phone. She borrows the money from her father who agrees to charge her an interest rate of 8% per annum compounded annually. She agrees to pay the loan off by repaying R2 000 after 1 year, a further R3 000 after 2 years and the remaining amount of R4 504,90 after 3 years. How much did Lerato borrow from her father to buy her phone?

Solutions to the Mixed Exercise number 12 are on page **354**

Chapter 8 – Financial Mathematics (Solutions)

Exercise 1 Solutions: Assume simple interest is used for all calculations in this exercise.

1)

a) $A = P(1+in)$

$= 8000[1+0{,}09(8)]$

$= R13\,760$

b) $\text{interest} = 13\,760 - 8\,000$

$= R5\,760$

$\therefore \text{yearly interest} = \dfrac{5\,760}{8}$

$= R720$ per year

Exercise 1 number 1 is on page 346

2)

a) $A = P(1+in)$

$\therefore 50\,000 = P[1+0{,}12(5)]$

$= R31250$

b) $\text{interest} = 50\,000 - 31250$

$= R18\,750$

$\therefore \text{yearly interest} = \dfrac{18\,750}{5}$

$= R3\,750$ per year

Exercise 1 number 2 is on page 346

3) $A = P(1+in)$

$\therefore 2P = P(1+0{,}2n) \rightarrow$ divide both sides by p

$\therefore 2 = 1 + 0{,}2n$

$\therefore 1 = 0{,}2n$

$\therefore n = 5$ years

Exercise 1 number 3 is on page 346

4) $A = P(1+in)$

$\therefore 4\,500 = 3\,000(1+5i)$

$\therefore 1{,}5 = 1 + 5i$

$\therefore i = 0{,}1$

$\therefore i = 10\%$

Exercise 1 number 4 is on page 346

5) Percy: $A = P(1+in)$

$= 1000(1+0.15n)$

Jane: $A = P(1+in)$

$= 1300(1+0{,}1n)$

$\therefore 1000(1+0{,}15n) = 1300(1+0{,}1n)$

$\therefore 1000 + 150n = 1300 + 130n$

$\therefore 20n = 300$

$\therefore n = 15$ years

Exercise 1 number 5 is on page 346

Exercise 2 Solutions:

1)

a) $A = P(1+in)$

$= 2\,500[1+0{,}18(2)]$

$= R3\,400$

$\therefore \text{Repayments} = \dfrac{3400}{24} = R141{,}67$ p/m

b) Interest = total paid – cash price

$= 3400 - 2500$

$= R900$

Exercise 2 number 1 is on page 346

2)

a) $\text{Deposit} = 6\,000(0{,}1) = 600$

$\therefore \text{amount owing} = 6\,000 - 600$

$= R5\,400$

$A = P(1+in)$

$= 5\,400[1+0{,}125(2)]$

$= R6\,750$

$\text{Insurance} = 50(24) = R1200$

$\therefore \text{Total paid} = 600 + 6\,750 + 1200$

$= R8\,550$

b) $\text{Monthly payments} = \dfrac{6750+1200}{24}$

$= R331{,}25$ p/m

Exercise 2 number 2 is on page 346

3) Total paid = 290(36) = R10 440

$A = P(1+in)$

$\therefore 10\,440 = P[1+0{,}15(3)]$

$\therefore P = R7\,200$

Exercise 2 number 3 is on page 346

4) $\text{Monthly payment total} = 185(6)$

$A = P(1+in)$

$\therefore 1\,110 = 1000[1+i(0{,}5)]$

$\therefore 1{,}11 = 1 + 0{,}5i$

$\therefore i = 0{,}22$

$\therefore i = 22\%$ per annum

Exercise 2 number 4 is on page 346

Exercise 3 Solutions:

1) $A = P(1+i)^n$

$= 8000(1+0{,}085)^{16}$

$= R29\,509{,}77$

Exercise 3 number 1 is on page **346**

2)

a) $A = P(1+i)^n$

$= 12\,000[1+0{,}11(7)]$

$= R21\,240$

b) $A = P(1+i)^n$

$= 12\ 000(1+0{,}11)^7$

$= R24\ 913{,}92$

Exercise 3 number 2 is on page **346**

3) $A = P(1+i)^n$

$\therefore 7\,000 = 6\,000(1+i)^5$

$\therefore \frac{7}{6} = (1+i)^5$

$\therefore \sqrt[5]{\frac{7}{6}} = 1+i$

$\therefore i = 0{,}0313$

$\therefore i = 3{,}13\%$ p.a.

Exercise 3 number 3 is on page **346**

4)

a) $A = P(1+i)^n$

$= 9{,}50(1+0{,}09)^8$

$= R18{,}93$

b) $A = P(1+i)^n$

$\therefore 9{,}50 = P(1+0{,}09)^7$

$\therefore P = \frac{9{,}50}{(1+0{,}09)^7}$

$\therefore P = R5{,}20$

Exercise 3 number 4 is on page **347**

5) Assume Rx is invested:

simple: $A = P(1+in)$

$\therefore A = x[1+0{,}13(5)]$

$\therefore A = R1{,}65x$

Compound: $A = P(1+i)^n$

$\therefore A = x[1+0{,}12]^5$

$\therefore A = R1{,}76x$

$\therefore$ 12% Compounded is better.

Exercise 3 number 5 is on page **347**

6)

a) $A = P(1+i)^n$

$\therefore 9\,200 = 5\,000(1+i)^6$

$\therefore 1{,}84 = (1+i)^6$

$\therefore \sqrt[6]{1{,}84} = 1+i$

$\therefore i = 0{,}1070$

$\therefore i = 10{,}7\%$ p.a.

b) $A = P(1+i)^n$

$\therefore 9\,200 = 5\,000(1+6i)$

$\therefore 2 = (1+6i)$

$\therefore 1{,}84 = 1+6i$

$\therefore i = 0{,}14$

$\therefore i = 14\%$ p.a.

Exercise 3 number 6 is on page **347**

7)

a) $A = P(1+in)$

$\therefore 2P = P(1+5i)$

$\therefore 2 = (1+5i)$

$\therefore 2 = 1+5i$

$\therefore i = 0{,}2$

$\therefore i = 20\%$ p.a.

b) $A = P(1+i)^n$

$\therefore 2P = P(1+i)^5$

$\therefore 2 = (1+i)^5$

$\therefore \sqrt[5]{2} = 1+i$

$\therefore i = 0{,}1487$

$\therefore i = 14{,}87\%$ p.a.

Exercise 3 number 7 is on page **347**

8) $A = P(1+i)^n$

$= 12\,000(1+0{,}16)^4$

$= R21\,727{,}67$

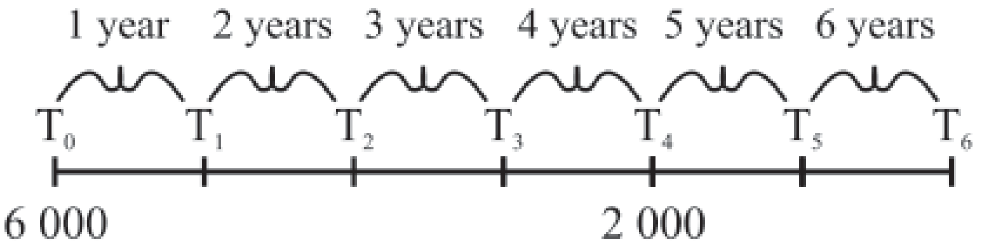
Exercise 3 number 7 is on page **347**

Exercise 4 Solutions:

1) $A = P(1+i)^n$

$= 6000(1+0,12)^6 + 2000(1+0,12)^2$

$= R14351,74$

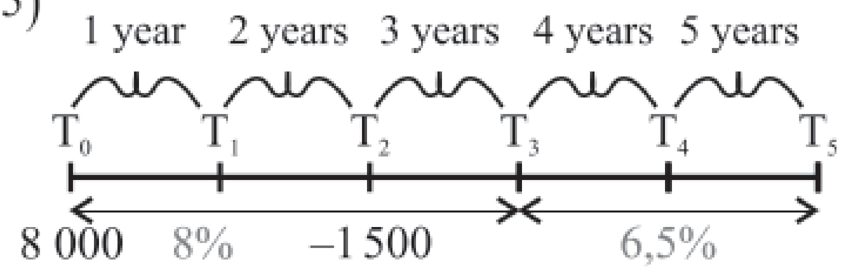

1 year
2 years
3 years
4 years
5 years
6 years
T₀
T₁
T₂
T₃
T₄
T₅
T₆
6 000
2 000

Exercise 4 number 1 is on page 347

2) $A = P(1+i)^n$

$= 8000(1+0,08)^3(1+0,065)^2 - 1500(1+0,08)^1(1+0,065)^2$

$= R9592,93$

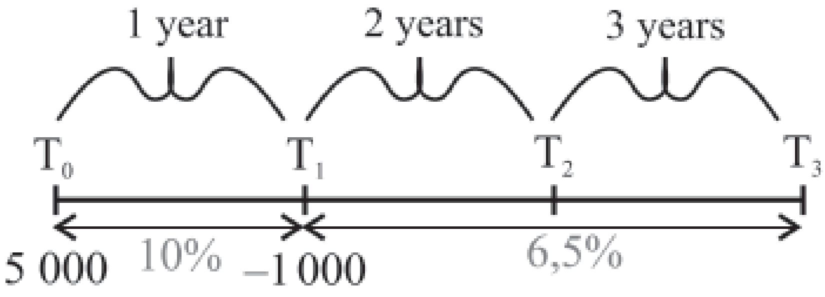

1 year
2 years
3 years
4 years
5 years
T₀
T₁
T₂
T₃
T₄
T₅
8 000
8%
–1 500
6,5%

Exercise 4 number 2 is on page 347

3) $A = P(1+i)^n$

$= 5000(1+0,1)^1(1+0,065)^2 - 1000(1+0,065)^2$

$= R5104,01$


1 year
2 years
3 years
T₀
T₁
T₂
T₃
5 000
10%
–1 000
6,5%

Exercise 4 number 3 is on page 347

4)

a) 2^{nd} year: $A = P(1+i)^n$

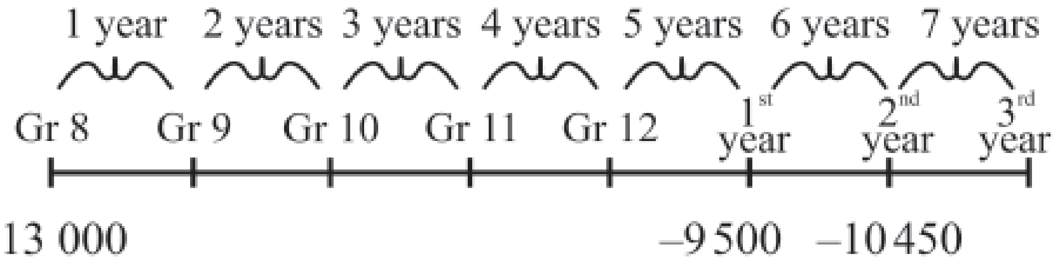
$\therefore A = 9500(1+0,1)^1$

$= R10450$

3^{rd} year: $A = P(1+i)^n$

$\therefore A = 10450(1+0,1)^1$

$= R11495$


1 year
2 years
3 years
4 years
5 years
6 years
7 years
Gr 8
Gr 9
Gr 10
Gr 11
Gr 12
1st year
2nd year
3rd year
13 000
–9 500
–10 450

b) $A = P(1+i)^n$

$= 13000(1+0,135)^7 - 9500(1+0,135)^2 - 10450(1+0,135)^1$

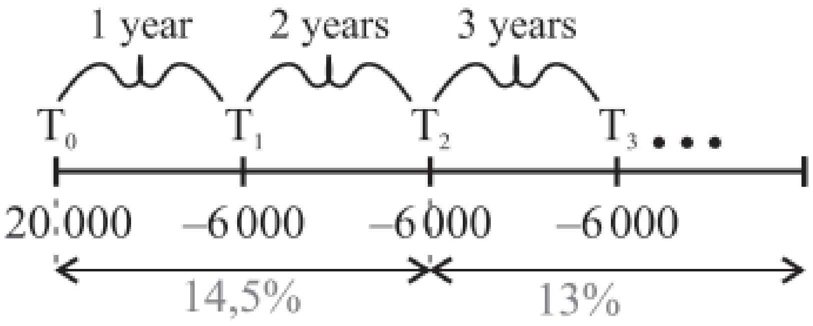
$= R7444,94$

$\therefore$ additional amount $= 11495 - 7444,94$

$= R4050,06$

Exercise 4 number 4 is on page 347

5)


1 year
2 years
3 years
T₀
T₁
T₂
T₃ • • •
20 000
–6 000
–6 000
–6 000
14,5%
13%

$T_0 \to T_1 : P = 20000 \quad i = 0,145$

$A = 20000(1+0,145)^1$

$= R22900$

$T_1 \to T_2 : P = 22900 - 6000 \quad i = 0,145$

$A = 16900(1+0,145)^1$

$= R19350,50$

$T_2 \to T_3 : P = 19350.5 - 6000 \quad i = 0,13$

$A = 13350,50(1+0,13)^1$

$= R15086,07$

$T_3 \to T_4 : P = 15086.07 - 6000 \quad i = 0,13$

$A = 9086,07(1+0,13)^1$

$= R10267,25$

$T_4 \to T_5 : P = 10267.25 - 6000 \quad i = 0,13$

$A = 4267,25(1+0,13)^1$

$= R4822$

$\therefore$ The savings would maintain his car for 4 years and he would have to pay additional funds in the 5th year.

Exercise 4 number 5 is on page 347

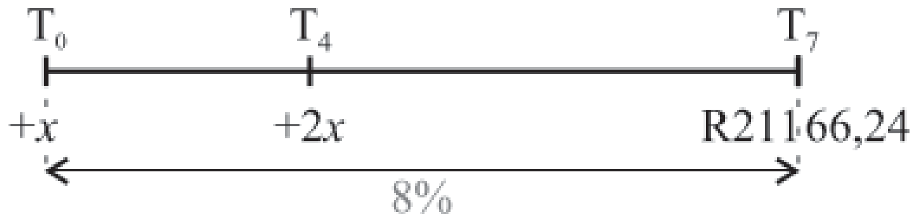
6) $A = P(1+i)^n$

$\therefore 21\,166,24 = x(1+0,08)^7 + 2x(1+0,08)^3$

$\therefore 21\,166,24 = x\left[(1+0,08)^7 + 2(1+0,08)^3\right]$

$$\therefore x = \frac{21\,166,24}{(1+0,08)^7 + 2(1+0,08)^3}$$

$\therefore x = \text{R}5\,000$



Exercise 4 number 6 is on page **347**

Exercise 5 Solutions:

1)

a) $\frac{10\,000}{11,23} = £890,47$

b) $150 \times 11,23 = \text{R}1684,50$

c) $200 \times 9,8 = \text{R}1960$

2) $\frac{6\,400}{0,32} = ¥20\,000$

Exercise 5 is on page **348**

3) London Price:

$220 \times 12,50 = \text{R}2\,750$

Frankfurt Price:

$310 \times 9,80 = \text{R}3\,038$

$\therefore$ he should buy it in London

4)

a) commission: $7000 \times 0,013 = \$91$

$91 \times 8,72 = \text{R}793,52$

b) $7\,000 \times 8,72 = \text{R}61\,040$

$\text{Total} = 61\,040 + 793,52 = \text{R}61\,833,52$

c) $1\,200 \times 9,2 = \text{R}11\,040$

Mixed Exercise Solutions:

1) Kim:

$A = P(1+in)$

$= 5\,000\left[1+0,15(5)\right]$

$= \text{R}8\,750$

Stephen:

$A = P(1+i)^n$

$= 5\,000(1+0,12)^5$

$= \text{R}8\,811,71$

$\therefore$ Stephen's Bank offers the better rate

The Mixed Exercise number 1 is on page **348**

2) $A = P(1+i)^n$

$\therefore 150\,000 = P(1+0,1185)^5$

$\therefore P = \text{R}85\,686,29$

The Mixed Exercise number 2 is on page **348**

3) $A = P(1+i)^n$

$2x = x(1+i)^{6,5}$

$\therefore 2 = (1+i)^{6,5}$

$\therefore \sqrt[6,5]{2} = 1+i$

$\therefore i = 0,1125$

$\therefore i = 11,25\%$

The Mixed Exercise number 3 is on page **348**

4) Deposit: $25\,000(0,1) = \text{R}2\,500$

Hire Purchase: $P = 25\,000 - 2\,500 = \text{R}22\,500$

$A = P(1+in)$

$= 22\,500\left[1+0,138(3)\right]$

$= \text{R}31815$

$\text{Monthly payments} = \frac{31815}{36} + 63,50$

$= \text{R}947,25$

The Mixed Exercise number 4 is on page **348**

5) Total payment = 450(36) = R16 200

$A = P(1+in)$

$\therefore 16\,200 = 12\,400(1+3i)$

$\therefore \frac{81}{62} = 1+3i$

$\therefore i = 0,1022$

$\therefore i = 10,22\%$

The Mixed Exercise number 5 is on page **348**

6) $A = P(1+i)^n$

$\therefore 185\,300 = P(1+0.061)^{10}$

$$\therefore P = \frac{185\,300}{(1+0,061)^{10}}$$

$\therefore P = \text{R}102\ 499,46$

The Mixed Exercise number 6 is on page **348**

7)

a) $1 \text{ barrel} = 56(6,8455) = \text{R}383,348$

$\therefore 7\,500 \text{ barrels} = 7\,500(383,348) = \text{R}2\ 875\ 110$

b) $1 \text{ barrel} = 60(7,2531) = \text{R}435,186$

$\therefore 7\,500 \text{ barrels} = 7\,500(383,348) = \text{R}3\,263\,895$

c) $\% \text{ increase} = \dfrac{\text{new price} - \text{old price}}{\text{old price}} \times 100\%$

$= \dfrac{435,186 - 383,348}{383,348} \times 100\%$

$= 13,52\%$

The Mixed Exercise number 7 is on page 348

8)

a) $\dfrac{100\,000}{6,78} = \$14\,749,26$

b) $30\,000(0,14285) = \text{R}4\,285,50$

The Mixed Exercise number 8 is on page 349

9) $\text{R}6,2287 = ¥100$

$\therefore \text{R}1 = ¥16,05471447$

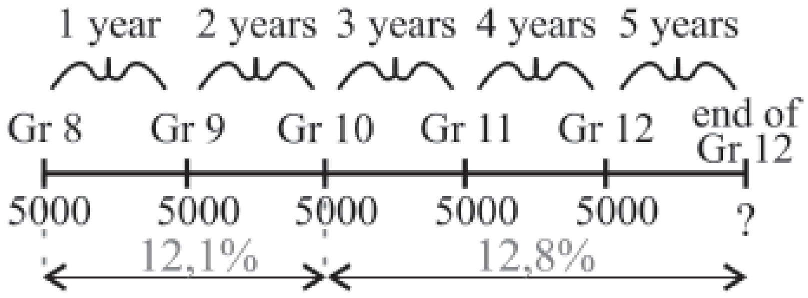
$\therefore \text{R}5,1094 = ¥82,0299581$

$1 \text{ AUD} = \text{R}5,1094$

$\therefore 1 \text{ AUD} = ¥82,0299581$

The Mixed Exercise number 9 is on page 349

10)


1 year
2 years
3 years
4 years
5 years
Gr 8
Gr 9
Gr 10
Gr 11
Gr 12
end of Gr 12
5000
5000
5000
5000
5000
?
12,1%
12,8%

$A = P(1+i)^n$

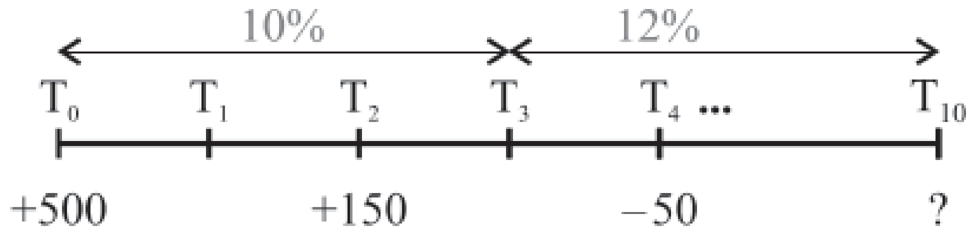
$= 5\,000(1+0,121)^2(1+0,128)^3 + 5\,000(1+0,121)^1(1+0,128)^3 + 5\,000(1+0,128)^3$

$+5\,000(1+0,128)^2 + 5\,000(1+0,128) \rightarrow$ note that this sum runs over two lines

$= \text{R}36\,240,70$

The Mixed Exercise number 10 is on page 349

11)


10%
12%
T_0
T_1
T_2
T_3
T_4 ...
T_{10}
+500
+150
−50
?

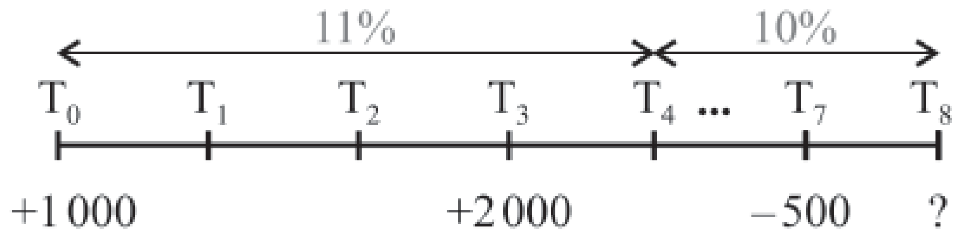
$A = P(1+i)^n$

$= 500(1+0,1)^3(1+0,12)^7 + 150(1+0,1)^1(1+0,12)^7 - 50(1+0,12)^6$

$= \text{R}1737,28$

The Mixed Exercise number 11 is on page 349

12)


11%
10%
T_0
T_1
T_2
T_3
T_4 ...
T_7
T_8
+1 000
+2 000
−500
?

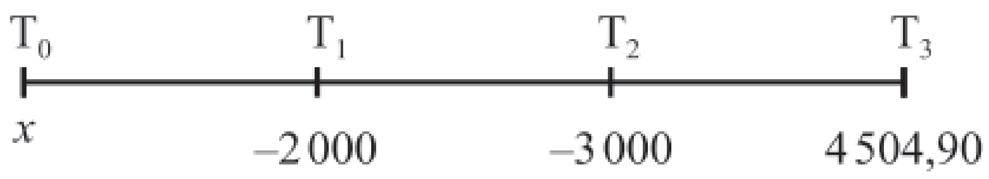
$A = P(1+i)^n$

$= 1000(1+0,11)^4(1+0,1)^4 + 2\,000(1+0,11)^1(1+0,1)^4 - 500(1+0,1)^1$

$= \text{R}4\,922,91$

The Mixed Exercise number 12 is on page 349

13)


T_0
T_1
T_2
T_3
x
−2 000
−3 000
4 504,90

$A = P(1+i)^n$

$4\,504,90 = x(1+0,08)^3 - 2\,000(1+0,08)^2 - 3\,000(1+0,08)^1$

$\therefore 10\ 077,70 = x(1+0,08)^3$

$\therefore x = \dfrac{10\ 077,70}{(1+0,08)^3}$

$\therefore x = \text{R}8\,000$

$\therefore$ Lerato borrowed R8 000.

The Mixed Exercise number 13 is on page 349

Chapter 9 – Data Handling and Statistics

Representation of Data

Exercise 1: (Theory on pages 180 - 182)

1) Using the data in the tables below:

Table A:

29	23	31	34	33	28	28	17	23	32
15	12	28	24	30	23	34	24	15	13

Table B:

14	37	38	15	27	24	15	42	26	44
25	39	28	41	29	29	38	36	35	25

a) Complete the frequency table below for Table A:

Interval	Tally	Frequency
$10 \leq x < 15$		
$15 \leq x < 20$		
$20 \leq x < 25$		
$25 \leq x < 30$		
$30 \leq x < 35$		

b) Draw a stem and leaf plot for the data in table B:

Solutions to Exercise 1 number 1 are on page **360**

2) Wild dogs are classified as an endangered species with fewer than 500 left in the wild in South Africa. In order to judge if populations are increasing in a game reserve, tourist sightings are recorded on a daily basis for a two year period. The results for 4 weeks are shown in the table below. Construct a frequency table using intervals of two to represent the data in the table.

0	0	2	5	3	7	0	8	4	9	3	3	0	7
8	1	2	5	4	2	2	1	0	0	1	3	9	6

Solutions to Exercise 1 number 2 are on page **360**

3) A farmer conducts an experiment using two fertilisers on a variety of different plants under the same conditions. He measures and records the heights of all of the plants 3 weeks after planting. Using his results which are tabulated below construct a back to back stem and leaf plot.

Fertilizer A:

29	23	31	35	33	28	28	17	23	39
15	12	28	24	30	23	34	24	15	13

Fertilizer B:

14	24	35	41	15	25	36	42	15	25
39	44	26	38	27	38	28	39	29	29

Solutions to Exercise 1 number 3 are on page **360**

Measures of Central Tendency

Exercise 2: (Theory on pages 183 - 188)

1) Calculate the mean, median and mode (if it exists) of the following data:

a) 5 ; 8 ; 9 ; 12 ; 28 ; 12

b) 10 ; 13 ; 14 ; 14 ; 17; 16 ; 18 ; 19 ; 22 ; 20; 22 ; 20

c) 11 ; 20 ; 23 ; 0 ; 18; 5; 34 ; 17 ; 25

Solutions to Exercise 2 number 1 are on page **360**

2) Lesedi plays club soccer and is the leading goal scorer in his team. During the season he played 13 matches and the number of goals he scored was recorded. The results are tabulated below.

3	2	0	0	1	2	2	1	0	4	3	1	2

a) Determine the average (mean) number of goals that Lesedi scored per game for the season correct to one decimal place.

b) Determine the median score for the above data.

c) Determine the mode of the above data.

Solutions to Exercise 2 number 2 are on page **360**

3) The results for a test out of 30 of 3 Grade 10 classes are recorded below:

Class 1	16	23	32	39	16	21	13	32	21	11	37	11	19	34	12	11	14	22
Class 2	17	13	17	25	27	22	23	24	29	25	20	28	22	27	12	17	30	-
Class 3	20	25	18	12	16	12	26	20	26	25	21	13	14	26	27	27	22	19

a) Calculate the mean for each class.

b) Calculate the mode for each class.

c) Calculate the median for each class.

Solutions to Exercise 2 number 3 are on page **360**

4) The total of Mr Gumede's water and electricity bill for 2011 is shown in the table below:

January	R590,66	**April**	R723,46	**July**	R985,55	**October**	R732,05
February	R700,53	**May**	R780,34	**August**	R850,55	**November**	R724,48
March	R750,89	**June**	R888,88	**September**	R725,22	**December**	R550,82

a) Determine Mr Gumede's average monthly cost of water and electricity.

b) Determine the median of Mr Gumede's monthly water and electricity bill.

c) Is the median a good indication of Mr Gumede's monthly cost of water and electricity? Justify your answer.

Solutions to Exercise 2 number 4 are on page **361**

5) The mean (average) number of hours that each of 9 employees of a company work in a week is 63 hours.

a) What is the total number of hours that all 9 employees work in a week?

b) If the time the owner of the company worked was not included in the above number of hours, what is the new mean if he is included and he worked a total of 84 hours in the week?

Solutions to Exercise 2 number 5 are on page **361**

6) Thabiso currently has an average of 78% for 5 maths tests that he has written in term 1 and there are 6 maths tests in a term which all count equally.

a) If Thabiso gets 84 for his last test, will he get an average of 80% for term 1?

b) What is the minimum mark Thabiso must get to achieve an average of 80%?

Solutions to Exercise 2 number 6 are on page **361**

7) The mean mass of a school rugby player is 80kg. The heaviest player in the team receives a red card and is sent off the field. Determine the mean mass of the remaining members of the team if the player sent off has a mass of 120kg and there are 15 players in a team.

Solutions to Exercise 2 number 7 are on page **361**

8) In a set of four numbers, one number is repeated twice. The mean is 16,75, the mode is 15 and the median is 16. What are the numbers in the set?

Solutions to Exercise 2 number 8 are on page **361**

Measures of Dispersion (Theory on pages 189 - 193)

Exercise 3:

1) The results for a test out of 100 of two Grade 10 classes are recorded below:

Class 1	68	52	75	66	73	48	38	75	78	82	63	58	44	25	88	75	88	52
Class 2	46	46	47	49	54	55	57	61	63	63	66	69	72	75	75	78	84	88

a) Calculate the mean for each class.

b) Determine the median for each class.

c) Determine the upper and lower quartile for each class.

d) Determine the inter-quartile range for each class.

e) Determine the 70th percentile of class 1 and the 40th percentile of class 2.

f) Explain what is meant by the 70th percentile of class 1 and the 40th percentile of class 2.

Solutions to Exercise 3 number 1 are on page **361**

2) The owner of a small supermarket records the number of cartons of milk purchased daily over a two week period. The data is tabulated below:

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of cartons purchased	25	80	34	26	21	65	28	39	21	30	34	28	35	22

a) Determine the mean number of milk cartons purchased.

b) Determine the median number of milk cartons purchased.

c) Determine the mode(s) for the set of data.

d) Determine the range for the set of data.

e) Determine the 60th percentile of the data set.

f) What does the 60th percentile represent?

Solutions to Exercise 3 number 2 are on page **362**

3) The stem and leaf plot below shows the weights of a Grade 12 class rounded to the nearest kilogram:

a) What is the median weight of the data set?
b) What is the lower quartile weight of the data set?
c) What is the upper quartile weight of the data set?
d) Justify whether the mean or median should be used to describe the data.
e) Is it likely that there are more boys than girls in the class? Justify your answer.

Stem	Leaf
4	4; 5; 5; 9
5	4; 5; 5; 5; 6; 7; 8; 9
6	0; 0; 0; 2; 2; 6; 8; 9
7	2; 3; 5
8	1; 2; 5
9	1
$4\|5 = 45$kg	

Solutions to Exercise 3 number 3 are on page **362**

4) Given the following frequency table:

a) What is the median of the data set?
b) What is mean of the data set?
c) What is the lower quartile of the data set?
d) What is the upper quartile of the data set?

x	Frequency (f)
152	4
156	3
163	7
166	1
168	1
169	1

Solutions to Exercise 3 number 4 are on page **362**

Working with Grouped Data (Theory on pages 193 - 195)

Exercise 4:

1) A paddle boat rental company records the number of hours that their boats are rented for in 1 month. The results are recorded in the frequency table below:

a) Determine the approximate mean number of hours that the boats are rented for each month.
b) Determine the approximate median number of hours the boats are rented for each month.
c) Determine the modal interval of the above data.

Time (h)	Frequency (f)
$0 < x \leq 0{,}5$	12
$0{,}5 < x \leq 1$	27
$1 < x \leq 1{,}5$	35
$1{,}5 < x \leq 2$	48
$2 < x \leq 2{,}5$	20
$2{,}5 < x \leq 3$	8

Solutions to Exercise 4 number 1 are on page **363**

2) Complete the following table and calculate:

Interval	Midpoint (X)	Frequency	$f \times x$	Cumulative frequency
$1\,800 < x \leq 2\,000$		6		
$2\,000 < x \leq 2\,200$		11		
$2\,200 < x \leq 2\,400$		16		
$2\,400 < x \leq 2\,600$		10		
$2\,600 < x \leq 2\,800$		4		
			Total:	

a) The approximate mean for the data set.
b) The interval in which the median lies.
c) The interval in which the upper quartile lies.

Solutions to Exercise 4 number 2 are on page **363**

The Five Number Summary (Theory on pages 196 - 197)

Exercise 5:

1) The data in the table below represents the percentages obtained in a Maths test for 7 learners.

% obtained	**72**	**90**	**80**	**55**	**60**	**30**	**40**

a) Write down the five number summary for the above data.
b) Hence, draw a box and whisker diagram for the given data.

Solutions to Exercise 5 number 1 are on page **364**

2) 32 students in each of 3 Grade 10 History classes wrote a test out of 60. Their marks are recorded on the stem and leaf plots below:

Stem	Grade 10A
1	2; 3; 4; 5; 6; 6; 7
2	0; 0; 1; 2
3	0; 1; 1; 2; 5; 7; 9
4	4; 6; 8; 9; 9
5	0; 2; 2; 6; 6; 7; 8; 8
6	0

4|4 = 44 marks

Stem	Grade 10B
1	2; 2; 2; 3; 3; 5; 7; 9; 9; 9; 9; 9
2	0; 0; 1; 2; 2; 3; 3; 5; 6; 7; 8; 9
3	1; 5; 7; 9
4	0; 0; 0; 0
5	
6	

4|0 = 40 marks

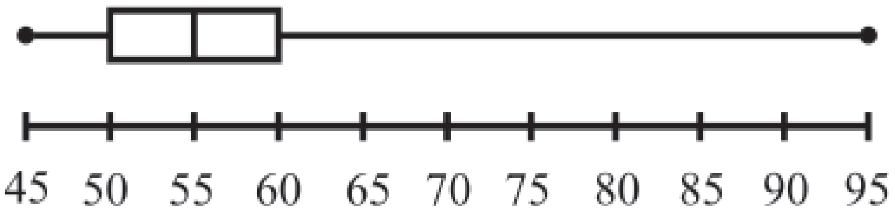
Stem	Grade 10C
1	4
2	0; 7
3	1; 3
4	0; 0; 1; 1; 1; 1; 2; 2; 6; 6; 9
5	1; 1; 1; 2; 3; 4; 4; 6; 6; 7; 8; 9
6	0; 0; 0; 0

4|0 = 40 marks

a) Calculate the five number summary for each of the classes.
b) Draw a box and whisker plot for each of the classes on the same set of axes.

Solutions to Exercise 5 number 2 are on page **364**

3) The box and whisker plot below shows the end of year results of a Grade 10 Mathematics class.



a) What is the median mark for the class?
b) Between which two marks do the lower 25% lie?
c) Between which two marks do the lower 50% lie?
d) Between which two marks do the lower 75% lie
e) The teacher claims to have an average of 65%. Is the mean a good indication of his class's overall performance? Justify your answer and state what measure of dispersion you think should be used to judge the overall performance of his class.

Solutions to Exercise 5 number 3 are on page **364**

Mixed Exercise: **(Chapter 9 is on pages 179 – 197)**

1) A manufacturing plant that produces fans outsources the motors for the fans. They have a choice of two companies to choose from. The data for each of the company's motors is shown below. The data represents the number of hours that each motor can run for without failing. Which company should the manufacturer choose? Explain your answer.

	Company A	Company B
Mean	6000 hours	6000 hours
Minimum hours	800 hours	800 hours
Maximum hours	7500 hours	7500 hours
Median	4000 hours	4000 hours
Interquartile range	150 hours	80 hours
Price per motor	R44	R65

Solutions to the Mixed Exercise Number 1 is on page **365**

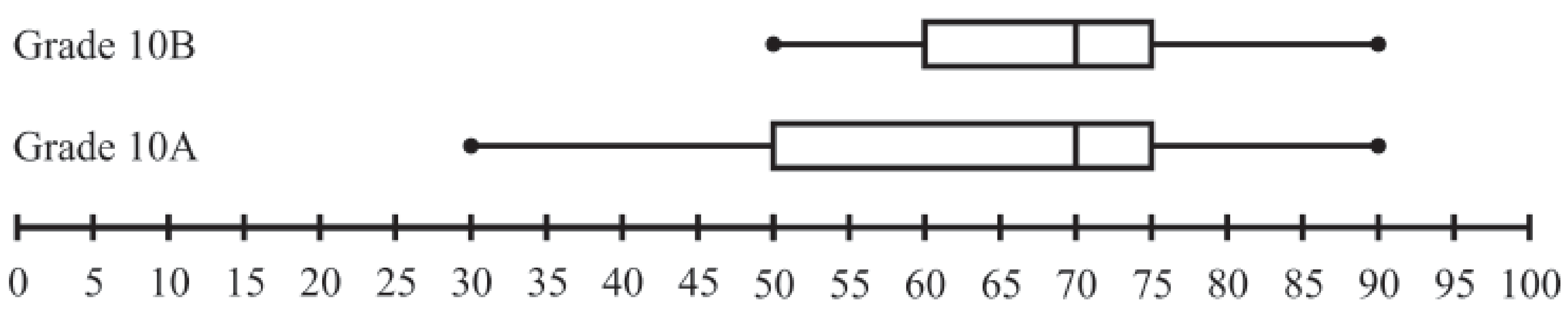
2) Due to complaints by residents, the local city council is considering constructing speed humps in a residential area where there is a speed limit of 60km/h. A local resident is given the task of recording the speeds of 10 cars outside her house during the course of the day. The results of the study are tabulated below.

Speed (km/h)	63	67	80	83	77	87	98	69	54	73

a) Determine the five number summary for the data.
b) Determine the mean speed of the cars.
c) From the above results should the city erect speed humps in the area? Justify your answer.
d) Do you think that the above statistics are reliable?

Solutions to the Mixed Exercise Number 2 is on page **365**

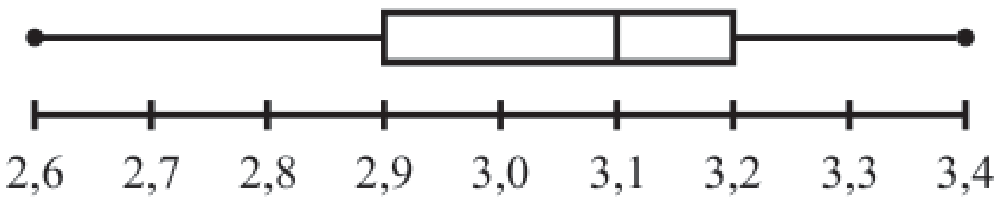
3) The box and whisker diagrams below show the end of year results of two grade 10 Mathematics classes:



a) What is the median mark for each class?
b) What was the lowest mark in class B?
c) What percentage of the class marks lie between the maximum mark and the median?

Solutions to the Mixed Exercise Number 3 is on page **365**

4) The following box and whisker plot represents the mass of 60 babies born at the Johannesburg Hospital. Solve for *w*, *x*, *y* and *z* in the frequency table below using the box and whisker diagram.



Mass interval	Frequency
$2{,}6 \leq x < 2{,}7$	w
$2{,}7 \leq x < 2{,}8$	6
$2{,}8 \leq x < 2{,}9$	7
$2{,}9 \leq x < 3{,}0$	7
$3{,}0 \leq x < 3{,}1$	x
$3{,}1 \leq x < 3{,}2$	y
$3{,}2 \leq x < 3{,}3$	z
$3{,}3 \leq x < 3{,}4$	2

Solutions to the Mixed Exercise Number 4 is on page **365**

Chapter 9 – Statistics (Solutions)

Exercise 1 Solutions:

1)

a)

Interval	Tally	Frequency
$10 \le x < 15$	||	2
$15 \le x < 20$	|||	3
$20 \le x < 25$	卌	5
$25 \le x < 30$	||||	4
$30 \le x < 35$	卌 |	6

b)

Stem	Leaf
1	4, 5, 5
2	4, 5, 5, 6, 7, 8, 9, 9
3	5, 6, 7, 8, 8, 9
4	1, 2, 4
Key: 1|4 represents the number 14	

Exercise 1 number 1 is on page 355

2)

Sightings (x)	Frequency
$0 \le x < 2$	9
$2 \le x < 4$	8
$4 \le x < 6$	4
$6 \le x < 8$	3
$8 \le x < 10$	4

3)

Fertilizer B	Stem	Fertilizer A
5, 5, 4	1	2, 3, 5, 5, 7
9, 9, 8, 7, 6, 5, 5, 4	2	3, 3, 3, 4, 4, 8, 8, 8, 9
9, 8, 8, 7, 6, 5	3	0, 1, 3, 4, 5, 9
4, 2, 1	4	
Key: 1|2 represents the number 12		

Exercise 1 numbers 2 and 3 are on page 355

Exercise 2 Solutions:

1) As you are required to determine the median, arrange the data in ascending order:

a) 5; 8; 9; 12; 12; 28

$$\text{median} = \frac{9+12}{2} = 10{,}5$$

$$\text{mean} = \frac{74}{6} = 12{,}33$$

$$\text{mode} = 12$$

b) 10; 13; 14; 14; 16; 17; 18; 19; 20; 20; 22, 22

$$\text{median} = \frac{17+18}{2} = 17{,}5$$

$$\text{mean} = \frac{205}{12} = 17{,}08$$

$$\text{modes} = 14,\ 20 \text{ and } 22$$

c) 0; 5; 11; 17; 18; 20; 23; 25; 34

$$\text{median} = 18$$

$$\text{mean} = \frac{153}{9} = 17$$

$$\text{modes} = \text{none}$$

Exercise 2 number 1 is on page 355

2) As you are required to determine the median, arrange the data in ascending order:

0; 0; 0; 1; 1; 1; 2; 2; 2; 2; 3; 3; 4

a) $\text{mean} = \frac{21}{13} = 1{,}62$ **b)** $\text{median} = 2$ **c)** $\text{mode} = 2$

Exercise 2 number 2 is on page 355

3) As you are required to determine the median, arrange the data in ascending order:

Class 1	11	11	11	12	13	14	16	16	19	21	21	22	23	32	32	34	37	39
Class 2	12	13	17	17	17	20	22	22	23	24	25	25	27	27	28	29	30	-
Class 3	12	12	13	14	16	18	19	20	20	21	22	25	25	26	26	26	27	27

a)

$$\text{class 1: mean} = \frac{384}{18} = 21{,}33$$

$$\text{class 2: mean} = \frac{378}{17} = 22{,}24$$

$$\text{class 3: mean} = \frac{369}{18} = 20{,}5$$

b)

class 1: mode = 11

class 2: mode = 17

class 3: mode = 26

c)

$$\text{class 1: median} = \frac{19+21}{2} = 20$$

$$\text{class 2: median} = 23$$

$$\text{class 3: median} = \frac{20+21}{2} = 20{,}5$$

Exercise 2 number 3 is on page 355

4) As you are required to determine the median, arrange the data in ascending order:

550,82	590,66	700,53	723,46	724,48	725,22	732,05	750,89	780,34	850,55	888,88	985,55

a) $\text{mean} = \dfrac{9003,43}{12} = \text{R}\,750,29$

b) $\text{median} = \dfrac{725,22 + 732,05}{2} = \text{R}\,728,64$

c) No, electricity and water bills vary between different seasons. During winter, electricity costs are larger than during summer. The median value is in the middle of winter which usually has the largest cost of electricity.

Exercise 2 number 4 is on page **356**

5)

a) $\text{mean} = \dfrac{\text{total hours worked}}{\text{number of people}}$

$\therefore 63 = \dfrac{\text{total hours worked}}{9}$

$\therefore \text{total hours worked} = 567 \text{ hours}$

b) $\text{new mean} = \dfrac{567 + 84}{10}$

$= 65,1 \text{ hours}$

Exercise 2 number 5 is on page **356**

6)

a) $\text{mean}_{5 \text{ tests}} = \dfrac{\text{sum of tests}}{5}$

$\therefore 78 = \dfrac{\text{sum of tests}}{5}$

$\therefore \text{sum of tests} = 390$

$\therefore \text{mean}_{final} = \dfrac{390 + 80}{6} = 79\%$

$\therefore$ he does not achieve an 80% average

b) $80 = \dfrac{390 + x}{6}$

$\therefore 480 = 390 + x$

$\therefore x = 90\%$

$\therefore$ He must obtain a minimum mark of 90%

Exercise 2 number 6 is on page **356**

7) $\text{mean} = \dfrac{\text{total mass}}{\text{number of people}}$

$\therefore 80 = \dfrac{\text{total mass}}{15}$

$\therefore 1200 = \text{total mass}$

$\therefore \text{mean}_{new} = \dfrac{1200 - 120}{14}$

$= 77,14 \text{ kg}$

Exercise 2 number 7 is on page **356**

8) 15 is repeated twice as it is the mode. If the two 15's were the middle numbers the median would be 15. Therefore, the 15's must be the first two numbers.

$\therefore \dfrac{15 + x}{2} = 16$

$\therefore x = 17$

$\text{mean} = \dfrac{15 + 15 + 17 + y}{4}$

$\therefore 16,75 = \dfrac{15 + 15 + 17 + y}{4}$

$\therefore 67 = 47 + y$

$\therefore y = 20$

$\therefore$ the numbers are: 15, 15, 17 and 20

Exercise 2 number 8 is on page **356**

Exercise 3 Solutions:

1) As you are required to determine the median, arrange the data in ascending order:

Class 1	25	38	44	48	52	52	58	63	66	68	73	75	75	75	78	82	88	88
Class 2	46	46	47	49	54	55	57	61	63	63	66	69	72	75	75	78	84	88

a) $\text{class 1: mean} = \dfrac{1148}{18} = 63,78\%$

$\text{class 2: mean} = \dfrac{1148}{18} = 63,78\%$

b) $\text{class 1: median} = \dfrac{66 + 68}{2} = 67\%$

$\text{class 2: median} = \dfrac{63 + 63}{2} = 63\%$

c) class 1: upper quartile = 75%
class 2: upper quartile = 75%
class 1: lower quartile = 52%
class 2: lower quartile = 54%

d) class 1: interquartile range = 23%
class 2: interquartile range = 21%

e) class 1: 70th percentile

$\text{position} = \dfrac{\text{percentile}}{100} \times \text{number of values in data set}$

$\therefore \text{position} = 0,7 \times 18$

$= 12,6 \approx 13$

$\therefore 70^{\text{th}} \text{ percentile} = 75\%$

f) It means that in class 1, 70% of the marks are less than or equal to 75% and in class 2, 40% of the marks are less than or equal to 57%

Exercise 3 number 1 is on page **356**

2) As you are required to determine the median, arrange the data in ascending order:

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of cartons purchased	21	21	22	25	26	28	28	30	34	34	35	39	65	80

a) $\text{mean} = \frac{488}{14} = 34{,}86 \text{ cartons/day}$

b) $\text{median} = \frac{28+30}{2} = 29 \text{ cartons}$

c) modes = 12 and 28 and 34

d) range = 59

e) $\text{position} = \frac{\text{percentile}}{100} \times \text{number of values in data set}$

$\therefore \text{position} = 0{,}6 \times 14$

$= 8{,}4 \approx 8$

$\therefore 60^{\text{th}}$ percentile = 30 cartons

f) It means that on 60% of days up to 30 cartons are purchased.

Exercise 3 number 2 is on page **356**

3)

Stem	Leaf
4	4, 5, 5, 9
5	4, 5, 5, 5, 6, 7, 8, 9
6	0, 0, 0, 2, 2, 6, 8, 9
7	2, 3, 5
8	1, 2, 5
9	1
4|5 = 45kg	

a) median = 60kg (position 14)

b) lower quartile = 55kg (position 7)

c) upper quartile = 72kg (position 21)

d) range = 59

e) There are outliers in the data set which could skew the mean. Therefore, median should be used.

f) No. 75% of learners are below 72kg. It is more likely that the class has more girls than boys.

Exercise 3 number 3 is on page **357**

4) Add a total row to the frequency column

x	Frequency (*f*)
152	4
156	3
163	7
166	1
168	1
169	1
	Total: 17

a) $\text{median position} = \frac{17+1}{2} = 9$

Looking down the table the 9th data point is 163.

$\therefore \text{median} = 163$

b)
$$\text{mean} = \frac{152 \times 4 + 156 \times 3 + 163 \times 7 + 166 + 168 + 169}{17}$$
$$= \frac{2720}{17}$$
$$= 160$$

c) There are 8 data points in the lower quartile.

$\therefore \text{lower quartile position} = \frac{8+1}{2} = 4{,}5$

Looking down the table the $4{,}5^{\text{th}}$ data point is between 152 and 156.

$\therefore \text{lower quartile} = \frac{152+156}{2} = 154$

d) There are 8 data points in the upper quartile.

$\therefore \text{upper quartile position} = 9 + \frac{8+1}{2} = 13{,}5$

Looking down the table the $13{,}5^{\text{th}}$ data point is between 163 and 163.

$\therefore \text{upper quartile} = \frac{163+163}{2} = 163$

Exercise 3 number 4 is on page **357**

Exercise 4 Solutions:

1) When determining the mean you need to add a midpoint and (frequency × midpoint) column to the table:

a)

Time (h)	Midpoint (X)	Frequency (f)	$f \times X$
$0 \leq x < 0,5$	$\frac{0+0,5}{2} = \mathbf{0,25}$	12	3
$0,5 \leq x < 1$	$\frac{0,5+1}{2} = \mathbf{0,75}$	27	20,25
$1 \leq x < 1,5$	$\frac{1+1,5}{2} = \mathbf{1,25}$	35	43,75
$1,5 \leq x < 2$	$\frac{1,5+2}{2} = \mathbf{1,75}$	48	84
$2 \leq x < 2,5$	$\frac{2+2,5}{2} = \mathbf{2,25}$	20	45
$2,5 \leq x < 3$	$\frac{2,5+3}{2} = \mathbf{2,75}$	8	22
	Total:	**150**	**218**

$\therefore$ approximate mean $= \frac{218}{150} = 1,45$ hours

b) To determine the median add a cumulative frequency column to the table:

Time (h)	Frequency	Cumulative frequency
$0 \leq x < 0,5$	12	12
$0,5 \leq x < 1$	27	39
$1 \leq x < 1,5$	35	74
$1,5 \leq x < 2$	48	122
$2 \leq x < 2,5$	20	142
$2,5 \leq x < 3$	8	150

position of median $= \frac{150+1}{2} = 75,5$

This lies in the interval of 1,5 to 2 hours. Therefore the approximate median is the midpoint of the interval.
$\therefore$ the approximate median is **1,75 hours**.

c) The modal interval **1,5 to 2 hours.**

Exercise 4 number 1 is on page 357

2)

Interval	Midpoint (X)	Frequency (f)	$f \times X$	Cumulative frequency
$1\,800 < x \leq 2\,000$	1900	6	11 400	6
$2\,000 < x \leq 2\,200$	2100	11	23 100	17
$2\,200 < x \leq 2\,400$	2300	16	36 800	33
$2\,400 < x \leq 2\,600$	2500	10	25 000	43
$2\,600 < x \leq 2\,800$	2700	4	10 800	47
			Total: 107 100	

a) approximate mean $= \frac{107100}{47} = 2\,278,72$

b) position of median $= \frac{47+1}{2} = 24$

$\therefore$ The median lies in the interval $2\,200 < x \leq 2\,400$

c) There are 23 data points in the upper quartile:

$\therefore$ position of upper quartile $= 24 + \frac{23+1}{2} = 36$

$\therefore$ the upper quartile lies in the interval $2\,400 < x \leq 2\,600$

Exercise 4 number 2 is on page 357

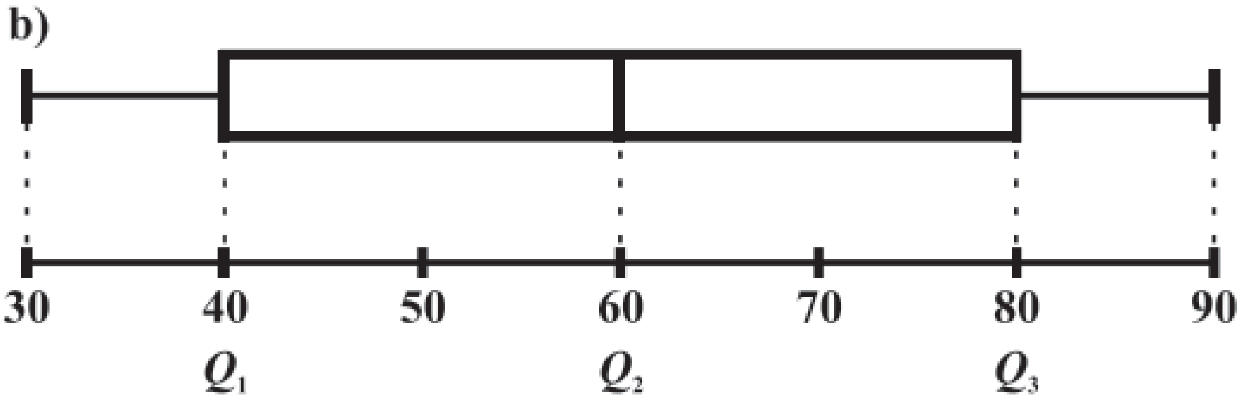
Exercise 5 Solutions:

1) Write the data in ascending order as you have to determine the five number summary:

% obtained	30	**40**	55	**60**	72	**80**	90

a) Lowest value = 30
Lower quartile = 40
Median = 60
Upper quartile = 80
Maximum value = 90

b)



Exercise 5 number 1 is on page 357

2)

a) Class 10A:

lowest value = 12

median position $= \frac{33}{2} = 16,5$

$\therefore$ **median** $= \frac{35+37}{2} = 36$

lower quartile position $= \frac{16+1}{2} = 8,5$

$\therefore$ **lower quartile** $= \frac{20+20}{2} = 20$

upper quartile position $= 16 + \frac{16+1}{2} = 24,5$

$\therefore$ **upper quartile** $= \frac{50+52}{2} = 51$

maximum value = 60

Class 10C:

lowest value = 14

median position $= \frac{33}{2} = 16,5$

$\therefore$ **median** $= \frac{49+51}{2} = 50$

lower quartile position $= \frac{16+1}{2} = 8,5$

$\therefore$ **lower quartile** $= \frac{41+41}{2} = 41$

upper quartile position $= 16 + \frac{16+1}{2} = 24,5$

$\therefore$ **upper quartile** $= \frac{56+56}{2} = 56$

maximum value = 60

Class 10B:

lowest value = 12

median position $= \frac{33}{2} = 16,5$

$\therefore$ **median** $= \frac{22+22}{2} = 22$

lower quartile position $= \frac{16+1}{2} = 8,5$

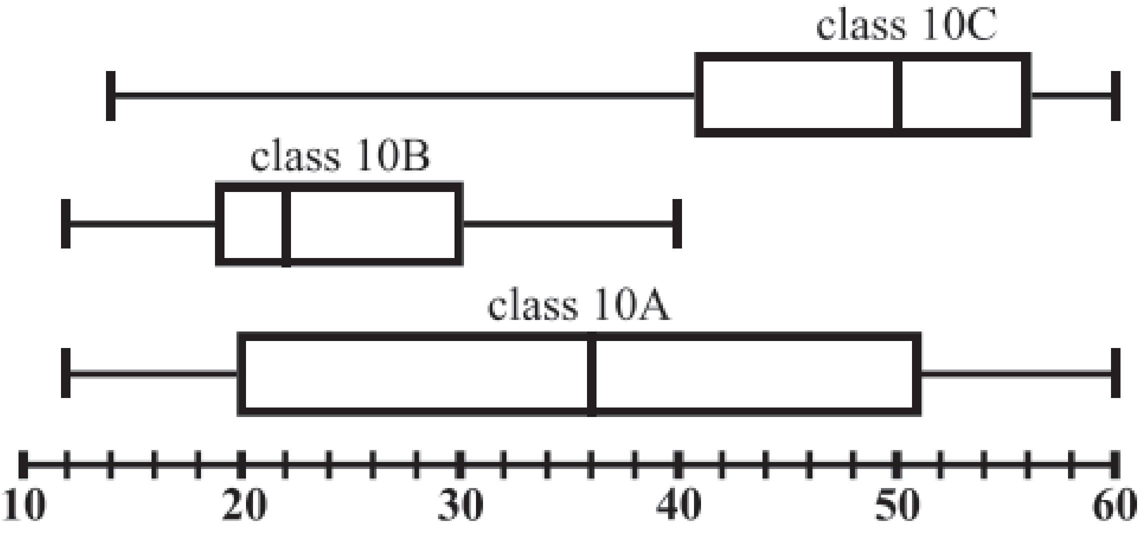
$\therefore$ **lower quartile** $= \frac{19+19}{2} = 19$

upper quartile position $= 16 + \frac{16+1}{2} = 24,5$

$\therefore$ **upper quartile** $= \frac{29+31}{2} = 30$

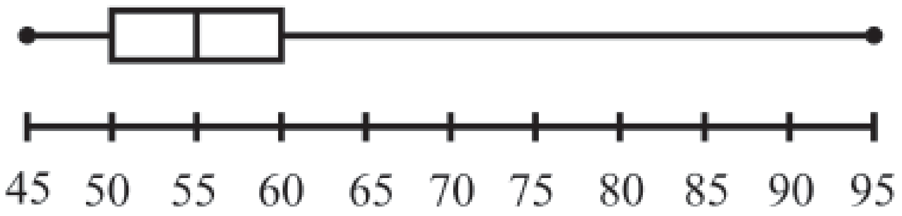
maximum value = 40

b)



Exercise 5 number 2 is on page 358

3)



a) 55%
b) 45% and 50%
c) 45% and 55%
d) 45% and 60%

e) It can be seen from the box and whisker plot that 75% of the class's marks are less than or equal to 60%. Clearly there are outliers in the class which are skewing the mean. The median would provide a more accurate representation of the class's performance as it is not affected by outliers.

Exercise 5 number 3 is on page 358

Mixed Exercise Solutions:

1) Although on average, both companies' motors last for the same amount of time, the motor from company B is more expensive than that of company A. However, it also has a much lower interquartile range. This means that the data is less spread around the median and is therefore more consistent in the time that they last. The manufacturer should therefore choose company B.

The Mixed Exercise number 1 is on page **358**

2) Arrange the data in ascending order to find the five number summary.

Speed (km/h)	54	63	67	69	73	77	80	83	87	98

a) Minimum value = 54 km/h

Lower quartile = 67 km/h

Median $= \dfrac{73+77}{2} = 75$ km/h

Upper quartile = 83 km/h

Maximum value = 98 km/h

b) mean $= \dfrac{751}{10} = 75{,}1$ km/h

c) On average the cars are breaking the speed limit. It can also be seen that at least 75% of cars are travelling above the speed limit (the lower quartile represents the 25th percentile). Based on the results speed humps are required on this road.

d) The sample space of the data is too small. The speed of a large number of cars should have been recorded on many roads in the area. Also, the person conducting the survey should have been independent and not a resident in the area.

The Mixed Exercise number 2 is on page **358**

3)

a) 70%

b) 50%

c) 50%

The Mixed Exercise number 3 is on page **359**

4) As there are 60 values in the data set, there are 15 values between the minimum value and lower quartile.

$\therefore w+6+7=15$

$\therefore w=2$

There are 15 values between the lower quartile and the median.

$\therefore 7+x=15$

$\therefore x=8$

There are 15 values between the median and the upper quartile. The interval of $3{,}1 < x \leq 3{,}2$ consists of the median through to the upper quartile.

$\therefore y=15$

There are 15 values between the upper quartile and the maximum value.

$\therefore z+2=15$

$\therefore z=13$

The Mixed Exercise number 4 is on page **359**

Chapter 10 – Shapes and Measurement

Surface Area and Volume of Rectangular Prisms

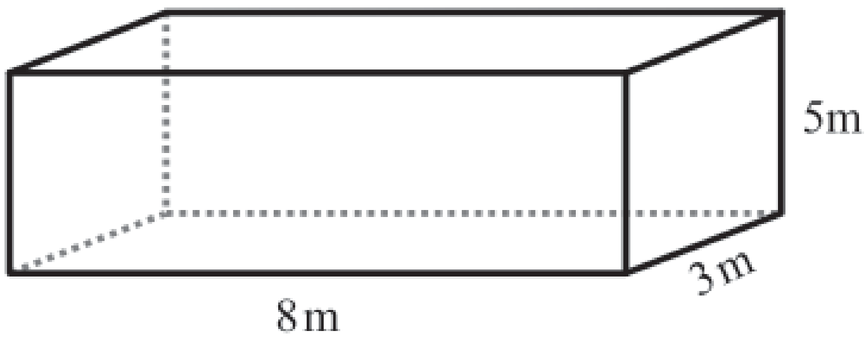
Exercise 1: (Theory on pages 199 - 201)

1) Calculate the surface area and volume of the following closed rectangular prisms with dimensions:

a) Length = 10cm Breadth = 5cm Height = 8cm
b) Length = 6cm Breadth = 10cm Height = 5cm
c) Length = 3cm Breadth = 7cm Height = 9cm
d) Length = 12cm Breadth = 5cm Height = 6cm

Solutions to Exercise 1 number 1 are on page **370**

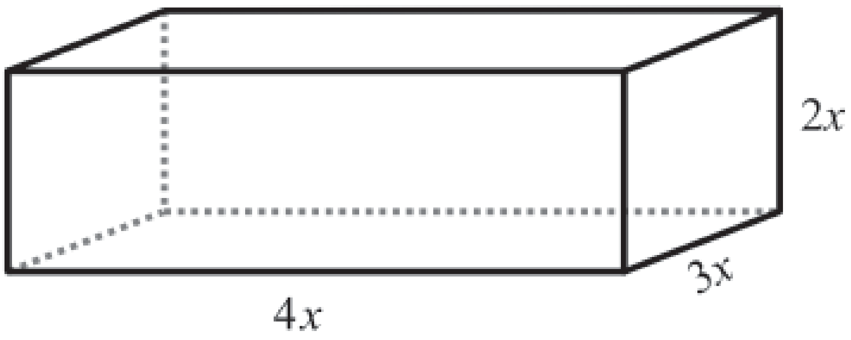
2) Given the rectangular prism below:



a) Calculate the surface area of the prism.
b) Calculate the surface area of the prism if the length is doubled.
c) Calculate the surface area of the prism if the width is doubled.
d) Calculate the volume of the prism.
e) Calculate the volume of the prism if the length is doubled.
f) Calculate the volume of the prism if the height is doubled and the length is doubled.
g) Calculate the volume of the prism if the length is doubled and the breadth is halved.

Solutions to Exercise 1 number 2 are on page **370**

3) Given the Rectangular Prism below:



a) Calculate the surface area of the prism in terms of x.
b) Calculate the volume of the prism in terms of x.
c) Calculate the surface area of the prism in terms of x if the height is doubled.
d) Calculate the volume of the prism in terms of x if the height is doubled.
e) Calculate the volume in terms of x and k if the length and breadth are multiplied by a scale factor of k.

Solutions to Exercise 1 number 3 are on page **370**

Surface Area and Volume of Cylinders

Exercise 2: (Theory on pages 202 - 204)

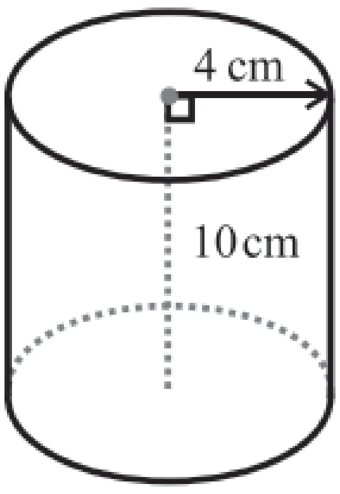
1) Calculate the surface areas of the following cylinders with dimensions:

a) The cylinder is closed and has a radius of 3cm and a height of 4cm.
b) The cylinder is open at one end and has a radius of 3cm and a height of 4cm.
c) The cylinder is open at both ends and has a radius of 6cm and a height of 5cm.
d) The cylinder is open at the top and has a radius of 6cm and a height of 5cm.

Solutions to Exercise 2 number 1 are on page **371**

2) Given the cylinder alongside:

a) Calculate the volume of the cylinder.
b) Calculate the volume of the cylinder if the radius is doubled.
c) Calculate the volume of the cylinder if the height is doubled.



Solutions to Exercise 2 number 2 are on page **371**

Surface Area and Volume of Spheres, Cones and Pyramids

Exercise 3: **(Theory on pages 205 - 209)**

1) Determine the volume and surface area of the following shapes:

a) A sphere with a radius of 6cm.

b) A solid hemisphere with a radius of 3m.

Solutions to Exercise 3 number 1 are on page **371**

2) Determine the volume of the following shapes:

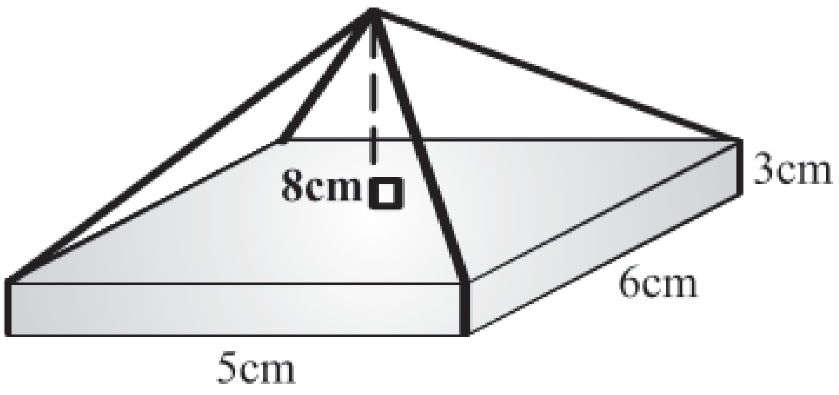
a) A triangular pyramid that has a height of 2cm and a slant height of 8cm, if the area of the base is 6cm^2.

b) A square pyramid that has a length of 8m and a height of 3m.

c) A cone that has a slant height of 500cm and radius of 3m.

Solutions to Exercise 3 number 2 are on page **371**

3) A glass decorative ornament is made from a hollow pyramid attached to a solid rectangular base with dimensions as shown in the diagram below. The ornament is then filled with 2 immiscible liquids of different colours.



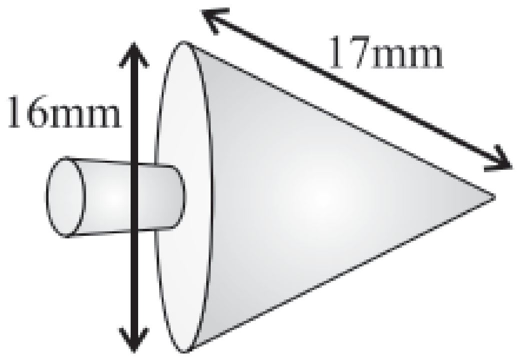
a) What volume of liquid would be required to fill the above ornament?

b) What volume of glass is needed to construct the base?

c) If the base of the ornament was changed to a hollow, open aluminium base, how much aluminium would be needed for the construction of the base?

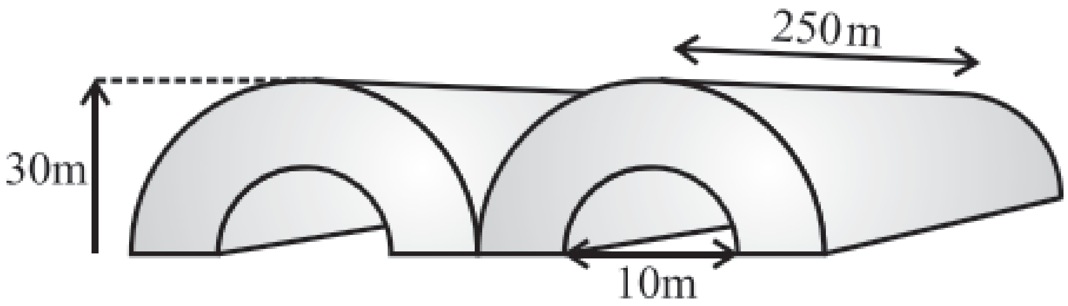
Solutions to Exercise 3 number 3 are on page **371**

4) A company uses the conical drill bit shown below to drill 12 holes into a block of plastic in order to make an ice tray. Calculate the volume of water that the ice tray can hold and give your answer to the nearest cubic millimetre. Assume that the drill bit fits exactly into the holes.



Solutions to Exercise 3 number 4 are on page **371**

5) Two tunnels each 250m in length are constructed through a mountain and reinforced using concrete shells as shown below.



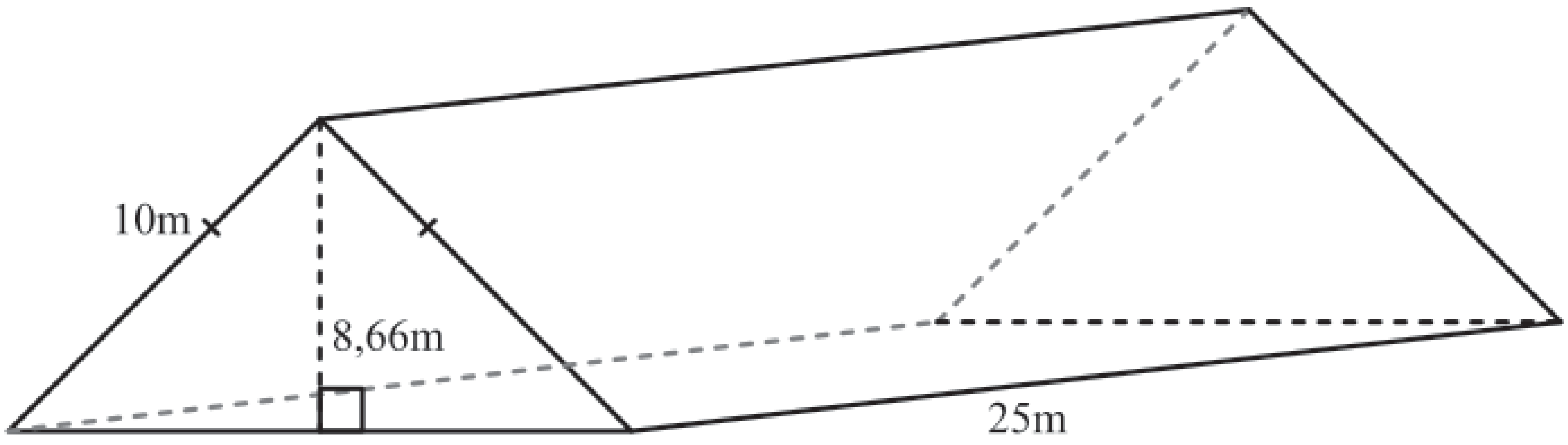
a) What volume of concrete is required to produce the shells?

b) If the inside of the tunnels are to be coated in white paint, how many tins of paint would be required if one tin can coat an area of 350m^2.

Solutions to Exercise 3 number 5 are on page **372**

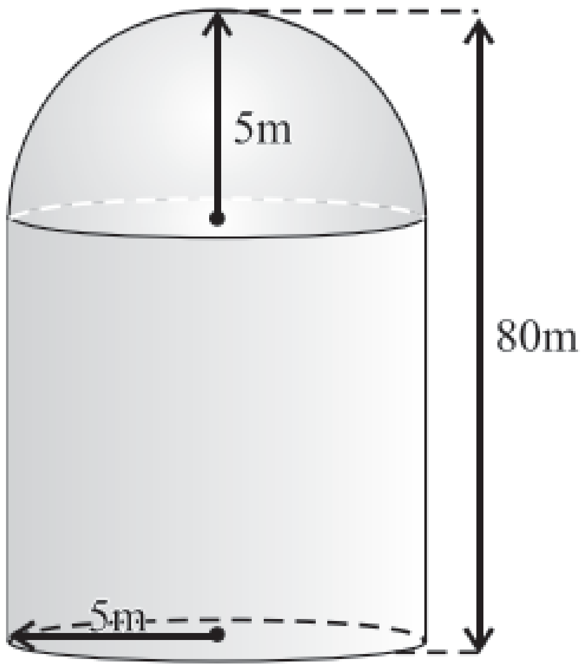
Mixed Exercise: **(Chapter 10 is on pages 198 – 209)**

1) A company is manufacturing a shade tent with the dimensions shown below. Calculate the amount of material used to manufacture the tent (assume there is no wastage and that the floor isn't covered).



Solutions to The Mixed Exercise number 1 are on page **372**

2) After harvesting, farmers often store their harvest in large storage silos. Such a silo is made up of a cylinder and a hemisphere. The height of the silo is 80m and the radius of the cylinder and hemisphere is 5m as shown below:



a) If the farmer paints his silo, how much paint will he need? Give your answer in m^2.

b) What volume of grain can the silo hold?

Solutions to The Mixed Exercise number 2 are on page **372**

3) Given the dimensions of cylinders A to D below, complete the table and answer the questions that follow:

Cylinder	r (m)	h (m)	Volume (m^3)
A	5	6	
B	5	12	
C	10	6	
D	10	12	

Give all answers in their simplest form:

a) What is the ratio of the volume of cylinder A to B?

b) What is the ratio of the volume of cylinder C to A?

c) What is the ratio of the volume of cylinder D to A?

d) If the radius is doubled what effect does it have on the volume of a cylinder?

e) If the height is doubled what effect does it have on the volume of a cylinder?

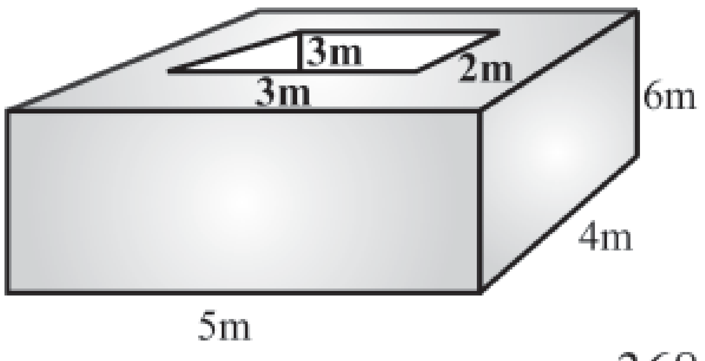
f) If the radius and height are double what effect does it have on the volume of a cylinder?

Solutions to The Mixed Exercise number 3 are on page **372**

4) A splash pool is constructed from a 5m by 4m by 6m rectangular prism of granite. In order to make the pool, a smaller 3m by 2m by 3m rectangular prism is cut out of the larger prism.

a) What is the volume of the splash pool?

b) What is the surface area of the remaining granite of the pool?

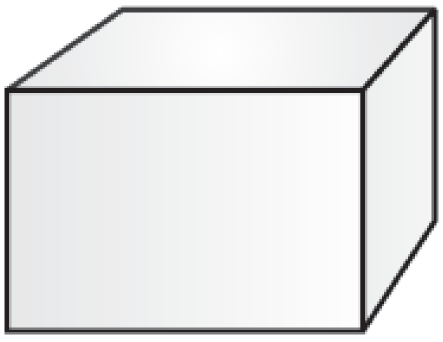


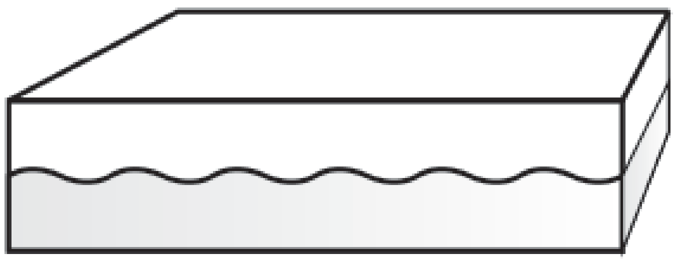
Solutions to The Mixed Exercise number 4 are on page **373**

5) **A cube with sides 12m in length is filled with water. The water in the cube is then emptied into a rectangular prism with a length of 18m and a width of 8m.**

a) What is the volume of water in the cube?

b) What would the height of the water be in the rectangular prism?





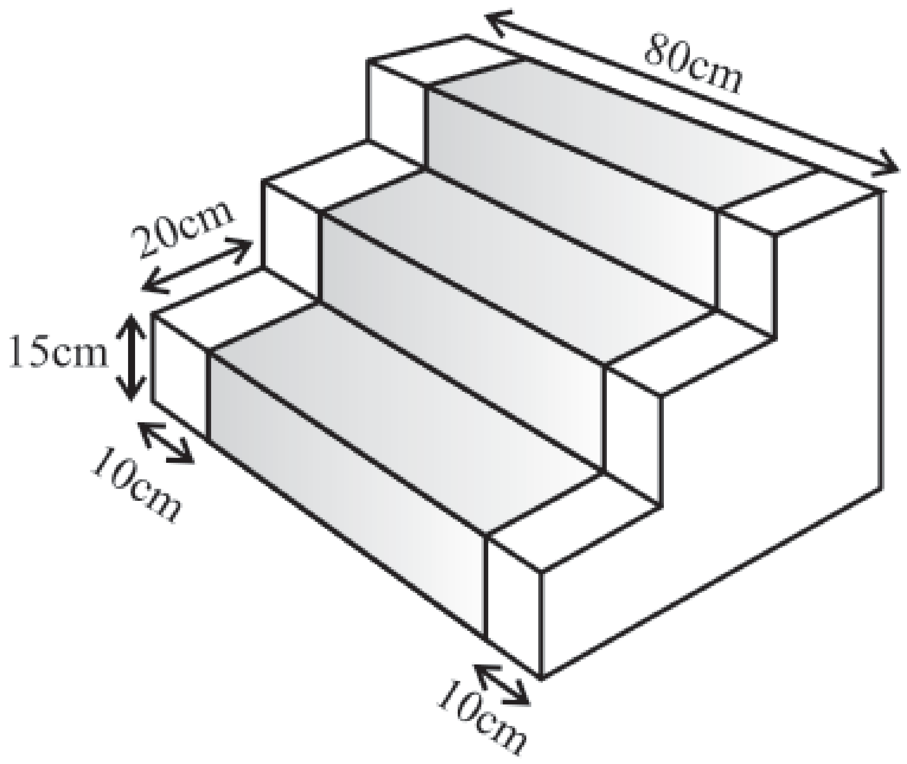
Solutions to The Mixed Exercise number 5 are on page **373**

6) **A rug is used to cover the middle and front section of a set of stairs as shown in the diagram below:**

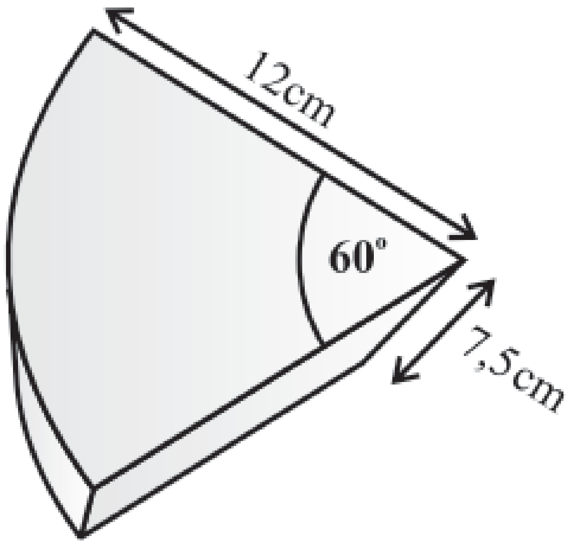
a) **Determine the surface area of the rug.**

b) **What is the volume of the staircase?**

Solutions to The Mixed Exercise number 6 are on page **373**

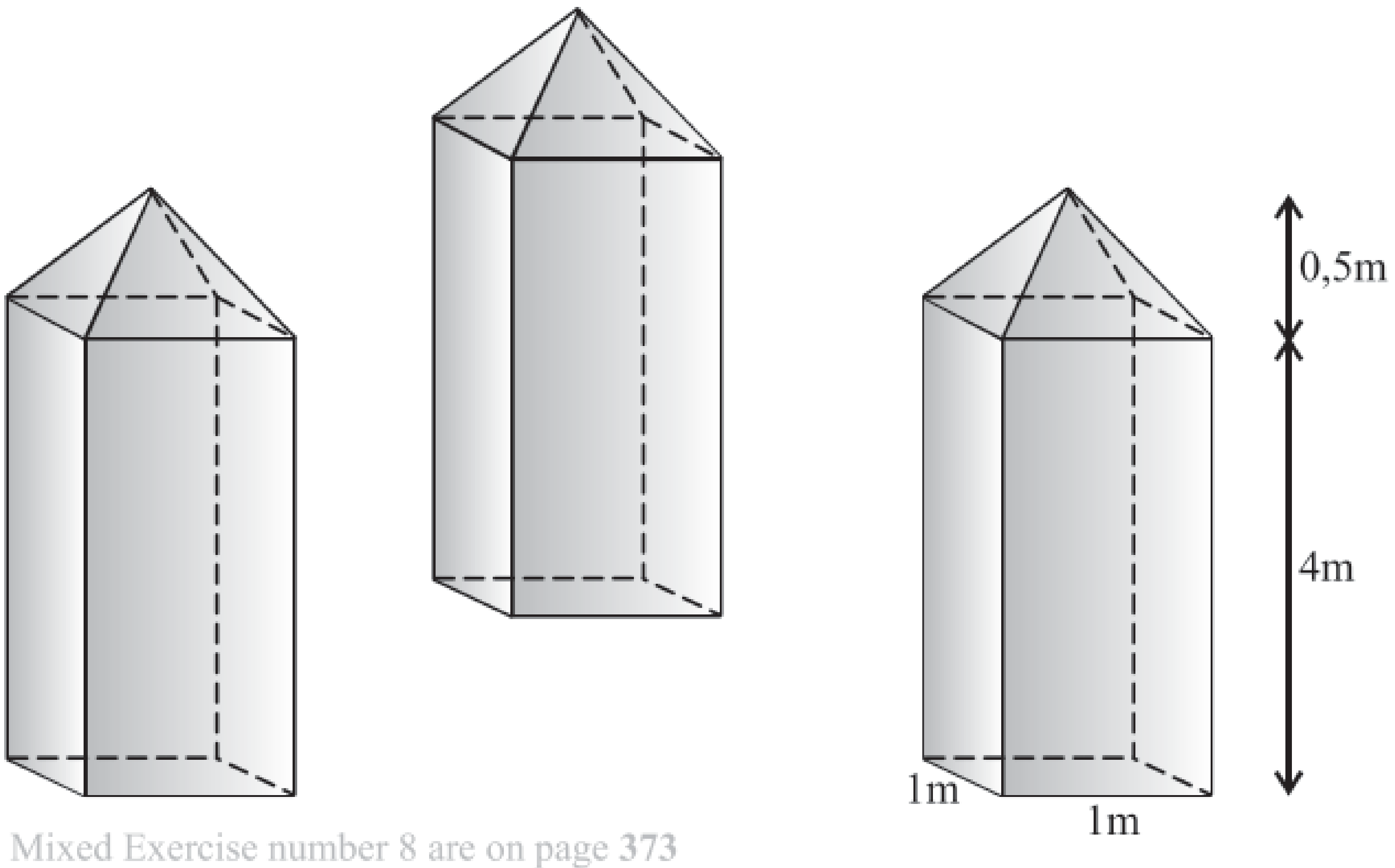


7) **A cake factory produces cakes with a height of 7,5cm and a radius of 12cm. The cakes are then cut at 60° angles and packaged in a box as shown below. What volume of cake does each box hold?**



Solutions to The Mixed Exercise number 7 are on page **373**

8) **An architect is designing a memorial tribute to soldiers that were killed in World War II. He decides to make 3 identical pillars out of concrete and have the names of every soldier that died engraved on the pillars. Each pillar will be constructed from a rectangular prism and a square right pyramid with dimensions as shown below. Determine the volume of concrete needed to construct all of the pillars.**



Solutions to The Mixed Exercise number 8 are on page **373**

Chapter 10 – Shapes and Measurement (Solutions)

Exercise 1 Solutions:

1)

a) $\text{SA} = (2)(10)(5) + (2)(10)(8) + (2)(8)(5)$

$= 340\,\text{cm}^2$

$\text{V} = 10(5)(8) = 400\,\text{cm}^3$

b) $\text{SA} = (2)(6)(10) + (2)(6)(5) + (2)(10)(5)$

$= 280\,\text{cm}^2$

$\text{V} = 10(6)(5) = 300\,\text{cm}^3$

c) $\text{SA} = (3)(7)(2) + (3)(9)(2) + (7)(9)(2)$

$= 222\,\text{cm}^2$

$\text{V} = 3(7)(9) = 189\,\text{cm}^3$

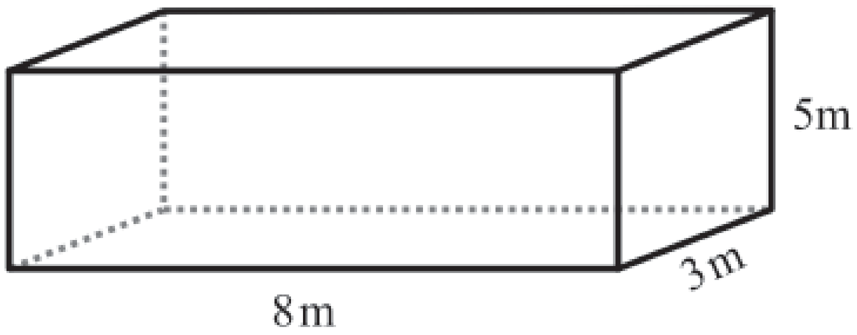
d) $\text{SA} = (2)(12)(5) + (2)(12)(6) + (2)(5)(6)$

$= 324\,\text{cm}^2$

$\text{V} = 12(5)(6) = 360\,\text{cm}^3$

Exercise 1 number 1 is on page 366

2)



a) $\text{SA} = (8)(3)(2) + (8)(5)(2) + (5)(3)(2)$

$= 158\,\text{m}^2$

b) $\text{SA} = (16)(3)(2) + (16)(5)(2) + (5)(3)(2)$

$= 286\,\text{m}^2$

c) $\text{SA} = (8)(6)(2) + (8)(5)(2) + (5)(6)(2)$

$= 236\,\text{m}^2$

d) $\text{V} = 8(3)(5)$

$= 120\,\text{m}^3$

e) $\text{V} = 120(2)$

$= 240\,\text{m}^3$

f) $\text{V} = 120(2)(2)$

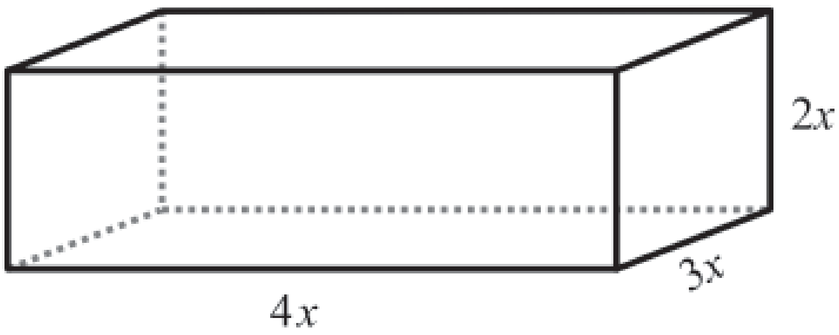
$= 480\,\text{m}^3$

g) $\text{V} = 120(2)(0{,}5)$

$= 120\,\text{m}^3$

Exercise 1 number 2 is on page 366

3)



a) $\text{SA} = (4x)(3x)(2) + (4x)(2x)(2) + (3x)(2x)(2)$

$= 52x^2\ \text{units}^2$

b) $\text{V} = 4x(3x)(2x)$

$= 24x^3\ \text{units}^3$

c) $\text{SA} = (4x)(3x)(2) + (4x)(4x)(2) + (3x)(4x)(2)$

$= 80x^2\ \text{units}^2$

d) $\text{V} = 24x^3(2)$

$= 48x^3\ \text{units}^3$

e) $\text{V} = 24x^3\left(k^2\right)$

$= 24k^2x^3\ \text{units}^3$

Exercise 1 number 3 is on page 366

Exercise 2 Solutions:

1)

a) $$\begin{aligned}\text{SA} &= 2\pi r^2 + 2\pi rH\\ &= 2\pi(3)^2 + 2\pi(3)(4)\\ &= 42\pi\ \text{cm}^2\end{aligned}$$

b) $$\begin{aligned}\text{SA} &= \pi r^2 + 2\pi rH\\ &= \pi(3)^2 + 2\pi(4)(3)\\ &= 33\pi\ \text{cm}^2\end{aligned}$$

c) $$\begin{aligned}\text{SA} &= 2\pi rH\\ &= 2\pi(6)(5)\\ &= 60\pi\ \text{cm}^2\end{aligned}$$

d) $$\begin{aligned}\text{SA} &= \pi r^2 + 2\pi rH\\ &= \pi(6)^2 + 2\pi(6)(5)\\ &= 96\pi\ \text{cm}^2\end{aligned}$$

Exercise 2 number 1 is on page **366**

2)

a) $$\begin{aligned}\text{V} &= \pi r^2 H\\ &= \pi(4)^2(10)\\ &= 160\pi\ \text{cm}^3\end{aligned}$$

b) $$\begin{aligned}\text{V} &= \pi r^2 H\\ &= \pi(8)^2(10)\\ &= 640\pi\ \text{cm}^3\end{aligned}$$

c) $$\begin{aligned}\text{V} &= \pi r^2 H\\ &= \pi(4)^2(20)\\ &= 320\ \text{cm}^3\end{aligned}$$

Exercise 2 number 2 is on page **366**

Exercise 3 Solutions:

1)

a) $$\begin{aligned}\text{SA} &= 4\pi r^2\\ &= 4\pi(6)^2\\ &= 144\pi\ \text{cm}^2\end{aligned}$$

$$\text{V} = \frac{4}{3}\pi(6)^3 = 288\pi\ \text{cm}^3$$

b) $$\begin{aligned}\text{SA} &= 3\pi r^2\\ &= 3\pi(3)^2\\ &= 27\pi\ \text{m}^2\end{aligned}$$

$$\text{V} = \frac{2}{3}\pi(3)^3 = 18\pi\ \text{m}^3$$

Exercise 3 number 1 is on page **367**

2)

a) $$\begin{aligned}\text{V} &= \frac{1}{3}(A_{base})h\\ &= \frac{1}{3}(6)(2)\\ &= 4\ \text{cm}^3\end{aligned}$$

b) $$\begin{aligned}\text{V} &= \frac{1}{3}(A_{base})h\\ &= \frac{1}{3}(8\times 8)(3)\\ &= 64\ \text{m}^3\end{aligned}$$

c) $$\begin{aligned}\text{V} &= \frac{1}{3}\pi r^2 h\\ &= \frac{1}{3}\pi(3)^2(0{,}5)\\ &= 1{,}5\pi\ \text{m}^3\end{aligned}$$

Exercise 3 number 2 is on page **367**

3)

a) $$\begin{aligned}\text{V} &= \frac{1}{3}(A_{base})h\\ &= \frac{1}{3}(5\times 6)(8) = 80\ \text{cm}^3\end{aligned}$$

b) $$\begin{aligned}\text{V} &= l\times b\times h\\ &= 5\times 6\times 3\\ &= 90\ \text{cm}^3\end{aligned}$$

c) $$\begin{aligned}\text{SA} &= (5)(6) + 2(5\times 3) + 2(6\times 3)\\ &= 96\ \text{cm}^2\end{aligned}$$

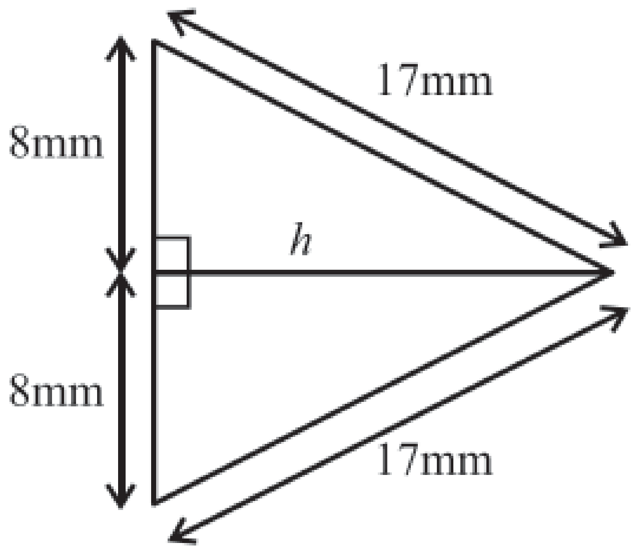
Exercise 3 number 3 is on page **367**

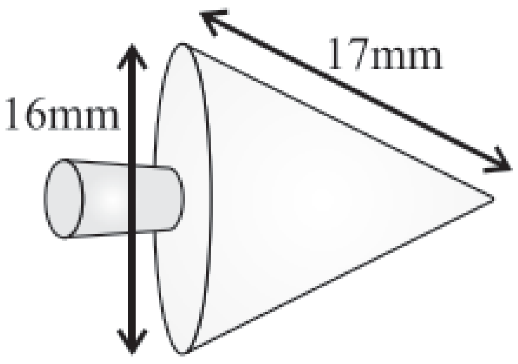
4) Volume of each hole: $\text{V} = \frac{1}{3}\pi r^2 h$

$$\begin{aligned}h^2 &= {h_s}^2 - r^2 \rightarrow \text{pythag}\\ &= 17^2 - 8^2\\ &= 225\end{aligned}$$

$\therefore h = 15\ \text{mm}$

$$\begin{aligned}\therefore \text{V} &= \frac{1}{3}\pi r^2 h\\ &= \frac{1}{3}\pi(8)^2(15)\\ &= 1005{,}31\ \text{mm}^3\ \text{per hole}\end{aligned}$$





$\therefore$ Total volume = $1005{,}31\times 12 \approx 12\,064\ \text{mm}^3 \rightarrow$ correct to the nearest mm^3

Exercise 3 number 4 is on page **367**

5)

a) $V_{tot} = V_{\text{large shell}} - V_{\text{small shell}}$

$$V_{\text{large shell}} = \pi r^2 h$$
$$= \pi(30)^2(250)$$
$$= 706\,858{,}35\,\text{m}^3$$

$$V_{\text{small shell}} = \pi r^2 h$$
$$= \pi(5)^2(250)$$
$$= 19\,634{,}95\,\text{m}^3$$

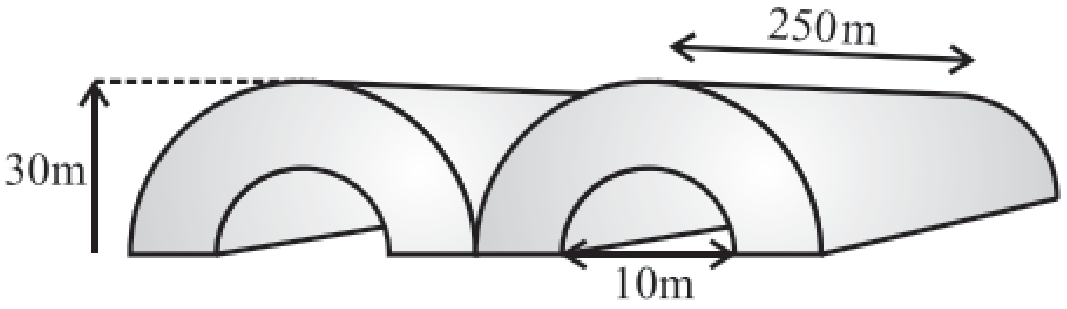
$$\therefore V_{\text{tot}} = 706\,858{,}35 - 19\,634{,}95$$
$$= 687\,223{,}4\,\text{m}^3$$

b) $SA = 2\pi r H$

$$= 2\pi(5)(250)$$
$$= 7\,853{,}98\,\text{m}^2$$

$7\,853{,}98 \div 350 = 22{,}44$

$\therefore$ 23 tins of paint are required



Exercise 3 number 5 is on page **367**

Mixed Exercise Solutions:

1) $SA = 2A_{\blacktriangle} + 2A_{\text{sides}}$

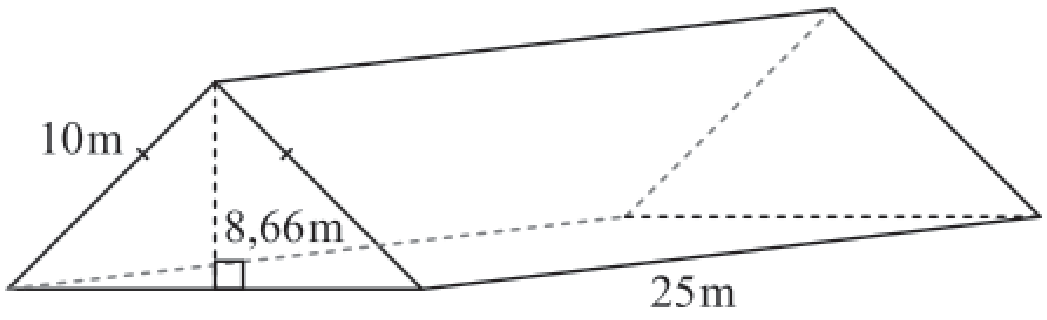
Base of triangle:

$b^2 = (10)^2 - (8{,}66)^2 \rightarrow$ pythag

$\therefore b = 5\text{m}$

$\therefore$ the base of the triangular face is 10 m long

$$\therefore SA = 2\left(\frac{1}{2}\right)(10)(8{,}66) + 2(25)(10)$$
$$= 586{,}6\,\text{m}^2$$



The Mixed Exercise number 1 is on page **368**

2)

a) $SA_{cylinder} = 2\pi rh$

$$= 2\pi(5)(75)$$
$$= 750\pi\,\text{m}^2$$

$$SA_{hemisphere} = 2\pi r^2$$
$$= 2\pi(5)^2$$
$$= 50\pi\,\text{m}^2$$

$$\therefore SA_{\text{total}} = 750\pi + 50\pi = 800\pi\ \text{m}^2$$

b) $V_{cylinder} = \pi r^2 h$

$$= \pi(5)^2(75)$$
$$= 1875\pi\,\text{m}^3$$

$$V_{hemisphere} = \frac{2}{3}\pi r^3$$
$$= \frac{2}{3}\pi(5)^3$$
$$= \frac{250}{3}\pi\,\text{m}^3$$

$$\therefore V_{\text{total}} = 1875\pi + \frac{250}{3}\pi = \frac{5875}{3}\pi\ \text{m}^3$$

The Mixed Exercise number 2 is on page **368**

3)

Cylinder	r (m)	h (m)	Volume (m³)
A	5	6	**150π**
B	5	12	**300π**
C	10	6	**600π**
D	10	12	**1200π**

Give all answers in their simplest form:

a) $\frac{150\pi}{300\pi} = \frac{1}{2}$

b) $\frac{600\pi}{150\pi} = 4$

c) $\frac{1200\pi}{150\pi} = 8$

d) The volume is 4 times greater if the radius is doubled. (From cylinder A to C)

e) The volume is 2 times greater if the height is doubled. (From cylinder A to B)

f) The volume is 8 times greater if the radius and height is doubled. (From cylinder A to D)

The Mixed Exercise number 3 is on page **368**

4)

a)

$$\begin{aligned} V &= l \times b \times h \\ &= (3)(2)(3) \\ &= 18\,\text{m}^3 \end{aligned}$$

b)

$$\begin{aligned} SA_{top} &= (5)(4) - (3)(2) \\ &= 14\,\text{m}^2 \\ \therefore SA_{tot} &= 14 + 2(5 \times 6) + 2(4 \times 6) + (5 \times 4) \\ &= 142\,\text{m}^2 \end{aligned}$$

The Mixed Exercise number 4 is on page **368**

5)

a)

$$\begin{aligned} V_{cube} &= l \times b \times h \\ &= (12)(12)(12) \\ &= 1728\,\text{m}^3 \end{aligned}$$

b)

$$\begin{aligned} &V_{water\ rect\ prism} = 1728\,\text{m}^3 \\ &V_{water\ rect\ prism} = l \times b \times h \\ &\therefore 1728 = (18)(8)(h) \\ &\therefore h = 12\,\text{m} \end{aligned}$$

The Mixed Exercise number 5 is on page **369**

6)

a)

$$\begin{aligned} SA_{rug} &= 3SA_{side\,step} + 3SA_{top\,step} \\ &= 3(60 \times 15) + 3(60 \times 20) \\ &= 6\,300\,\text{cm}^2 \end{aligned}$$

b)

$$\begin{aligned} V_{sc} &= V_{prism} - 3V_{step} \\ &= (80 \times 60 \times 45) - 3(80 \times 20 \times 15) \\ &= 144\,000\,\text{cm}^3 \end{aligned}$$

The Mixed Exercise number 6 is on page **369**

7)

$$\begin{aligned} V_{whole\ cake} &= \pi r^2 H \\ &= \pi (12)^2 (7{,}5) \\ &= 3\,392{,}92\,\text{cm}^3 \end{aligned}$$

If the cake is cut into 60° segments there will be 6 slices per cake $\rightarrow \left(\frac{360^\circ}{60^\circ}\right)$

$$\begin{aligned} \therefore V_{slice} &= \frac{3\,392{,}92}{6} \\ &= 565{,}47\,\text{cm}^3 \end{aligned}$$

The Mixed Exercise number 7 is on page **369**

8)

$$\begin{aligned} V_{\text{prisms}} &= l \times b \times h \\ &= (1)(1)(4) \\ &= 4\ \text{m}^3 \end{aligned}$$

$$\begin{aligned} V_{\text{pyramids}} &= \frac{1}{3}(A_{\text{base}})(\text{height}) \\ &= \frac{1}{3}(1)(1)(0{,}5) \\ &= \frac{1}{6}\ \text{m}^3 \end{aligned}$$

$$\begin{aligned} \therefore V_{\text{total}} &= 3\left(\frac{1}{6} + 4\right) \\ &= 12{,}5\,\text{m}^3 \end{aligned}$$

The Mixed Exercise number 8 is on page **369**

Chapter 11 – Probability

Calculating Probability

Exercise 1: **(Theory on pages 211 - 214)**

1) **In an experiment an 8 sided die is tossed:**
   - **a)** Write down the sample space for this experiment.
   - **b)** Determine the probability of getting a 6.
   - **c)** Determine the probability of getting an odd number.
   - **d)** Determine the probability of getting a number greater than 5.

Solutions to Exercise 1 number 1 are on page 377

2) **A letter is chosen at random from the word PROBABILITY:**
   - **a)** What is the probability that the letter chosen is a B?
   - **b)** What is the probability that the letter chosen is a vowel?
   - **c)** What is the probability that a consonant is chosen?

Solutions to Exercise 1 number 2 are on page 377

3) **A random card is drawn from a fair pack of playing cards:**
   - **a)** What is the probability of drawing a queen?
   - **b)** What is the probability of drawing a red card?
   - **c)** What is the probability of drawing a black 7?
   - **d)** If the first card drawn is a red card and then another card is randomly selected without replacing the first card, what is the probability that the second card drawn is black?

Solutions to Exercise 1 number 3 are on page 377

4) **In an experiment two fair die are tossed and the sum of the die faces are added together:**
   - **a)** Draw a table to represent the sample space for this experiment.
   - **b)** What is the probability of getting a sum of 8?
   - **c)** What is the probability of getting a sum of 12?
   - **d)** What is the probability of getting a sum less than 8?
   - **e)** What is the probability of getting a sum of more than 12?

Solutions to Exercise 1 number 4 are on page 377

5) **John is assigned to toss a coin at a cricket match to see who bats first. He wants to ensure that the coin he is using is fair. He sets up an experiment and tosses the coin ten times and records the results. He finds that the coin lands heads up 7 times and tails up 3 times. He concludes that the coin is not fair and selects another coin and repeats the experiment.**
   - **a)** Why would John conclude that the coin is not a fair coin?
   - **b)** Would you expect that John could find a fair coin using his experiment? Justify your answer.

Solutions to Exercise 1 number 5 are on page 377

Mutually Exclusive Events, Complementary Events and Venn Diagrams

Exercise 2: **(Theory on pages 215 – 221)**

1) **10 balls numbered 1 to 10 are placed in a bag and one is chosen at random:**
   - **a)** Use the letter O to represent the event of getting an odd number and represent it in set notation.
   - **b)** Use the letter E to represent the event of getting an even number and represent it in set notation.
   - **c)** Draw a Venn diagram to represent the sample space (S) and the above events.
   - **d)** Determine the probability of getting an even number.
   - **e)** Determine the probability of getting an odd number.
   - **f)** Are events O and E mutually exclusive? Justify your answer.
   - **g)** Are events O and E complementary events? Justify your answer.

Solutions to Exercise 2 number 1 are on page 377

2) **Numbers from 1 to 10 are written on cards and placed face down on a table and one card is chosen at random:**
   - **a)** Use the letter A to represent the event of getting a number less than 3 and represent it in set notation.
   - **b)** Use the letter B to represent the event of getting a number greater than 7 and represent it in set notation.

Solutions to Exercise 2 number 2a and 2b are on page 378, Exercise 2 number 2 is continued on the next page.

- **c)** Draw a Venn diagram to represent the sample space (S) and the above events.
- **d)** Determine P(A).
- **e)** Determine P(B).
- **f)** Are events A and B mutually exclusive? Justify your answer
- **g)** Are events A and B complementary events? Justify your answer.

Solutions to Exercise 2 number 2c to 2d are on page **378**

3) In a grade of 76 learners, 48 take Science, 60 take Biology and 8 don't take either subject.

- **a)** Draw a Venn diagram to represent the above information.
- **b)** If a learner is chosen at random determine the probability of the learner taking science or biology.
- **c)** If a learner is chosen at random determine the probability of the learner not taking science.
- **d)** If a learner is chosen at random determine the probability of the learner taking science and biology.
- **e)** Are the events of a learner taking science and a learner taking biology mutually exclusive?

Solutions to Exercise 2 number 3 are on page **378**

Mixed Exercise: (Chapter 11 is on pages 210 – 221)

1) In a container there are 18 blue spoons and 16 red spoons. The container is then shaken. Giving all your answers to the nearest percentage:

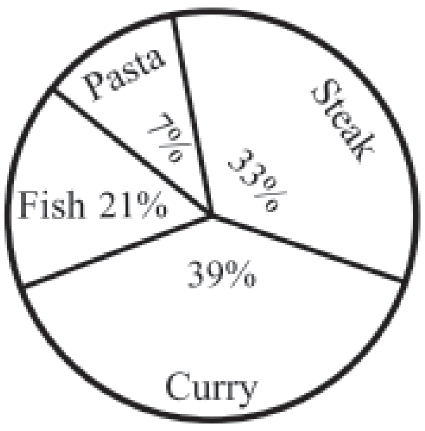
- **a)** Determine the probability of drawing a red spoon?
- **b)** Determine the probability of drawing a blue spoon?
- **c)** If a blue spoon is drawn and then replaced and another spoon is drawn, what is the probability of getting a blue spoon on the second attempt?
- **d)** If a blue spoon is drawn without replacement and another spoon is drawn, what is the probability of getting a blue spoon on the second attempt?

Solutions to The Mixed Exercise number 1 are on page **378**

2) Arshad throws a dart at the board below to decide what he should have for supper. The percentage shown is the percentage that each sector occupies on the board. Assuming Arshad always hits the board, Determine:

- **a)** The probability of landing on steak.
- **b)** The probability of landing on steak or pasta.
- **c)** The probability of not landing on curry.
- **d)** The probability of not landing on pasta or curry.

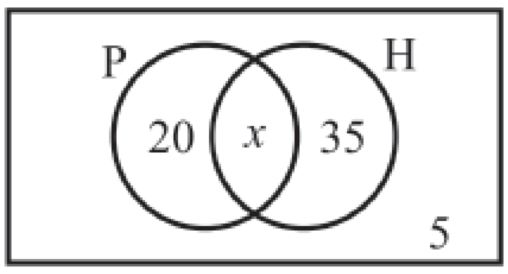
Solutions to The Mixed Exercise number 2 are on page **378**



3) 160 Grade 10 learners are selected at random and asked to indicate which of the following fast foods they eat:

- **Pizza**
- **Hamburgers**

The information gathered is summarised in the Venn diagram below where P = {eat pizza} and H = {eat hamburgers}. Give all probabilities as percentage correct to two decimal places.



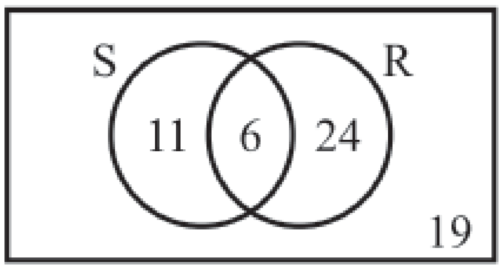
- **a)** How many learners do not eat any of the above fast foods?
- **b)** Determine the value of x, the number of learners that eat both types of fast foods?
- **c)** If a learner is selected at random what is the probability that the learner eats both types of fast food?
- **d)** If a learner is selected at random what is the probability that the learner eats none of the fast foods?
- **e)** What is the probability that a randomly selected learner eats at least one of the fast foods listed above?
- **f)** What is the probability that a randomly selected learner eats pizza?

Solutions to The Mixed Exercise number 3 are on page **378**

4) 60 Grade 10 learners are selected at random and asked if they participate in the following sports:

- **Rugby (R)**
- **Swimming (S)**

The results are shown in the Venn diagram below. Give all answers in simplest fraction form:



a) If a learner is selected at random what is the probability that they don’t play any of the sports?
b) If a learner is selected at random what is the probability that they play rugby?
c) If a learner is selected at random what is the probability that they swim and play rugby?
d) If a learner is selected at random what is the probability that they swim or play rugby?
e) If a learner is selected at random what is the probability that they don’t swim?

Solutions to The Mixed Exercise number 4 are on page **379**

5) At a school there are 200 learners in grade 10. Out of the students:

- **107 learners take history**
- **63 learners take science**
- **147 learners take history or science**
- ***x* learners take history and science**

a) Draw a Venn diagram to represent the information above. Let S be science and H be history.
b) How many learners do not take any of the subjects listed above?
c) Determine the value of x.
d) What is the probability that a learner chosen at random does science but not history?
e) What is the probability that learner chosen at random does not do any of the subjects listed above?
f) What is the probability that learner chosen at random does science?

Solutions to The Mixed Exercise number 5 are on page **379**

6) A supermarket conducted a survey on its customers buying milk. 400 Customers were asked which brands of milk they use at home. They were given the choice of brand A or brand B. The results of the survey are listed below:

- **2 customers did not use either of the brands**
- **248 customers used brand B**
- **100 customers used both brands A and B**

a) Draw a Venn diagram to represent the information above.
b) How many customers use only brand A?
c) What is the probability that a customer chosen at random uses brand A?
d) What is the probability that a customer chosen at random uses brand A and brand B?
e) What is the probability that a customer chosen at random does not use brand A?
f) What is the probability that a customer chosen at random does not use brand B?
g) What is the probability that a customer chosen at random uses brand A but not brand B

Solutions to The Mixed Exercise number 6 are on page **379**

7) Malaika writes MATHSMATHSMATHSMATHS… on a piece of paper and continues the patterns until she has written down 2001 letters.

a) What is the last letter she writes down?
b) How many time does she write the letter S?
c) What is the probability if a letter is randomly chosen that the letter selected is an M?

Solutions to The Mixed Exercise number 7 are on page **379**

8) Given the Venn diagrams below choose the option which best represents the shaded regions.

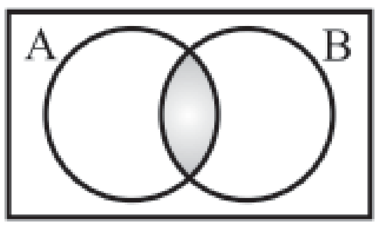


Diagram 1

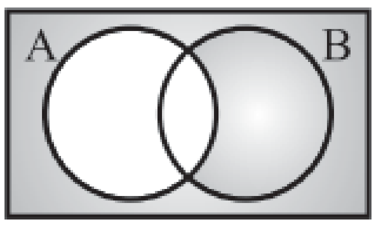


Diagram 2

	A	**B**	**C**	**D**
Diagram 1	A and B	A or B	(A and B)′	A and B
Diagram 2	B′	(A and B)′	A	A′

Solutions to The Mixed Exercise number 8 are on page **379**

Chapter 11 – Probability (Solutions)

Exercise 1 Solutions:

1)

a) $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$

b) $P(6) = \frac{1}{8} = 0{,}125 = 12{,}5\%$

c) $P(\text{Odd number}) = \frac{1}{2} = 0{,}5 = 50\%$

d) $P(\text{Number} > 5) = \frac{3}{8} = 0{,}375 = 37{,}5\%$

Exercise 1 number 1 is on page **374**

2)

a) $P(B) = \frac{2}{11} = 0{,}1818 = 18{,}18\%$

b) $P(\text{Vowel}) = \frac{4}{11} = 0{,}3637 = 36{,}37\%$

c) $P(\text{consonant}) = \frac{7}{11} = 0{,}6364 = 63{,}64\%$

Exercise 1 number 2 is on page **374**

3)

a) $P(\text{Queen}) = \frac{4}{52} = 0{,}0769 = 7{,}69\%$

b) $P(\text{Red}) = \frac{26}{52} = 0{,}5 = 50\%$

c) $P(\text{Black } 7) = \frac{2}{52} = 0{,}0385 = 3{,}85\%$

d) $P(\text{2nd being black}) = \frac{26}{51} = 0{,}5098 = 50{,}98\%$

Exercise 1 number 3 is on page **374**

4)

a)

1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12
	1	**2**	**3**	**4**	**5**	**6**

b) $P(8) = \frac{5}{36} = 0{,}1389 = 13{,}89\%$

c) $P(12) = \frac{1}{36} = 0{,}0278 = 2{,}78\%$

d) $P(\text{less than } 8) = \frac{21}{36} = 0{,}5833 = 58{,}33\%$

e) $P(\text{more than } 12) = \frac{0}{36} = 0 = 0\%$

Exercise 1 number 4 is on page **374**

5)

a) John must have been expecting that a fair coin would result in the same number of heads and tails.

b) It generally takes a large number of trials before the relative frequency of a coin falling heads up approaches 0,5. John's number of trials is too small to prove that a coin is fair.

Exercise 1 number 5 is on page **374**

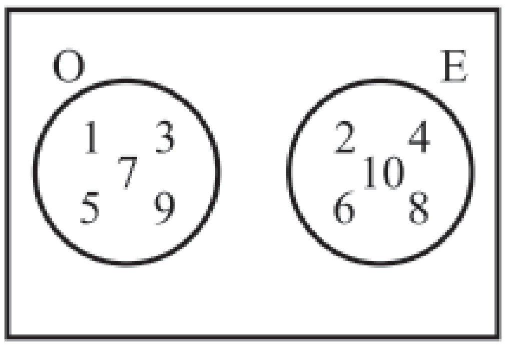
Exercise 2 Solutions:

1)

a) $O = \{1, 3, 5, 7, 9\}$

b) $E = \{2, 4, 6, 8, 10\}$

c)



d) $P(E) = \frac{5}{10} = 0{,}5 = 50\%$

e) $P(O) = \frac{5}{10} = 0{,}5 = 50\%$

f) Yes as $P(O \text{ and } E) = 0$

g) Yes as $P(O) = 1 - P(E)$

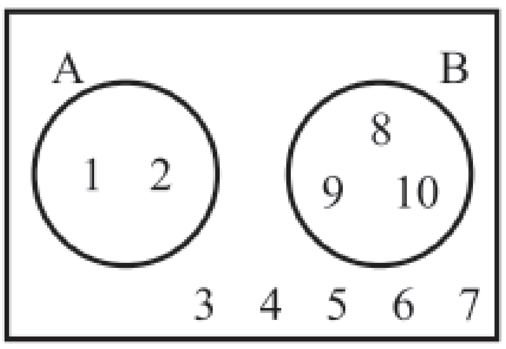
Exercise 2 number 1 is on page **374**

2)

a) A = {1, 2}

b) B = {8, 9, 10}

c)



d) $P(A)=\frac{2}{10}=0,2=20\%$

e) $P(B)=\frac{3}{10}=0,3=30\%$

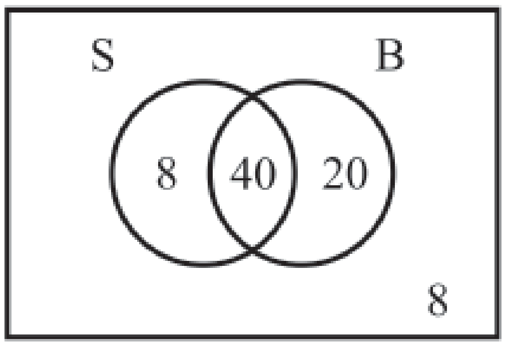
f) Yes as $P(A \text{ and } B)=0$

g) No as $P(A)\neq 1-P(B)$

Exercise 2 number 2*a* and 2*b* are on page **374** and numbers 2*c* to 2*g* are on page **375**

3)

a) Learners that do Science and Biology $=(48+60)-(76-8)=40$



b) $P(S \text{ or } B)=\frac{8+40+20}{76}=\frac{17}{19}=0,8947=89.47\%$

c) $P(\text{not } S)=\frac{20+8}{76}=\frac{7}{19}=0.3684=36.84\%$

d) $P(S \text{ and } B)=\frac{40}{76}=\frac{10}{19}=0,5263=52,63\%$

e) No as the $P(S \text{ and } B)\neq 0$

Exercise 2 number 3 is on page **375**

Mixed Exercise Solutions:

1)

a) $P(R)=\frac{16}{34}=0,47=47\%$

b) $P(B)=\frac{18}{34}=0,53=53\%$

c) 53%

d) $P(B)=\frac{17}{33}=0,52=52\%$

The Mixed Exercise number 1 is on page **375**

2)

a) $P(S)=33\%$

b)
$$\begin{aligned}P(S \text{ or } P)&=33\%+7\%\\&=40\%\end{aligned}$$

c)
$$\begin{aligned}P(C')&=1-P(C)\\&=1-0,39\\&=0,61\\&=61\%\end{aligned}$$

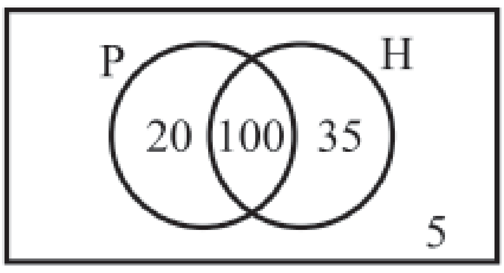
d)
$$\begin{aligned}P(P' \text{ or } C')&=1-P(P \text{ or } C)\\&=1-(0,07+0,39)\\&=0,54\\&=54\%\end{aligned}$$

The Mixed Exercise number 2 is on page **375**

3)

a) 5

b) $20+x+35+5=160$
$\therefore\ x=100$



c) $P(P \text{ and } H)=\frac{100}{160}=0,625=62,5\%$

d) $P(\text{none})=\frac{5}{160}=0,03125=3,13\%$

e) $P(\text{at least } 1)=1-P(\text{none})=1-0,03125=0,96875=96,88\%$

f) $P(P)=\frac{120}{160}=0,75=75\%$

The Mixed Exercise number 3 is on page **375**

4)

a) $P(\text{none}) = \frac{19}{60} = 0,3167 = 31,67\%$

b) $P(R) = \frac{30}{60} = 0,5 = 50\%$

c) $P(\text{S and R}) = \frac{6}{60} = 0,1 = 10\%$

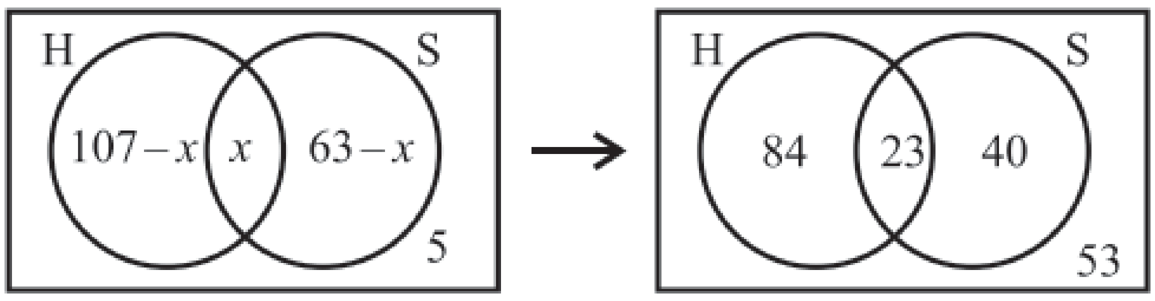
d) $P(\text{S or R}) = \frac{41}{60} = 0,6833 = 68,33\%$

e) $P(S') = \frac{24+19}{60} = \frac{43}{60} = 0,7167 = 71,67\%$

The Mixed Exercise number 4 is on page **376**

5)

a)



b) $200 - 147 = 53$

c) $107 - x + x + 63 - x + 53 = 200$
$\therefore x = 23$

d) $P(\text{S only}) = \frac{40}{200} = 0,2 = 20\%$

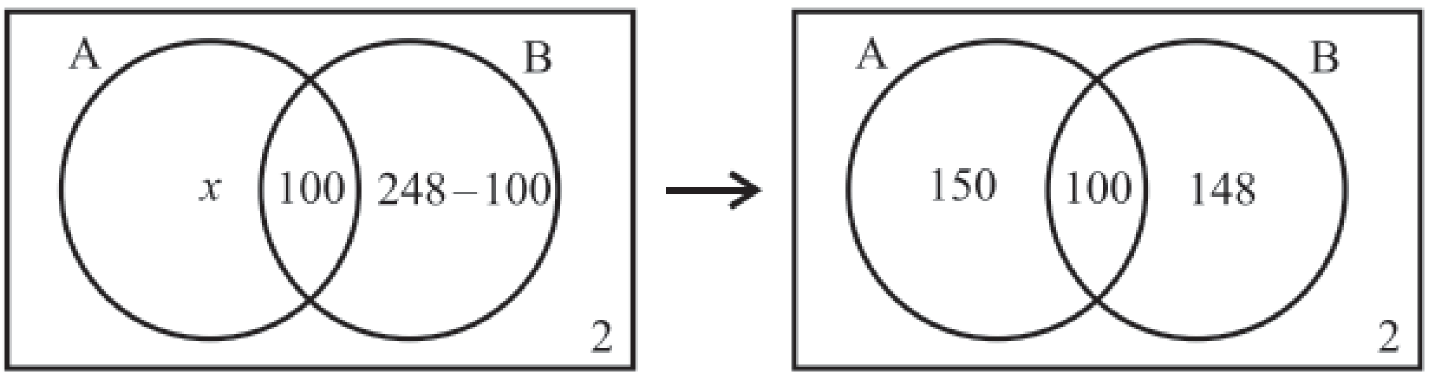
e) $P(\text{none}) = \frac{53}{200} = 0,265 = 26,5\%$

f) $P(S) = \frac{40+23}{200} = 0,315 = 31,5\%$

The Mixed Exercise number 5 is on page **376**

6)

a)



b) $x + 100 + 148 + 2 = 400$
$\therefore x = 150$

c) $P(A) = \frac{150+100}{400} = \frac{250}{400} = 0,625 = 62,5\%$

d) $P(\text{A and B}) = \frac{100}{400} = 0,25 = 25\%$

e) $P(A') = \frac{148+2}{400} = \frac{150}{400} = 0,375 = 37,5\%$

f) $P(B') = \frac{150+2}{400} = \frac{152}{400} = 0,38 = 38\%$

g) $P(\text{A only}) = \frac{150}{400} = 0,375 = 37,5\%$

The Mixed Exercise number 6 is on page **376**

7) The pattern repeats itself every 5 letters. This means that every 5th letter is an S.

a) There are 2001 letters in the pattern. As 2000 is divisible by 5 the 2000th letter will be an S. Therefore, the last letter is an M.

b) The letter S is repeated once every 5 letters. There are 2000 letters that is repeated in. Therefore, the letter S is written 400 (2 000 ÷ 5) times.

c) The letter M is written 400 times up to the 2 000th letter. Therefore, the letter M is written a total of 401 times out of 2001 letters.

$\therefore P(M) = \frac{401}{2001} = 0,2004 = 20,04\%$

The Mixed Exercise number 7 is on page **376**

8) D

The Mixed Exercise number 8 is on page **376**

CAPS - The tables below show the curriculum as prescribed by the National Department of Education for examination purposes.

CAPS NUMBER AND EXPLANATION	CHAPTER
1. Functions	
Work with relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae). Include linear and some quadratic polynomial functions, exponential functions, some rational functions and trigonometric functions.	1
Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence generalise the effect of the parameter which results in a vertical shift and that which results in a vertical stretch and /or a reflection about the x axis.	1
Problems solving and graph work involving the prescribed functions.	1
2. Number Patterns, Sequences and Series	
Investigate number patterns leading to those where there is constant difference between consecutive terms, and the general term is therefore linear.	4
3. Finance, Growth and Decay	
Use Simple and compound growth formulas $A = P(1+in)$ and $A = P(1+i)^n$ to solve problems (including interest, hire purchase, inflation, population growth and other real life problems).	8
Work with and understand the implications of fluctuating foreign exchange rates.	8
4. Algebra	
(a) Understand that real numbers can be irrational or rational.	1
(a) Simplify expressions using the laws of exponents for rational exponents. (c) Round real numbers to an appropriate degree of accuracy (to a given number of decimal digits). (b) Establish between which two integers a given simple surd lies.	1 and 3
Manipulate algebraic expressions by: • multiplying a binomial by a trinomial; • factorising trinomials; • factorising the difference and sums of two cubes; • factorising by grouping in pairs; and • simplifying, adding and subtracting algebraic fractions with denominators of cubes (limited to sum and difference of cubes).	2
Solve: • linear equations; • quadratic equations; • literal equations (changing the subject of a formula); • exponential equations; • linear inequalities; • system of linear equations; and • word problems.	4
5. Differential Calculus	
Not covered in Grade 10.	
6. Probability	
(a) Compare the relative frequency of an experimental outcome with the theoretical probability of the outcome. (b) Venn diagrams as an aid to solving probability problems. (c) Mutually exclusive events and complementary events. (d) The identity for any two events A and B: $P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$	11
7. Euclidean Geometry Measurement	
(a) Revise basic results established in earlier grades. (b) Investigate line segments joining the midpoints of two sides of a triangle. (c) Properties of special quadrilaterals	7
Solve problems involving volume and surface area of solids studied in earlier grades as well as spheres, pyramids and cones and combinations of those objects	10
8. Trigonometry	
(a) Definitions of the trigonometric ratios sin θ, cos θ and tan θ in a right-angled triangles. (b) Extend the definitions of sin θ, cos θ and tan θ to $0^\circ \leq \theta \leq 360^\circ$. (c) Derive and use values of the trigonometric ratios (without using a calculator for the special angles $\theta \in \{0^\circ; 30^\circ; 45^\circ; 60^\circ; 90^\circ\}$ (d) Define the reciprocals of trigonometric ratios.	5
Solve problems in two dimensions.	5
9. Analytical Geometry	
Represent geometric figures in a Cartesian coordinate system, and derive and apply, for any two points $(x_1 ; y_1)$ and $(x_2 ; y_2)$, a formula for calculating: • the distance between the two points; • the gradient of the line segment joining the points; • conditions for parallel and perpendicular lines; and • the co-ordinates of the mid-point of the line segment joining the points.	7
10. Statistics	
(a) Collect, organise and interpret univariate numerical data in order to determine: • measures of central tendency • five number summary • box and whisker diagrams • measures of dispersion	9

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Recommended Teaching Plan by the National Department of Education

Term 1

Weeks	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11
Topics	Algebraic expressions			Exponents		Number patterns	Equations and inequalities		Trigonometry		
Chapter	1 and 2			3		4	4		5		
Assessment	Investigation or Project						Test				
Date Completed											

Term 2

Weeks	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11
Topics	Functions				Trigonometric functions	Euclidean Geometry			MID-YEAR EXAMINATION		
Chapter	6				6	7					
Assessment	Assignment / Test										
Date Completed											

Term 3

Weeks	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10
Topics	Analytical Geometry		Finance and growth		Statistics		Trigonometry		Euclidean Geometry	Measurement
Chapter	7		8		9		6		7	10
Assessment	Test						Test			
Date Completed										

Term 4

Weeks	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10
Topics	Probability		Revision							Admin
Chapter	11									
Assessment	Test						FINAL EXAMINATIONS			
Date Completed										

Paper 1: 2 hours	
Algebraic Expressions and Equations (and Inequalities) Exponents	30
Number Patterns	10
Functions and graphs	30
Finance and growth	15
Probability	15
Total marks	100

Paper 2: 2 hours	
Euclidean geometry and Measurement	20
Analytical geometry	15
Trigonometry	50
Statistics	15
Total marks	100

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ANNUAL TEACHING PLAN
Summary - Grade 10

	Subject	Time	Chapter/s	Paper
Term 1	Algebraic Expressions	3 Weeks	1 and 2	1
	Exponents	2 Weeks	3	1
	Number Patterns	1 Week	4	1
	Equations and Inequalities	2 Weeks	4	1
	Trigonometry	3 Weeks	5	2
Term 2	Functions	4 Weeks	6	1
	Trigonometric Functions	1 Week	6	1/2
	Euclidean Geometry	3 Weeks	7	2
	Exams	3 Weeks		–
Term 3	Analytical Geometry	2 Weeks	7	2
	Finance, Growth and Decay	2 Weeks	8	1
	Statistics	2 Weeks	9	2
	Trigonometry	2 Weeks	6	2
	Euclidean Geometry	1 Week	7	2
	Measurement	1 Week	10	2
Term 4	Probability	2 Weeks	11	1
	Revision	4 Weeks	–	–
	Exams	3 Weeks	–	–

The teaching plan above is recommended by the National Department of Education from the CURRICULUM AND ASSESSMENT POLICY STATEMENT (CAPS)

Notes

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Date: 4 January 2021

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